

**RATIONAL AMBIGUITY AND MONITORING THE CENTRAL BANK**

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**Working Paper No. 04-W04**

February 2004

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# Rational Ambiguity and Monitoring the Central Bank\*

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February 10, 2004

## Abstract

In this paper we examine the consequences of having a Central Bank whose preferences are state contingent. This has been identified in the literature as a Central Bank that is ‘rationally inattentive’ or ‘constructively ambiguous’. The new feature in this paper is that we show how the private sector is likely to react. There are two possibilities: the public can form rational expectations and internalise the uncertainty about the Central Bank’s preferences in full. Alternatively, and if this process of internalisation is costly, it can form a ‘best’ guess regarding those preferences and use that. This implies a certainty equivalence strategy. We examine the magnitude of the resulting error in inflation and output, following the assumption of certainty equivalence. Under all reasonable levels of uncertainty this error turns out to be small. But it involves trading a deflation bias against the cost of gathering the information needed for the full rational expectations solution.

J.E.L Classification: E52, E58

Keywords: Central Bank Transparency, Certainty Equivalence, Rational Expectations, Ambiguity and Rational Inattention

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\*Views expressed are our own and do not necessarily reflect those of the institutions we are affiliated with.

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# 1 Introduction

There are circumstances in which the work of Central Banks will be conducted under a regime of incomplete transparency for the private sector. Incomplete transparency may occur either because the bank itself chooses not to reveal certain aspects of what it knows; or because it does not have firm information itself, and is therefore unable to reveal that information precisely. The empirical implications of the different sources of intransparency are shown to be diverse (Demertzis and Hughes Hallett, 2002, 2003).

The first case is the traditional one, in which the private sector faces preference uncertainty or control errors (incomplete political and economic transparency, respectively) and has been studied by many authors [*inter alia*, Faust and Svensson (2001, 2002), Geraats (2002), Muscatelli (1998), Sibert (2002), Walsh (1999)].

The second type of intransparency can be associated with potentially time inconsistent preferences in which the parameters applied by the Central Bank (CB) are state contingent, randomised or otherwise varied. In that case the Central Bank is unable to announce in advance exactly what values might be taken at any specific time. This kind of model has not been widely studied, but examples may be found in the *Rational Inattention* model of Sims (2003) and the *Constructive Ambiguity* approach of Cukierman-Metzler (1986).

In this paper, we assume that there is uncertainty about what the Central Bank preferences really are, and examine the way the private sector handles this lack of transparency, irrespective of how it arises. In most of the studies undertaken, it is assumed that the private sector is unable to internalise the full effects of the uncertainty it faces. This may happen either because it does not realise that the Central Bank may rationally be uncertain itself; or because it may not have sufficient information to characterise that uncertainty fully; or because it is too expensive to do so accurately, and forms a first order certainty equivalent (FOCE) estimate of what the Bank is likely to deliver instead. The consequence of such simplifications, realistic as they may be in terms of how people actually act, is systematic errors in private sector forecasts of inflation or the output gap. In order to evaluate these costs we compare the resulting inflation rate to that achieved when the private sector is fully rational and the costs of any remaining lack of transparency are fully internalised. We find that under reasonable levels of intransparency, the costs of following a regime of certainty equivalence are small. If the costs of acquiring the extra information that will allow rational expectations to be formed are relatively high, then a regime of certainty equivalence may remain the optimal strategy.

We start from the observation that, unless there are very distinct gains from investing the resources needed to estimate (or obtain information on) the exact distribution of the parameters of the Central Bank's preferences, it may be simpler for the private sector to adopt FOCE estimates of the relevant parameters. Section 2 provides a summary of how this is justified in the literature, with the aid of a standard set-up, and section 3.1 presents its solution. We then

look at the scope for, and the consequences of, internalising the effects of the uncertainty the private sector faces, as that leads to the rational expectations solution in section 3.2. We compare the two in section 3.3. Section 4 concludes.

## 2 The Set Up

We adopt a simple Rogoff (1985) type model of a conservative central banker in which the Central Bank has the following objective function<sup>1</sup>:

$$L = \frac{1}{2} E \left[ \pi^2 + b (y - k)^2 \right] \quad (1)$$

constrained by a simple Lucas supply function:

$$y = \pi - \pi^e + \varepsilon \quad (2)$$

where  $y$  and  $\pi$  are measured as deviations from their steady state paths<sup>2</sup>. The model produces the following solutions for the policy variables in question:

$$\begin{aligned} \pi &= bk - \frac{b}{1+b} \varepsilon \\ \pi^e &= bk \\ y &= \frac{1}{1+b} \varepsilon \end{aligned} \quad (3)$$

For simplicity, we assume that there is only one (monetary) policy authority in the game. There is therefore, no fiscal policy and we assume for simplicity that there is no uncertainty about the policy transmission mechanism.

But what would happen if the Bank itself was unsure about the precise value of  $b$  at any particular moment; or was unable to commit itself to one particular value of  $b$  for all circumstances? This may fall under the heading of rational inattention or constructive ambiguity. In fact the literature contains a number of models which explain why policy makers may be uncertain about which priorities to set for any particular problem. The first is when their relative priorities (the marginal rates of substitution between  $\pi$  and  $y$ ) are state dependent. That would happen if the true preferences (over the entire policy space) were more complicated than those represented by a simple quadratic loss function. In that case, (1) represents a local approximation for the current position of the economy; and, being dependent on the uncertain values of  $\pi$  and  $y$ , the marginal

<sup>1</sup>This system of objectives in inflation and output gaps is based on the utility-based approach by Rotemberg and Woodford (1998) and Gali and Monacelli (2002).

<sup>2</sup>The microfoundations of this model were originally derived by Lucas, (1972, 1973). Extensions to his model, to incorporate fiscal policy appear in Hughes Hallett and Viegi (2003). To justify the presence of  $k$ , a target deviation from the steady state, see the discussions by Rogoff (1985), Blinder (1997). Furthermore, as the literature remains ambiguous about the use of  $k$ , Demertzis and Hughes Hallett (2002) recognise it explicitly as a source of uncertainty. For our purposes however, it is sufficient to note that  $k$  might be either positive or zero.

rate of transformation between them (and hence the desired value of  $b$ ) will become a random variable (Hughes Hallett 1979). The same will happen if policy makers wish to retain an element of risk aversion in their preferences since the strict linear-quadratic framework adopted here will generate risk neutral decisions. There is also a possibility that policy makers cannot always specify their relative priorities in advance, but have to uncover them iteratively by revealed preference (van Eijck and Sandee, 1959).

Recent theory has provided several other reasons why  $b$  might be uncertain. The first is to preserve risk sensitivity in a linear-quadratic approach. This can be done by minimising the mean and variance of (1),  $\alpha E(L) + (1 - \alpha)V(L)$ , where  $\alpha$  is a coefficient of risk sensitivity and  $V(L)$  is fourth order in  $\pi$  and  $y$  (Hughes Hallett, 1984a). That in turn is a truncation of the full risk averse decision making solution devised by Whittle (1982). From here, it is straight forward to show that risk sensitive decision making is equivalent to having solved the linear-quadratic problem (1) subject to (2), where  $b$  is replaced by

$$b \left[ \frac{\alpha + (1 - \alpha) b \sigma_2^2}{\alpha + (1 - \alpha) b \sigma_1^2} \right]$$

with  $\sigma_1^2 = V(y)$  and  $\sigma_2^2 = V(\pi)$ . If the degree of risk sensitivity  $\alpha$  (the curvature of the ‘true’ preferences) is state dependent, then the new value of  $b$  will be uncertain too.

A second version of this idea is to note that the ‘robust control’ approach used by Hansen and Sargent (2002), Hansen *et al* (1999), Basar and Bernhard (1995) and others, is equivalent to minimising (1) minus a term in the variance in the state variable  $y$ . Again, if the degree of risk sensitivity (the Lagrange multiplier attached to that extra variance term) is state dependent, the implied change to the value of  $b$  will be uncertain *ex ante*. But that, as Kasa (2002) points out, is also identical to the model of ‘rational inattention’ introduced by Sims (2003). Here the problem is one in which policy makers have a limited capacity to monitor all the variables in the economy. They will rationally reduce the effort made to forecast and control the most volatile of them in order to concentrate on those that can be controlled effectively. As Sims shows, that is a problem which can be solved by minimising (1), less a term in the variance of the state variable which is subject to a monitoring constraint. That implies a one-to-one correspondence with our robust control problem. Hence rational inattention is an other reason for supposing that  $b$  would be uncertain *ex ante*.

Finally, the best known model of preference uncertainty is one in which policy makers deliberately retain ‘randomised’ preferences in order to exploit the effects of ambiguity (Cukierman and Meltzer, 1986). Normally this is modelled as a process of control errors in monetary policy. But, in order to generate those errors systematically and to their advantage, policy makers need to create shifts in their relative preferences for output stabilisation and inflation in a favourable manner and with suitable timing. Because that shift is random but with persistence, policy makers can affect the speed with which the private sector becomes aware of it. That then allows the Central Bank to choose its mo-

ment for monetary policy changes. For example, it may plan a positive surprise to stimulate the economy when output is down; but with expectations lagging and not being certain what the Bank's true intentions are, the private sector will not anticipate a rise in inflation. The stimulus can therefore, be achieved at lower cost in terms of inflation. Conversely, negative surprises can be timed to reduce inflation without the private sector anticipating a loss in output. But to gain these advantages, the Bank must allow the precise value of  $b$  to remain uncertain.

We model the existence of incomplete transparency as uncertainty in the Central Bank's true preference for output stabilisation, as follows:

$$b = \beta + u \text{ with } E(u) = 0 \text{ and } V(u) = \sigma_u^2 \quad (4)$$

Full transparency (or full clarity) will require both  $E(u) = 0$  as well as  $\sigma_u^2 = 0$ . From here we can distinguish two cases:

- The first is where, for lack of accurate information on the characteristics of the entire probability distribution being used by the Central Bank to generate  $b$ , the private sector considers the mean of that distribution,  $E(b) = \beta$ , to be the actual preference parameter. This constitutes its best guess or 'consensus estimate' when it cannot evaluate the distribution of  $b$  more precisely. Equally, this estimate can be used to generate *first order certainty equivalent* approximations to the Bank's best decisions when the higher order moments of the  $b$  distribution are unknown; or are considered difficult or too expensive to estimate accurately. This will be identified as the Certainty Equivalence case.
- The second case is where the private sector can and does make the effort to evaluate the higher moments of the  $b$  distribution. In that case, the private sector is able to evaluate the Bank's optimal decision rules accurately. Similarly, the Bank, which also knows the distribution of  $b$ , conditions its decisions on the private sector's improved information. We suppose that they both do that<sup>3</sup> and call this the Rational Expectations case.

The Central Bank is fully aware of (4) in either case, and can correctly identify the regime that the private sector follows.

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<sup>3</sup>So long as we deal with the linear-quadratic case as specified here, we only need the first two moments of the  $b$  distribution to do all that. A non-linear set-up however, would require the private sector to have access to all the higher order moments of the distribution of  $b$  (Hughes Hallett, 1984b, p39).

### 3 The Consequences of Ambiguity in Monetary Policy

#### 3.1 A Certainty Equivalent Solution

As argued above, if the private sector does not have accurate information on the moments of the distribution of  $b$  or finds it too expensive (relative to the potential benefits in terms of improved decision making) to obtain that information, then adopting a certainty equivalent approach is the best it can do. In that case, taking the mean of that distribution  $E(b) = \beta$  as its certainty equivalent ('best estimate') and using (3), the private sector will expect the Central Bank to implement the following policy rule:

$$\pi = \beta k - \frac{\beta \varepsilon}{1 + \beta} \quad (5)$$

which implies  $\pi_{CE}^e = \beta k$  is the inflation expectation in the markets. In fact, in view of (7) and (8) below, we can see that  $\pi_{CE}^e$  is the **first order** certainty equivalent estimate of the optimal policy rule when  $b$  and  $\varepsilon$  are uncorrelated. But it is also a biased estimate, as (9) will show. The Central Bank now optimises the following loss function

$$\min_{\pi} L = \frac{1}{2} E \left[ \pi^2 + b (\pi - \beta k + \varepsilon - k)^2 \right] \quad (6)$$

Since  $\pi$  is a choice entirely within the Bank's control, the result is:

$$\pi_{CE}^* = \frac{b(1 + \beta)k}{1 + b} - \frac{b\varepsilon}{1 + b} \quad (7)$$

But, notice that  $E(\pi_{CE}^*) \neq bk$  or  $\beta k$ , since

$$E \left( \frac{b}{1 + b} \right) \neq \frac{\beta}{1 + \beta} \quad (8)$$

In fact, taking expectations, we get (approximately),

$$E(\pi_{CE}^*) = \beta k - \frac{\sigma_u^2 k}{(1 + \beta)^2} < \beta k = \pi_{CE}^e \quad (9)$$

assuming, again,  $b$  and  $\varepsilon$  to be independent. Equation (9) has **four** immediate implications: a) the average level of inflation is affected by imperfect transparency; b) greater transparency will *increase* average inflation; c) the private sector will consistently overestimate inflation; d) the Bank will have an incentive to preserve some ambiguity (or incomplete transparency) since that will deliver lower inflation than anticipated, on average. Unfortunately it will also reduce average output, as we show below. Nevertheless, a conservative central bank ( $b < 1$ ) and a conservative population would regard this lower inflation as more important than the loss in output. Hence we may say that ambiguity is *constructive* in this case.

To see exactly what happens to output, we substitute (7) into (2) where  $\pi_{CE}^e = \beta k$ :

$$y_{CE}^* = \frac{b(1+\beta)k}{1+b} - \frac{b\varepsilon}{1+b} - \beta k + \varepsilon = \frac{uk + \varepsilon}{1+b} \quad (10)$$

This implies, that the average level of output will also be affected by a lack of clarity:

$$E(y_{CE}^*) = -\frac{k\sigma_u^2}{(1+\beta)^2} \quad (11)$$

In fact there is a deflation bias here,  $E(y^*) < 0$ , which *decreases* with greater transparency. This acts as a restraint on the incentive to create lower average inflation rates by retaining some ambiguity in the Central Bank's preferences. Moreover, it implies that there will be an *optimal* level of ambiguity if  $b < 1$  and  $k^2/\sigma_\varepsilon^2$  is not too small<sup>4</sup>.

### 3.2 The Rational Expectations Solution

We assume next that the private sector now recognises that its certainty equivalent approximation to the Bank's behaviour leads to biased forecasts of inflation. Given information on the distribution of  $b$ , as supplied by the Bank or obtained from its own research, the private sector knows the Bank will solve (1) subject to (2) to obtain

$$\pi = \frac{b}{1+b}(\pi^e - \varepsilon + k) \quad (12)$$

The difference now is in the way that the private sector forms its expectations. This is done by taking expectations through (12), and solving for the new value of  $\pi^e$ . This yields the rationally expected inflation rate as

$$\pi_{RE}^e = \left[ \frac{(1+\beta)^2 - \sigma_u^2/\beta}{(1+\beta)^2 + \sigma_u^2} \right] \beta k = \theta \beta k \quad (13)$$

Notice that  $\theta < 1$  if  $\sigma_u^2 \neq 0$ ; but that  $\theta \rightarrow 1$  if  $\sigma_u^2 = 0$  and as  $\beta$  increases without limit. But  $\theta$  may turn negative if  $\beta$  becomes very small ( $\beta \rightarrow 0$ ): although that can only happen if  $\sigma_u^2 \neq 0$ <sup>5</sup>. This allows us to measure the proportional error in expected inflation<sup>6</sup> that the private sector would obtain in the certainty equivalence case,

$$\frac{(1-\theta)\beta k}{\beta k} = \frac{\sigma_u^2(1+\beta)}{\beta[(1+\beta)^2 + \sigma_u^2]} > 0 \quad (14)$$

<sup>4</sup>If  $k^2/\sigma_\varepsilon^2$  were small, there would be little space for reducing inflation and little cost in output. The gains from ambiguity would be rather small in that case.

<sup>5</sup>In fact,  $\theta < 0$  only if  $\sigma_u^2 > \beta(1+\beta)^2$ . However, if this happens because  $\beta$  is small, instead of  $\sigma_u^2$  being large, then  $\pi_{CE}^e \rightarrow \pi_{RE}^e \rightarrow 0$ .

<sup>6</sup>This is calculated as  $\frac{\pi_{CE}^e - \pi_{RE}^e}{\pi_{CE}^e}$ .



which vanishes if  $\sigma_u^2 \rightarrow 0$  but increases if  $\beta \rightarrow 0$ . This implies  $\pi_{RE}^e < \pi_{CE}^e$ , as noted earlier.

To obtain the actual inflation outcome in this case, we substitute (13) into (12)

$$\pi_{RE}^* = \frac{b(1+\theta\beta)k}{1+b} - \frac{b\varepsilon}{1+b} \quad (15)$$

and hence obtain

$$E(\pi_{RE}^*) = \left[ \beta - \frac{\sigma_u^2}{(1+\beta)^2} \right] \left[ \frac{1+\theta\beta}{1+\beta} \right] k \quad (16)$$

It is straight forward to show that (16) is indeed equal to (13), consistent with rational expectations. Given (9), the proportional error in actual inflation in the certainty equivalence solution, compared the rational expectations equilibrium, now is<sup>7</sup>

$$\frac{(1-\theta)\beta}{1+\beta} = \frac{\sigma_u^2}{(1+\beta)^2 + \sigma_u^2} > 0 \quad (17)$$

on average. Hence  $E(\pi_{RE}^*) < E(\pi_{CE}^*)$ <sup>8</sup>. Notice that this error also vanishes as  $\sigma_u^2 \rightarrow 0$  or as  $\beta$  increases, but rises if  $\beta$  becomes smaller. Finally, and most important, the fact that (17) and (14) are both positive means that the **proportional** forecasting error - and hence the incentive for the private sector to change from the certainty equivalent solution to this one - is the difference between the two:  $\sigma_u^2 / \left[ \beta \left( (1+\beta)^2 + \sigma_u^2 \right) \right]$ .

As regards output, using (15) and (2), we have

$$y_{RE}^* = \frac{uk + (1-\theta)\beta k + \varepsilon}{1+b} \quad (18)$$

and hence, using (17), that

$$E(y_{RE}^*) = 0 \quad (19)$$

This implies  $E(y_{CE}^*) < E(y_{RE}^*) = 0$ : the deflation bias has been removed.

In the next section, we provide some idea of the numerical importance of the difference between these two solutions, and hence of the incentive to shift from certainty equivalence to a full rational expectations solution with its more exacting information requirements. It will also give an indication of the incentive, faced by the Central Bank, to provide full information about the likely value of  $b$ , in the interest of better economic performance. It turns out both incentives

<sup>7</sup>This is calculated as  $\frac{E(\pi_{CE}^*) - E(\pi_{RE}^*)}{E(\pi_{CE}^*)}$ .

<sup>8</sup>Note that the errors in the expectations go in the same direction as the 'errors' in the outcomes, but are bigger.

are pretty small. Constructive ambiguity, with its implication that expectations can be ‘manipulated’ to produce lower inflation rates (at the cost of some deflation bias), may therefore, seem attractive. That has been the traditional argument<sup>9</sup>. However, although inflation can indeed be restrained in this way, it can be reduced even further with full information or transparency, and at no cost in terms of a deflationary bias. So constructive ambiguity is a false expedient - except in so far as it may cost too much to gather or circulate the relevant information on  $b$ . In that situation, the RE solution is not realistic and ambiguity may well be a useful expedient for the Central Bank. That is the real message of this paper.

### 3.3 How Large are the Errors if the Private Sector Assumes Certainty Equivalence?

Certainty Equivalence, in which  $\beta = E(b)$  is taken as a fixed value representing Central Bank preferences, is the standard assumption in the existing literature on constructive ambiguity. It says that, in the absence of accurate information on the entire distribution which the Central Bank may decide to use for  $b$ , the private sector will take a first order certainty equivalent estimate of the full rational expectations equilibrium. That is a sensible strategy if the expectational errors ( $\pi_{CE}^e \neq E(\pi_{CE}^*)$ ) are small in relation to the cost of gathering the information necessary to support the full rational expectations equilibrium.

How big might the errors be under Certainty Equivalence?

**Case a).** Systematic errors in expectations are the key determinant of which solution would be chosen in practice, since the private sector has no interest beyond being able to forecast the Central Bank’s decisions accurately. As the private sector leads, once it has decided which expectations solution to follow, the Central Bank is locked in - except in so far as the Bank may decide to supply the private sector with enough information to get an equilibrium which, in its own estimate, it thinks is superior or more stable.

Suppose the Central Bank chooses  $b$  to be distributed uniformly on the unit interval. This implies a certain degree of conservatism, with  $\beta = 0.5$  and  $\sigma_u^2 = 1/12$ . Compared to the correct rational expectation ( $\pi_{RE}^e$ ),  $\pi_{CE}^e$  will be too high by 7.5 *per cent*. As a result, the Central Bank will end up choosing inflation rates that are higher, on average, than those in the full rational expectations solution by 3.6 *per cent*<sup>10</sup>. Consequently the private sector will find itself making systematic errors in its forecasts of inflation of just 3.9 *per cent* on average. By way of an example, if the correct expectations,  $\pi_{RE}^e$ , were 2 *per cent*, the private sector would be expecting an inflation rate of 2.15 *per cent* and the Central Bank would end up choosing to have average inflation 0.07 percentage points higher than it needed to have. The forecasting error then observed by the private sector would 0.08 percentage points. Errors of that scale are hardly

<sup>9</sup>Issing (1999), Cuckierman and Meltzer (1986).

<sup>10</sup>These figures are derived from (14) and (17) respectively and reflect proportional errors - not percentage points.

likely to generate great pressures for a switch to the expense of computing the full rational expectations solution.

Even in a country where the correct underlying inflation rate is 10 *per cent*, the private sector and the Central Bank would get  $\pi$  values that were too high by 0.4 percentage points. However, if the underlying rate were 20 *per cent*, then their errors begin to approach 1 percentage point. Incomplete transparency and ambiguity may therefore become more of a contentions issue in middle-to-high inflation countries. The Central Bank may see greater advantages in ambiguity because of the ability to manipulate expectations and average inflation downwards; and also to build its reputation for discipline by creating errors in which inflation outcomes consistently come out lower than expectations. It is natural that the private sector will see disadvantages in its own persistent errors and the deflation biases that they imply. However, the Central Bank will lead in this case since the process of learning is of little immediate value.

**Case b).** Suppose more realistically, that the Central Bank chooses  $b$  from a normal distribution such that it can be 99 *per cent* certain to remain inflation averse (conservative) at all times. Assuming the same mean, this implies  $b \sim N(0.5, 0.028)$ , with 99 *per cent* of the distribution in the unit interval. In this case, private sector expectations,  $\pi_{CE}^e$ , will be in error (too high) by 3.7 *per cent* and  $E(\pi_{CE}^*)$  higher than the rational equilibrium solution by 1.2 *per cent*. The private sector then faces systematic errors in the inflation forecasts of just 2.4 *per cent*.

These results show that, if the Central Bank were persuaded that it should be inflation averse under all circumstances, then the difference between the certainty equivalent and the full rational expectations solutions would be rather small: probably 3-4 *per cent* in the private sector expectations; and one half to one third of that in the Central Bank's assessment of the inflation rates it can achieve on average. That means the forecasting errors made by the private sector would be small: typically 2-3 *per cent* of the actual inflation outcomes. These errors are almost certainly small enough to be ignored. Indeed if the costs of monitoring the ECB or 'Fed Watching' were greater than 2.4 *per cent* of the private sector's research budget - which seems highly likely - then it might actually be rational to do so. In that case, it is unlikely that the private sector will be under any real pressure to adopt the rational expectations solution - unless the Central Bank decides, out of self-interest, to provide the necessary information on the distribution of  $b$  for free.

**Case c).** With respect to output, the potential deflation bias (output loss) in the certainty equivalence solution, as a proportion of the target level for stabilising output, is

$$\frac{E(y_{RF}^*) - E(y_{CE}^*)}{k} = \frac{\sigma_u^2}{(1 + \beta)^2} > 0$$

which shows that the bias is eliminated in the rational expectations solution. Evaluating this expression for the case of uniformly distributed  $b$  values, we find

the certainty equivalent solution leads to a deflation bias of 0.22 *per cent* in output target units. If preferences are normally distributed, the bias would be 0.074 *per cent*.

## 4 Conclusions

Full transparency is unequivocally the desired result. Inflation, is at its lowest (and the deflation bias is altogether eliminated) if the Central Bank supplies all the necessary information about its intentions to the private sector; or if it could persuade the private sector to invest the resources to collect that information. Problems arise however, if the following occur: a) the gains from using a full information/full transparency solution over the certainty equivalent case with limited information are relatively small; the private sector would then be unlikely to invest the money to gather sufficient information; b) even if it did attempt to gather the necessary information, this may not be sufficiently accurate (error free); c) finally, even if the Central Bank supplied the information, its credibility may not be perfect - in which case the private sector may not accept that information without question. In any of these cases, the Central Bank would be forced onto the certainty equivalence solution. The losses, for either party, would be small and can be minimised by maintaining ambiguity. Thus, if the costs of ‘monitoring the Central Bank’ exceed the gains of a full transparency solution, constructive ambiguity may indeed be the rational strategy.

1. Following this, our analysis then shows that strict inflation targeters and very conservative Central banks will be tempted to be less than fully transparent, and to maintain a degree of ambiguity in their decision making. That will decrease average inflation  $E(\pi_{CE}^*)$  and its associated expectations  $\pi_{CE}^e$ , at the cost of a permanent deflation bias. The latter will be relatively unimportant to a genuinely conservative Central Bank.
2. We have also found that  $E(\pi_{RE}^*) = \pi_{RE}^e < E(\pi_{CE}^*) < \pi_{CE}^e$ . But  $E(y_{CE}^*) < E(y_{RE}^*) = 0$ . Hence conservative Central Banks face no trade-off: full disclosure leads to lower inflation rates but no deflationary bias. However, if the deflationary bias is unimportant to them, and if they are continuously and very carefully monitored (because they are independent of the political authorities), they may nevertheless face a temptation to employ constructive ambiguity since that will produce a better inflation performance than expected ( $E(\pi_{CE}^*) < \pi_{CE}^e$ ) at an unimportant cost, for as long as the private sector refrains from trying to learn. They may also think it safer to do this if they think the private sector’s information gathering is likely to be error-prone. That in practice, may be the more persuasive case for retaining an element of ambiguity, or incomplete transparency, in policy making.
3. A liberal Central Bank faces a much greater conflict because a) the deflation bias would be more important to them (and larger if  $k$  is larger); b)

the expectations error it could generate to bolster its reputation would be smaller; c) the gap between  $E(\pi_{RE}^*)$  and  $E(\pi_{CE}^*)$  will be larger, meaning that supplying fuller information to the private sector is more worthwhile.

On this view, strict inflation targeters and conservative Central Bankers are likely to argue for constructive ambiguity (as they do, Issing 1999). But more liberal Central Banks, or those with an asymmetric inflation target (such as the Bank of England), or an explicit stabilisation mandate (the Federal Reserve), would typically aim to become more transparent.

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