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## The Relationship Between Offshoring, Growth and Welfare

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# The Relationship Between Offshoring, Growth and Welfare

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June, 2019

**Abstract**

A dynamic model of offshoring is studied which permits the analysis of how offshoring can affect economic and welfare outcomes. Firm owners make location decisions based on the future returns from locating in either foreign or domestic markets, or ceasing operations altogether. It is shown that increased offshoring can raise growth and welfare in *both* the domestic and foreign economies. Imposing a tax on firms that relocate abroad can make firms delay this move, but at the cost of lowering both domestic and foreign welfare, as well as growth. A tax on domestic profits has an ambiguous impact on growth, while lowering domestic welfare. The effect that these policy or parameter changes have on domestic income inequality, and international wage inequality is also studied. In contrast to the view that the economic impact of outsourcing is equivalent to that of admitting more immigrants, the present model implies that these policies are nearly the *opposite* of each other. Immigration reduces growth, and lowers the welfare of both foreign and domestic agents.

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# 1 Introduction

A dynamic model of offshoring is studied to investigate how the prospect of foreign offshoring can influence domestic innovation, and therefore the growth rate. The existence of a foreign labor market, to which domestic firms can relocate in order to take advantage of the lower costs, can influence the production and innovation decisions of domestic firms. It is also possible to assess how various polices, such as changes in taxes or immigration policy, can act in conjunction with this foreign labor market, to affect the growth rate, as well as welfare. Because the focus here is on the dynamics and growth aspects of outsourcing, this analysis stands in stark contrast with the existing literature which largely reticent on the dynamic decision-making that may underlie offshoring.

The topic of "offshoring", or foreign-outsourcing, seems to have aroused increasing concern in recent years. Politicians frequently seem to become exorcised over the fact that US firms occasionally relocate domestic operations in order to take advantage of lower costs overseas. It is a frequently stated presumption that offshoring is obviously bad for the domestic economy because it results in lower domestic output and employment. Such an outlook would seem to presume a "zero sum game" in that if welfare foreign workers is improving, that of domestic workers must be eroding. Missing from such a superficial view is any analysis of the *dynamics* of such an issue. In particular, left unstudied is how *prohibiting* such movement might influence the prior innovation activity. Is it possible that permitting such movement, or offshoring, might be good for the domestic economy as well? On the surface, it might be possible for the following reason. The incentive to innovate must be the expected discounted profit that the innovator expects to receive. The higher is this profit, the greater will the innovation activity. By permitting firm-owners to relocate their firms to a lower cost, foreign location, after a period of domestic production, this may raise the prospective profit from innovating, and facilitate an increase the overall level of innovation. On the other hand, it could be that offshoring could possibly be harmful for growth. This could happen if offshoring itself employed resources that might otherwise be used for innovation, which might then result in lower growth.

There is a multitude of additional reasons why the study of offshoring is important. For example, one might like to know how offshoring affects economic growth and wages in both the domestic and foreign economies. If offshoring raises growth rates everywhere, then it seems like there might be a "free lunch", in that agents in all economies might benefit. One might also wish to investigate how this phenomenon would affect measures of cross-country inequality. If, for example, all agents were to benefit from offshoring, but that global inequality would rise, then this raises the question as to whether this trade-off is nevertheless worthwhile. Similarly, one might also wish to understand how offshoring would affect domestic income inequality.

There is a limited amount of existing research into the issue of offshoring. Much of the empirically-oriented literature focuses on the effect that this has on domestic and foreign wages. Hummels, Jorensen, Munch, and Xiang [13] study the empirical relationship between wages and offshoring for Danish workers. Feenstra and Hanson [6] study the effect that wages has had on US wages from 1979-1990.[1] They find that while offshoring certainly affects relative wages, the impact seems to be much smaller than that of the introduction of computers. Ottaviano, Peri and Wright [22] find that offshoring may actually be beneficial for domestic firms and wages because it can result in higher productivity for the domestic firms. Mills [20] suggests that the movement of firms may not be attributable just to lower labor costs, or factor prices generally, but more importantly firms might be escaping more onerous domestic regulations as well. Jackson [19] finds that much of this offshoring occurs in the manufacturing industry, but there is also some in a variety of service industries. Also, he finds that US multinational corporations are employing an increasing share of

[1] See also Feenstra [5].

foreign labor.

In addition to this empirical literature there is also a more theoretical literature that studies the decision-making underlying offshoring, but much of this research is of a static nature, and so does not study the dynamic character of the issue. Antràs and Helpman [3] analyze a model in which intermediate goods can be produced either domestically or abroad. Grossman and Helpman [10] use a model in which firms search for partners, in order to study the relationships that are formed when firms outsource some production. Like Grossman and Helpman [8], these papers form a literature that shows how outsourcing can contribute to an understanding of industrial structure, in which the organization of production is endogenous. Grossman and Rossi-Hansberg [11] propose a model of offshoring in which there are "tradeable tasks", and this contrasts with much of the existing literature in which the focus is on *goods*. Because most of these models in this existing literature are essentially static in nature, they are not useful for studying dynamic phenomenon. Therefore, these models cannot shed much insight into domestic macroeconomic conditions, and specifically into such phenomena as the growth rate. The present paper employs a dynamic growth model to fill this lacuna.

The goal here is to study a simple dynamic model in which innovation takes place in the domestic economy, but the firm has the option to locate either domestically, or abroad. Locating in the domestic location has the benefit of providing the option of improving a relatively new technology, while the foreign location provides cheaper inputs (i.e. labor). There are interesting questions that then arise. First, when is it optimal to move the firm abroad from the domestic location? Secondly, what are the factors that influence this location decision? Third, when is it optimal to shut down the firm in the foreign location? That is, what is the optimal stopping rule for this problem? Fourth, does the presence of the foreign economy, as well as its many features, change the incentive for innovation in the domestic economy? That is, does the possibility of offshoring raise the incentive to innovate, and therefore alter the growth rate, or are there countervailing factors that reduce this incentive? Fifth, how does the introduction of a tax in either the domestic or foreign economy influence the location decision, as well as the incentive to innovate? Sixth, if the domestic government were to introduce some penalty to firms who moved their production facilities abroad, is it possible this would reduce domestic innovation, or welfare? Seventh, how do these various policies, such as an increase in offshoring or an increase in the profit tax, influence the level of domestic income inequality, or international wage inequality? Lastly, this is an ideal environment to investigate whether offshoring is merely a substitute for immigration of foreign workers.

Within the context of the model studied below, it may turn out that permitting a firm to re-locate abroad can *benefit* the domestic economy in several ways. First, by generating a bigger potential pool of profit as the return to innovation, this may result in more innovation activity, and also raise wages.[2] Secondly, having firms re-locate abroad "frees-up" inputs (i.e. labor) in the domestic economy, which results in higher profit, and this again may result in more innovation activity.[3]

The model studied here will be rather simple, which will emphasize the impact that factor offshoring can have on both foreign and domestic factor prices. That is, absent from the analysis will be such things as technological or international spillovers. These features may well be integral to our understanding of the universal impact of technological change, but they will also complicate the simple channels that offshoring can have in the absence of these features.

The present analysis is interesting for another reason. If the ability to offshore lowers domestic growth, then if there are international spillovers that result from innovation, this could lower growth

[2]This is exactly what is found by Fritsch and Görg [7]: they find a significant empirical relationship between offshoring and R&D spending.

[3]It seems to be an open question within the empirical literature as to whether offshoring serves as a complement or substitute for domestic employment. A priori, the model studied here is agnostic on this issue. However, if the impact on the growth rate is sufficiently positive then offshoring can certainly complement domestic employment.

internationally as well. Furthermore, this is likely to result in lower political support for offshoring and could give rise to government adopting policies that make offshoring less attractive. Conversely, if offshoring results in higher domestic growth, then this suggests that there may be considerable benefits from countries behaving cooperatively in encouraging the international movement of firms and technology.

The model employed below has the flavor of the "product cycle models", say in Vernon [24], or Grossman and Helpman [8].[4] However, instead of product cycles there are cycles in the nature of which managers are operating frontier technologies or firms. In contrast with this existing literature, here the focus will be on inherently dynamic economies, which then can facilitate the explicit study of growth rates, as well as the study of decisions regarding innovation, location, as well as firm destruction.

The model presented below will also have an explicit "creative destruction" feature, in that new firms or technologies are continually being discovered, while older firms are shut down voluntarily as their profitability diminishes.[5] However, in contrast with the existing literature (see, for example Aghion and Howitt Aghion [2]), in this model there autonomous processes determining the "creative" and "destructive" processes. That is, these decisions are made by different agents, instead being one amalgamated mechanism.[6] The model studied here is related to that studied in Huffman [12], in that there are new firms being created while older firms are ceasing operations. In an equilibrium there are different agents (or firms) making somewhat independent innovation and exit decisions (although these decisions influence each other through market prices.)

There are many innovative results that are shown below. First, in the benchmark model it is shown that both the foreign and domestic growth rates, as well as welfare measures, can be increasing in the amount of offshoring (or, the size of the foreign labor force). An increase in the cost of offshoring may increase the time a firm spends in the domestic market, but also in the foreign market as well. An increase in the cost of offshoring can result in reduced growth and welfare for both the foreign and domestic economies. A tax on domestic profit has a complicated impact, but can cause firms to offshore sooner, and to stay abroad longer. This tax can also lower domestic welfare, and raise the welfare of foreign workers. It is also shown that the effect of offshoring is quite different from just permitting foreign workers to emigrate into the domestic economy. For example, offshoring can result in a higher growth rate for both economies, while increased immigration can lower the growth rate. Lastly, in contrast with a popular view, there are many important differences between increased immigration and offshoring. The latter can have beneficial effects on welfare and growth while the former can have the opposite effect.

The structure of the paper is as follows. The economic environment is described in Sections 2 and 3. The dynamic optimization problems of firm-owners are studied in Section 4. The equations characterizing the steady state growth path are summarized in Section 6. Some features of the equilibria are described in Section 7. The main features of the model are studied using numerical

[4] Specifically, chapter 12 of the latter.

[5] There are other papers in which incumbent firms exit an industry, while newer firms enter. For example, Luttmer [17] presents a model that is used to characterize the size distribution of firms. In his paper, firms face exogenous variations in productivity, which eventually leads to exit from the market when they can no longer cover their costs. However, Luttmer does not study many of the issues addressed here, such as why the exit decision may not be made in a socially optimal manner, or how this decision affects the incentives for innovation, or how government policies might alter this decision to achieve a better outcome. There are other models such as firms exit at a random, exogenous rate (Jones and Kim [15]),

[6] As explained in Huffman [12], in many papers in this literature, such as the Aghion and Howitt paper, there are no separate "creative" and "destructive" channels or decision. Instead, when one new good or technology is invented, the incumbent must be retired. In the present paper this is not the case and, in fact, there are many firms or technologies operating in a competitive market simultaneously. The decision to retire a firm is a decision made by the firm owner, and is the result of an optimal decision. Furthermore, this is done independently of the innovation decisions, which are made by other agents in the economy. Of course, these decisions interact, or influence each other through market prices.

methods, and this is presented in Section 9.

# 2 Description of the Environment

Time is continuous, and is generally indexed by $t$. There will be two economies, or production locations: the domestic economy ($d$), and the foreign economy ($f$). In each location there will be labor to be employed. In an equilibrium, the wages in the domestic economy will be higher than in the foreign economy. The workers will do nothing but work. These workers will have a trivial non-dynamic problem: they supply a unit of labor inelastically, and do not innovate or conduct any other activity.

There will be another class of agents, who will be termed *entrepreneurs*. These agents will try to innovate a new technology, and when they are successful, this will permit them to operate or mange a firm, which will employ labor. They can only manage one firm at a time, and it is located either in the domestic or foreign economy. When an innovator discovers a new technology, he may locate either domestically or abroad. However, there is a built-in advantage to initially locating in the domestic market in that there is "learning-by-doing" that takes place in the domestic market, which gradually shifts the production function upward. This process has diminishing returns and this, combined with the fact that wages are rising over time, will mean that at some future date it will be advantageous for the firm to cease domestic operations and instead produce in the foreign market.

In summary, at any date there will be three types of entrepreneurs. There will be those operating firms in the domestic market, those in the foreign market, and those who are not operating firms but who instead are trying to discover a new technology will can be used to operate a new firm. These are potential innovators.

## 2.1 The Foreign Economy

The foreign economy is simple. It has $N^f$ workers, who are all identical, and they receive a wage of $w_t^f$ at date $t$, which is determined in a competitive labor market. Firms that are operating in this economy pay the market wage for labor, which is the only input into production. Output net of labor costs may be consumed by the firm-owners. To insure that there is a balanced growth path, it will be necessary to insure that this foreign wage grows at some constant rate.

## 2.2 The Domestic Economy

In the domestic economy there are $N^d$ workers, and they receive a wage of $w_t^d$ at date $t$. Again, firms that are operating in this economy pay the market wage for labor, which is the only input.[7]

There is also a pool of other agents, which is normalized to one, who are termed entrepreneurs. These individuals can sacrifice some utility so that they can potentially innovate a new technology, which will permit them to become a firm-owner, or manager, which will enable them to consume the profits. Individuals who have not innovated, or are not in possession of such an operational technology, will consume nothing but are permitted to innovate. These managers will have preferences described as follows:

$$\int_0^\infty e^{-rt} \left(c_t - h\left(z_t, \bar{\lambda}_t\right)\right) dt,$$

[7] The reader will eventually realize that there is nothing in the framework that necessarily means identifying this (foreign or domestic) resource as labor, since there really is not any intensive or extensive labor decision. It is only necessary that there be some resource, that is in limited supply, which is a factor of production, and will have its value priced in a competitive market. Since much of the literature on offshoring considers the effect that this activity has on employment and wages, it seems sensible label it here as labor.

where $c_t$ is the amount of consumption, and $z_t$ is the amount of innovation activity, while $h\left(z_t, \bar{\lambda}_t\right)$ is the disutility from this activity.[8,9] Also, $\bar{\lambda}_t$ is the leading or frontier technology at date $t$. These agents can either manage a firm, which is located either domestically or abroad, or, if they do not do so they can choose to innovate.

## 2.3 The Problem of a Firm-Owner

Each firm owner is in possession of a technology, which is indexed by the parameter $\lambda$, and the production function is the $\lambda\left[(n_t^\alpha) + q(s)\right]$, where $n_t$ is the level of employment for this firm, and $s$ is the age of the firm. The term $(q(s))$ is a feature is termed "learning by doing". This feature is assumed to be an increasing, concave function. An example of such a function would be $q(s) = \gamma_1\left[1 - e^{\gamma_2 s}\right]$, for positive constants $\gamma_1, \gamma_2$. This might be interpreted as a form of (unsophisticated) learning by doing, so that production becomes more efficient as it ages (independent of its scale of production). The technology for each firm is proprietary in that other producers cannot simply copy or mimic the production technology of their more productive counterparts. For each firm with a fixed technology level ($\lambda$), there is another firm that is arbitrarily close to this one.

Then the optimization problem for a typical firm-owner, whose firm or technology is of age $s$, is given as follows:

$$\pi_{t,s}^d = \max_{n_t}\left\{\lambda\left(n_t^\alpha\right) - w_t^d n_t + \lambda q(s)\right\}. \tag{1}$$

Here $w_t^d n_t$ is the wage bill. What will happen here is that the firm will face a path for domestic wages that is rising, and this will inevitably reduce profit. However, a young firm can naturally raise its level of output for a while (or, alternatively, lower its costs) through this learning-by-doing process. However, since this process has diminishing marginal returns (because $q(s)$ is concave), then eventually the effect of the higher wages will overwhelm the benefits of being an aging firm. At some point however, it may benefit the firm to switch locations, and when it does so, the learning-by-doing process ceases.[10]

Another interpretation of the term $q(s)$ is that this captures the "spillovers" of technology from the domestic economy to the foreign economy. In this way the foreign economy benefits from having the firm operate in its domestic home for a period prior to moving abroad.[11]

[8] Alternatively, it could be assumed that at any date $t$, the individual has preferences over the consumption goods which deliver a range of *services*. At date $t$ this range is defined over the interval $\left[0, \bar{\lambda}_t\right]$. Individuals then care about the services yielded by those commodities, and preferences are defined as follows: $\int_0^\infty e^{-rt}\left(\int_0^{\bar{\lambda}_t} c_{s,t} ds\right) dt$. As time progresses, some older goods are no longer produced, while some new goods are introduced. An innovation in $\lambda$ could then be interpreted as a new technology for producing a new good. Each commodity then has the same production function $c_{s,t} = n_s^\alpha$, and so it will be optimal to devote more labor to the production of more advanced technologies or commodities. This approach is similar to that employed by Grossman and Helpman.

[9] The analysis could be conducted with other non-linear preferences as well. The current approach may be the simplest line of attack, and this also has the advantage that welfare is proportional to output. In addition, with linear preferences, credit markets operate in a primitive manner since everyone can borrow or lend at the fixed rate of interest. It is also the case that there is no point in a firm-owner selling his firm to to someone with a lower level of wealth or consumption.

[10] There is an alternative approach to describing the model that does not rely on this specific type of "learning by doing". In this case, there would be some factor in the production function that is accumulated through investment. The initial accumulation of this factor raises output, but ultimately the returns to this deteriorate because of diminishing marginal returns, and this contributes to choosing to move the firm abroad. However, this feature entails studying another layer of optimal decision-making related to the optimal accumulation of this factor. The alternative approach used in this paper seems to be a much simpler way to proceed. If the term $q(s)$ is interpreted as further "innovation" undertaken by existing firms, then this would bring the model into line with the findings of Decker, Haltiwanger, Jarmin and Miranda [4], who find that a non-trivial fraction of innovation is conducted by existing firms.

[11] There is still another interpretation of the learning-by-doing technology. This could be interpreted as further secondary innovations undertaken by the firm owner subsequent to the initial drastic innovation that led to the start-up

The demand for labor by the domestic firm is then given by

$$n_t(\lambda) = \left(\frac{\alpha\lambda}{w_t^d}\right)^{\frac{1}{1-\alpha}}.$$

Let the distribution of *domestic* technologies be described as $F_\lambda^d()$. Since there are $N^d$ domestic workers, equating the supply and demand for workers then implies that

$$N^d = \int n_t(\lambda)\, dF_\lambda^d(\lambda) = \int \left(\frac{\alpha\lambda}{w_t^d}\right)^{\frac{1}{1-\alpha}} dF_\lambda^d(\lambda). \tag{2}$$

If the firm instead chooses to operate abroad, it cannot engage in learning-by-doing (i.e. this process ceases).[12] The profit function for a firm that chooses to operate abroad, after operating for $t_1$ periods in the domestic market, is written as

$$\pi_{t,t_1}^f = \max_{n_t} \left\{\lambda\left(n_t^\alpha\right) - w_t^f n_t + q(t_1)\right\}, \tag{3}$$

where $t_1$ is the amount of time this firm was operational in the domestic economy, and this is now fixed.[13] Equating the supply and demand for labor in the foreign economy then implies that

$$N^f = \int n_t(\lambda)\, dF_\lambda^f(\lambda) = \int \left(\frac{\alpha\lambda}{w_t^f}\right)^{\frac{1}{1-\alpha}} dF_\lambda^f(\lambda), \tag{4}$$

where $F_\lambda^f()$ is the distribution of foreign technologies.

The primitive nature of the learning-by-doing process in the profit function ensures that labor demand is independent of this process.

In general, it will be the case that the foreign economy will have lower wages ($w_t^f < w_t^d$), so that locating abroad has a cost advantage. On the other hand, the "learning-by-doing" process only occurs domestically, so there is an advantage to locating in the domestic economy. The approach presented below will consider the case in which new firms will initially choose to locate in the domestic economy, but that they will then eventually choose to move abroad.

It will be the case that the support of the operational technologies will be rising over time due to more innovations. That is, the support for the distributions $F_\lambda^d(\lambda)$ and $F_\lambda^f(\lambda)$ will be shifting to the right. Since the domestic and foreign wages are determined as follows:

$$\left(w_t^d\right)^{\frac{1}{1-\alpha}} = \left(\frac{1}{N^d}\right)\int (\alpha\lambda)^{\frac{1}{1-\alpha}}\, dF_\lambda^d(\lambda) \tag{5}$$

$$\left(w_t^f\right)^{\frac{1}{1-\alpha}} = \left(\frac{1}{N^f}\right)\int (\alpha\lambda)^{\frac{1}{1-\alpha}}\, dF_\lambda^f(\lambda). \tag{6}$$

of the firm. Decker, Haltiwanger, Jarmin, and Miranda [4] documents that although a non-trivial portion of innovation is conducted by new entrants, existing firms also continue to innovate as well. This alternative interpretation would capture some of these features associated with non-drastic innovations. In other contexts, these observations have led to the study of models where innovation is performed by both entrants and incumbents (see Acemoglu and Cao [1], Luttmer [16]).

[12] An alternative approach is to assume that the "learning-by-doing" process continues when firm relocates, but that the returns to doing this are quite low. It would seem the assumption that the process stops when relocation occurs is the simplest approach. In any event, it seems necessary that there be some advantage to locating in the domestic economy initially, to offset the higher wage costs.

[13] It is not critical that the learning-by-doing cease in the foreign market, only that the returns diminish. However, this approach seems simplest.

If the supports of these distributions are shifting to the right at some constant rate, then the wages will also be rising at this same rate, which will then be consistent with having a balanced growth path. Along this path it will be necessary that both the foreign and domestic wages grow at the same rate, and so the distributions of technologies will then be shifting at the same rate as well.

To make this exercise interesting it will have to be the case that the domestic wage is above the foreign wage ($w_t^d > w_t^f$), and this will be the motivation for relocation. This will result if $N^d$ is sufficiently large relative to $N^f$.[14] Also, it must be the case that the expected profit for a new firm locating in the domestic economy would be greater than the expected profit to locating it abroad, for otherwise it might be difficult to find a reason why a new firm would locate in the domestic economy. This is the reason for having the "learning-by-doing" feature.

The model will have the property that as the leading technology for an economy rises together with the market wage, while the technology for any specific firm is fixed, this will imply that employment at the firm will fall over time. This is consistent with the findings of Monarch, Park and Sivadasan [21] who find that offshoring firms exhibit declining employment.

# 3 The Distribution of Firms or Technologies

The distribution of available technologies becomes important because it affects so many features of the economy, from factor prices to location decisions, to innovation activities, and many more. There are a continuum of firms, with technologies denoted by the parameter $\lambda$. A firm that has technology $\lambda$, has this parameter fixed forever, until it ceases operation. It would then seem imperative to keep the analysis as simple as possible and therefore some structure will be put on this distribution. Henceforth, we will let $\theta_t \equiv \left(\lambda/\bar{\lambda}_t\right)$ denote the "relative technology" of a particular firm, which possesses technology parameter $\lambda$, when the best, or *frontier*, technology is $\bar{\lambda}_t$ at that date. Obviously $\theta_t$ ranges between $\underline{\theta}= \left(\underline{\lambda}_t/\bar{\lambda}_t\right)$ and unity. On a balanced growth path, the distribution of $\theta_t$ will assumed to be time-invariant. It can then be shown, through the use of the Kolmogorov forward equation that the density must satisfy $f_\theta = (1/\theta)$ over the interval $[\underline{\theta}, 1]$. This implies that the distribution $G_t(\lambda)$ will be a truncated reciprocal distribution.[15]

In the analysis of the equilibrium below it will be the case that new firms will choose to produce in the domestic market initially, and then subsequently they will move abroad. With this in mind, let $\bar{\theta}$ denote the relative technology of the lowest productivity firm located in the domestic market. Similarly, let $\underline{\theta}$ denote the relative technology of the lowest productivity firm located in the *foreign* market. Hence in the domestic market, at any date, the distribution of relative technologies is distributed with cdf $(\ln(\theta))$, over the interval $\left[\bar{\theta}, 1\right]$, while in the foreign the distribution has the same cdf, but is over the interval $\left[\underline{\theta}, \bar{\theta}\right]$. Since there is a one-to-one correspondence between managers and firms (or technologies), this will then imply the following two equilibrium conditions:

$$M^d = \int_{\bar{\theta}}^{1} \left(\frac{1}{\theta}\right) d\theta = -\ln(\bar{\theta}),$$

[14] Alternatively, it could be assumed that the foreign workers are not as productive as the domestic workers.

[15] Note that the reciprocal distribution is what the Pareto distribution converges to as the latter's shape parameter approaches zero. The truncated reciprocal distribution has the convenient property that, as you raise the lower and upper limit by the same proportion, the density on the overlapping section is unchanged. In other words, the mass lost on the left side exactly equals the mass gained on the right side. The reciprocal distribution is limit of the Pareto distribution converges to as the latter's shape parameter approaches zero. Fortunately, there is empirical support for employing this distribution. Luttmer [16], [17] finds that the size distribution of firms can be closely approximated by Pareto distribution. This has led to many other researchers to construct growth models which give rise to such a distribution (for example, Acemoglu and Cao [1], and Luttmer [18]). Obviously the "truncated" nature of the distribution employed here is a simplification used for convenience.

$$M^f = \int_{\underline{\theta}}^{\bar{\theta}} \left(\frac{1}{\theta}\right) d\theta = \ln(\bar{\theta}/\underline{\theta}) = -\ln(\underline{\theta}) - M^d$$

Of course $\bar{\theta}$ is not a parameter, and instead is determined by the optimal decisions of firm-owners, and when they choose to relocate abroad. Similarly, $\underline{\theta}$ is determined by when foreign firm-owners choose to shut down their operations. Since all firms are identical, up to the scaling factor of $\bar{\lambda}$, they will make the same location decisions.

# 4 Optimization Problem of a Firm-Owner, or Innovator

Workers do not have an optimization problem to consider. This leaves three types of agents to study: there are firm-owners who operate domestically, firm-owners who operate abroad, and innovators who do not manage a firm, but who are attempting to innovate a new technology. Let $M^d$ denote the number of domestic managers, $M^f$ will be the quantity of foreign managers, and $M^I$ will be the quantity of innovators. The number of entrepreneurs is normalized to unity, and so the following must hold

$$M^d + M^f + M^I = 1. \tag{7}$$

Let $V^d$, $V^f$, $V^I$ be the value functions associated with the domestic firm-owner, the foreign firm-owner, and the innovator, respectively.

## 4.1 Domestic Producers

The domestic producer, who owns a technology ($\lambda$) of age $s$, will receive an instantaneous profit $\pi^d_{t,s}$ at date $t$. For any such producer, the state variables consist of the current leading technology ($\bar{\lambda}_t$), which will help determine the equilibrium domestic wage ($w^d_t$), the firm's own technology parameter ($\lambda$), as well as the age of the firm ($s$). If at any date $t$ the firm-owner decides to shut down domestic operations and move abroad, he can do so by paying a cost $K_t$.[16] Therefore, suppressing the explicit expression of this state vector and using some short-hand notation, the value function of this agent can then be written as:

$$rV^d_t = \max\left\{\pi^d_{t,s} + \dot{V}^d_t, \quad r\left(V^f_t - K_t\right)\right\}, \tag{8}$$

where the indirect profit function (equation (1)) is given by the following

$$\pi^d_{t,s} = (1-\alpha)\left(\lambda^{\frac{1}{1-\alpha}}\right)\left(\alpha^{\frac{\alpha}{1-\alpha}}\right)\left(w^d_t\right)^{\frac{-\alpha}{1-\alpha}} + q(s). \tag{9}$$

After a brief period of initial production, the value of the firm would be falling because of the increasing domestic wages, and so then $\dot{V}^d_t < 0$. Of course, at the date at which $V^d_t = V^f_t - K_t$, it will be optimal for the firm to switch to producing in the foreign location. If the domestic wage is rising, then clearly the profit will be falling since $\lambda$ is constant. It will be important that $q'(s)$ not be too large, relative to the growth rate of the domestic wage, for otherwise the domestic firm would never choose to re-locate abroad.

## 4.2 Foreign Producers

Consider a foreign producer, who owns a technology ($\lambda$), and who has already operated in the domestic economy for $t_1$ periods. This producer will receive an instantaneous profit $\pi^f_{t,t_1}$ at date $t$. For such a foreign producer, the state variables consist of the current leading technology ($\bar{\lambda}_t$), which

[16] In order to preserve the stationary nature of the economy, it will be assumed that this cost will grow at the same rate as the frontier technology at each date. Also, it should be noted that to the extent that $K_t$ is low, or perhaps even negative, this may reflect lower costs or perhaps a weaker regulatory environment in the foreign economy.

will help determine the foreign wage ($w_t^f$), the firm's own technology parameter ($\lambda$), as well as the age of the firm when it departed the domestic economy ($t_1$). The firm-owner who is producing abroad has a value function that satisfies the following:

$$rV_t^f = \max\left\{\pi_{t,t_1}^f + \dot{V}_t^f,\ \ rV_t^I\right\}, \tag{10}$$

where indirect profit function (equation (3)) is written as follows

$$\pi_{t,\bar{s}}^f = (1-\alpha)\left(\lambda^{\frac{1}{1-\alpha}}\right)\left(\alpha^{\frac{\alpha}{1-\alpha}}\right)\left(w_t^f\right)^{\frac{-\alpha}{1-\alpha}} + q(t_1), \tag{11}$$

where $t_1$ is now fixed, and so $q(t_1)$ should be interpreted as just a constant. Again, if the foreign wage ($w_t^f$) is rising, then clearly the profit will be falling. Again, the value of the firm would be falling because of the increasing foreign wages, and so then $\dot{V}_t^f < 0$.

### 4.3 Innovators

Lastly, there are innovators who do not produce but who spend their time trying to discover a new technology. These agents should be treated as identical, irrespective of their past history. These agents expend effort ($z$) that results in the discovery of a new frontier technology. These innovators have discoveries or innovations that arrive according to a Poisson arrival rate. Let $\mu(\cdot)$ be the probability of locating such a technology, and it will be convenient to assume that $\mu(z)$, is merely a function of $z$, and this function is increasing, differentiable, and concave.

The innovator then has a value function that satisfies the following Hamilton-Jacobi-Bellman equation:[17]

$$rV_t^I = \max_{z\geq 0}\left\{-h\left(z_t,\bar{\lambda}_t\right) + \mu\left(z_t\right)\left[V_t^d - V_t^I\right]\right\}. \tag{12}$$

This last problem then also has the following optimization condition:

$$\frac{dh\left(z_t,\bar{\lambda}_t\right)}{dz_t} = \mu'\left(z_t\right)\left[V_t^d - V_t^I\right]. \tag{13}$$

Since all innovators face the same problem, they will all choose the same value of $z$.

One could interpret this "research sector" as being an informal, or non-market, sector within which all innovation conducted. For example, it could be that these innovators are always spending their time engaged in puttering around informally, and there is some prospect this activity will turn out something very profitable.[18] The amount of effort expended by such an agent in discovering a new technology ($z$) cannot be observed by other agents, and so it is not possible to engage in contracts contingent on the amount of effort ($z$), or the outcome from such effort. The effect this innovative process is fully internalized by the individual.

[17] To be formally correct here this expression should also include the option of the innovator to take is new firm abroad immediately, and skip the process of locating domestically. This would be equivalent to letting $t_1 = t_2$ in the analysis below.

[18] This is not entirely ad-hoc, as it has its motivation in economic history. Many of the most historic inventions were produced by individuals who were not employed in research labs, or universities, but instead were people tinkering around in their spare time, and ultimately made historic discoveries. For example, the Wright brothers were merely two capable mechanics who had bicycle shop but who, in their spare time, loved to play around with things that might fly. This is also (or perhaps especially) true of the electronic revolution over the past century. Issacson [14] describes the multitude of inventions that have given rise to electronic, computer, internet, and IT revolutions. In his book, Issacson repeatedly refers to people making or discovering things in their garage in their spare time. The word "garage" seems to arise recurrently in this narrative, especially so when talking about the history of Silicon Valley. Reading this narrative one gets the impression that most of the discoveries were made by people, many of whom would never graduate college, working long hours in their garages, and that the company offices or laboratories were merely places where the inventors went to the next day to brief others on the progress of their research effort.

Figure 1 gives a visual representation of how the producers and innovators interact. When an innovator discovers a new technology, he is immediately, but temporarily, located at the technological frontier of the domestic economy. However, as other innovations take place he moves further back in the domestic distribution, until such time as he decides to move his operations to the foreign economy. The arrows in the diagram illustrate the movement of these innovators and producers amongst the distributions.

### 4.3.1 Example

An example or sample path for such an economy is shown in Figure 2. This figure shows how the dynamic paths for these value functions would interact.[19] On the horizontal axis is time, and the vertical axis if the log of the value function for a manager. This person begins as a potential innovator, and discovers a new technology at date $T_1$. This manger operates the firm domestically until date $T_2$, at which time he moves production abroad. As can be shown the firm benefits from the learning-by-doing process while it is located domestically, and this is why the value function ($V_t^d$) may be increasing initially. At date $T_3$ this manager decides to shut down foreign operations, and then becomes a potential innovator again. At some future date he then strikes-it-rich again at date $T_3$, and the cycle begins anew.

## 4.4 Interaction of These Value Functions

It is important to understand the linkages between these different value functions, since the decisions of agents at one date are a function of decisions that will be made in the future. Consider a firm-owner who develops a new frontier technology at date $t = 0$. The true value function of a new firm owner would then be described as follows (with some abusively cryptic notation):

$$V^d = \int_0^{t_1} \left(e^{-rs}\pi_{s,s}^d\right) ds - e^{-rt_1}K_t + \int_{t_1}^{t_2} \left(e^{-rs}\pi_{s,t_1}^f\right) ds + e^{-r(t_2)}V_{t_2}^I, \tag{14}$$

where $t_1$ and $t_2$ are the dates at which the firm ceases operations in the domestic and foreign economy, respectively. This is the sum of the discounted profit from operating in the domestic market, less the cost of moving abroad, plus the profit from operating abroad, and then the terminal value from shutting down the firm. This last equation could be written in the following alternative manner:

$$V_t^d = \int_t^{t_1} \left(e^{-r(s-t)}\pi_{s,s}^d\right) ds + e^{-r(t_1-t)}\left(V_{t_1}^f - K_{t_1}\right) \tag{15}$$

where

$$V_t^f = \int_t^{t_2} \left(e^{-r(s-t)}\pi_{s,t_1}^f\right) ds + e^{-r(t_2-t)}V_{t_2}^I. \tag{16}$$

Note that $V^f$ is actually a function of $t_1$, since this influences the learning-by-doing feature of the technology for the future. These equations satisfy the value matching property that at date $t_1$, $V_{t_1}^d = \left(V_{t_1}^f - K_{t_1}\right)$, and at date $t_2$, $V_{t_2}^f = V_{t_2}^I$. It is also shown in the Appendix that these value functions satisfy the smooth-pasting condition, which is also necessary at an optimum.[20]

[19] This figure is drawn for the case in which $K_t = 0$.

[20] There is a detail that is omitted here. There is the prospect that the firm might elect to go straight from domestic production to shutting down, and not outsource at all. This would happen if $V_{t_1}^f - K_{t_1} < V_{t_2}^I$, for $t_2 \leq t_1$. Strictly speaking, on the right side of equations (8) and (15), the term $\left(V_{t_1}^f - K_{t_1}\right)$ should be replaced by $\max\left\{V_{t_1}^f - K_{t_1}, V_{t_1}^I\right\}$. The marginal conditions, equations (18) and (19) would then have to be modified as well.

The optimal value of $t_2$ can be determined by maximizing equation (14) wrt $t_2$, which then yields the following condition:

$$\pi^f_{t_2,t_1} + \left(\frac{\partial V^I_{t_2}}{\partial t_2}\right) = rV^I_{t_2}. \tag{17}$$

This condition recognizes the fact that the value $V^I_{t_2}$ is an implicit function of the shutdown date $t_2$. Similarly, using equation (15) the optimal value of $t_1$ is then determined by the following equations:

$$\pi^d_{t_1,t_1} = r\left(V^f_{t_1} - K_{t_1}\right) - \left(\frac{\partial V^f_t}{\partial t_1}\right) + \left(\frac{\partial K_{t_1}}{\partial t_1}\right) \tag{18}$$

This is the usual margin condition adjusted for the fact that in this case the terms $V^f_t$ and $K_{t_1}$are functions of $t_1$, which means that the two last terms on the right side must appear. Equation (16) can be used to derive an expression for $\left(dV^f_t/dt_1\right)$. In the technical appendix it is shown that the result, in conjunction with equation (18) then can be used to yield the following condition:

$$\pi^d_{t_1,t_1} + \int_{t_1}^{t_2}\left(e^{-rs}\left(\frac{\partial \pi^f_{s,t_1}}{\partial t_1}\right)\right)ds - \left(\frac{\partial K_{t_1}}{\partial t_1}\right) = \pi^f_{t_1,t_1} \tag{19}$$

This equation has the following interpretation. The right side represents the instantaneous benefit from switching to producing abroad, net of fixed costs. The left side is the instantaneous benefit from producing domestically, and this is composed of three terms: the first is the profit that it produces $\pi^f_{t_1,t_1}$, while the second term measures the added benefit that producing domestically would have on the technology when subsequently employed abroad. The third term measures the change in the cost of relocation. Prior to the optimal shutdown date, the left side of this expression will be greater than the right side. At the moment when these two expressions are equal, it will be optimal for the firm to move operations from the domestic economy to the foreign location. Since the integral term is positive, at the moment of switchover it will be the case, if $\left(\frac{dK_{t_1}}{dt_1}\right)$ is not too large, that the instantaneous domestic profit will be less than the foreign profit $\left(\pi^f_{t_1,t_1} > \pi^d_{t_1,t_1}\right)$.

Let $A^d_\pi(\theta_{t_1}) = \pi^d_{t_1,t_1} - q(t_1)$ and $A^f_\pi(\theta_{t_1}) = \pi^f_{t_1,t_1} - q(t_1)$ denote the non-learning-by-doing components of the domestic and foreign profit functions respectively, at date $t_1$. These expressions are produced explicitly below in equations, and derived in the technical appendix. Substituting these into expressions into equation (19) yields the following useful expression:[21]

$$A^d_\pi(\theta_{t_1}) + \int_{t_1}^{t_2}\left(e^{-rs}\left(\frac{\partial \pi^f_{s,t_1}}{\partial t_1}\right)\right)ds - \left(\frac{\partial K_{t_1}}{\partial t_1}\right) = A^f_\pi(\theta_{t_1})$$

This is further illustrated in Figure 3, which characterizes a case in which $K_t = 0$. Furthermore, since the integral term in equation (19) is the future impact of the learning-by-doing technology, let us denote it as "LBD". Figure 3 illustrates the behavior of $A^d_\pi$ , $V^f_t$ and LBD. In this diagram, the terminal date $t_2$ is held constant, and $t_1$ is on the horizontal axis. In the figure, $A^f_\pi$ always exceeds $A^d_\pi$ because foreign wages are always lower than domestic wages. But initially the left side of equation (19) is greater than the right side. It is only when this equation holds with equality that it will optimal to move the firm abroad.

In the technical appendix it is verified that the value matching, and smooth pasting conditions are satisfied.

[21] The point of this manipulation is that the two terms involving $q(t_1)$ cancel out from both sides of equation (19).

# 5 Functional Forms and Parameter Values

Also, suppose that the other functions are written as follows:

$$\mu(z) = \mu \cdot z.$$

In conjunction with the assumption about the distribution of technologies (see Section 3), this will imply that the growth rate is determined as

$$g = M^I \mu \cdot z = \frac{\dot{\bar{\lambda}}_t}{\bar{\lambda}_t}. \tag{20}$$

This means that the growth rate is then determined by two factors that are determined within the model: The number of people engaged in research ($M^I$), and the effort ($z$) each of these agents devotes to producing new technology. Both of these variables are influenced by the incentives determined within the model.

The cost of relocation will be characterized as $K_t = \kappa\bar{\lambda}_t$, although in the benchmark economy this will be set to zero.

It is assumed that the function characterizing the disutility of research effort is given as follows:

$$h\left(z_t, \bar{\lambda}_t\right) = \bar{\lambda}_t \cdot \phi \left(\frac{z_t^{1+\omega}}{1+\omega}\right). \tag{21}$$

This form of the function insures that the value functions are homogeneous of degree one in $\bar{\lambda}_t$. Next, let us assume that the "learning by doing" process is described as follows:

$$q(s) = \gamma_1 \left(1 - e^{-\gamma_2 s}\right). \tag{22}$$

So $\gamma_1$ will be the scale parameter in this function that will determine how important this feature is, relative to the other part of the production technology, and this parameter will operate to influence the relative wage differences in the two economies (with $\gamma_1 = \kappa = 0$ implying $w_t^d = w_t^f$).

## 5.1 Parameter Values

In assessing the quantitative results it is necessary to use specific parameter values for the model. Many of the parameters in the model do not have any obvious counterparts in the existing literature. In such an unusual model it is not straightforward to do this using any guidance from the existing literature. Since the goal here is gain a qualitative assessment of the features of the model, the model will not be used to mimic any specific actual economy. Instead, some rough parameter values will be used in a benchmark model, and then variations of these parameters will be studied.

With this in mind, the following parameter values will be used in the benchmark model. The domestic workforce will be normalized to $N^d = 1$, while the size of the foreign workforce will initially be set to $N^f = 5.0$. For the remaining parameters, the following values will be employed: $\mu = 0.10$, $\alpha = .65$, $r = .07$, $\gamma_1 = .80$, $\gamma_2 = .20$, $\kappa = 0$, $\omega = 1.0$, $\phi = 1.01$.These values will produce a growth rate of 3% in the benchmark model.

# 6 Analysis of a Balanced Growth Path

With the above assumption about the distribution of technologies, it is then straightforward to verify that the equilibrium wage in the domestic market can then be written as $w_t^d = \bar{\lambda}_t A_w^d$ and the wage in the foreign market will be $w_t^f = \bar{\lambda}_t A_w^f$, where the counterparts to equations (5) and (6) are then written as

$$A_w^d = \alpha \left[ \frac{1}{N^d} \int_{\underline{\bar{\theta}}}^{1} (\theta)^{\frac{1}{1-\alpha}} f_\theta(\theta)\, d\theta \right]^{1-\alpha} = \alpha \left[ \left( \frac{1-\alpha}{N^d} \right) \left( 1 - \bar{\theta}^{\frac{1}{1-\alpha}} \right) \right]^{1-\alpha},$$

and

$$A_w^f = \alpha \left[ \frac{1}{N^f} \int_{\underline{\theta}}^{\bar{\theta}} (\theta)^{\frac{1}{1-\alpha}} f_\theta(\theta)\, d\theta \right]^{1-\alpha} = \alpha \left[ \left( \frac{1-\alpha}{N^f} \right) \left( \bar{\theta}^{\frac{1}{1-\alpha}} - \underline{\theta}^{\frac{1}{1-\alpha}} \right) \right]^{1-\alpha}.$$

It is natural to refer to $A_w^d$ and $A_w^f$ as the domestic and foreign productivity-normalized wage. It should be noted that terms like $\left( \frac{1-\bar{\theta}^{\frac{1}{1-\alpha}}}{N^d} \right)$ and $\left( \frac{\bar{\theta}^{\frac{1}{1-\alpha}} - \underline{\theta}^{\frac{1}{1-\alpha}}}{N^f} \right)$ are the ratio of the number of firms to the number of workers, and this is obviously analogous to the capital-labor ratio.

Next, the profit function for a domestic producer with relative technology $\theta_t$, which is the counterpart to equation (9) is written as $\pi_{t,s}^d = \bar{\lambda}_t A_\pi^d \left( \theta_t^{\frac{1}{1-\alpha}} \right) + \bar{\lambda}_t \theta_t q(s)$, where

$$A_\pi^d = (1-\alpha) \left[ \left( \frac{1-\alpha}{N^d} \right) \left( 1 - \bar{\theta}^{\frac{1}{1-\alpha}} \right) \right]^{-\alpha}.$$

Similarly, the profit function for a foreign producer is written as $\pi_{t,t_1}^f = \bar{\lambda}_t A_\pi^f \left( \theta_t^{\frac{1}{1-\alpha}} \right) + \bar{\lambda}_t \theta_t q(t_1)$, where

$$A_\pi^f = (1-\alpha) \left[ \left( \frac{1-\alpha}{N^f} \right) \left( \bar{\theta}^{\frac{1}{1-\alpha}} - \underline{\theta}^{\frac{1}{1-\alpha}} \right) \right]^{-\alpha}.$$

In the Appendix a derivation of the value functions is conducted. Because of the linear nature of preferences, in conjunction with the form of the production functions and the distribution of technologies, the value functions are shown to be homogeneous of degree one in the leading or frontier technology. This turns out to be a convenient property because the value functions and decision rules are then relatively easy to describe. In the Appendix it is shown that an equilibrium is then characterized by an equal number of equations and unknowns.[22]

A *competitive equilibrium* on a balanced growth path for this economy consists of a list of the time-invariant variables $(A_w^d, A_w^f, A_\pi^d, A_\pi^f, \bar{\theta}, \underline{\theta}, t_1, t_2, z, V^d, V^I, V^f, M^I, M^f, M^d, g)$ which satisfy these equations. All firms and workers behave competitively, and maximize utility or profit while treating market prices parametrically.

# 7 Properties of an Equilibrium

It is possible to establish some properties of an equilibrium without going through too much formal analysis. To do this, it seems proper to list these properties as a series of statements.

**Claim 1** If the relocation cost $K_t$ is sufficiently large, then domestic firms will never wish to move operations to the foreign market. A sufficient condition for this to happen is if this cost is greater than the discounted stream of profits when the foreign wages are zero, over any period of time. Since this is not a particularly interesting case, it is assumed henceforth that this situation does not prevail.

[22] There are some non-linear equations in this system, and it is then an open question as to whether there can be multiple equilibria. It is impossible at this time to rule this out, but multiple equilibria have not yet been discovered.

**Claim 2** It will be the case that there will be firms operating in both the domestic and foreign markets. If there were no firms operating in one of the markets, then since there is unemployed labor there while wages are zero, a firm could make infinite profit from moving into this market.

**Claim 3** Suppose that there is no learning-by-doing ($q(s) = \kappa = 0, \forall\ s$). It follows that wages in the two economies must be equalized ($A_w^f = A_w^d$). Then in this case there is no distinction between the foreign and domestic economy, and $t_1$ is not determined.

**Claim 4** Suppose that the learning-by-doing technology is operational ($q(s) > 0$, $\forall\ s > 0$, and $q'(0) > 0$), and $K_t = 0$. Then it cannot be that domestic wages are lower than the foreign ones ($A_w^f > A_w^d$) because this imply that there could never be offshoring, even when the foreign wages were zero (because no firms locate there). But this is inconsistent with Claim 1, and cannot be profit maximizing. Hence it must be that ($A_w^f < A_w^d$).

**Claim 5** Then it is optimal for a new firm to spend at least some time (however short) in the domestic economy ($0 < t_1 < t_2$). This is because if $t_1 \nearrow t_2$, then the firm would not be maximizing profit because it would always benefit by moving just prior to $t_2$ to benefit from the lower foreign wages. The question then is whether $t_1 = 0$. Once again, if $t_1 \searrow 0$, this would imply that $A_w^f \gg A_w^d \searrow 0$ and $A_\pi^f \ll A_\pi^d \nearrow +\infty$, because there are virtually no domestic firms.

**Claim 6** One plausible measure of inequality in the domestic economy is the ratio $\left(A_\pi^d / A_w^d\right)$, which is the ratio of the highest to the lowest economy in the domestic economy. A little algebra reveals that this ratio is increasing in $\bar{\theta}$. This will mean that this measure of inequality will be inversely related the productivity-adjusted wage. Since $\bar{\theta}$ can also be interpreted as inversely related to the rate of firm destruction, this implies that this measure of inequality is inversely related to the measure of firm destruction. Since firm destruction likely accompanies growth in this environment, the growth rate might seem related to the degree of inequality.

**Claim 7** If $h'(0) > h(0) = 0$, or $\mu(0) = 0$, then there may exist an equilibrium in which $g = 0$. This might be one reason why one might observe certain economies not growing or growing at very low rates: these economies have higher costs of acquiring new technologies.[23]

**Claim 8** It is also straightforward to verify that the length of time that a firm will operate in the domestic market will be

$$t_1 = \frac{-\ln\left(\bar{\theta}\right)}{g} = \frac{M^d}{g},$$

while the length of time a firm will operate abroad will be

$$t_2 - t_1 = \frac{\ln\left(\bar{\theta}/\underline{\theta}\right)}{g} = \frac{M^f}{g}.$$

The numerator in these expressions is the number of firms operating in the respective location, while the denominator represents the speed with which these firms enter and exit that economy. If the growth rate were to approach zero, then the firms are gradually spending an arbitrarily long time operating in that location. A plausible measure of business destruction in such a model is the inverse of time spent operating, and so $(1/t_1)$ and $(1/(t_2 - t_1))$ are one reasonable measure of business destruction. Another measure is the number of firms that are operating in each locality, which would then be $(1 - \bar{\theta})$ and $(\bar{\theta} - \underline{\theta})$ for domestic and foreign business destruction, respectively.

[23] Here "acquiring new technologies" should be interpreted broadly. Some economies do this by engaging in research, while other lesser developed economies acquire new technologies by learning or acquiring the knowledge from foreign counterparts.

# 8 Measuring Welfare

In some of the examples below it will be useful to assess the welfare benefits or costs that result from certain parameter or policy changes. Since agents have linear utility functions, the value functions will give a useful measure of the change in welfare from various policy changes. Since workers care only about the discounted value of their wages, the welfare functions for domestic and foreign workers will be characterized as $\frac{A_w^d}{r-g}$ and $\frac{A_w^f}{r-g}$, respectively.

The remaining agents are the domestic firm-owners and innovators. By giving equal weight to each of these agents, it seems reasonable to write this welfare function as

$$\int_{\bar{\theta}}^{1} V^d(\theta) f_\theta(\theta) d\theta + \int_{\underline{\theta}}^{\bar{\theta}} V^f(\theta) f_\theta(\theta) d\theta + M^I V^I.$$

It turns out that it is possible to write out this expression explicitly, but it is quite long and complicated. Alternatively, one could study each of these value functions individually. However, since these functions are linked, or functions of each other, they tend to move in tandem, and so studying the aggregate welfare function seems appropriate.

It is also important to recognize that not all of these firm-owners have utility that is increasing over time, or increasing in the growth rate. As Figure 2 shows, a firm-owner can have declining utility (or a falling value function) because factor (labor) prices are causing profit to fall. This is where the "creative destruction" nature of the model has a unique impact. This means that policy or parameter changes might raise the utility of some firm-owners, while reducing that of others. However, the owner of a new firm can have utility that is increasing due to the learning-by-doing feature. Furthermore, a policy change, such as a profit tax, can have the opposite effect on the welfare of an owner of a new firm ($V^d(1)$), as compared with the owner of an existing firm ($V^d(\bar{\theta})$). For this reason, it will be the case in some of the experiments conducted below that policy or parameter changes that increase the growth rate, will not necessarily increase the welfare measure, as calculated above.

# 9 Characterization of Model

Characterizing the equilibrium of such an economy using analytical methods is beyond challenging. It is then useful to vary some parameters to see how this would affect the equilibrium values of this economy. This will be done in a series of different experiments.

## 9.1 What is the Effect of Offshoring Itself?

Perhaps the most obvious question to answer is, what is the effect of offshoring itself? One way to answer this question is to vary the size of the foreign worker population ($N^f$). One could interpret an increase in this parameter as a sudden permanent change in government policy that permits firms to relocate to a foreign economy to which they previously did not have access.

Figures 4 and 5 shows the effect that varying the size of the foreign labor market, above and below the benchmark of $N^f = 5.0$, can have on various features of the economy. In this case increasing ($N^f$) raises the foreign and domestic growth rate. This is because the lower cost of foreign labor makes innovation more attractive. Although growth increases, the length of time each firm spends in the domestic economy falls as ($N^f$) increases. Actually, the lifespan of foreign-located firms also declines. In other words, increased access to foreign labor *reduces* the amount of time firms will spend operating abroad. Firm owners have utility that is strictly increasing in ($N^f$) for obvious reasons.

The fact that increased offshoring raises the growth rate might seem like an obvious result that must necessarily take place. Some investigation reveals that this is certainly not the case. Equation

(20) shows that the growth rate is determined by the number of people engaged in research ($M^I$), as well as how much effort they put forth ($z$). Increased offshoring leads to firms operating longer, because they can relocate their operations offshore to take advantage of the lower cost inputs. The fact that firms stay in operation longer should result in a lower value for $M^I$, which should *lower growth*. The increased growth rate then derives from the fact that the level of innovation effort ($z$) rises when there is increased incentives arising from the prospect from offshoring, and this causes the growth rate to rise. Of course, equation (13) shows that the increased research ($z$) can only result from the increased difference in the value functions $V_t^d$ and $V_t^I$.

Figure 5 shows that an increase in ($N^f$) lowers the wages of both domestic and foreign workers in the short-term. However, the *value function* of these workers is strictly increasing in ($N^f$) because the growth effects overwhelm the short-term fall in wages. In other words, *an increase in $N^f$ increases the welfare of all agents.* As "free lunches" go, this one may come as a bit of a surprise. It might then be useful to investigate the magnitude of these welfare gains. In the case of raising $N^f$ from 4.0 to 5.0, this results in an increase in welfare of domestic and foreign workers of 3.6% and 2.8%, respectively. Furthermore, the welfare gains to the firm-owners and innovators amount to 9.5%. These would seem to be non-trivial welfare gains for all parties, and so this is certainly an argument for permitting offshoring.

It bears emphasizing that this result - that all agents can seem to benefit from increased offshoring - would seem to be unique, and depends on the dynamic nature of the economy under study. If one were to consider a static version of this economy it would seem inevitable that the domestic workers would be harmed by an increase in offshoring.

It is also worth noting that the increase in offshoring also results in an increase in domestic inequality, as measured by the ratio $\left(A_\pi^d/A_w^d\right)$. Although the richest domestic firm-owners seem to benefit more from offshoring, even the domestic workers benefit, in spite of the fact that their wages do not respond as much. This would seem to be a good reason for not necessarily considering how offshoring would influence domestic inequality when considering this policy change..

There is one additional aspect of the model that is of interest. There are features of the model that might be regarded as measures of the rate of firm destruction. These variables are $(1/t_1)$ ,which is a measure of how little time firms spend in the domestic economy, or the growth rate ($g$) which is also related to how rapidly firms exit the economy. It is then of interest to note that Figures 4 and 5 show that both these measures of firm destruction are directly linked to the amount of offshoring. Again, since welfare is also increasing, this suggests that, at least for this exercise, the rate of firm destruction is positively related to welfare.

## 9.2 Varying the Cost of Relocation

In this model when a firm relocates abroad, it can do so instantaneously. However, one could also imagine that there are actual costs associated with such movement, and so it is useful to understand how varying this cost might affect the equilibrium. Additionally, it is natural to ask what would happen if the government were to impose some cost on such relocation - perhaps under the guise of dissuading firms from doing this and lowering domestic employment.

Figures 6 and 7 show how varying this cost, as measured by $\kappa$, above zero would affect the equilibrium. Figure 6 shows that the growth rate is decreasing in this cost, as are the value functions for the firm-owners. It is interesting to see that as this cost rises, the lifespan of a domestic firm rises, but the length of time the firms spend operating abroad also increases. It is because firms are spending more time operating that the growth rate falls, since there are now fewer innovators ($M^I$).

It should be noted that there is perhaps one perceived marginal benefit from this policy. Firms react by spending more time operating in the domestic economy. This raises the benefit from the learning-by-doing technology, and this raises the benefit from operating abroad. This will partially offset the increased relocation cost.

Figure 7 shows that increasing this relocation cost would raise the immediate cost of domestic labor, because there would be a greater number of domestic firms. This has the effect of lowering the cost of foreign labor. Therefore, the relative wage disparity between countries ($w_t^d/w_t^f$) increases. It is interesting to see that both foreign and domestic workers would have lower value functions as a result of raising this relocation cost, mostly due to the lower growth rate. Presumably then, if given the option, all individuals would vote against having the government introduce any further costs of relocation.

In summary, a government might think by taxing firms that are moving abroad, it would be helping domestic labor. Instead, it would be reducing the welfare of not only those workers, but all other agents as well. If one were to design a government policy with the intension of hurting everyone (including foreign workers!), this policy would be an ideal candidate. In fact, the opposite policy would be expedient: the government could subsidize firms that wish to locate abroad.

This also has implication for foreign governments that might do well to adopt policies, such tax abatements or dispensations from regulations, to attract businesses.

### 9.3 Changing the Learning-By-Doing Technology

The learning-by-doing technology plays a role here in enticing the new firms to locate in the domestic economy. Raising the productivity of this feature benefits agents in both the domestic and foreign economies. It is then of interest to see how altering this particular feature affects the equilibrium.

Figures 8 and 9 show how the equilibrium is affected by changing the parameter $\gamma_1$. Since this parameter reflects the productivity of this technology, one would expect higher values to be better. Figure 8 shows that raising this parameter actually *lowers* the growth rate, but raises the value functions of all firm-owners and innovators. In other words, this parameter change that makes the domestic economy more productive relative to the foreign location, actually reduces the equilibrium growth rate. Raising $\gamma_1$ also raises the lifespan of both domestic and foreign firms.

Figure 9 shows that raising $\gamma_1$ lowers the cost of foreign labor, but raises the wages in the domestic economy. Hence, the relative wage disparity between countries increases. This latter effect is because there are more domestic firms. It is interesting so see that the value function of domestic workers has an "inverted-U" shape. The initial rising portion is due to the higher domestic wages, while the falling portion is because of the lower growth rate.

Getting back to the growth rate: It is falling in this case due to the fact that as $\gamma_1$ rises, firm-owners are letting their firms operate for a longer period, and consequently there are fewer innovators ($M^I$). It is these individuals who are determining the growth rate. Nevertheless, this is yet another example of a model in which it is conceivable for welfare (of domestic agents) to rise even as the growth rate falls.

### 9.4 The Effect of a Domestic Profit Tax

It is of interest to see how the introduction of a tax on domestic profits will affect this economy. Of course, this should be interpreted as a tax *relative to the tax on foreign profits*, which are assumed to be untaxed. Also, assume that the government revenue from this tax is thrown away.

Figures 10 and 11 illustrate the impact of raising the profit tax from zero to 40%. The growth rate initially falls a modest amount, but subsequently rises as the tax rate increases beyond 15%. The reason for this is as follows. As the tax rate rises, less is spent on innovation (i.e. $z$ falls), and this reduces growth. However, since innovation is less attractive, and lifespan of domestic firms is reduced, more entrepreneurs are engaged in innovation, rather than managing firms (i.e. $M^I$ rises). This has a positive impact on growth. The latter effect eventually dominates the effect of lower $z$ as the tax rate rises.

As the tax rate rises, the value functions of the entrepreneurs fall for obvious reasons. In addition, the lifespan of domestic firms falls, while the lifespan of foreign firms rises, and then eventually falls as well.

Figure 11 also shows the impact that this tax has on the wages of workers. Foreign workers benefit from a domestic profit tax, while domestic workers are harmed. Therefore, the relative wage disparity between countries *decreases*, although this hardly seems like a good reason for introducing the tax. In terms of welfare, the domestic workers are certainly harmed, while the foreign workers benefit from the tax.

It bears noting that the profit tax results in an *increase* in domestic inequality, as measured by the ratio $\left(A_{\pi}^{d}/A_{w}^{d}\right)$. This might come as a surprise, since such a tax would seem to impinge primarily on the firm owners. Although the richest domestic firm-owners will pay the tax, the domestic workers will certainly bear some of the brunt of this because they will ultimately have fewer employers in the domestic economy, and therefore this will result in lower wages. This result, of the tax influencing domestic income inequality, is not one that is usually considered when contemplating such a policy.

One interesting conclusion from this exercise is that the beneficiaries of the domestic profit tax appear to be the foreign workers, both in terms of higher initial wages, and possibly a higher growth rate. Obviously, this implies that a reduction in the domestic profit tax would benefit all domestic agents, and hurt foreign workers.

This analysis suggests that these domestic taxes might also influence *where* the firm might wish to locate abroad. Consider a related environment in which there are two potential foreign economies to which the domestic firms might relocate, given some initial level of domestic taxation. Suppose that in the first foreign economy the wages are relatively low, but the cost of moving there $(K_t)$ is relatively high. In contrast, suppose that in the second economy these conditions are switched. Assume that a firm is close to indifferent about which of these locations to choose. Now suppose that in the domestic economy the profit tax $(\tau_{\pi})$ is raised. Figure 10b suggests that this may make the firm want to spend a longer time operating abroad. However, doing so would then mean it should have a stronger preference for locating where the wages are lower. That is, the firm should then prefer locating in the first foreign economy, rather than the second. In other words, it could be that a change in the domestic profit tax rate can change the *location* as well as the *timing* of the firm's offshoring-location decision. So a change in the domestic tax rate can potentially create winners and losers amongst the foreign economies.

There is one additional feature is of note. Once again, as mentioned above, the variables $(1/t_1)$ and $(g)$ are potential measures of firm destruction, and these variables tend to be increasing in the tax rate. It is then of interest to note that a higher profit tax can increase the rate of firm destruction in this economy.

## 9.5 The Optimal Amount of Offshoring?

It is then interesting to see if there is such a thing as "the optimal amount of offshoring " That is, suppose that it were possible to impose some law or mandate as to how much time a firm must spend abroad after relocating there. What amount of time would be best? Moreover, what would the growth rate be? It might be a natural reaction to think that reducing the amount of time firms are permitted to spend abroad would reduce the welfare of firm-owners, but raise the welfare of domestic workers.

Figures 12 and 13 show the results from this experiment. The growth rate is decreasing in the value of $t_2 - t_t$. Forcing firms to spend more time abroad lowers growth because there will be fewer entrepreneurs (i.e. $M^I$) trying to innovate. On the other hand forcing firms to curtail the amount of time they spend operating abroad will raise the growth rate.

As the value of $t_2 - t_t$ rises, the amount of time that firms wish to spend operating in the domestic economy ($t_1$) increases. This is because although the value of operating a foreign firm ($V^f$) increases, the value of operating a domestic firm ($V^d$) increases even more.

Figure 13 shows that the foreign wages are increasing, and also the domestic wages are initially increasing in the value of $t_2 - t_t$.This is because there are more firms operating in both economies, as $t_2 - t_t$ increases. It is interesting to note that the welfare, or value function, of both the domestic and foreign workers is decreasing in the amount of time spent abroad. In spite of the rise in wages, this effect is overwhelmed by the fact that the growth rate is falling.

It may be puzzling that the welfare of the firm-owners is increasing in the size of the mandated value of $t_2 - t_t$. This is interesting because the equilibrium of this benchmark economy has a lifespan of a foreign firm to be just over 25 periods. This is in spite of the fact that as $t_2 - t_t$ increases, the growth rate falls, and wages are rising. The reason for this rise in welfare is as follows. First, as $t_2 - t_t$ increases, a rather severe constraint on the firm's operations is being moderated. Secondly, for small values of $t_2 - t_t$, the foreign firms are forced to shut down, and these managers then consume *nothing* while they try to innovate. This must reduce the welfare of all entrepreneurs.

In sum, mandating a reduced time for firms to spend abroad raises the welfare of both domestic and foreign workers, by raising both their wages and the growth rate. However, it also lowers the welfare of entrepreneurs.

### 9.6 Forced Domestic Production

It is possible to study what would happen if the domestic government that forced domestic firms to spend more time producing in the domestic economy. This does not sound like a good idea.

Imagine beginning from the benchmark economy, and just mandating a higher value of $t_1$. This would certainly reduce the growth rate, and lowers the welfare of firm owners, since this is an added constraint on their behavior. Although this policy would raise the wage of domestic workers, it lowers their welfare because of the reduction in the growth rate. Additionally, it also reduces the welfare of foreign workers, by both lowering their wages and lowering the growth rate. In short, everyone would seem to be harmed by such a policy.

### 9.7 How About Permitting Immigration of Foreign Workers?

It is then of interest to see how the results shown above compare with what would happen if the some of the foreign workers are permitted to permanently emigrate into the domestic economy. From a superficial perspective, it might seem that offshoring and increased immigration might be perfect substitutes, in that they both result in foreign workers gaining more employment. Fortunately, the current model is ideally suited to compare the effects of these two phenomenon.

Let us assume that there are no other changes to the features of the economy, other than raising the domestic worker population ($M^d$) and lowering the foreign population ($M^f$) by an identical amount. One might initially speculate that this would lower domestic wages and raise foreign wages, thus making offshoring less attractive.

The results from this experiment are shown in Figures 14 and 15. Figure 14 shows that increased immigration actually lowers the growth rate. The reason for this is that although research spending ($z$) rises, there is a decrease in the number of researchers ($M^I$), and this reduces the growth rate. The reason for the fall in $M^I$ is that firm-owners are choosing to keep their firms operating longer. The same figure shows that firms will be operational in the domestic economy for a longer period. For modest levels of immigration, firms will also spend more time in the foreign economy as well. It is interesting to note that the welfare of the firm-owners is decreasing for larger increases in immigration. This is due to the reduced growth rate.

Figure 15 shows that both domestic and foreign workers are hurt by the immigration, since wages in both economies fall at first, and the growth rate also falls. Moreover, as there is more immigration, the foreign wages begin to rise since there is a reduced supply of foreign workers. In addition, the foreign workers who actually do emigrate would certainly be helped by this policy since they receive higher wages. Hence, the effect of this immigration on the relative wage inequality across the economies is rather complicated.

It is also worth noting that the increase in immigration also results in an increase in domestic inequality, as measured by the ratio $\left(A_\pi^d/A_w^d\right)$, which need not even consider the new immigrants to be the lowest income-earners. The richest domestic firm-owners seem to benefit more from immigration, while the domestic workers are harmed. Perhaps this result is not too surprising.

In sum then, permitting increased immigration of foreign workers does not seem to provide any obvious welfare benefits to foreign or domestic workers (other than the foreign workers who actually do move), and if there are benefits to the domestic agents, they accrue to some of the firm-owners.

This experiment is of special interest because it if frequently thought that offshoring is actually an alternative to, or a substitute for, immigration of workers. According to this view, these two alternatives amount to asking whether you want to have foreign workers do their work in the foreign economy, or in the domestic economy. To see if this is in fact the case, it is then appropriate to compare these results of increased immigration to those in Section 9.1. In other words, let us compare Figures 14 and 15 with Figures 4 and 5 respectively. An inspection of these results reveals some important differences. First, increased offshoring raises the growth rate, while immigration lowers it. Secondly, offshoring raises the welfare of firm owners, while immigration does not. Third, offshoring causes firms to spend less time in the domestic economy (lower $t_1$) while immigration raises this time. Fourth, immigration causes the firms to spend more time operating in the foreign economy, while offshoring reduces this time. Fifth, although both offshoring and immigration both lower domestic wages (at least for small levels of immigration), offshoring raises the welfare of workers (because of the higher growth rate) while immigration lowers the welfare of existing domestic workers. Six, although the welfare of the emigrating agents would be improved, the welfare of foreign workers is reduce by increased immigration, while their welfare is enhance through offshoring. In summary, there seem to be very few similarities between these two alternative policies.

The key to understanding the differences between these experiments is that offshoring results in the foreign workers being employed at the foreign, rather than the domestic wage. Since the latter is greater than the former, this beneficial to the firm owners, and so this raises their profit profile, results in increased innovation and growth. In contrast, when the foreign workers emigrate to the domestic economy, they are then employed at the higher domestic wage. This is not beneficial to the firm owners since it raises their costs, and results in reduced creased innovation and growth.

## 9.8 Multiple Types of Labor

Labor is not homogeneous. The frequently stated motivation for much of the offshoring is to take advantage of the relatively low wages of low-skilled labor found in developing many economies. It is then of interest to study this issue within the context of a model in which there is also some high-skilled labor as well.[24]

Therefore, consider the same benchmark model as above, except where there is both high and low-skilled labor in the domestic economy, and both types of labor enter into the production function. Once again, these laborers do nothing but supply their unit of labor inelastically, and consume the resulting wage. Suppose that the firm can, at some future date, cease using the domestic low-skilled labor and instead employ the foreign labor instead. When they decide to offshore this employment, they can continue to use the domestic high-skilled labor. In this instance the behavior of the low-skilled labor would be qualitatively (but not quantitatively) the same as in

[24] This is obviously related to the issues studied by Grossman and Rossi-Hansberg [11], and Ottaviano, Peri and Wright [22].

the model already studied. However, the high-skilled labor would behave like some fixed factor of production, and its wage would closely track the growth rate in the domestic economy.

Under these circumstances, what would be the impact of permitting increased offshoring, as in Section 9.1? An increase in $N^f$ would lead to an increase in welfare of all agents. Additionally, it would lead to a current decrease in the wage of domestic unskilled labor (Figure 5a), but an increase in the wage of the domestic skilled labor. In other words, it would lead to increased wage inequality.

Suppose that there was an increase in the cost of relocation (as in Section 9.2). This would harm both types of labor because the growth rate would fall. Domestic wages of both types of agents would rise because firms would be spending more time in the domestic economy, but the welfare effects would be negative.

The effect of an increased profit tax (Section 9.4) would not be so clear-cut, since this has an ambiguous effect on the growth rate (Figure 10a). It might be possible that the skilled workers could be the one group that benefits from offshoring. Similarly, admitting some of the foreign labor into the domestic economy as new immigrants (Section 3) could also have an ambiguous impact on the domestic skilled labor. The reason is that increasing this resource could raise the marginal product of the skilled labor. This may or may not be offset by the reduced growth rate. Nevertheless, domestic wage inequality is likely to rise in this case, since unskilled wages are falling.

## 10 Final Remarks

Like most interesting economic phenomenon, the economics of offshoring would seem to be inherently a dynamic phenomenon. Firms must decide not just if it appropriate to do so, but also when it is optimal to do so. The model studied here focuses on the economics of these decisions. The prospect of offshoring must inevitably influence the profitability of a firm or technology, and therefore influence the innovation activity. It follows that offshoring is likely to influence the growth rate of economies. The analysis presented here shows that there are a multitude of interesting results that follow from this model, which do not seem to be found in the existing analyses of this topic.

Perhaps it is best to conclude by summarizing some of the more important results:

1. The growth rate can be *increasing* in the amount of offshoring, or the size of the foreign workforce.

2. The value functions, or welfare, of entrepreneurs can be *increasing* in the amount of offshoring.

3. Although offshoring results in lower foreign and domestic wages, the value function of both types of workers can also be increasing in the amount of offshoring. In sum, in conjunction with the previous point, all individuals, foreign and domestic, can have welfare that is increasing in the amount of offshoring.

4. Not only can everyone gain from increased offshoring, but also the welfare gains to all parties that result from this can be substantial, with the major portion of these gains accruing to domestic agents.

5. An increase in the cost of offshoring (i.e. moving a firm abroad) can lead to an increase in the length of time firms spend in the domestic market, and also to an increase in the length of time firms spend in the foreign market.

6. An increase in the cost of offshoring can lead to a *decrease* in the welfare of both domestic workers and firm owners, and to foreign workers as well. Some of this welfare effect is due to the lower growth rate.

7. A tax on domestic profit can have a non-monotonic effect on the growth rate. Even when it raises the growth rate, the tax an lower the welfare of entrepreneurs.
8. A tax on domestic profit can cause domestic firms to offshore earlier and stay abroad longer.
9. A tax on domestic profit can *reduce* the wages and welfare of domestic workers, while raising those of foreign workers. In sum, all domestic agents are hurt by the profit tax, while foreign workers benefit.
10. When a mandated degree of offshoring (i.e. length of time a firm must spend abroad) is imposed, it turns out that both domestic and foreign workers prefer a greater amount of offshoring. All workers benefit from this.
11. When a mandated degree of offshoring (i.e. length of time a firm must spend abroad) is imposed, it turns out that limiting foreign operations can raise the growth rate, but reduce the welfare of entrepreneurs.
12. A limit on the time firms can operate abroad can raise both the wages and welfare of both domestic and foreign workers.
13. Permitting immigration of some foreign workers into the domestic economy can lower growth, and lower the welfare of all domestic agents, as well as workers who remain in the foreign economy.
14. In contrast with a popular view, there are many important differences between increased immigration and offshoring.

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## 11 Appendix

In this Appendix the equations determining the equilibrium of the balanced growth path are presented. These equations are explicitly derived in the technical appendix. In this section the assumed functional forms of equations (20) through (22) will be employed.

Let the actual profit functions for a domestic or foreign firm be written in the following manner

$$\pi^d_{t,t}(\theta_t)\bar{\lambda}_t = \bar{\lambda}_t A^d_\pi (\theta_t)^{\frac{1}{1-\alpha}} + \gamma_1 \lambda \left(1 - e^{-\gamma_2 t}\right),$$

$$\pi^f_{t,t_1}(\theta_t)\bar{\lambda}_t = \bar{\lambda}_t A^f_\pi (\theta_t)^{\frac{1}{1-\alpha}} + \gamma_1 \lambda \left(1 - e^{-\gamma_2 t_1}\right),$$

With the assumed nature of the distribution of the technologies, the terms $A^d_\pi$ and $A^d_\pi$ can be written as

$$A^d_\pi = (1-\alpha)\left[\frac{1}{N^d}\int_{\bar{\theta}}^{1} (\theta)^{\frac{\alpha}{1-\alpha}} \, d\theta\right]^{-\alpha} = (1-\alpha)\left[\left(\frac{1-\alpha}{N^d}\right)\left(1 - \bar{\theta}^{\frac{1}{1-\alpha}}\right)\right]^{-\alpha} \tag{23}$$

$$A^f_\pi = (1-\alpha)\left[\frac{1}{N^f}\int_{\underline{\theta}}^{\bar{\theta}} (\theta)^{\frac{\alpha}{1-\alpha}} \, d\theta\right]^{-\alpha} = (1-\alpha)\left[\left(\frac{1-\alpha}{N^f}\right)\left(\bar{\theta}^{\frac{1}{1-\alpha}} - \underline{\theta}^{\frac{1}{1-\alpha}}\right)\right]^{-\alpha} \tag{24}$$

Next, the wage function in the domestic economy

$$A^d_w = \alpha\left[\frac{1}{N^d}\int_{\underline{\bar{\theta}}}^{1} (\theta)^{\frac{1}{1-\alpha}} f_\theta(\theta) \, d\theta\right]^{1-\alpha} = \alpha\left[\left(\frac{1-\alpha}{N^d}\right)\left(1 - \bar{\theta}^{\frac{1}{1-\alpha}}\right)\right]^{1-\alpha}. \tag{25}$$

and the wage function in the foreign economy:

$$A^f_w = \alpha\left[\frac{1}{N^f}\int_{\underline{\theta}}^{\bar{\theta}} (\theta)^{\frac{1}{1-\alpha}} f_\theta(\theta) \, d\theta\right]^{1-\alpha} = \alpha\left[\left(\frac{1-\alpha}{N^f}\right)\left(\bar{\theta}^{\frac{1}{1-\alpha}} - \underline{\theta}^{\frac{1}{1-\alpha}}\right)\right]^{1-\alpha}. \tag{26}$$

The value function for a new firm locating domestically is written as follows is written as $V^d \cdot \bar{\lambda}_t$, where $V^d$ is a constant. This term is then written as follows:

$$V^d = \left[\frac{A^d_\pi}{r + \frac{\alpha g}{1-\alpha}}\right] + \left[\frac{\gamma_1\left[1 - e^{-(r+g)t_1}\right]}{r+g} - \frac{\gamma_1\left[1 - e^{-(r+g+\gamma_2)t_1}\right]}{(r+g+\gamma_2)}\right] + e^{-(r-g)t_1}\left[\left(V^f - \kappa\right) - \left(\frac{A^d_\pi}{r + \frac{\alpha g}{1-\alpha}}\right) e^{-\left(\frac{g}{1-\alpha}\right)t_1}\right] \tag{27}$$

This equation has the following interpretation. The first term on the right side is the value of a new firm or technology, exclusive of learning-by-doing, if the manager wishes to operate it *forever* in the domestic economy. If we were to set $t_1 = \infty$ in the remaining terms, then these terms would be zero, and so $V^d$ would just equal the first term, which again means that this is the value of operating the firm forever. The second term is the value of the learning-by-doing if he manager operates within the domestic economy for $t_1$ periods. The third term, which is positive, is the value that the manger gets from switching from the domestic economy to the foreign economy at date $t_1$. The manager receives $\left(V^f - \kappa\right)$, but gives up the remaining profit from operating in the domestic economy.

The optimal value of $t_1$ is written as the following condition

$$A^d_\pi\left(\theta_{t_1}^{\frac{1}{1-\alpha}}\right)\bar{\lambda}_t + \theta_{t_1} q(t_1) + (r-g)\kappa \cdot \bar{\lambda}_{t_1} = A^f_\pi\left(\theta_{t_1}^{\frac{1}{1-\alpha}}\right)\bar{\lambda}_{t_1} + \theta_{t_1} q(t_1) - \int_{t_1}^{t_2} e^{-r(s-t_1)}\left(\frac{d\pi^f_{s,t_1}}{dt_1}\right) ds.$$

Now using equation (22) this latter equation can be re-written as follows:

$$A^d_\pi\left(\theta_{t_1}^{\frac{1}{1-\alpha}}\right) + (r-g)\kappa = A^f_\pi\left(\theta_{t_1}^{\frac{1}{1-\alpha}}\right) - \left[\gamma_1\gamma_2\left(e^{-\gamma_2 t}\right)\right]\left[\frac{1 - e^{-r(t_2-t_1)}}{r}\right] \tag{28}$$

The value function for a new foreign firm can be written as follows:

$$V^f = \frac{A_\pi^f (\theta_{t_1})^{\frac{1}{1-\alpha}}}{\left(r + \frac{\alpha g}{1-\alpha}\right)} + \left[\frac{\gamma_1 \left[1 - e^{-\gamma_2 t_1}\right] (\theta_{t_1})}{r+g} \left[1 - e^{-(r+g)(t_2-t_1)}\right]\right] \tag{29}$$

$$+e^{-(r-g)(t_2-t_1)} \left[V^I - \left(\frac{A_\pi^f (\theta_{t_1})^{\frac{1}{1-\alpha}}}{\left(r + \frac{\alpha g}{1-\alpha}\right)}\right) e^{-\left(\frac{g}{1-\alpha}\right)(t_1-t_1)}\right] \tag{30}$$

This equation has the following interpretation. The first term on the right side is the value of a new firm or technology, exclusive of learning-by-doing, if the manager wishes to operate it forever in the foreign economy. The second term is the value of the learning-by-doing if he manager operates within the foreign economy for $t_1 - t_2$ periods. The third term, which is positive, is the value that the manger gets from shutting down the firm in the foreign economy to the foreign economy at date $t_2$.

The optimal value of $t_2$ is written as the following condition

$$A_\pi^f (\theta_{t_2})^{\frac{1}{1-\alpha}} + e^{-g(t_2-t_1)} \gamma_1 \left[1 - e^{-\gamma_2 t_1}\right] = (r-g) V^I \tag{31}$$

Then there is the condition for the innovators. The value function is written as follows:

$$rV^I = \max_{z \geq 0} \left\{-h(z_t) + \mu(z_t) \left[V^d - V^I\right]\right\}. \tag{32}$$

There is also the condition for the optimal research effort:

$$h'(z_t) = \mu'(z_t) \left[V_t^d - V_t^I\right]. \tag{33}$$

The relationship between the exit dates ($t_1$ and $t_2$) and the values $\underline{\theta}$ and $\bar{\theta}$ of is the following:

$$e^{-gt_1} = \bar{\theta} \text{ or } t_1 = \frac{-\ln(\bar{\theta})}{g} \tag{34}$$

and

$$e^{-gt_2} = \underline{\theta} \text{ or } t_2 = \frac{-\ln(\underline{\theta})}{g} \tag{35a}$$

Then there are the equilibrium conditions for the number of innovators, managers, and firms in each location:

$$M^d = -\ln(\bar{\theta}) \tag{36}$$

$$M^f = \ln(\bar{\theta}/\underline{\theta}) = -\ln(\underline{\theta}) - M^d \tag{37}$$

$$M^d + M^f + M^I = 1. \tag{38}$$

Lastly, there is the equation that determines the equilibrium growth rate, which is written as follows:

$$g = M^I \mu(z) \tag{39}$$

Equations (23) - (39) now constitute 16 equations in the following 16 unknowns:

$$A_w^d, A_w^f, A_\pi^d, A_\pi^f, \bar{\theta}, \underline{\theta}, t_1, t_2, z, V^d, V^I, V^f, M^I, M^f, M^d, g.$$

The following parameters will need to be fixed prior to calculating the equilibrium from these equations:

$$\alpha, r, N^d, N^f, \gamma_1, \gamma_2, \mu, \omega, \phi.$$

Figure 1

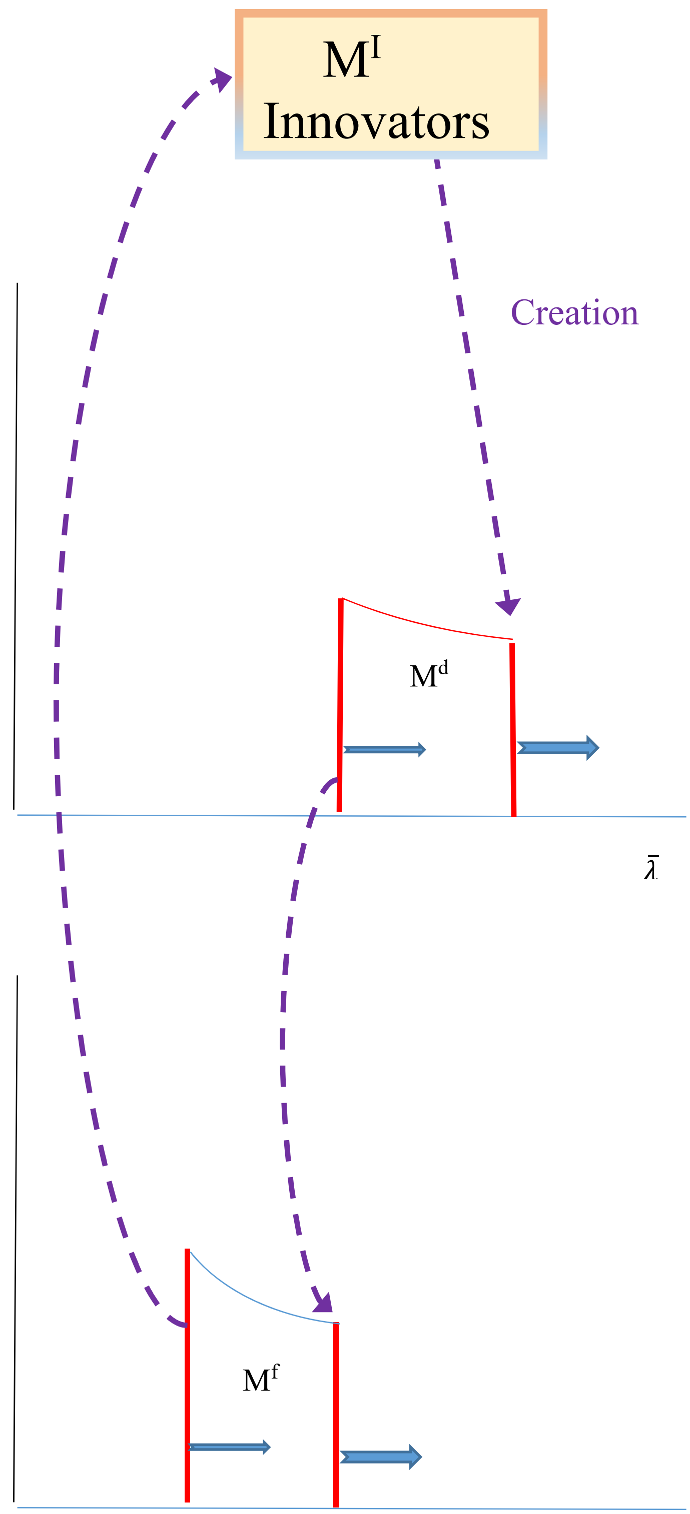


**Figure 2**

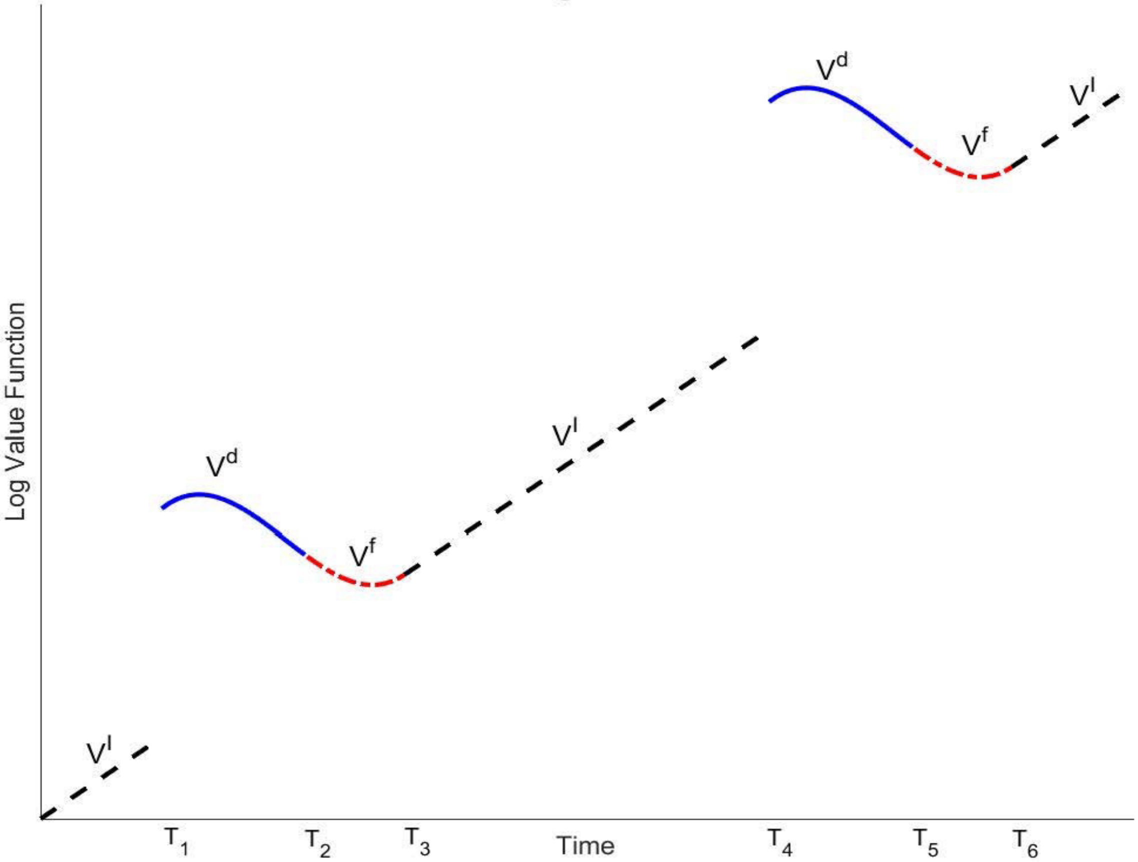
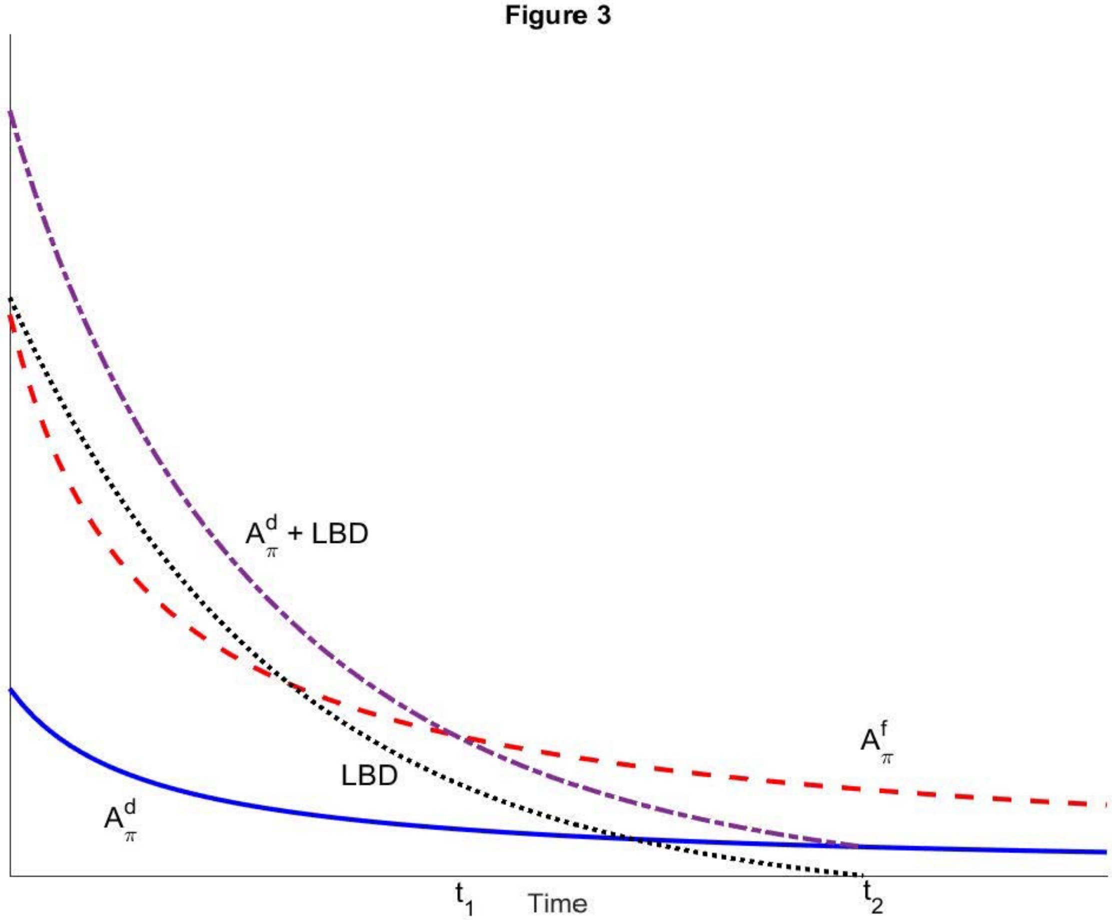



**Figure 3**



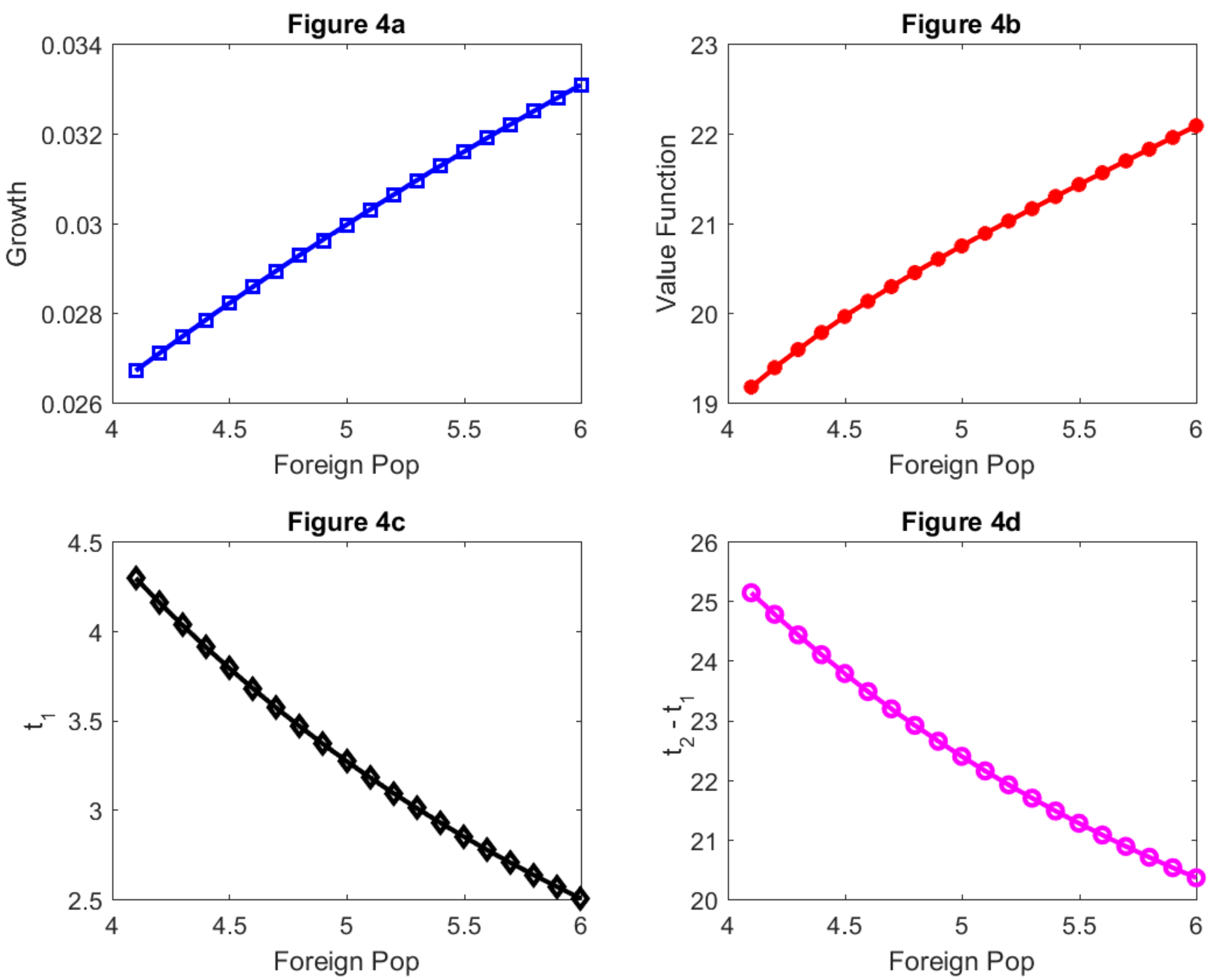


Figure 4a
Growth
0.034
0.032
0.03
0.028
0.026
4
4.5
5
5.5
6
Foreign Pop
Figure 4b
Value Function
23
22
21
20
19
4
4.5
5
5.5
6
Foreign Pop
Figure 4c
$t_1$
4.5
4
3.5
3
2.5
4
4.5
5
5.5
6
Foreign Pop
Figure 4d
$t_2 - t_1$
26
25
24
23
22
21
20
4
4.5
5
5.5
6
Foreign Pop

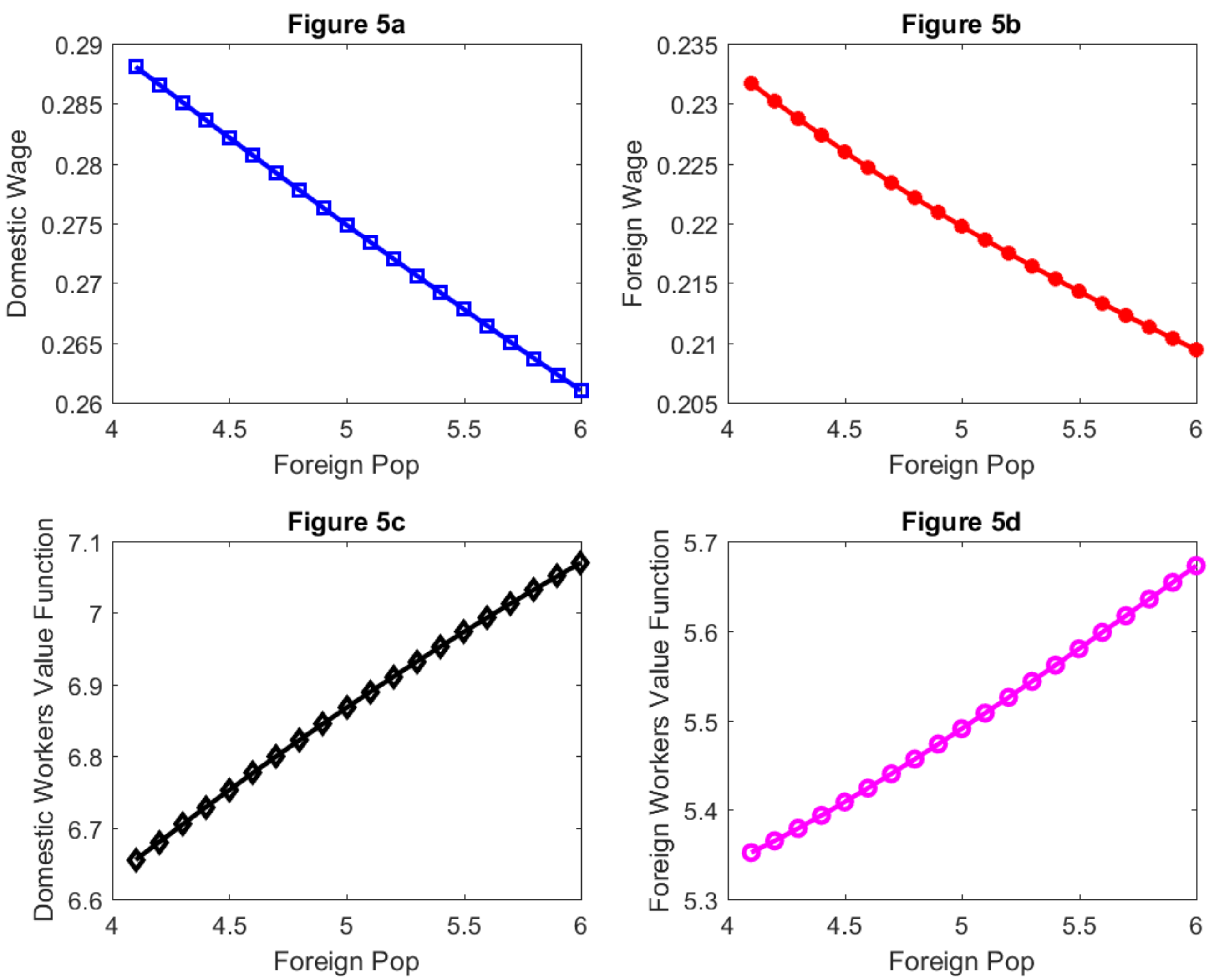


Figure 5a
Domestic Wage
0.29
0.285
0.28
0.275
0.27
0.265
0.26
4
4.5
5
5.5
6
Foreign Pop
Figure 5b
Foreign Wage
0.235
0.23
0.225
0.22
0.215
0.21
0.205
4
4.5
5
5.5
6
Foreign Pop
Figure 5c
Domestic Workers Value Function
7.1
7
6.9
6.8
6.7
6.6
4
4.5
5
5.5
6
Foreign Pop
Figure 5d
Foreign Workers Value Function
5.7
5.6
5.5
5.4
5.3
4
4.5
5
5.5
6
Foreign Pop

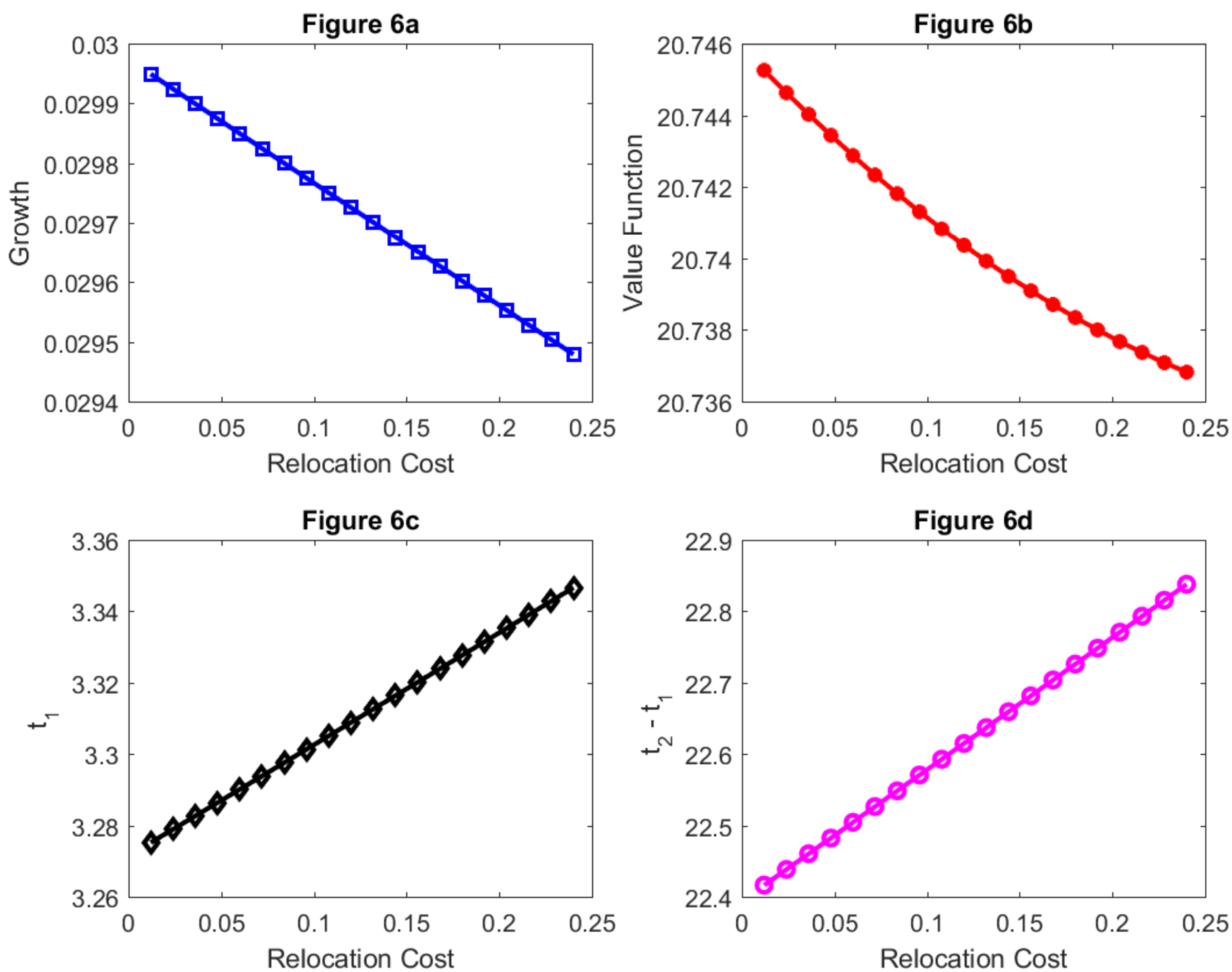


Figure 6a
Growth
0.03
0.0299
0.0298
0.0297
0.0296
0.0295
0.0294
0
0.05
0.1
0.15
0.2
0.25
Relocation Cost
Figure 6b
Value Function
20.746
20.744
20.742
20.74
20.738
20.736
0
0.05
0.1
0.15
0.2
0.25
Relocation Cost
Figure 6c
t1
3.36
3.34
3.32
3.3
3.28
3.26
0
0.05
0.1
0.15
0.2
0.25
Relocation Cost
Figure 6d
t2 - t1
22.9
22.8
22.7
22.6
22.5
22.4
0
0.05
0.1
0.15
0.2
0.25
Relocation Cost

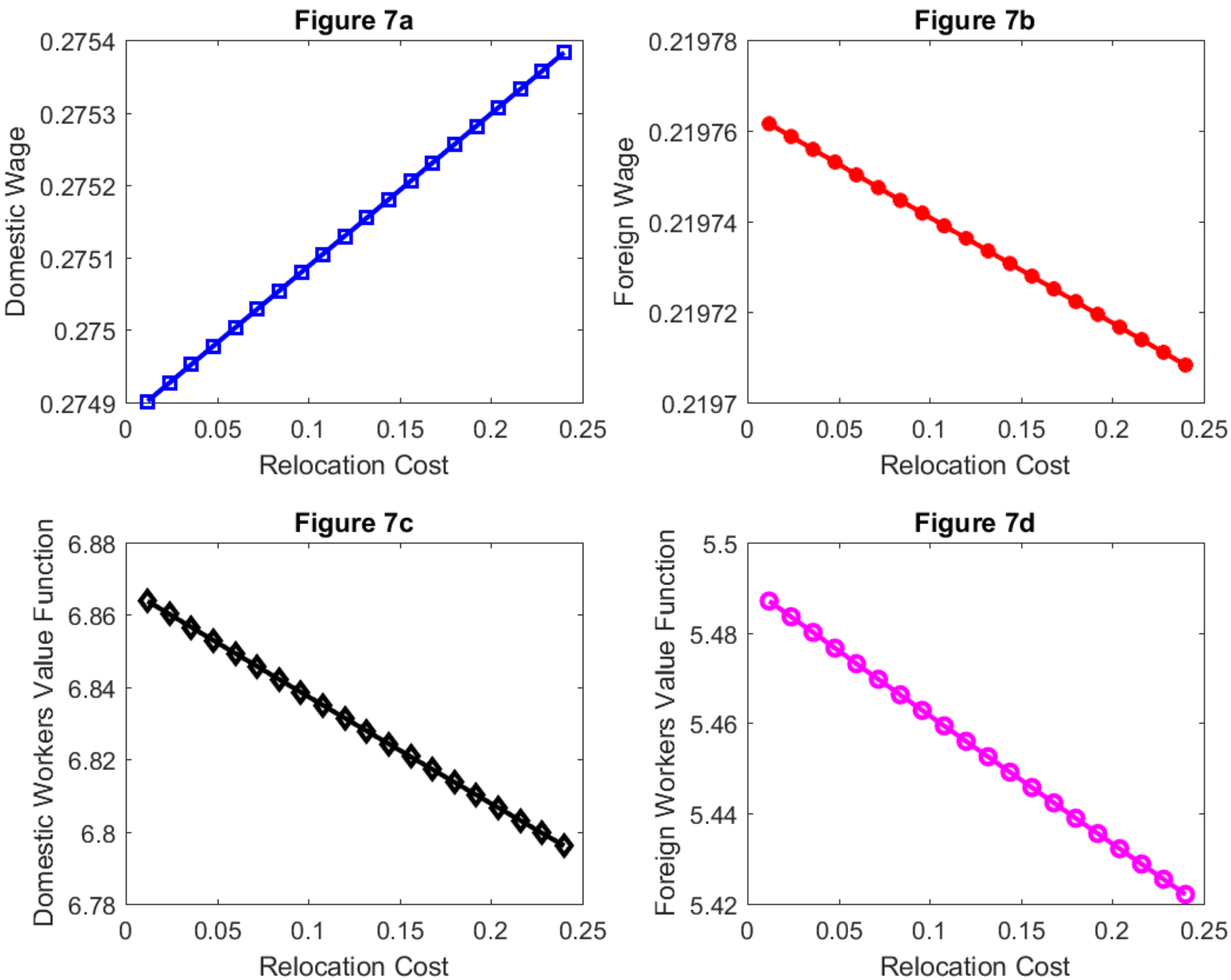


Figure 7a
Domestic Wage
0.2754
0.2753
0.2752
0.2751
0.275
0.2749
0
0.05
0.1
0.15
0.2
0.25
Relocation Cost
Figure 7b
Foreign Wage
0.21978
0.21976
0.21974
0.21972
0.2197
0
0.05
0.1
0.15
0.2
0.25
Relocation Cost
Figure 7c
Domestic Workers Value Function
6.88
6.86
6.84
6.82
6.8
6.78
0
0.05
0.1
0.15
0.2
0.25
Relocation Cost
Figure 7d
Foreign Workers Value Function
5.5
5.48
5.46
5.44
5.42
0
0.05
0.1
0.15
0.2
0.25
Relocation Cost

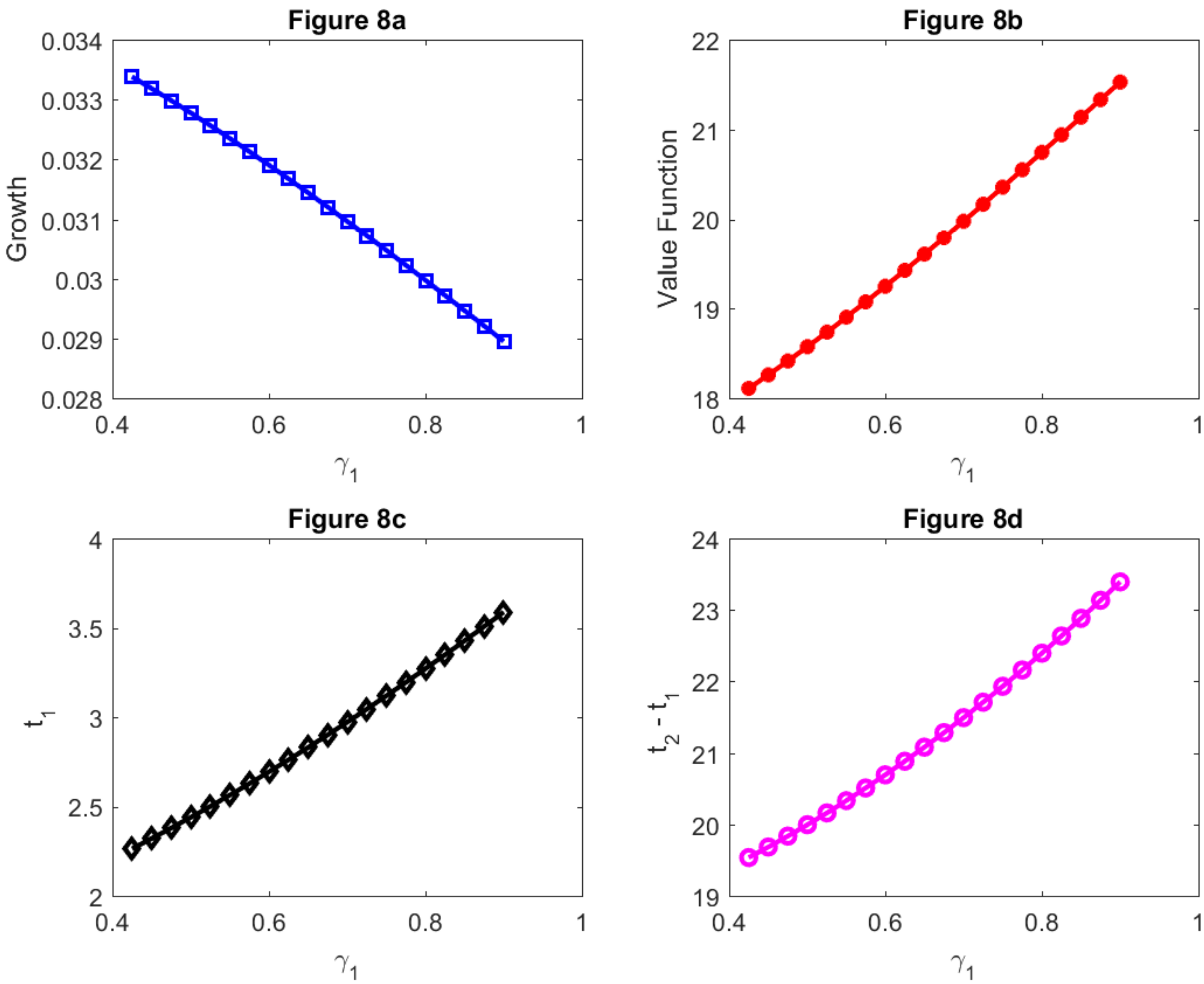


Figure 8a
Growth
0.034
0.033
0.032
0.031
0.03
0.029
0.028
0.4
0.6
0.8
1
$\gamma_1$
Figure 8b
Value Function
22
21
20
19
18
0.4
0.6
0.8
1
$\gamma_1$
Figure 8c
$t_1$
4
3.5
3
2.5
2
0.4
0.6
0.8
1
$\gamma_1$
Figure 8d
$t_2 - t_1$
24
23
22
21
20
19
0.4
0.6
0.8
1
$\gamma_1$

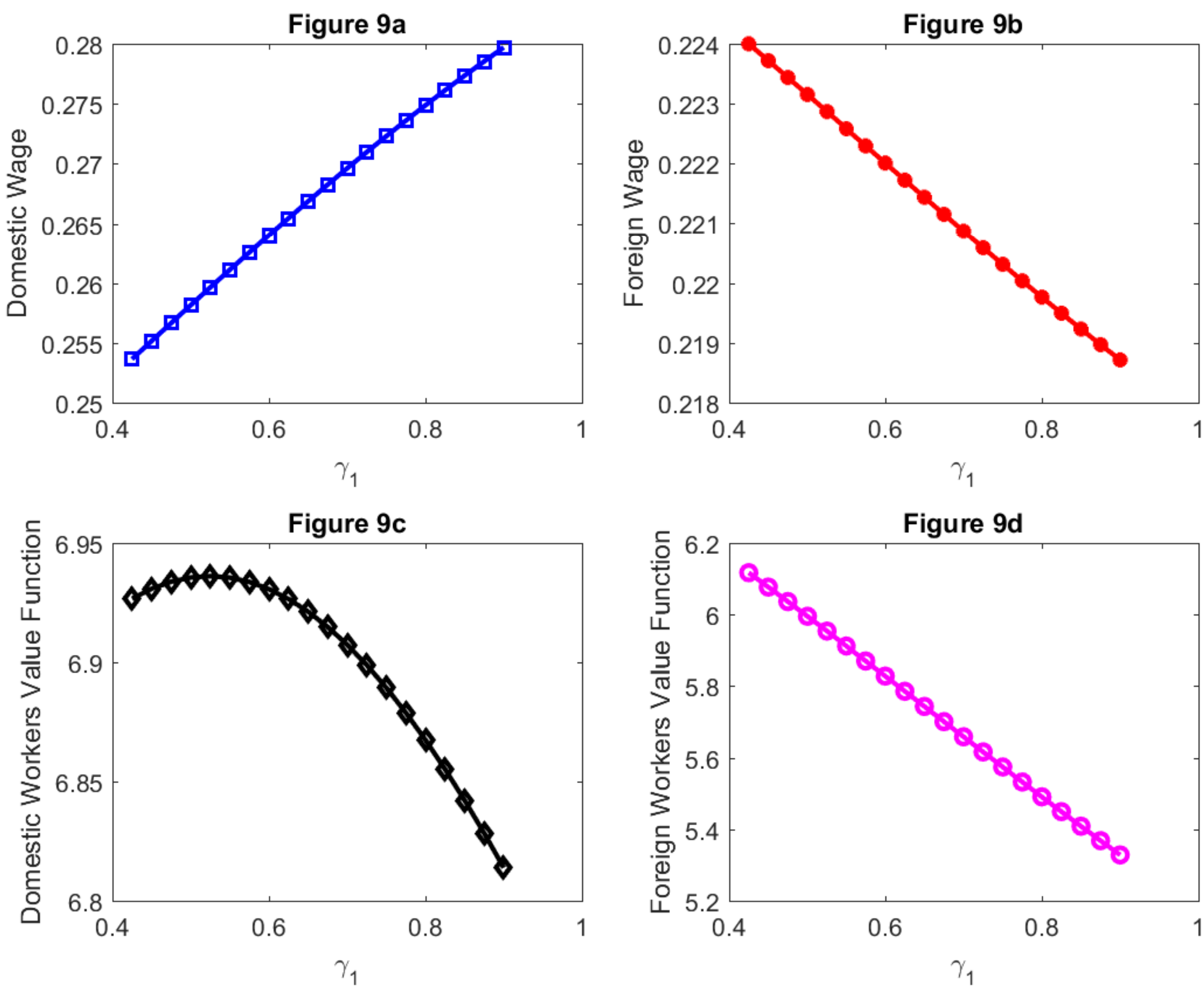


Figure 9a
Domestic Wage
0.28
0.275
0.27
0.265
0.26
0.255
0.25
0.4
0.6
0.8
1
$\gamma_1$
Figure 9b
Foreign Wage
0.224
0.223
0.222
0.221
0.22
0.219
0.218
0.4
0.6
0.8
1
$\gamma_1$
Figure 9c
Domestic Workers Value Function
6.95
6.9
6.85
6.8
0.4
0.6
0.8
1
$\gamma_1$
Figure 9d
Foreign Workers Value Function
6.2
6
5.8
5.6
5.4
5.2
0.4
0.6
0.8
1
$\gamma_1$

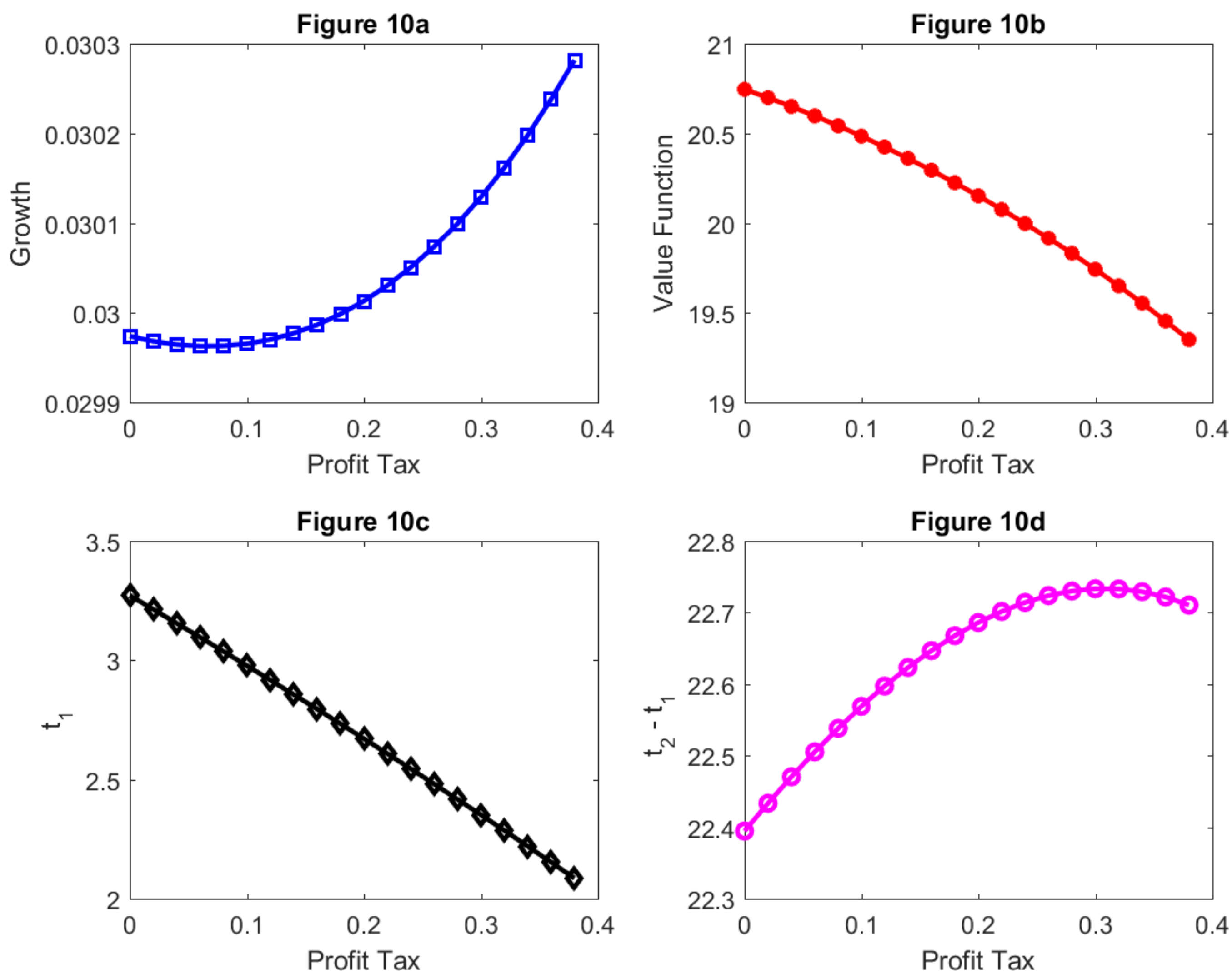


Figure 10a
0.0303
0.0302
0.0301
0.03
0.0299
Growth
0
0.1
0.2
0.3
0.4
Profit Tax
Figure 10b
21
20.5
20
19.5
19
Value Function
0
0.1
0.2
0.3
0.4
Profit Tax
Figure 10c
3.5
3
2.5
2
$t_1$
0
0.1
0.2
0.3
0.4
Profit Tax
Figure 10d
22.8
22.7
22.6
22.5
22.4
22.3
$t_2 - t_1$
0
0.1
0.2
0.3
0.4
Profit Tax

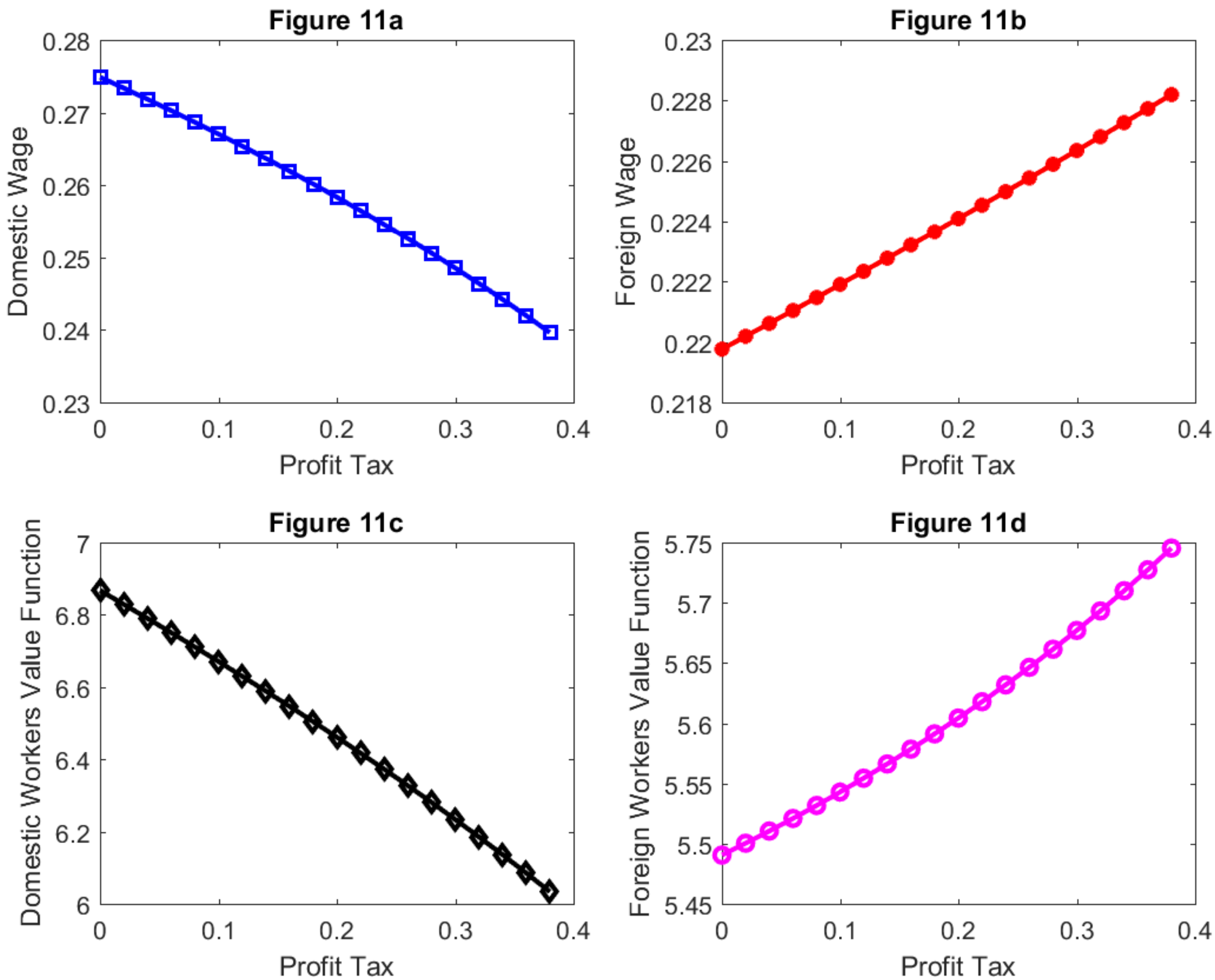


Figure 11a
Domestic Wage
0.28
0.27
0.26
0.25
0.24
0.23
0
0.1
0.2
0.3
0.4
Profit Tax
Figure 11b
Foreign Wage
0.23
0.228
0.226
0.224
0.222
0.22
0.218
0
0.1
0.2
0.3
0.4
Profit Tax
Figure 11c
Domestic Workers Value Function
7
6.8
6.6
6.4
6.2
6
0
0.1
0.2
0.3
0.4
Profit Tax
Figure 11d
Foreign Workers Value Function
5.75
5.7
5.65
5.6
5.55
5.5
5.45
0
0.1
0.2
0.3
0.4
Profit Tax

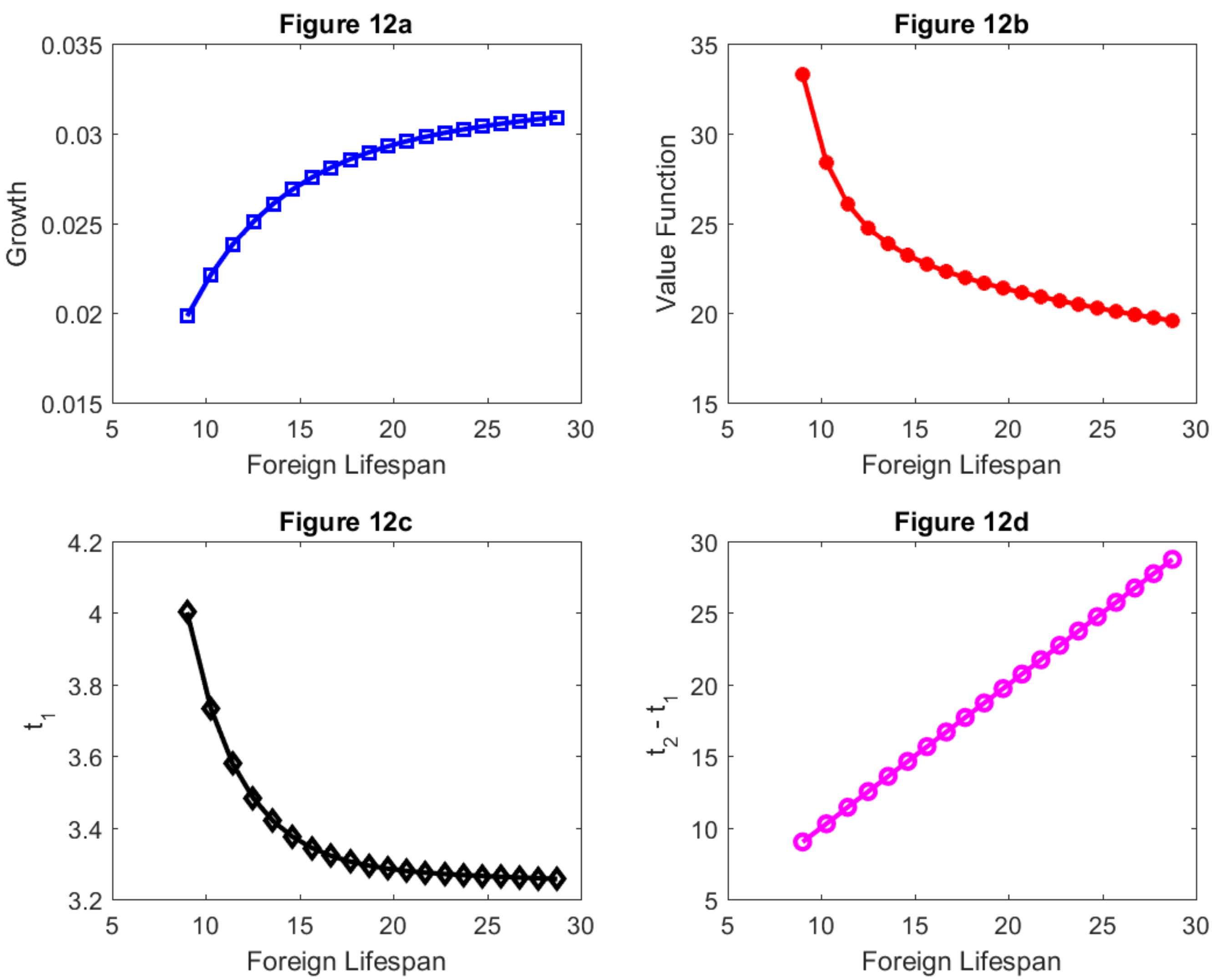


Figure 12a
Growth
Foreign Lifespan
Figure 12b
Value Function
Foreign Lifespan
Figure 12c
t1
Foreign Lifespan
Figure 12d
t2 - t1
Foreign Lifespan

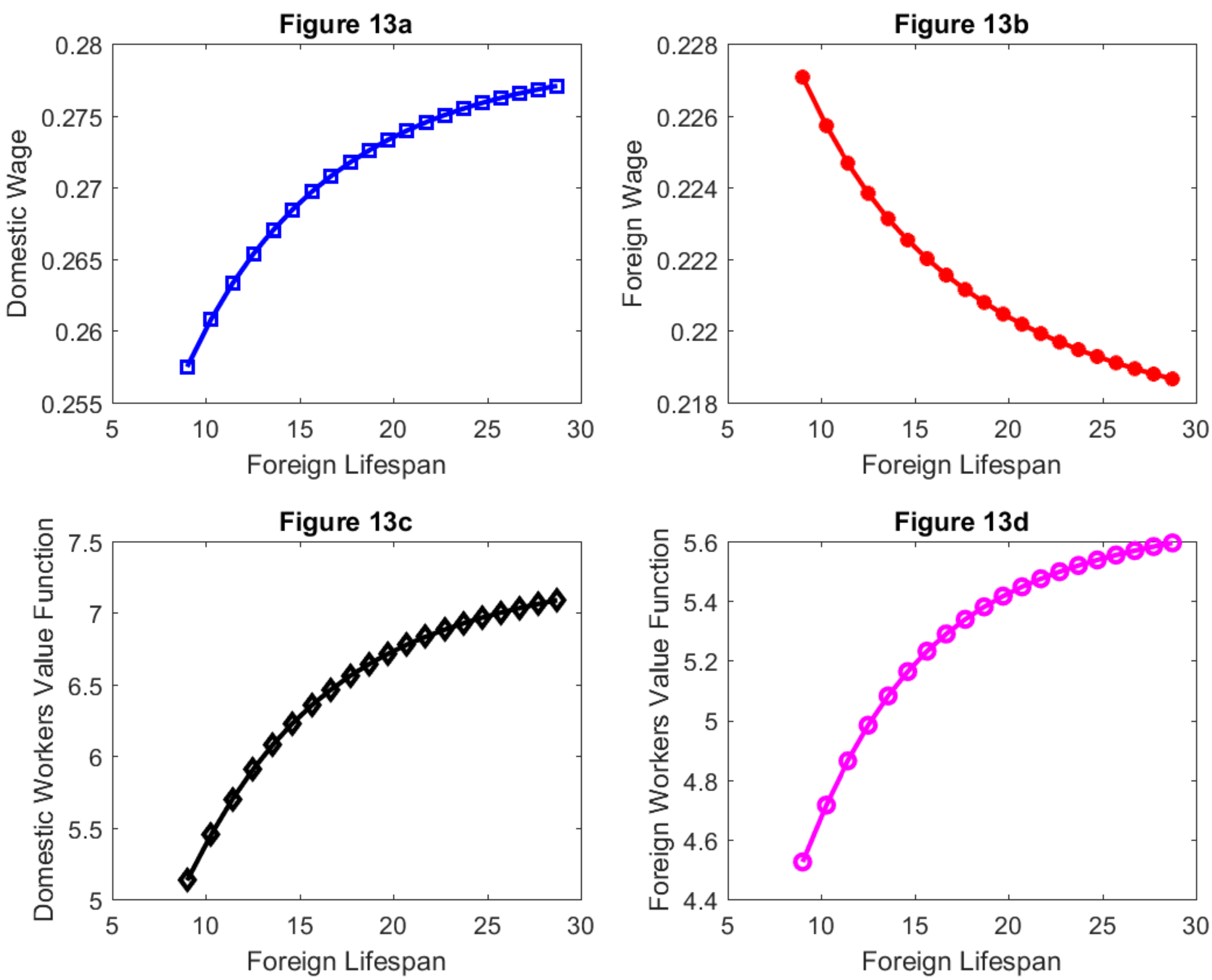


Figure 13a
Domestic Wage
Foreign Lifespan
0.28
0.275
0.27
0.265
0.26
0.255
5
10
15
20
25
30
Figure 13b
Foreign Wage
Foreign Lifespan
0.228
0.226
0.224
0.222
0.22
0.218
5
10
15
20
25
30
Figure 13c
Domestic Workers Value Function
Foreign Lifespan
7.5
7
6.5
6
5.5
5
5
10
15
20
25
30
Figure 13d
Foreign Workers Value Function
Foreign Lifespan
5.6
5.4
5.2
5
4.8
4.6
4.4
5
10
15
20
25
30

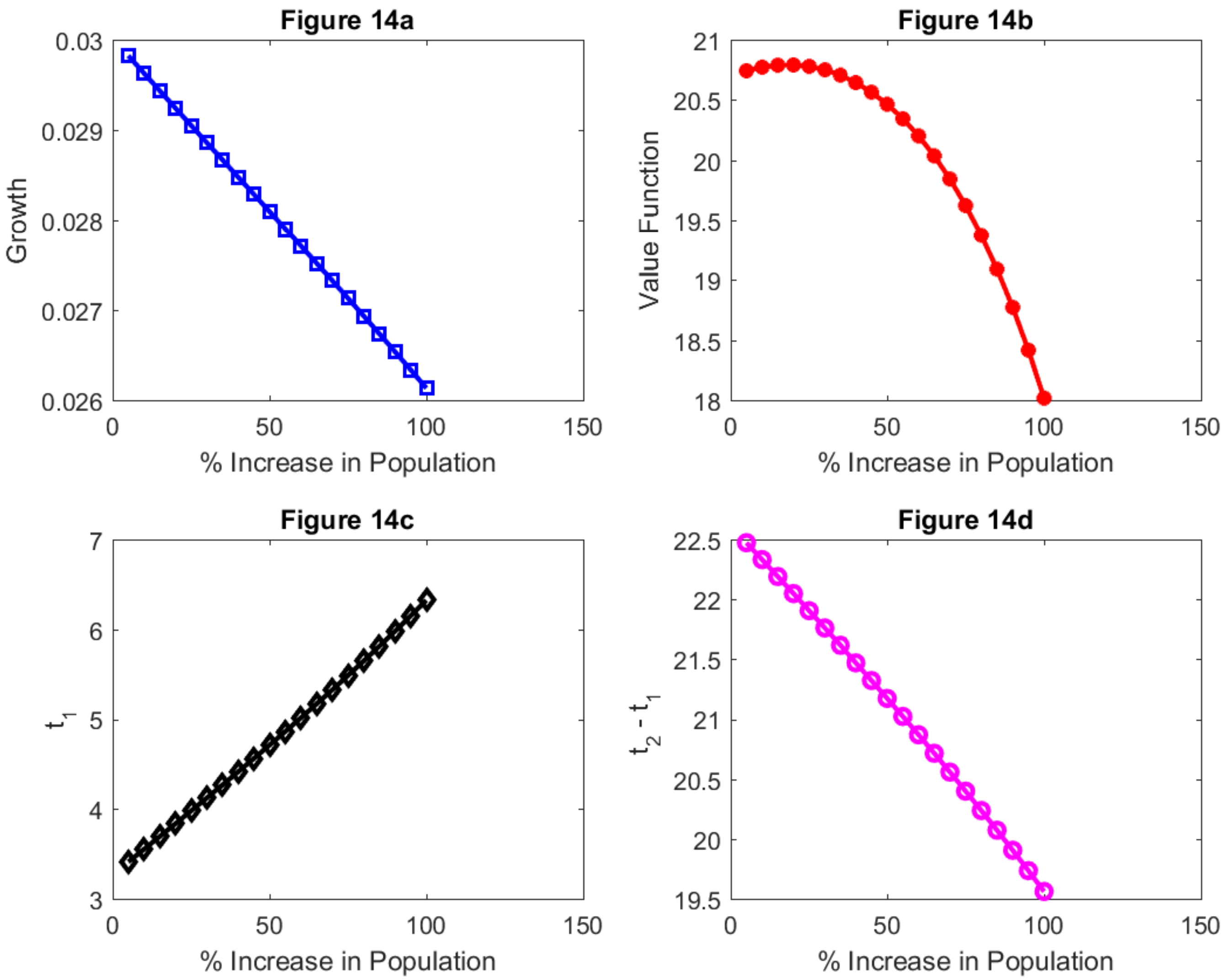


Figure 14a
Growth
% Increase in Population
Figure 14b
Value Function
% Increase in Population
Figure 14c
t1
% Increase in Population
Figure 14d
t2 - t1
% Increase in Population

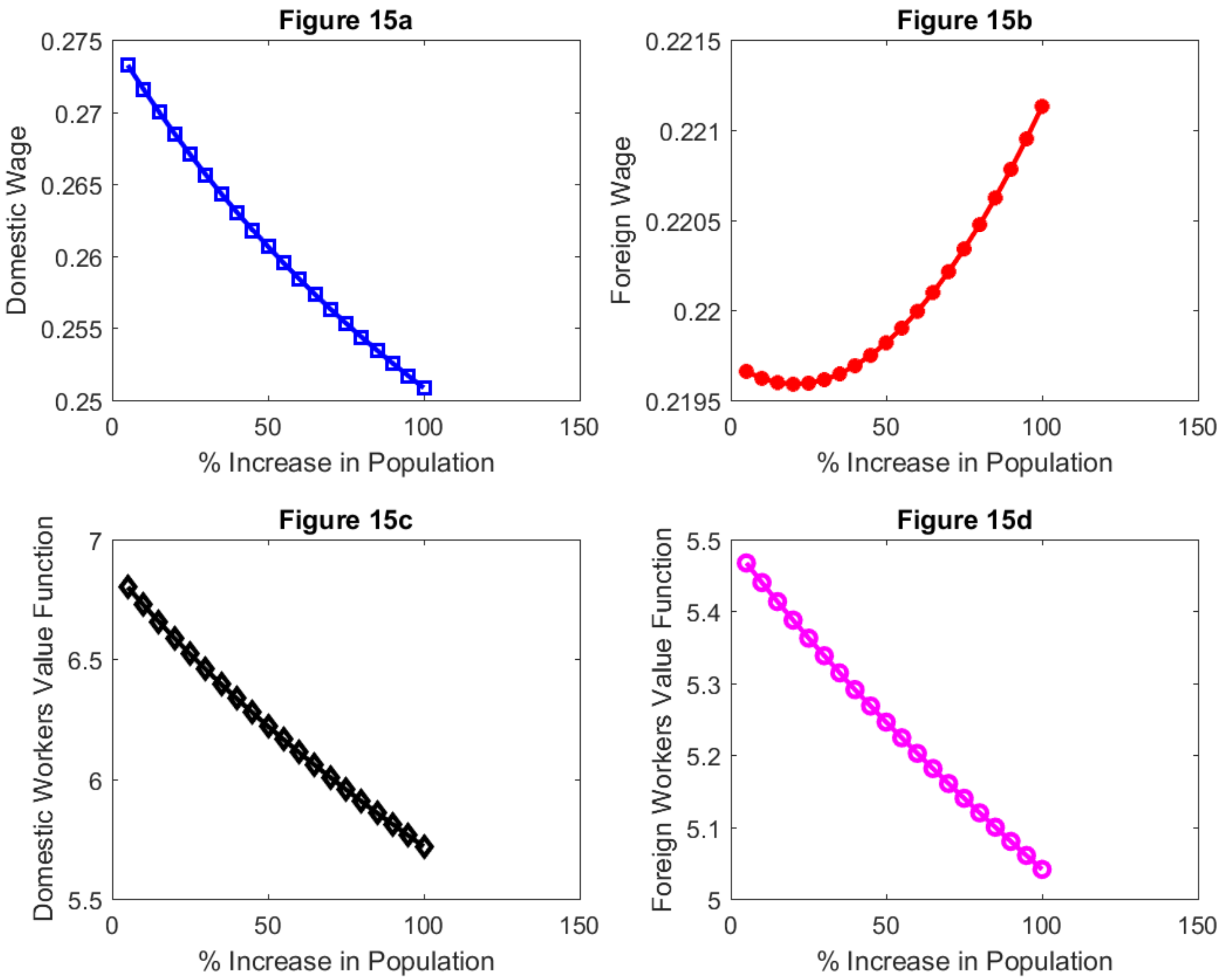


Figure 15a
Domestic Wage
0.275
0.27
0.265
0.26
0.255
0.25
0
50
100
150
% Increase in Population
Figure 15b
Foreign Wage
0.2215
0.221
0.2205
0.22
0.2195
0
50
100
150
% Increase in Population
Figure 15c
Domestic Workers Value Function
7
6.5
6
5.5
0
50
100
150
% Increase in Population
Figure 15d
Foreign Workers Value Function
5.5
5.4
5.3
5.2
5.1
5
0
50
100
150
% Increase in Population