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REAL INTEREST POLICY AND THE HOUSING CYCLE

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Key Words: Housing-cycles, Interest Rate, Bubbles, Government loans. JEL Codes: E32, E60

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1 INTRODUCTION

"Home prices do indeed go through years of price increases and then years of price decreases. So, the random walk model of home price behavior is just not even close to being true for home prices". (Shiller's Nobel lecture, 2014, page 1502).

In an earlier paper Shiller (2007) observed that housing prices in the US rose 86% in real terms between 1996-2006. During this period real rent has been extremely stable and labor cost experienced a slight decline. To explain these observations he proposes "a psychological theory that represents the boom as taking place because of a feedback mechanism or social epidemics that encourages a view of housing as an important investment opportunity". [1]

Here I use standard theory of rational bubbles to discuss the role of government in bubbles. I use an overlapping generations economy with two assets: Government bonds and houses. The young in the model allocate their labor between the production of a perishable good and houses. Houses yield services in addition to their use as a store of value.

I distinguish between two cases. In the first, the government is willing to lend and borrow at the announced real interest rate. In the second, the government borrows but does not lend. In both cases, the real interest rate on government bonds is a policy choice. (It will be shown that the government can indeed peg the real interest rate.) I also consider the case in which the government supplies mortgages but does not supply consumption loan. This more realistic case is similar to the no borrowing case if there is a downpayment requirement.

Cycles are possible in the first case, when borrowing from the government is allowed. In the second case, cycles are not possible if the interest rate is sufficiently high. The affordability issue discussed in Scheinkman (1980) and Tirole (1985) plays a key role. When borrowing is not allowed and the interest rate is sufficiently high, the young may not be able to buy the housing stock. The level of the interest rate is relevant because the growth in the value of the housing stock depends on it. When the interest is high, the young will not be able to buy the housing stock if the bubble lasts for a long time. Since rational bubbles require market clearing even when the bubble lasts for a long time, a high interest rate will rule out rational bubbles in the no borrowing case.

The level of interest that is required to prevent the formation of bubbles (in the no borrowing case) depends on the rate of technological change in the perishable good industry relative to the rate of technological change in the housing sector. To see why here the relative rather than the absolute rate of growth is relevant, note that in the no borrowing case, the young cannot spend on houses more than the amount of the perishable good he produces. When the output in the perishable good industry rise at y%, the young will not be able to afford (in the long run) a housing stock that its value increases by x% > y%. Therefore, when y is large a high interest rate that will make x large is required to discourage bubbles.

[1] For a model in which agents change their views as a result of "social dynamics", see Burnside, Eichenbaum and Rebelo (2016).

A high interest rate may discourage bubbles but may have some welfare cost. I compare welfare across deterministic steady states and show that allowing two interest rates, one on government bonds and a lower one on mortgages, is enough to achieve the first best. The interest rates that maximize steady state welfare may not be sufficiently high to eliminate bubbles and this pose a dilemma for the policy maker.

Section 2 is about related literature. Section 3 provides some stylized facts. Section 4 is the model with no restrictions on borrowing. Section 5 provides deterministic-steady-state analysis. Section 6 is about deterministic non-steady state equilibria and section 7 is about cycles. Section 8 assumes no government loans. In section 9 the government supplies mortgages that require down payment. Section 10 discuss the ability of the model to account for the stylized facts, Section 11 allows for deviations from strict rationality and section 12 concludes.

2 RELATED LITERATURE

I start by elaborating on the affordability issue. Tirole (1985) argues that in an overlapping generations model, a bubble can emerge if there is sufficient growth in the economy. Otherwise, if the long run interest rate is positive, the asset bubble - which must grow at the interest rate - eventually becomes so big that the young generation cannot buy the asset.

Here relative rather than absolute growth is relevant. The affordability problem is "solved" if the technological change in the non housing sector is high relative to the technological change in the housing sector. Here absolute growth is not relevant for affordability. For example, here population growth is not relevant because it affects the long run growth in both sectors in a symmetric way. [2]

Government policy is also relevant for affordability. There are many ways in which the government can "solve" the "affordability problem". For example, the government can announce that in states of the world in which the young cannot afford the housing stock it will collect lump sum taxes from the owner of houses (the old in the model) and transfer them to the young.[3] Here the government "solves" the problem when it supplies loans to the young which are

[2] There are other differences form Tirole's model. Here there are two goods while in Tirole's model there is one. Here the interest rate is a policy choice while in Tirole it is an endogenous variable. The bubbles in Tirole (1985) crowd out productive capital. They may also improve welfare. Here bubbles are likely to cause overproduction of housing and reduce welfare. This is similar to Dupor (2005) but here agents are fully rational.

[3] The expectations that the government will step in when housing prices are too high, is not unfounded. Something close to that happened in Israel. In 2011 there were big demonstrations by young people who could not afford housing. To make the point that housing is not affordable, young people erected tents in parks in the middle of Tel Aviv and lived there for the entire summer. This turned out to be quite effective. The finance minister was elected on the promise to solve the housing problem. His proposed solution includes subsidies to the young who do not have apartments and taxes on those who have three or more apartments.

financed by the loan payments that the government receives from the old and by lump sum taxes.

Here the government can discourage bubbles. A policy maker who wants to disccourage bubbles may look for a policy that makes the affordability problem more "severe" so that bubbles will be ruled out on the ground that the young will not be able to afford the housing stock if the bubble lasts for a long time. I argue that this can be done by restricting borrowing from the government and setting a high interest rate.

This is different from Gali (2014) who argues that monetary policy cannot affect bubbles. An increase in the interest rate will be matched by an increase in the rate of return on the bubble. It seems that Gali reaches his conclusion because he ignores the affordability problem.[4]

In Gali's model there are no government bonds. Here there are government bonds and it makes a big difference if the young can borrow from the government or not. When borrowing is allowed, the young can always buy the housing owned by the old by taking loans from the government and as in Gali (2014), high interest rate will not discourage bubble formation. It is also true in my model (as in Gali's model) that when borrowing is not restricted housing prices will rise faster when the interest rate is high. But when borrowing from the government is not allowed a path of fast rising housing prices may not be possible because of the affordability problem discussed above: Eventually the young may not be able to buy the housing stock. It follows that high interest that leads to fast growth in housing prices, can rule out rational bubbles in the no borrowing case.

Other papers assume that monetary policy can affect bubbles but argue against it. Bernanke and Gertler (2001, 1999) advocate monetary policy reaction to changes in asset prices that affect the central bank's forecast of inflation. But once the predictive content for inflation has been accounted for, there should be no additional response of monetary policy to asset price fluctuations. Gilchrist and Leahy (2002) summarize the literature on monetary policy and asset prices. They also do not find a case for including asset prices in monetary policy rules. Here there is no money and therefore inflation is not a problem. Nevertheless, the government may wish to discourage bubbles.

The literature on bubbles is large. Allen and Gorton (1993), Allen and Gale (2000) and Barlevy (2014) model bubbles in economies with asymmetric information and agency problems. Using the terminology of Allen, Morris and Postlewaite (1993) the model here is about "strong bubbles" in which the lack of fundamentals is common knowledge as in Samuelson (1958) and Tirole (1985). There is a vast literature that asks under what conditions "strong bubbles" can exist. See Santos and Woodford (1997) for a survey of this literature. It seems that "strong bubbles" may arise in economies that are "close" to dynamically inefficient OG (overlapping generations) economies.[5] For early models that allow

[4]The equilibrium definition (on page 732) in Gali's model does not require that the budget constraint of the young (unnumbered equation on page 728) must be satisfied.

[5]In OG models the economy is dynamically inefficient whenever a planner can improve the terms in which the young can save. In the steady state the planner can promise a rate of return that is equal to the rate of population growth by taking goods from the young and

pops of “strong bubbles” see Blanchard and Watson (1982) and Weil (1987).

Recently there is a growing literature on the possibility of “strong bubbles” in economies with financial constraints. See for example, Caballero and Krishnamurthy (2006), Kocherlakota (2009), Farhi and Tirole (2012), Basco (2014) and Miao and Wang (2018). Woodford (1990) notes that some economies populated by infinitely-lived-liquidity-constrained agents are similar to an OG economy. For an excellent review of this literature, see Martin and Ventura (2018).

Part of the literature surveyed by Martin and Ventura assumes a small open economy and allows the young to borrow at the world interest rate. Bubbles may be used as a collateral and therefore they relax the borrowing constraint. The assumption of a "small open economy" does not completely solve the affordability problem. If the bubble does not pop for a long time (which is a small probability event in most models) then it becomes "big" in the sense that the world’s young generation will not be able to buy it.

Here the government intermediates between generations. In the baseline model, the government is willing to lend and borrow at a given interest rate. There is thus, no financial friction and therefore a bubble in the housing market does not relax financial constraints. In the version of the model in which government supply mortgages that require down payments, a rise in the price of housing exacerbates the liquidity constraint rather than relaxes it.

This paper focus on the role of government in bubbles and is therefore also related to the old literature on the role of government in money (which is the classical example of bubbles). For example, Lerner (1947) argued that general acceptability of almost anything can be established by the state if it is willing to accept it as tax payments. And the state can destroy a particular type of money if it declares that it will not accept it as tax payment. Here houses cannot be used to pay taxes but nevertheless the government may play an important role in supporting a housing bubble.

2.1 More on the policy debate

In his famous presidential address, Friedman (1968) argues that monetary policy cannot peg (a) the interest rate and (b) the rate of unemployment. Friedman starts by arguing that increasing the money supply will reduce interest rate in the short run but not in the long run. He then argue (on page 7) that

> "Paradoxically, the monetary authority could assure low nominal rates of interest-but to do so it would have to start out in what seems like the opposite direction, by engaging in a deflationary monetary policy. Similarly, it could assure high nominal interest rates by engaging in an inflationary policy and accepting a temporary movement in interest rates in the opposite direction."

He then cite Wicksell for the concept of the "natural" interest rate and argue that

transferring them to the old. A planner can improve matters when the rate of population growth is higher than the rate that the young can get in equilibrium that has no bubbles.

> " It will require not merely deflation but more and more rapid deflation to hold the market rate above the initial "natural" rate."

Based on Friedman's analysis many have concluded that in the long run both the real interest rate and the rate of unemployment must equal their respective "natural" levels. This is not true. Friedman argues that the long run levels of these variables cannot be affected by monetary policy defined as changes in the money supply that are the result of open market operations. But the level of both can be affected by other policies. The long run level of unemployment can be affected by unemployment insurance, for example. And it is possible to peg the real interest rate by the following policy. The government announces that it will lend and borrow at a given interest rate and that it will use taxes and transfers to make the difference between the revenue from selling bonds and the cost of retiring bonds. Usually pegging a relative price is a "bad" policy. The real interest may be different because it affects bubble formation.[6]

To get a better sense of the current policy debate, I now turn to discuss a thoughtful paper by Neel Kashkari the president of the Minneapolis Fed.[7] Kashkari initial remarks are about the importance of financial stability.

> " In 1977, Congress gave the Fed its dual mandate: stable prices and maximum employment. However, we can't ignore the implicit role the Fed also has to try to achieve financial stability. After all, when Congress first created the Fed in 1913, it did so in response to financial crises that repeatedly hammered the U.S. economy in the late 1800s and in the panic of 1907. The Board of Governors and 12 regional Federal Reserve Banks were specifically created with the goal of promoting financial stability. Price stability and maximum employment came almost 70 years later."

And

> "Achieving financial stability is hard—really hard. Human societies are prone to mass delusion and to bubbles; history has numerous examples, from the tulip bubble in Holland in the 1600s to the stock market bubble in the 1920s to the housing bubble in the 2000s. Future generations are exceptionally good at repeating past mistakes. Even if we focus just on the Fed's official dual mandate, financial crises can cause very high unemployment and low inflation or even deflation. My perspective is that whether it is officially acknowledged or not, whether we want the responsibility or not, the Fed has an important role to try to ensure financial stability."

[6]Pegging the real interest rate is similar to the pegging of any other relative price. For example, if the government wants to peg the real price of wheat it may have some shortages in case of excess demand and some surpluses in case of excess supply. It can distribute the surpluses in case of excess supply and tax the suppliers in case of excess demand.

[7]Published May 17, 2017
https://www.minneapolisfed.org/news-and-events/messages/monetary-policy-and-bubbles

Kashkari assumes mass delusion while here I focus on rational bubbles. The analysis of rational bubbles may serve as a useful benchmark. For example, he argues for increasing down payment requirements noting that:

> "Going into the financial crisis, people were putting little to nothing down with those infamous no-doc loans. Those loans were bundled into mortgage-backed securities, which were then bundled into collateralized debt obligations, and then banks bought them with yet more borrowed money. It was leverage on top of leverage with little equity supporting it all."

Here I show that having some down payment requirement can prevent the formation of rational bubbles. The argument is as follows. When the bubble lasts for a long time the young may not have enough funds to buy the housing stock and this cannot occur in equilibrium where markets are cleared. When the down payment requirement is low the probability that the bubble will last until the young generation will not be able to afford the housing stock is low. In reality agents are not completely rational and may ignore small probability events. When we increase the down payment requirement the probability that the young will not be able to buy the entire housing stock goes up and the chance that this possibility will be ignored goes down. A policy maker who thinks that agents are not fully rational may use this argument to increase the down payment requirement.

Kashkari distinguishes between two related policy issues: (a) should the Fed try to burst bubbles and (b) should it attempt to prevent the formation of future bubbles. He answers the first question in the negative because it is difficult to identify a bubble and even if the Fed can identify it the interest rate is not a good instrument to deal with it. Regarding the second question, he says the following.

> "Current estimates are that the neutral real rate (net of inflation) is currently around zero or perhaps slightly negative. Could it be that such low rates make bubbles more likely to form and, if so, what should we do about it? The truth is we don't have a good answer to this question. If inflation is low and there is slack in the labor market, how high should we raise rates to reduce the chances of bubbles forming? We don't have a good economic theory to analyze this scenario and offer policy guidance. It is a question that needs more research. Until we have such a theory that we have confidence in, I believe we should continue to focus on our dual mandate goals to set monetary policy and then keep our eyes open for potential bubbles and respond as best we can. The cost of keeping rates high to reduce the chances for future bubbles would be higher unemployment and a risk of unanchoring inflation expectations to the downside. Those are large economic costs."

Here, high interest does not lead to less employment. High interest actually increases employment, when the government lends money or when the "no borrowing constraint" is not binding. In the model the young work in the first period and consume only in the second. Therefore, an increase in the interest rate is equivalent to an increase in the real wage.[8]

3 STYLIZED FACTS

Figures 1 describes post world war data about housing prices and stock prices. The solid lines are the logs of the real price (right scale). The dotted lines are the rates of change in the real price in the last year.[9]

The real price of housing exhibits cycles. Housing prices decreased by about 7% from the peak of 1971 to the trough in 1974. They decrease by 11% from the peak of 1978 to the trough of 1982. And they decrease by roughly 40% from the peak of 2005 to the trough of 2011.

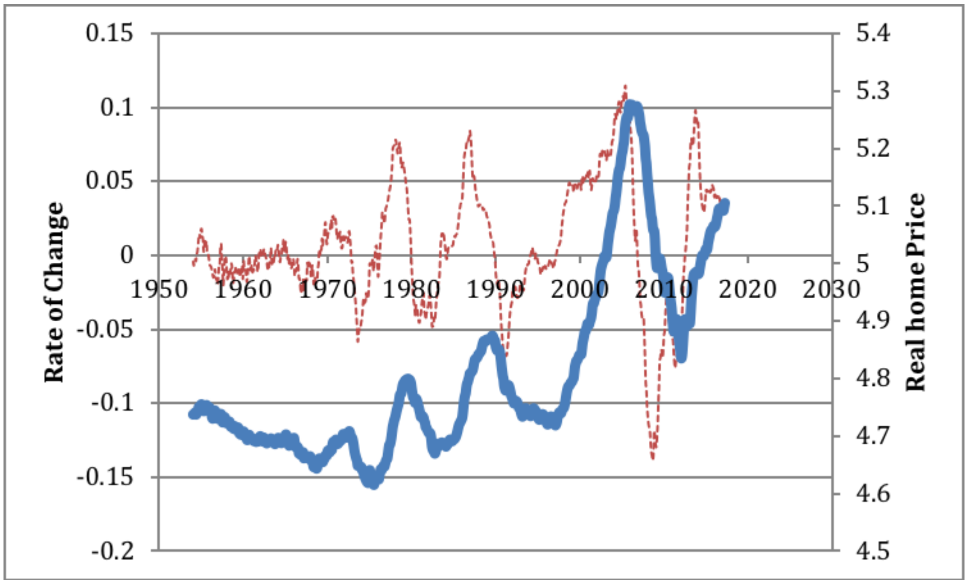
The rate of change in housing price can also be described by the use of business cycle language. The annual rate of change peaked around 1970 at 2%. This means that on average, someone who bought a house in 1969 experienced a 2% increase in the price of his house after a year. The rate of change peaked in 1977. On average, someone who bought a house in 1976 experienced a 7% increase in the price of his house after a year. It also peaked in 1988 at the same level of 7%, at 2005 at the level of 11% and at 2013 at the level of 9%.

The rate of return on housing fluctuates much less than the rate of return on stocks. It is in the range of 11 to -14 percent while the rate of return on stocks is in the range of 48 to - 42 percent. The correlation between the rate of change in housing real price and its one-year lag is 0.74. This says that the rate of change is likely to be high if it was high a year ago. The same correlation for stocks is 0.08. This says that for stocks the rate of change does not depend on the last year rate of change.

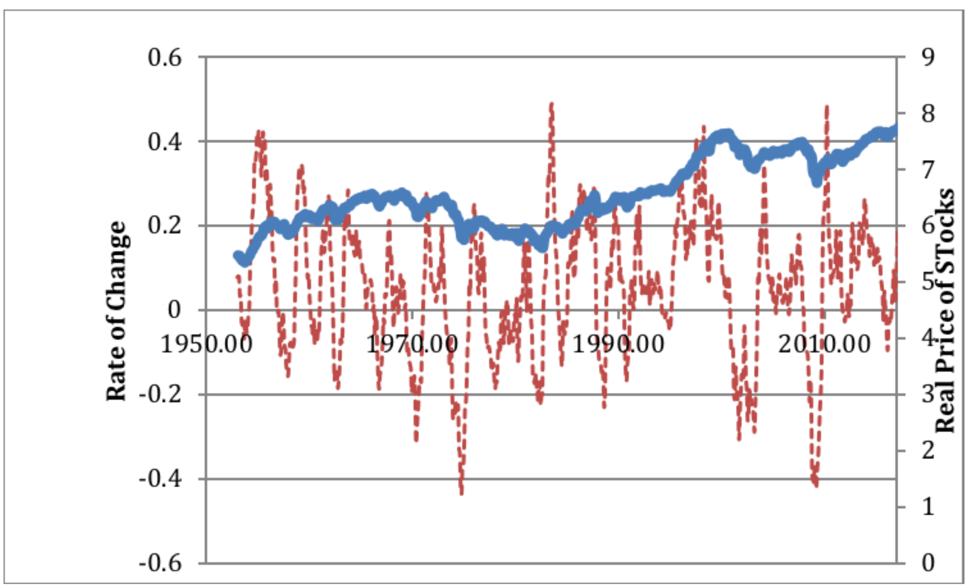
Although houses and stocks are both assets, Figure 1 suggests that their price behavior is very different and maybe we should not attempt to capture both behaviors in the same model.

[8] In reality, an increase in the interest rate is good for savings, including savings for retirement. Currently savings is subsidized (by tax deferral for 401Ks) so raising the interest rate and eliminating the 401Ks maybe a good idea.

[9] The data about real housing price and real stock prices is from Robert Shiller's web page. The employment share was calculated using data from the St. Louis Fed web site.



A: Real Home price (in logs, right axis, solid line) and the rate of price change from last year.



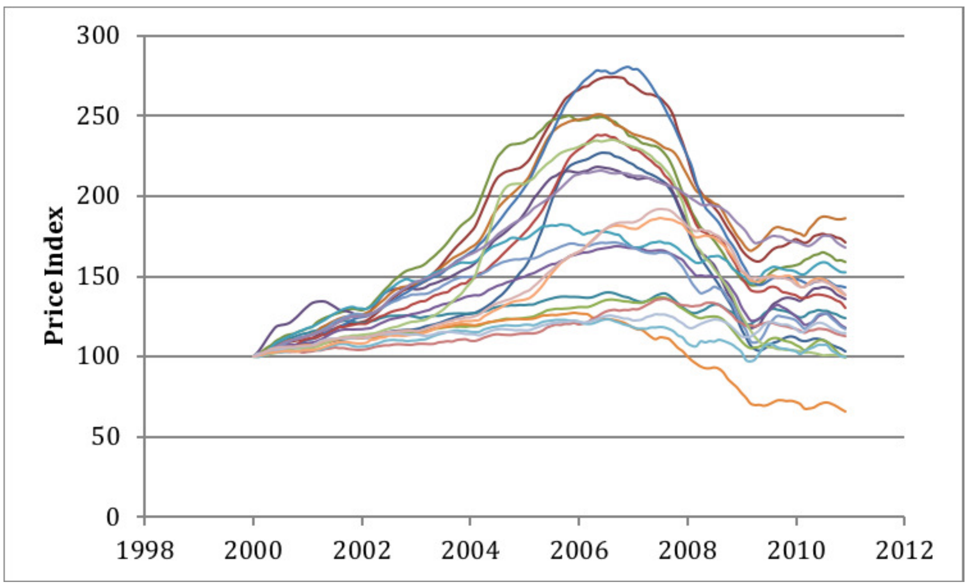
B: Real Stock Price (in logs, right axis, solid line) and their rate of change from last year.

Figure 1: Real Housing Prices and Real Stock Prices. Rate of change are calculated as the percentage change from the same month in the last year (12 months lag).

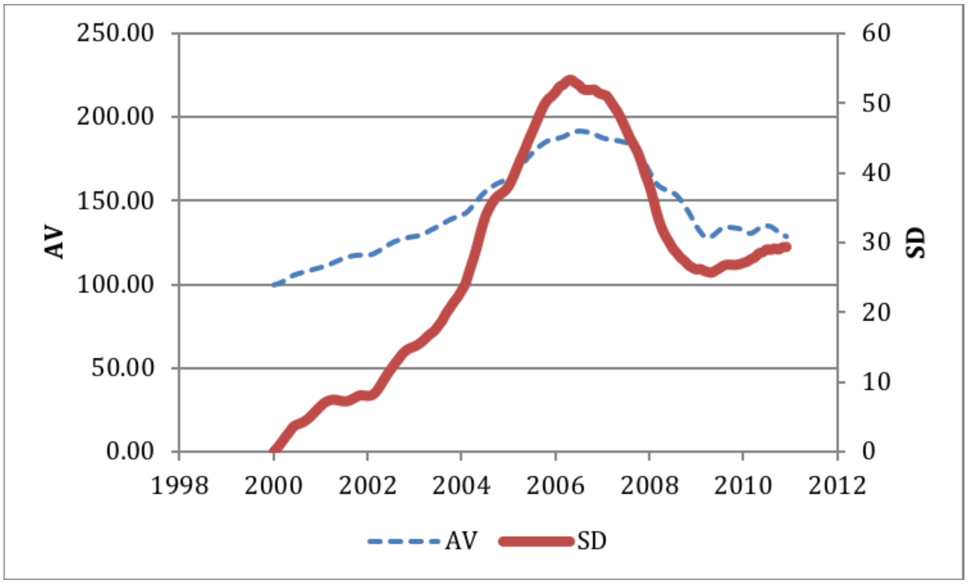
Figure 2 focuses on the last housing cycle. Figure 2A describes the Case-

Shiller index of housing prices in 20 major metropolitan areas across the US. The index is set at 100 in the year 2000 for all the 20 metropolitan areas. It then increased for all observations reaching a level above 250 in 2006 in some cases (273 for Los Angeles CA and 278 for Miami FL). By Dec. 2010 the indices were much lower (170 for Los Angeles and 143 for Miami). As can be seen from Figure 2B the cross sectional standard deviation of the Case-Shiller price indices fell by 50% during the period 2006-2009 from 52 to 26.

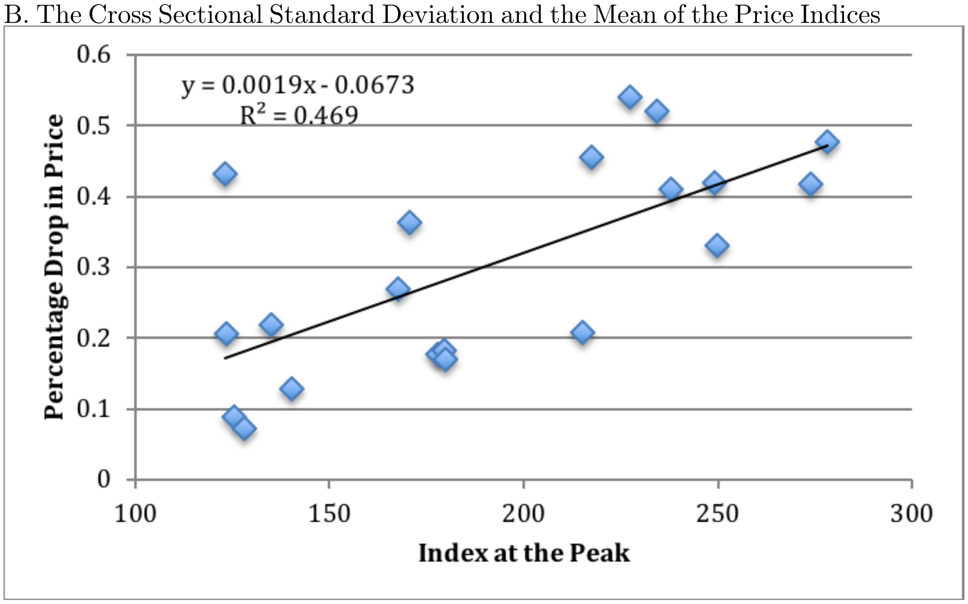
The fall in the standard deviation of the indices suggests that the fall in prices (from the peak to the trough) was larger for cities that experienced a large increase in price (from 2000 to 2006). Figure 2C plots the rate of decrease in price from the peak to the trough against the price index at the peak. It shows that on average the cities that experienced a large increase in prices also experienced a large fall in prices.



A. Price Indices for 20 metropolitan areas (1/1/2000=100)



B. The Cross Sectional Standard Deviation and the Mean of the Price Indices



C. The percentage drop in price from the peak of July 2006 to the trough of April 2009. The regression equation and the R squared are in the upper left corner of the graph.

Figure 2: The Case-Shiller Price Indices for 20 Metropolitan areas

Figure 3 is a plot of the employment share in the housing sector and real housing price.[10] The correlation between the two is close to zero. But this near

[10]The data about real housing price is from Robert Shiller's web page. The employment

zero correlation is the result of two opposite forces. The correlation between the trends is negative: The share of employment in the housing sector exhibits a negative trend while the real housing price exhibits a positive trend. At the cycle frequency the correlation is positive. For example, employment share has been growing from 4.2% in 1992 to 5.6% in 2007. During this period housing prices grew by 66%. The negative trend in the employment share that occurs in spite of the positive trend in the real price suggests a non-neutral technological change that reduces the marginal product of labor. The increase in housing price during the boom is strong enough to increase the employment share in spite of the technological changes that do not favor labor.

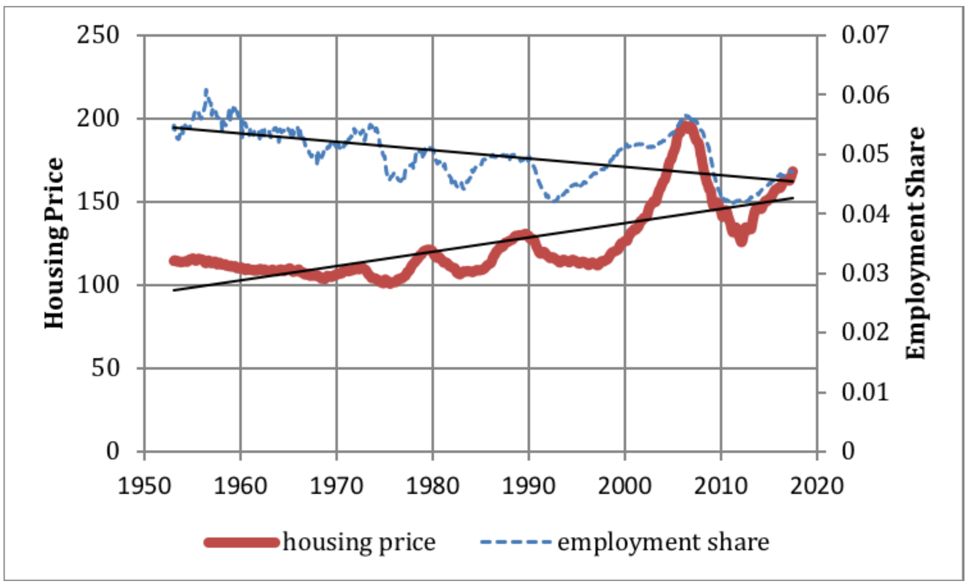


Figure 3: Employment in construction as a share of total non-farm employment and the real price of housing. Both series are seasonally adjusted. Trends lines are added.

Figure 4 is about household debt. The ratio of household debt to GDP reached a peak of close to 100% in 2008 and then declined to about 80% by 2015. Who did the household borrow from? It seems that Foreigners plaid an important role in buying mortgage-backed securities. But eventually the US government bought much of these securities. This brings the question of how to model Government implicit loan guarantees. Here I assume that the government gave the loans and implicitly promised to bailout the representative agent in the case of a crash.

share was calculated using data from the St. Louis Fed web site.

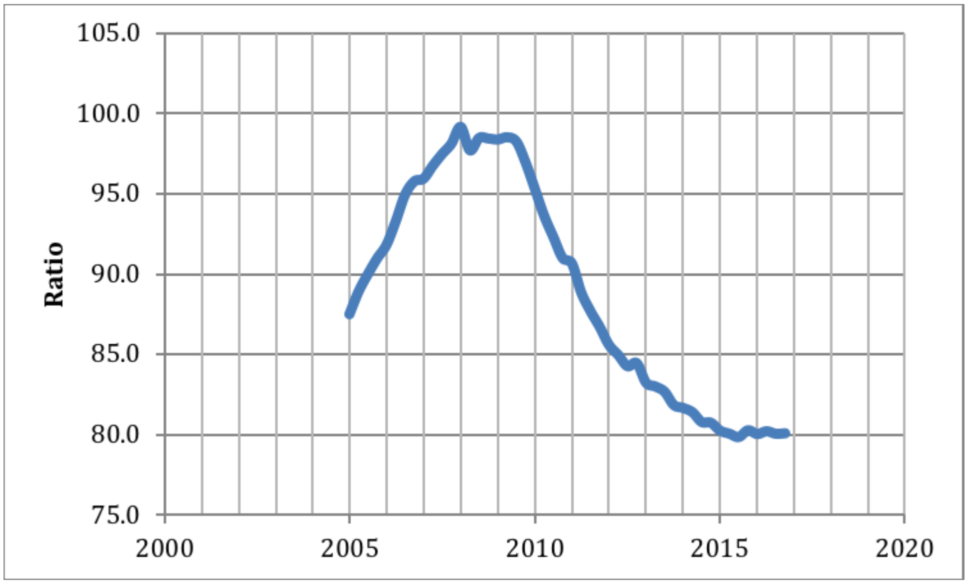


Figure 4: Household debt to GDP

I now turn to the model.

4 THE MODEL

I assume an overlapping generations economy. There is one agent per generation and the agent lives for two periods. In the Appendix, I consider the case in which the agent consumes in both periods. Here I assume the simpler case in which the agent works only in the first period and consumes only in the second.

There are two goods: A perishable good and houses. I use x to denote the amount of time allocated to the production of the perishable good and y to denote the amount of time allocated to the production of houses. The endowment of time (labor input) of the young is $\overline{L}$ and the total time devoted to both activities cannot exceed the endowment:

$$L_t = x_t + y_t \leq \overline{L} \tag{1}$$

The agent born at time t gets θ^t units of the perishable good for each unit of labor, where $\theta \geq 0$ is a productivity parameter. At time t, y_t units of labor in the housing sector yields $\psi^t f(y_t)$ units of housing where $\psi > 0$ is a productivity parameter, $f(y)$ has a maximum at $\overline{y}<\overline{L}$ and is differentiable with $f'(y) > 0$ and $f''(y) < 0$ for $y < \overline{y}$. These assumptions impose a limit on housing production.

Houses are homogeneous and the quantity of houses is measured by say, square feet. The productivity parameter ψ may be different from θ and because the quantity of land is fixed, it may be less than unity.[11]

[11]For example, in Manhattan NY, the cost of adding say 10,000 square foot of living space

In addition to producing houses, the young can also buy houses from the old. A young agent who buys H_t units of housing and devotes y_t units of labor for housing production will have at his old age:

$$H_{t+1} = (1 - \delta)\left(H_t + \psi^t f(y_t)\right) \tag{2}$$

units of housing, where $0 < \delta < 1$ is the depreciation rate. Thus, here new houses depreciate before they get used. The utility function of the agent born at t is:

$$\beta(c_{t+1} + \gamma H_{t+1}) - \theta^t v(x_t + y_t) \tag{3}$$

where $c_{t+1} = E(C_{t+1})$ is the expected consumption of the perishable good at time $t+1$, $\beta > 0, \gamma > 0$, are parameters and v is a monotone and strictly convex cost function. Unlike an Inada type condition, I assume $lim_{L \to \overline{L}} v'(L) = v'(\overline{L}) < \infty$. Note that the cost of labor grows over time at the rate of productivity growth in the non-housing sector reflecting the growth of productivity in leisure activities.[12]

Credit is central to the affordability issue. Other sectors can provide credit to the housing market but a bubble can survive in the long run only if the government steps in. I therefore focus on the credit provided by the government.

I start by assuming that the government lends and borrows at the gross real rate $R = 1+r$. There is no money and the payment is in terms of the perishable good.

The price of housing in terms of the perishable good evolves according to:

$$p_{t+1} = \{g_t p_t \text{ with probability } q \text{ and } I \text{ otherwise}\} \tag{4}$$

Thus, the price of houses grows at the rate g_t with probability q and it "crashes" to the price I with probability $1-q$. I assume that g_t may change over time but I and q are constants. This is of course not the only way to model bubbles. See

may grow over time because of the need to build taller buildings. In the 19 century you could still build a house on a vacant piece of land. Now you have to take out an old building and build a higher building in its place. The net gain in square footage may be small in spite of the high labor input.

[12]The model is rather abstract and does not capture many important aspects of the housing market. The main focus of the paper is on the role of government in making houses affordable. I do not think that a more realistic formulation will change the main results. The issue of consumption in both periods is in the Appendix. Another issue is the length of the period. A literal interpretation of the model may assume that the length of the period is 30 or 40 years but this is not the only possible interpretation. We may think of the agents in the model as entrepreneurs. The representative entrepreneur builds new houses and buys some old ones. He rents the houses for one period and then sell them. He may then use the funds he gets from selling the houses to start a new project. The young in our model are entrepreneurs who are in the buying and building phase. The old are entrepreneurs who are in the selling phase. An entrepreneur switches between roles over his lifetime and the length of the period maybe relatively short. We may also think of agents who buy houses to live in. An agent may buy a house and leave it as a bequest. And he may buy a house with the intention of eventually selling it and buying a different (possibly better) house. The young in the model attempt to capture some aspects of the buyer/investor type. The old in the model represents the sellers.

Blanchard and Watson (1982). Here I look at a special case that is consistent with the stylized facts.

The sequence of events within the period is as follows. First the old get the housing services which is a fraction γ of the stock of houses they own. Then the price of houses is announced and the young make labor choices (choosing x and y). At the end of the period the old sell their houses for the perishable good and settle debt with the government (the government redeems its bonds and collects lump sum taxes).

After the completion of trade, the young agent has $B_t = \theta^t x_t - p_t H_t$ units of government bonds and $H_t + \psi^t f(y_t)$ units of housing. When old he will get RB_t from the government in exchange for his bonds and $p_{t+1}H_{t+1}$ units in exchange for his houses.

The expected consumption of the perishable good is:

$$c_{t+1} = E(C_{t+1}) = R(\theta^t x_t - p_t H_t) + H_{t+1}E(p_{t+1}) - \overline{T}_{t+1} \tag{5}$$

where $E(p_{t+1}) = qg_t p_t + (1-q)I$ is the expected price calculated from (4) and $\overline{T}_{t+1}$ is the expected lump sum tax. The representative young agent chooses (H, y, x) by solving the following problem.

$$max_{H_t,y_t,x_t} \beta(c_{t+1} + \gamma H_{t+1}) - \theta^t v(x_t + y_t) \tag{6}$$

$$s.t. \text{ (1), (2) and (5).}$$

An interior solution to this problem must satisfy the following first order conditions:

$$\theta^t \beta R = \theta^t v'(x_t + y_t) = \beta(1-\delta)\psi^t f'(y_t)\left(qg_t p_t + (1-q)I + \gamma\right) \tag{7}$$

$$Rp_t = (1-\delta)\left(qg_t p_t + (1-q)I + \gamma\right) \tag{8}$$

The term on the left hand side of (7) is the discounted real wage in the perishable good sector: The agent will get $\theta^t R$ units of the perishable good when old for each unit of x. The first equality in (7) says that the discounted real wage in the perishable good sector must equal the marginal cost. The second equality in (7) says that the expected discounted benefit from an additional unit of y must equal the marginal cost. Condition (8) requires that the rate of return on housing is equal to the interest rate. The left-hand side of (8) is the cost of housing in terms of next period's consumption. The right-hand side of (8) is the expected consumption from buying an additional housing unit: After depreciation the agent will have $1-\delta$ units that will yield $(1-\delta)\gamma$ units of services and then will be sold at the expected price.

Under (8) the demand for housing is infinitely elastic and the housing market clears. I now turn to the perishable good market.

4.1 The clearing of the perishable good market

I show that the perishable good market clears if the government uses lump sum tax to finance the difference between debt retirement and the demand for new bonds.

The demand for the perishable good by the old is: $p_t H_t + RB_{t-1} - T_t$, where T_t is the realization of the lump sum tax. The clearing of the perishable good market requires:

$$p_t H_t + RB_{t-1} - T_t = \theta^t x_t \tag{9}$$

I assume that the government chooses the lump sum tax to satisfy (9) and therefore: $T_t = p_t H_t + RB_{t-1} - \theta^t x_t$. Note that since $\theta^t x_t > 0$, $p_t H_t + RB_{t-1} - T_t > 0$ and the old can pay the tax. (They do not go bankrupt).

I now show that choosing the tax to satisfy (9) is equivalent to using the tax to finance the difference between debt retirement and the demand for new bonds.

Claim 1. Choosing taxes to satisfy the market clearing condition (9) is equivalent to choosing taxes to satisfy:

$$T_t = RB_{t-1} - B_t \tag{10}$$

To see this claim, note that the demand for government bonds by the young is:

$$B_t = \theta^t x_t - p_t H_t \tag{11}$$

Using (10) we can write the left hand side of (9) as: $p_t H_t + RB_{t-1} - T_t = p_t H_t + B_t$. Using (11) we can write the left-hand side of (9) as: $p_t H_t + RB_{t-1} - T_t = p_t H_t + B_t = \theta^t x_t$. Thus. the government policy (10) insures the clearing of the perishable good market.

The claim says that the government's policy does not require a lot of information. It is enough that the government satisfies the demand for bonds by the young and use a lump sum tax on the old to make the difference between debt retirement and the revenue raised by selling newly printed bonds.

Claim 1 is extended in the Appendix to the case in which the agents consume in both periods. It implies that the government can peg the real interest rate by an appropriate fiscal policy. This does not contradicts Friedman (1968) who says that monetary policy cannot peg the real interest rate. See the discussion in section 2.1.

Note that the affordability issue does not arise in this case in which the government satisfies the demand for loans: The young can always buy the existing stock of houses with government loans.

4.2 Equilibrium

A housing cycle is the period between two consecutive crashes. The length of the cycle is a random variable $\widetilde{\Omega}$ and with small probability it may last for a long time. I consider a cycle that starts at time τ, where the price and the housing stock at the time of the crash are $p_\tau = I$ and H_τ .

Definition. Equilibrium for a cycle that starts at time τ with a stock of houses H_τ and a price $p_\tau = I$ requires that for each realization Ω of $\widetilde{\Omega}$ the sequence $\{p_t, g_t, y_t, x_t, H_t, B_t, T_t\}_{t=\tau}^{\Omega+\tau}$ satisfies non negativity constraints, (1),(2),(7),(8),(9),(10),(11) and $p_{t+1} = g_t p_t$.

Claim 2. In equilibrium, the production of housing is given by the solution to:

$$f'(y) = \left(\frac{\theta}{\psi}\right)^t \frac{1}{p_t} \tag{12}$$

To show this claim note that substituting (8) in (7) leads to (12). This Claim is extended in the Appendix to the case in which the agent consumes in both periods.

Note that when $\frac{\theta}{\psi} > g$ the right-hand side of (12) increases over time. We can therefore state the following.

Corollary 1. *The amount of labor allocated to the housing sector* (y) *decreases between time* $t-1$ *and time* t *if* $\frac{\theta}{\psi} > \frac{p_t}{p_{t-1}}$. *It increases if this inequality is reversed and does not change if* $\frac{\theta}{\psi} = \frac{p_t}{p_{t-1}}$.

To get the intuition note that when prices do not change and $\psi < \theta$ labor moves to the non-housing sector with relatively high productivity growth. In the data (Figure 3) the long run trend in housing prices is positive and the long run trend in the employment share of the housing sector is negative. This is consistent with $\psi < \theta$. In what follows I assume: $\psi < \theta$.

Claim 3. When $v'(\overline{y}) < R\beta$, there exists a unique interior solution $(y(p) > 0, x(R,p) > 0)$ to (7) and (8).

Proof. Let $y(p_t)$ denote the solution to (12). We can now use (7) to solve for x: □

$$\beta R = v'\left(x + y(p)\right) \tag{13}$$

Equation (13) has a unique and strictly positive solution because $v'\left(y(p)\right) < v'(\overline{y}) < \beta R$. Figure 5 illustrates. In the Figure, the total labor supply $L(R) = x + y$ is the solution to (13) where $y(p)$ is the solution to (12). The amount of labor devoted to the production of the perishable good is: $x(R,p) = L(R) - y(p)$.

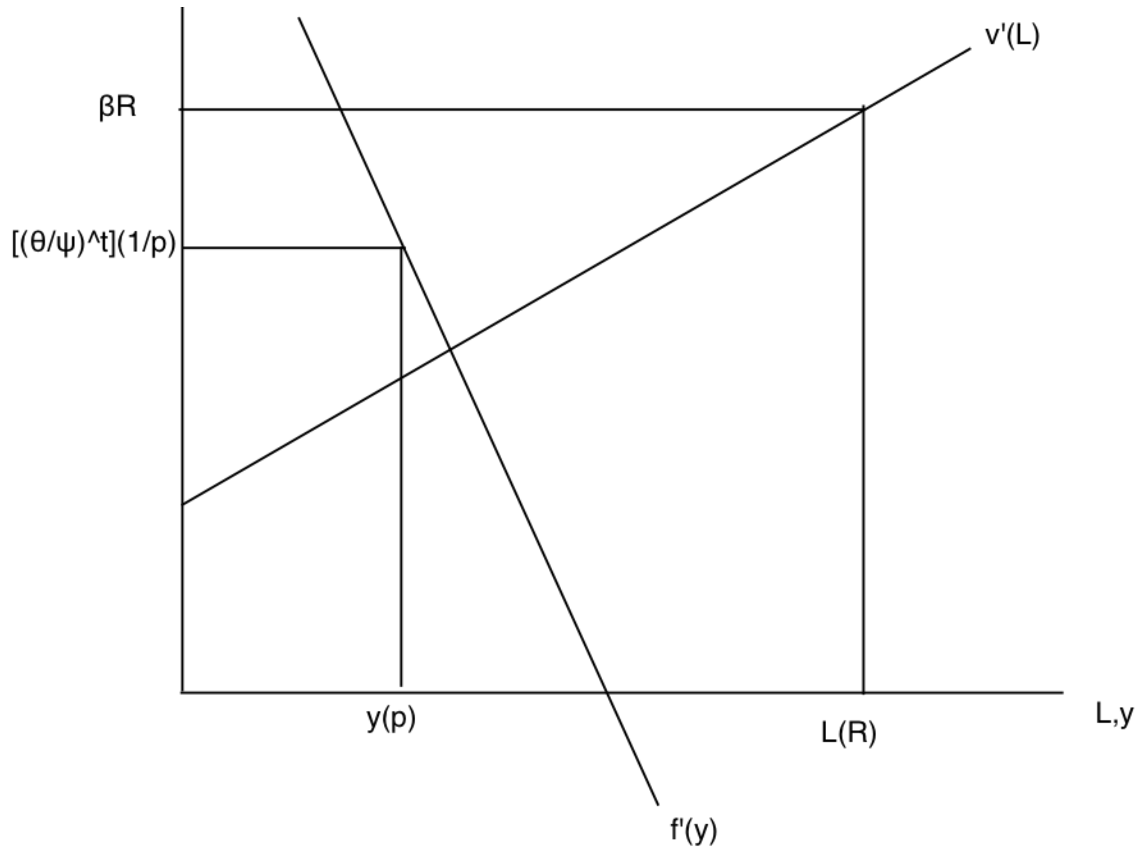


Figure 5: The solution to (12) and (13).

Claims 1 - 3 leads to:

Claim 4. When R is sufficiently high there exists a unique equilibrium for a cycle that starts at time τ.

Note that a policy of high interest rate is associated with higher employment. Here consumption occurs with a one period delay and therefore the real interest rate affects the real wage. This delay occurs in the cash in advance model and other models. In reality even long term interest rate affect the real wage because part of earnings goes to savings for retirement. This is different from the point of view expressed by Kashkari in his 2017 paper. Here investment in housing does not depend on the interest rate.

I now turn to discuss deterministic equilibria.

5 DETERMINISTIC STEADY STATE EQUILIBRIUM

A deterministic steady state equilibrium is equilibrium with $q = 1$, $g_t = g$, and $y_t = y$ for all t.

Claim 5. A deterministic steady state equilibrium exists if and only if $\theta = \psi$.

A deterministic steady state does not exist when $\theta \neq \psi$, because labor keeps moving to the sector that is becoming relatively more productive.

Proof. I substitute $q = 1$ and $g_t = g$ in (8) to get:

$$p_t = \frac{(1-\delta)\gamma}{R-(1-\delta)g} \tag{14}$$

Since (14) implies: $\frac{p_t}{p_{t-1}} = 1$, $g = 1$. Substituting $y_t = y$ in (12) leads to:

$$g = \frac{p_{t+1}}{p_t} = \frac{\theta}{\psi} \tag{15}$$

Therefore a deterministic steady state cannot exist when $\theta \neq \psi$.

When $\theta = \psi$ a steady state with stable prices exists. In this special case, $g = 1$ and

$$p = \frac{(1-\delta)\gamma}{r+\delta} \tag{16}$$

where $r = R - 1$ is the interest rate.

□

To characterize the steady state, I substitute (16) in (12) to get:

$$f'(y) = \frac{r+\delta}{(1-\delta)\gamma} = J(r, \delta, \gamma) \tag{17}$$

It can be shown that $y(r, \delta, \gamma)$ is decreasing in $r$and in δ and is increasing in γ.

5.1 Maximizing steady state welfare

In the steady state,

$$H_t = \theta^t H \tag{18}$$

Substituting (18) in (2) leads to $\theta^{t+1}H = (1-\delta)\left(\theta^t H + \theta^t f(y)\right)$ and

$$H = \frac{(1-\delta)f(y)}{\theta - 1 + \delta} \tag{19}$$

The steady state utility of an individual born at t is:

$$\theta^t \left(\beta(x+\gamma H) - v(x+y)\right) \tag{20}$$

Therefore, a planner that wants to maximize steady state welfare will solve the following problem:

$$max_{x,y,H}\theta^t \left(\beta(x+\gamma H) - v(x+y)\right) \tag{21}$$

$$s.t.\ (19)$$

The first order conditions for an interior solution to (21) are:

$$v'(x+y) = \beta = \beta\frac{\gamma(1-\delta)}{\theta+\delta-1}f'(y) \tag{22}$$

The second inequality in (22) implies:

$$f'(y) = \frac{\theta+\delta-1}{\gamma(1-\delta)} \tag{23}$$

Claim 6. (a) When $\psi = \theta = 1$, the steady state allocation solves the planner's problem (21) when $r = 0$; (b) When $\psi = \theta > 1$, the steady state allocation does not solve the planner's problem (21) regardless of the choice of r.

Proof. The first order condition (7) requires $v'(x+y) = \beta R$. To satisfy the first equality in (22) we must have $R = 1$ and $r = 0$. Substituting in the steady state solution (17) leads to: $f'(y) = \frac{\delta}{\gamma(1-\delta)}$ which coincides with (23) only when $\theta = 1$. □

This result is not surprising. The policy maker needs to determine two magnitudes: Total labor supply ($L = x + y$) and the allocation of labor to the housing sector (y). He therefore needs an additional policy instrument. This is further explored in Appendix B.

6 DETERMINISTIC EQUILIBRIUM

A deterministic equilibrium allows y and g to change over time but still requires $q = 1$. I now show the following Claim.

Claim 7. The following must hold in a deterministic equilibrium: (a) When the price is (16), the price does not change over time; (b) If $g_t > 1$ then $g_{t+1} > g_t$ and $lim_{t\to\infty} g_t = \frac{R}{1-\delta}$.

Thus if housing prices are increasing they must be increasing at an increasing rate and the rate of change converges to $\frac{R}{1-\delta}$.

Proof. I substitute $q = 1$ in (8) to get:

$$g_t = \frac{R}{1-\delta} - \frac{\gamma}{p_t} \tag{24}$$

Substituting (16) in (24) leads to $g = 1$. Thus, when the price is (16) and (8) is satisfied, the price does not change over time. To show (b) note that if $g_t > 1$ then $p_{t+1} > p_t$ and $g_{t+1} = \frac{R}{1-\delta} - \frac{\gamma}{p_{t+1}} > g_t$. Since prices are increasing in this equilibrium the right-hand side of (31) converges to $\frac{R}{1-\delta}$. □

To get the intuition note that we can write (24) as: $R = (1-\delta)\left(g_t + \frac{\gamma}{p_t}\right)$. This says that the expected real return on housing must equal the interest rate. When prices are increasing the importance of the constant dividend component in the return on housing, $\frac{\gamma}{p_t}$, diminishes and therefore g_t has to compensate for that. Thus, when the price of housing increases, the dividend component of the return decreases and the capital gains component increases. In the limit, when the price of housing is high, the capital gains component equals the interest rate.

Note that according to (12), $f'(y)$ grows at the rate $\frac{\theta}{\psi g_t}$. In the long run g_t converges to $R/(1-\delta)$ and $f'(y)$ grows at the rate $\frac{\theta(1-\delta)}{\psi R}$. In a deterministic equilibrium, the long run trend in the share of employment in the housing sector is therefore negative, if $\frac{\theta(1-\delta)}{\psi R} > 1$ and $\theta(1-\delta) > \psi R$.

7 CYCLES

I now allow $q < 1$ and use (8) to get:

$$g_t = g(p_t, I) = \frac{R}{(1-\delta)q} - \frac{(1-q)I + \gamma}{qp_t} \tag{25}$$

Note that we can write (25) as: $(1-\delta)\left(qg_t + (1-q)\frac{I}{p_t} + \frac{\gamma}{p_t}\right) = R$ which says that the expected rate of return on housing must equal the interest rate.

Immediately after the crash, $p = I$ and the (gross) rate of change is:

$$g(I, I) = \frac{R}{(1-\delta)q} - \frac{1-q}{q} - \frac{\gamma}{qI} \tag{26}$$

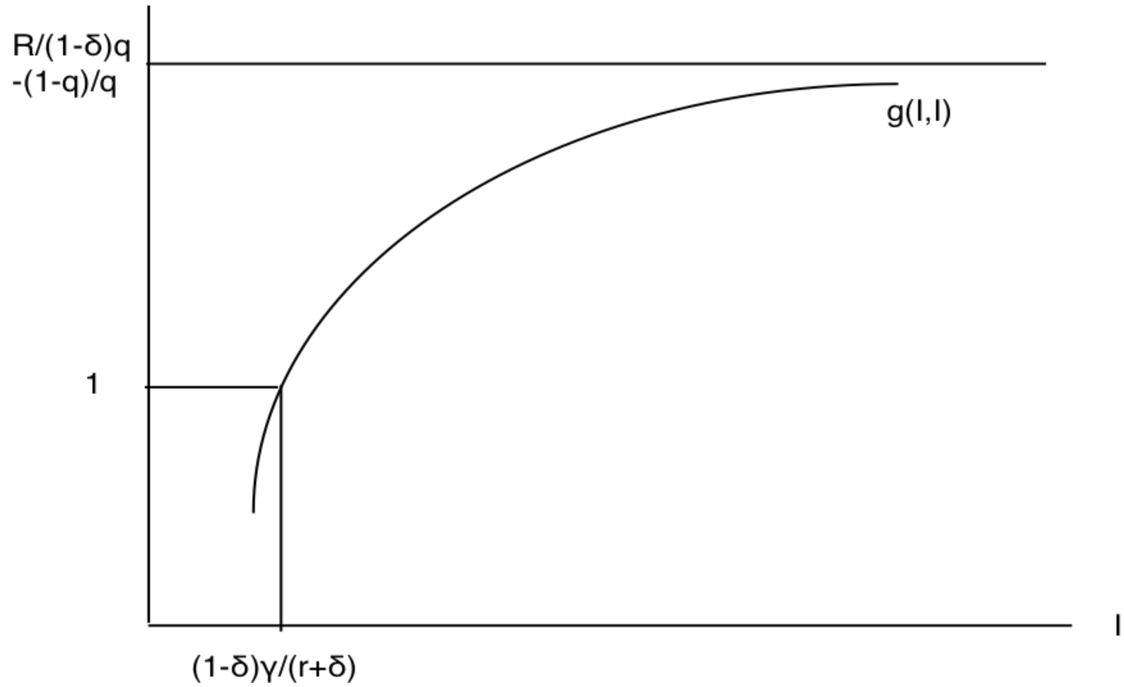
We may get price stability if the price after the crash satisfies: $g(I, I) = 1$. This leads to $I = \frac{(1-\delta)\gamma}{r+\delta}$ which is the same as (16). Note that if the price drops to (16) it will remain constant but the amount of labor devoted to housing will change if $\psi \neq \theta$. The function (26) is illustrated by Figure 6A.

When $g(I, I) > 1$ housing prices will increase over time at an increasing rate. (The rate of housing price change increases over time because $g(p, I)$ is increasing in its first argument). When $g(I, I) < 1$ housing prices decrease over time until they jump up. I focus here on the case in which housing price are either stable or increasing.

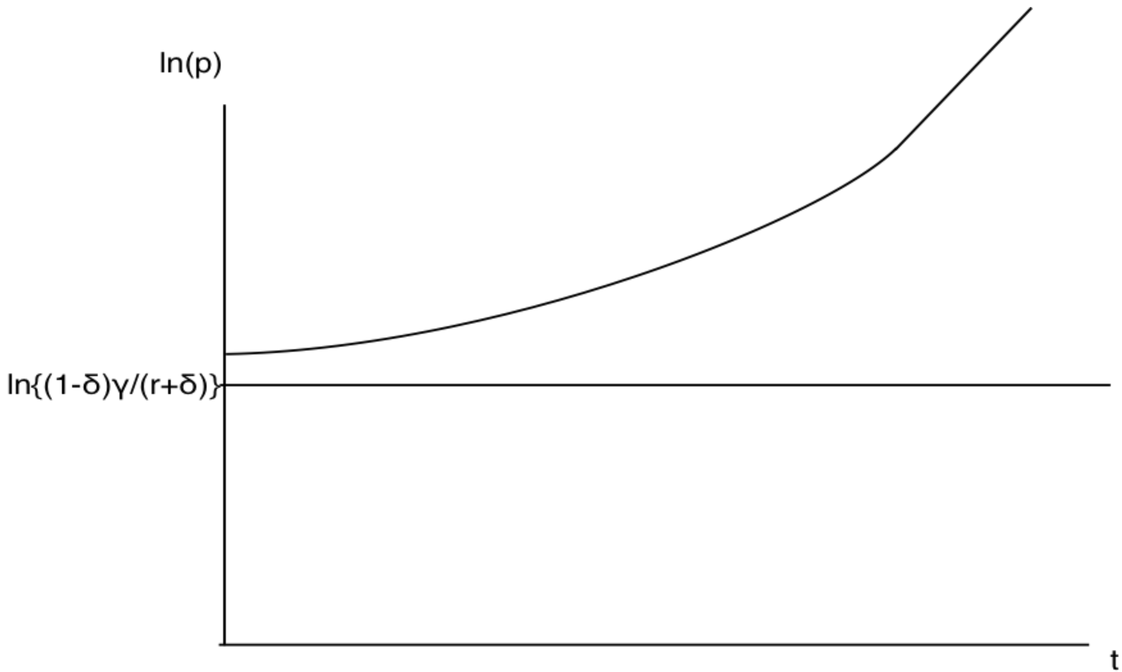
We have thus shown the following claim.

Claim 8. (a) if the price after the crash is greater than the deterministic steady state level (16), then housing price will rise at an increasing rate, until the next crush; (b) if the price after the crash is equal to (16) then housing prices will remain stable; (c) housing price growth rate is less than $\frac{R}{(1-\delta)q}$ and converges to this level when the cycle lasts for a long time.

Figure 6 illustrates.



A. The rate of change in housing price immediately after the crash as a function of the price immediately after the crash



B. The evolution of housing prices after a crash

Figure 6: The evolution of housing prices depends critically on the price immediately after the crash.

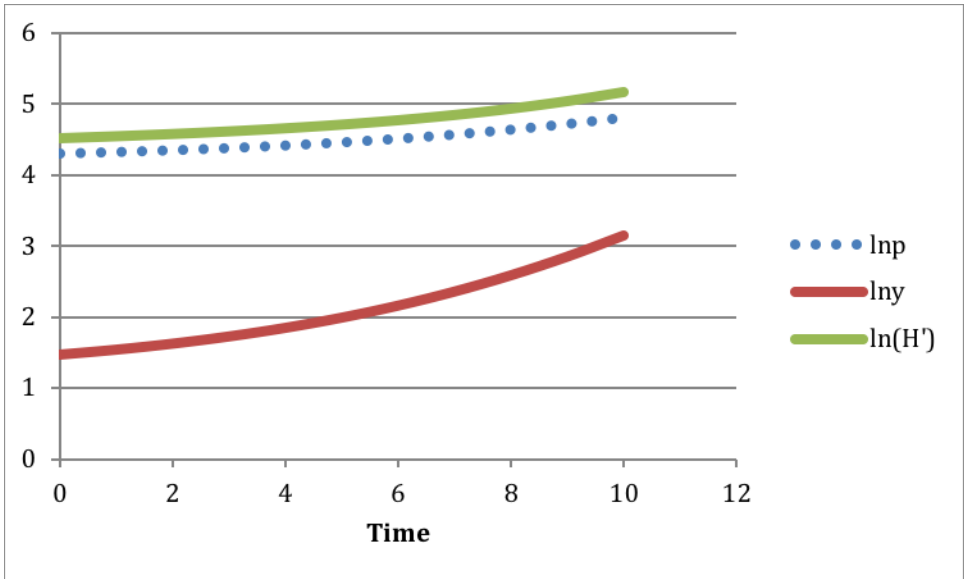
I now consider a numerical example that illustrates the effect of a shock to housing price. We may think of an auctioneer who announces the price of housing each period. The economy starts from a steady state in which the price is (16). The auctioneer then makes an unanticipated change and announce a price that is higher than (16). As a result, the economy moves to a different equilibrium. We may also follow the sunspot equilibria literature, and think of sunspots rather than an auctioneer who moves the economy from one equilibrium to another.[13]

The numerical example assumes: $f(y) = y^{0.7}$ when $y \leq \overline{y}$ and $f(y) \leq \overline{y}^{0.7}$ when $y > \overline{y}$. I also assume that $\overline{y}$ is large, $\psi = \theta = 1$, $\gamma = 0.05$ and $q = 0.9$.

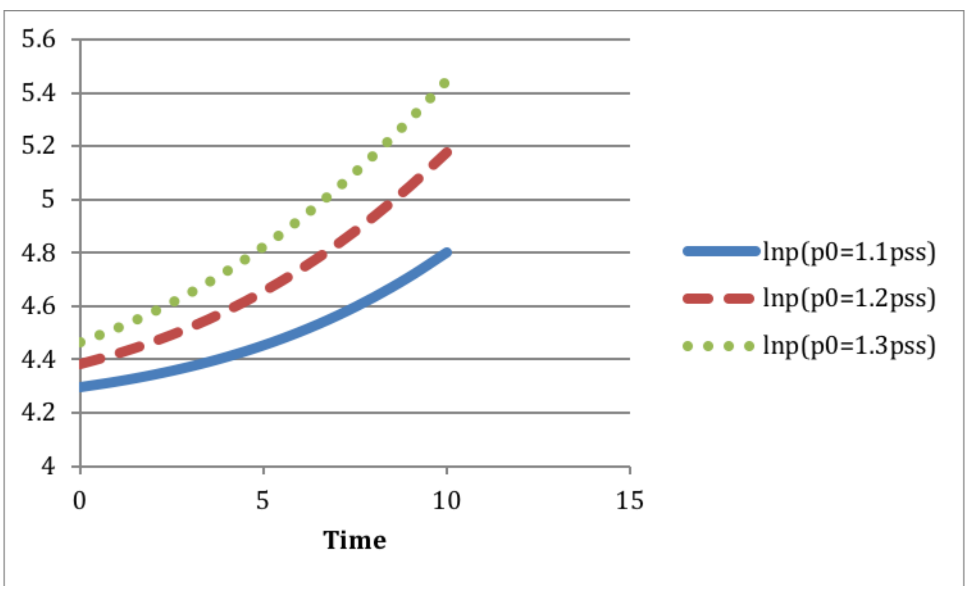
Figure 7A illustrates what happens to housing price, the amount of labor in construction and next period's housing stock in response to a shock of 10% in the housing price. All magnitudes increase in response to the increase in the price and the rate of change increases over time. Figure 7B compares 3 different shocks: An increase in the price by 10% above the steady state level, an increase by 20% and an increase by 30%. As we can see the variance of log prices increases over time. Assuming that different cities experienced different

[13]The assumption of a shock that is completely unanticipated is problematic and is used for illustrative purpose. We may think of zero (or small) probability events or of agents that are not fully rational.

shocks, this may explain why the cross-sectional variance of the index of housing prices increases during the boom and decreases after the crash as in Figure 2.



A. The log of Price (lnp), housing stock (lnH') and labor employed in construction (lny) when the price at time zero is 10% higher than the steady state level. Housing stock at time zero is at the steady state level. The number 5 was added to the logs to get positive magnitudes that look better on the graph. For example lnp is the log of price plus 5.



B. The log of Price when the price at time zero is 10% higher than the steady state level [lnp(p0=1.1pss)], 20% higher [lnp(p0=1.2pss)] and 30% higher [lnp(p0=1.3pss)].

Figure 7: Starting from the steady state there is a shock to the price of housing.

8 NO BORROWING

I now consider the case in which borrowing from the government is not possible and the young must satisfy the following no borrowing constraint:

$$\theta^t x_t \geq p_t H_t \tag{27}$$

I show that in this case, rational bubbles can be ruled out if the interest rate is sufficiently high. The intuition is as follows. When the interest rate is high housing prices and the value of the housing stock increase at a high rate and eventually the young generation will not be able to buy the housing stock. When the housing stock is constant we need to show only the relatively easy claim that housing price increases faster when the interest rate is high. Here the housing stock changes over time but when the bubble lasts for a long time it will eventually reach a rate of growth that is equal to the rate of technological change, ψ. In the long run the price of houses grow at the rate that is higher than $\frac{R}{(1-\delta)q}$ and the value of the houses stock grow at the rate that is higher than $\frac{R\psi}{(1-\delta)q}$. The young will not be able to afford the housing stocks if the rate of growth in the value of the stock is higher than the rate of growth in the production of the non-perishable good: If $\frac{R\psi}{(1-\delta)q} > \theta$ and $R > \frac{\theta(1-\delta)q}{\psi}$. It follows that $R > \frac{\theta(1-\delta)q}{\psi}$ is sufficient to rule out rational bubbles.

I assume that the solution to (6) is interior $(H > 0, y > 0, x > 0, y + x < \overline{L})$ and start by showing that Claim 1 holds even when borrowing is not allowed.

Lemma 1. *The production of housing must satisfy (12) when the solution to the young agent's problem (6) is interion and (27) is imposed.*

Proof: Since (1) is not binding the agent can increase x by p units and buy a unit of housing. The amount of labor required for doing it is p/θ^t units and the cost of doing it (the Pain) is $(p/\theta^t)\theta^t v'(x+y) = pv'(x+y)$. The benefit (the Gain) is: $\beta(1-\delta)\left(qg_t p_t + (1-q)I + \gamma\right)$. Since at the optimum $Pain \geq Gain$ we must have: $\beta(1-\delta)\left(qg_t p_t + (1-q)I + \gamma\right) \leq pv'(x+y)$. The agent can also reduce housing by a unit and reduce labor input by p/θ^t units. Therefore, at the optimum we must have: $\beta(1-\delta)\left(qg_t p_t + (1-q)I + \gamma\right) \geq pv'(x+y)$. It follows that the solution must satisfy:

$$\beta(1-\delta)\left(qg_t p_t + (1-q)I + \gamma\right) = pv'(x_t + y_t) \tag{28}$$

Since $y > 0$, the second equality in (7) must hold. That is:

$$\beta(1-\delta)\psi^t f'(y_t)\left(qg_t p_t + (1-q)I + \gamma\right) = \theta^t v'(x_t + y_t) \tag{29}$$

Substituting (28) in (29) leads to (12).

I now show that the lower bound of the rate of housing price change is increasing in the interest rate.

Lemma 2. *Under the no borrowing constraint (27) the rate of price growth in the boom phase must satisfy:*

$$g_t \geq \frac{R}{(1-\delta)q} - \frac{(1-q)I+\gamma}{qp_t} \text{ with equality if } B_t > 0 \tag{30}$$

Proof. Since $H > 0$, the young agent can sell houses and buy bonds. If he sells a unit of housing and invests in government bonds he will get Rp_t utils in the next period. The cost of doing that is $(1-\delta)(qg_tp_t + (1-q)I + \gamma)$ utils. The solution to (6) must therefore satisfy: $Rp_t \leq (1-\delta)(qg_tp_t + (1-q)I + \gamma)$. When $B_t > 0$ we can do the reverse and therefore:

$$Rp_t \leq (1-\delta)(qg_tp_t + (1-q)I + \gamma) \text{ with equality if } B_t > 0 \tag{31}$$

This leads to (30). □

The third Lemma is about the evolution of the housing stock when y is constant over time.

Lemma 3. *When labor input in the housing sector is constant over time ($y_t = y$ for all t), and the evolution of housing stock satisfies (2) then eventually (when t is sufficiently large), housing stock is given by*

$$H_t = \psi^t H, \text{ where } H = \frac{(1-\delta)f(y)}{\psi+\delta-1}. \tag{32}$$

Proof. Substituting (32) in (2) leads to:

$$\psi^{t+1}H = (1-\delta)\left(\psi^t H + \psi^t f(y)\right) \tag{33}$$

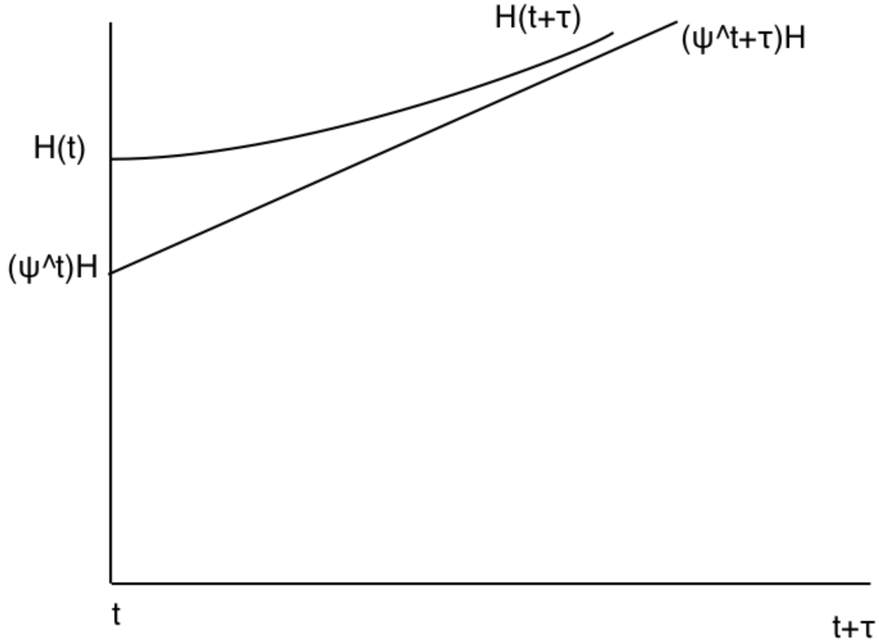
Thus if (32) holds at time t it will also hold at time τ for all $\tau > t$. Suppose now that

$$H_t > \frac{(1-\delta)\psi^t f(y)}{\psi+\delta-1} = \psi^t H \tag{34}$$

Then,

$$H_t\psi > (1-\delta)\left(\psi^t H + \psi^t f(y)\right) = H_{t+1} \tag{35}$$

which says that housing stock grows at a rate that is lower than ψ. Eventually, when τ is large enough, $H_{t+\tau}$ will converge to $\psi^{t+\tau}H$ as illustrated by Figure 8. A similar argument can be made when the inequality in (34) is reversed.



□

Figure 8: The convergence of the rate of change in the housing stock

I now show that bubbles can be ruled out when the interest rate is sufficiently high.

Claim 9. Bubbles are not possible if $R > \frac{\theta(1-\delta)q}{\psi}$.

Proof. If the bubble lasts for a long time, $p \to \infty$ and the right hand side of (30) goes to $\frac{R}{(1-\delta)q}$. Under the condition in the claim, $\frac{R}{(1-\delta)q} > \frac{\theta}{\psi}$ and therefore in the limit $g > \frac{\theta}{\psi}$ and $g\psi > \theta$. By Lemma 1, eventually labor input will be close to $\overline{y}$, and by Lemma 3 the stock of houses in the long run will grow at a rate close to ψ and the value of houses will grow at the rate close to $g\psi > \theta$. Therefore the value of the housing stock grows at a rate that is higher than θ and eventually the no borrowing constraint (27) will be binding. □

Corollary 2. *Bubbles are not possible if* $\frac{\psi}{\theta} > \frac{(1-\delta)q}{R}$.

This says that high relative technological change in the housing sector prevents rational bubbles. To prevent bubbles, the relative technological change in the housing sector should be high if the interest rate is low. The Corollary says that relative rather than absolute growth is relevant for bubbles. In Appendix C, I show that the rate of population growth is not relevant for bubbles in the housing sector because it affects absolute rather than relative growth.

9 SUBSIDIZED MORTGAGES

We have considered two extreme cases. In the first there are no restrictions on borrowing from the government. In the second borrowing from the government is not possible. The reality is somewhere in between these two extremes. The government does not provide unlimited loans but it has a role in mortgages.[14]

I now turn to the more realistic case in which the government provides mortgages that require down payments. The government supplies subsidized mortgages but no other loans. The interest on mortgages is $R^* \leq R$, where R is the interest on government bonds (holding a negative amount of government bonds is not allowed). The down payment is a fraction λ of the price of the house, where $0 < \lambda < 1$ and the down payment requirement is:

$$\lambda p_t H_t \leq \theta^t x_t \tag{36}$$

The amount of government bonds that the consumer buys is: $\theta^t x_t - \lambda p_t H_t \geq 0$. When old the payment on government bonds is $R(\theta^t x_t - \lambda p_t H_t)$ and the mortgage payment is: $(1-\lambda)R^* p_t H_t$. Expected consumption is now:

$$c_{t+1} = R(\theta^t x_t - \lambda p_t H_t) - (1-\lambda)R^* p_t H_t + H_{t+1}E(p_{t+1}) - \overline{T}_{t+1} \tag{37}$$

The problem of the young agent is:

$$max_{H_t, y_t, x_t} \beta(c_{t+1} + \gamma H_{t+1}) - \theta^t v(x_t + y_t) \tag{38}$$

$$s.t.\ (1),\ (2),\ (36) \text{ and } (37).$$

I assume that (36) may be binding but otherwise the solution to (38) is innterior and satisfies: $H > 0, y > 0, x > 0, y + x < \overline{L}$.

Lemma 4. *The solution to (38) satisfies:*

(a) $v'(L) \geq \beta R$ with equality when (36) is not binding;

(b) When (36) is binding, the amount of labor supplied to the housing sector must satisfy

$$f'(y_t) = \left(\frac{\theta}{\psi}\right)^t \frac{v'(L_t)}{p_t\left(\lambda v'(L_t) + \beta(1-\lambda)R^*\right)} \tag{39}$$

[14] The role of government in mortgages is large. The US congressional budget office (CBO, 2001) reports that at the end of 2000 Fannie Mae and Freddie Mac held or guaranteed 39% of all residential mortgages and 71% of all fixed-rate conforming mortgages. By one estimate, these Government Sponsored Enterprises (GSEs) backed half of the mortgages on the eve of the crisis and after the crisis many criticize these GSEs for providing mortgages with no sufficient guarantees and low down payments. During the wave of default in 2007 the Federal government stepped in. Fannie Mae and Freddie Mac were placed under the conservatorship of the Federal Housing Finance Agency (FHFA) in September 2008. The Fed also purchased toxic mortgage backed securities in its quantity easing policies. See, Frame and Wall (2002a, 2002b).

(c) When (36) is not binding, $v'(L) = \beta R$ and

$$f'(y_t) = \left(\frac{\theta}{\psi}\right)^t \frac{R}{R^{**} p_t} \tag{40}$$

where $R^{**} = (1-\lambda)R^* + \lambda R$.

Proof. To show (a) note that the young agent can increase work by a unit, get θ^t units of the perishable good and invest it in government bonds. This yields $\theta^t R$ in the next period. The cost of doing that is $\theta^t v'(L)$ and since at the optimum deviations cannot improve matters we must have: $\theta^t v'(L) \geq \beta \theta^t R$ and this leads to: $v'(L) \geq \beta R$. When (36) is not binding the young can reduce labor by a unit and reduce his holding of government bonds by θ^t units. Since deviations from the optimal solution do not increase welfare we must have $v'(L) \geq \beta R$ and $v'(L) \leq \beta R$ which leads to $v'(L) = \beta R$.

To show (b) note that when (36) is binding, the implicit wage (or the value of time) is: $\theta^t v'(L_t)$. The implicit price of a house is calculated as follows. Since (1) is not binding the agent can increase X by λp units and take a mortgage of $(1-\lambda)p$ to buy a unit of housing. The amount of labor required for doing it is $\frac{\lambda p}{\theta^t}$ units and the labor cost is $\frac{\lambda p}{\theta^t} \theta^t v'(L) = \lambda p v'(L)$. The mortgage cost in terms of future perishable good is $(1-\lambda)pR^*$. The utility cost of buying an additional unit of housing is therefore: $\lambda p v'(L) + \beta(1-\lambda)pR^*$. At the optimum the value of the marginal product must equal the wage: $(\lambda p v'(L) + \beta(1-\lambda)pR^*)\, \psi^t f'(y) = \theta^t v'(L)$. This leads to (39).

When (36) is not binding $v'(L) = \beta R$. Substituting this in (39) leads to (40).

$\square$

I now assume equilibrium with $H_t > 0$ and show the following Lemma.

Lemma 5. *The rate of price growth in the boom phase must satisfy:*

$$g_t \geq \frac{R^{**}}{(1-\delta)q} - \frac{(1-q)I + \gamma}{q p_t} \tag{41}$$

Proof. The young agent can reduce his holding of houses by one unit. If he does it, his down payment will go down by λp_t and he can reduce his labor input by: $\frac{\lambda p_t}{\theta^t}$ and cut his labor cost by $\frac{\lambda p_t}{\theta^t} \theta^t v'(L) = \lambda p_t v'(L)$. In addition, his mortgage payment will go down by $(1-\lambda)p_t R^*$ units. The gains from reducing the amount of housing that he buys by a unit are therefore: $\lambda p_t v'(L) + \beta(1-\lambda)p_t R^*$. A unit of housing yields $\beta(1-\delta)\,(q g_t p_t + (1-q)I + \gamma)$ utils. Since at the optimum he does not choose to do it we must have:

$$\lambda p_t v'(L) + \beta(1-\lambda)p_t R^* \leq \beta(1-\delta)\,(q g_t p_t + (1-q)I + \gamma) \tag{42}$$

This leads to:

$$g_t \geq \frac{\lambda v'(L) + \beta(1-\lambda)R^*}{\beta(1-\delta)q} - \frac{(1-q)I+\gamma}{qp_t} \geq \frac{R^{**}}{(1-\delta)q} - \frac{(1-q)I+\gamma}{qp_t} \tag{43}$$

The second inequality follows from the claim $v'(L) \geq \beta R$ in Lemma 4.

$\square$

Note that Lemma 3 holds also in this case. I now show the following Claim.

Claim 10. Bubbles are not possible if $R^{**} > \frac{\theta(1-\delta)q}{\psi}$.

Proof: If the bubble lasts for a long time, the right hand side of (41) goes to $\frac{R^{**}}{(1-\delta)q}$. Under the condition in the claim, $\frac{R^{**}}{(1-\delta)q} > \frac{\theta}{\psi}$ and therefore in the limit $g > \frac{\theta}{\psi}$ and $g\psi > \theta$. Since $v'() < \infty$, Lemma 4 implies that eventually labor input will be close to $\overline{y}$. Lemma 3 can therefore be used to show that the stock of houses in the long run will grow at a rate ψ and the value of houses will grow at the rate $g\psi > \theta$. Therefore total spending on houses grow at a rate that is higher than θ and eventually (36) will be violated.

Note that there is a difference between $\lambda = 0$ and $\lambda > 0$. When $\lambda = 0$ there is no constraint on borrowing and affordability is not an issue. When $\lambda > 0$ the affordability issue emerges when the bubble lasts for a long time and bubbles can be ruled out when the interest rate is sufficiently large.

10 THE STYLIZED FACTS

The model is rather abstract but can account for most of the stylized facts. In general, it seems that the conditions prior to the collapse of housing prices in 2007-2008 were conducive for bubble formation. The relative rate of technological change in the non-housing sector was high because of advancement in information technology and robotics. The interest was low, the involvement of the government in mortgages was large and downpayment requirements were relaxed.

The model can account for the puzzling observations in Shiller (2007). It can account for a period in which prices are increasing while production cost and rent are relatively stable. When the government provides loans, we get an equilibrium in which labor cost and rent are stable during the boom: the cost of labor is constant and is given by $v'(L) = \beta R$ and rent is γ.

In the model the rate of growth of housing prices increases during the boom phase and this is also the case in the data. In the model the fall in prices does not take time. In the data it does. It seems that the average time that a house is "on the market" is counter-cyclical. This suggests downward price rigidity that is not in our model. Indeed, Shiller (2007) note that home sellers tend to hold out for high prices when prices are falling and there was a 17% decline in the volume of US existing home sales since the peak in volume sales in 2005.

The example in Figure 7B can be used to account for the increase in the variance of housing prices across cities that occurs during the boom (Figure

2). In the model, the percentage difference between housing prices in different cities increases in the boom phase if they do not start from the same initial conditions. The cities that start the cycle with a modest overpricing (the price after the crash is above but close to the deterministic steady state level) will experience a modest cycle while the cities that start the cycle with a substantial overpricing will experience a more dramatic cycle.

The model allows for the case in which employment in the housing sector increases during the boom but exhibits downward trend (Figure 3). This will occur if the rate of techonological change in the housing sector is relatively low and $\psi < \theta$.

The case in which the government provides subsidized mortgages may help in understanding the behavior of household debt. Since mortgages is a fraction of the value of houses, the amount of mortgage debt behave in a way that is similar to the value of houses. Since the value of houses increases during the boom and then crashes, the value of mortgage debt will exhibit similar behavior. This is consistent with the pattern in Figure 4.

11 DEVIATIONS FROM RATIONALITY

Kahneman and Tversky (1979) argued that people edit prospects before evaluating them and that "A particularly important form of simplification involves the discarding of extremely unlikely outcomes". Bubbles are more likely if we discard extremely unlikely outcome.

The proof of Claim 9 argues that under certain conditions, bubbles are not possible because eventually the young may not have sufficient purchasing power to buy the stock of houses. The likelihood of the insufficient funds event may depend on the interest rate. Since the right hand side of (30) is decreasing in R, this will occur when (30) holds with equality. In this case, the higher R is, the higher is the rate of price increase and the shorter is the time it takes to get to the insufficient funds point.[15]

It is thus possible, that an increase in R increases the probability that the young will not be able to buy the housing stock and therefore when R is sufficiently high this event will not be ignored. Once this event is not ignored agents will see that the only equilibrium possible is the constant price equilibrium.

12 CONCLUDING REMARKS

Housing cycles can emerge when the government is ready to lend and borrow at a given real interest rate. This policy requires the use of lump sum taxes (or transfers) to make the difference between the amount the government pays to retire old bonds and the demand for new bonds.

[15] In detail, let τ denote the number of no pop periods that it takes to get to the insufficient fund point. Since the probability of "no pop in the next τ periods", $(1-q)^{\tau}$, is decreasing in τ and since τ is decreasing in R, it follows that the probability of getting to the insufficient funds point is increasing in R.

Housing cycles can also emerge when the government does not provide loans but the interest rate on its bonds is low and the rate of technological change in the housing sector is low relative to the rate of change in the perishable good sector. The intuition is as follows. When the interest rate is low housing prices and the value of the housing stock increase at a low rate. If on top of that the rate of technological change in the perishable good sector is relatively high, the young will be able to buy the housing stock even when the boom lasts for a long time and even when the government does not provide loans.

I also study the more realistic case in which the government does not offer consumption loans but offer mortgages that require down payments. When the bubble lasts for a long time, the young may not have enough to make the downpayment on the entire housing stock and therefore, as in the no borrowing case, high interest rates may rule out bubbles.

It follows that to discourage bubbles in housing the government should get out of mortgages and set a high interest rate. Discouraging bubbles is an important objective but it is not the only one. In general a high interest may discourage bubbles but may create other distortions. It may distort the labor\leisure choice and the allocation of consumption over time.

The analysis was extended in Appendix A, to the case in which the agent consumes in both periods. Also in this case the government can set the real interest by using lump sum taxes.

A deterministic steady state exists only when the rate of technological change is the same across the two sectors. The policy that maximizes steady state welfare requires two instruments. In Appendix B I show that one possibility is to subsidize mortgages. Another possibility is to exempt the housing sector from an income tax that is imposed on the other sector. The second alternative may be better because it does not encourage the formation of bubbles.

Population growth does not "help" bubbles (See, Appendix C). This is different from standard models of bubbles. The intuition is that population growth increases both the growth of the value of the housing stock and the growth of the perishable good output by the same amount and therefore it does not affect the condition that guarantee the affordability of the housing stock.

Appendix A: Allowing for Consumption in the First Period

I now replaces (3) by:

$$U(C_t^y) + \beta E\left(U(C_t^o)\right) + \beta Z(H_{t+1}) - v(x_t + y_t) \tag{44}$$

Where C_t^y is the amount consumed by the young born at t, C_t^o is the amount they will consume when old, E is the expectation operator, U is a strictly monotone and concave utility function from consumption, Z is a strictly monotone and concave utility function from housing services and v is a strictly convex disutility from labor. The agent can lend and borrow at the gross interest rate

R and chooses C_t^y, x_t, y_t, H_t to maximize (53) subject to (1), (2), non-negativity constraints and

$$C_t^o = R(\theta^t x_t - p_t H_t - C_t^y) + p_{t+1} H_{t+1} - T_{t+1} \tag{45}$$

The first order conditions for an interior solution to this problem are:

$$U'(C_t^y) = \beta R E\left(U'(C_t^o)\right) \tag{46}$$

$$\theta^t U'(C_t^y) = v'(x_t + y_t) \tag{47}$$

$$v'(x_t + y_t) = (1-\delta)\beta\psi^t f'(y_t)\left[E\left(p_{t+1}U'(C_t^o)\right) + Z'(H_{t+1})\right] \tag{48}$$

$$Rp_t E\left(U'(C_t^o)\right) = (1-\delta)\left[E\left(p_{t+1}U'(C_t^o)\right) + Z'(H_{t+1})\right] \tag{49}$$

I now show that Claim 2 holds also for this case.

Claim 11. The first order conditions (54)-(58) leads to (12).

From (57) and (58) we get:

$$v'(x_t + y_t) = \psi^t f'(y_t) p_t \beta R E\left(U'(C_t^o)\right) \tag{50}$$

From (56) and (55) we get:

$$v'(x_t + y_t) = \theta^t \beta R E\left(U'(C_t^o)\right) \tag{51}$$

From (59) and (60) we get:

$$\theta^t = \psi^t f'(y_t) p_t \tag{52}$$

Or,

$$f'(y_t) = \left(\frac{\theta}{\psi}\right)^t \frac{1}{p_t} \tag{53}$$

which is the same as (12).

The Clearing of the perishable good market

The clearing of the perishable good market requires:

$$C_t^y + C_{t-1}^o = \theta^t x_t \tag{54}$$

Since $C_{t-1}^o = p_t H_t + RB_{t-1} - T_t$, (63) is equivalent to:

$$p_t H_t + RB_{t-1} - T_t = \theta^t x_t - C_t^y \tag{55}$$

Therefore to satisfy (63) the government has to choose: $T_t = p_t H_t + RB_{t-1} - \theta^t x_t + C_t^y$. I now show that Claim 1 holds also in this case.

Claim 12. Choosing taxes to satisfy the market clearing condition (63) is equivalent to choosing taxes to satisfy:

$$T_t = RB_{t-1} - B_t \tag{56}$$

To see this Claim, note that the demand for government bonds by the young is:

$$B_t = \theta^t x_t - p_t H_t - C_t^y \tag{57}$$

Using (65) we write the left-hand side of (64) as: $p_t H_t + RB_{t-1} - T_t = p_t H_t + B_t$. Since (66) implies: $p_t H_t + B_t = \theta^t x_t - C_t$, (64) must hold and therefore (63) must hold. Thus. the government policy (65) insures the clearing of the perishable good market.

Temporary Equilibrium

Expectations about consumption when old depends on the amount of taxes that the old will have to pay. This depends on the production of the perishable good and the action of the next period's young.

$$C_t^o = \theta^{t+1} x_{t+1} - C_{t+1}^y \tag{58}$$

Temporary equilibrium is defined for a given expectations (67).

Definition. Given expectations about (67), a Temporary Equilibrium is a vector (p_t, y_t, x_t, C_t^y) that satisfies (55), (56) and (62).

To solve for a Temporary Equilibrium first use (55) and solve for C_t^y. Then use (56) to solve for $L_t(C_t^y) = x_t + y_t$. Then use (62) to solve for $y_t(p_t)$ and then solve for $x_t(C_t^y, p_t) = L(C_t^y) - y(p)$.

After solving for temporary equilibrium we can use (2) to get $H_{t+1} = (1 - \delta)(H_t + \psi^t f[y_t(p_t)])$ and then use (58) to solve for the distribution of p_{t+1}.

Appendix B: Adding a policy instrument

Achieving the first best requires preferential treatment to the housing sector. One way of doing it is to impose a tax on output in the perishable good sector so that the young agent gets $\Omega = 1 - \rho$ units for each unit produced, where ρ is the per unit tax. The expected consumption of the perishable good is now:

$$c_{t+1} = E(C_{t+1}) = R(\theta^t \Omega x_t - p_t H_t) + H_{t+1} E(p_{t+1}) - \overline{T}_{t+1} \tag{59}$$

$$= R(\theta^t \Omega x_t - p_t H_t) + H_{t+1}(q g_t p_t + (1-q)I) - \overline{T}_{t+1}$$

The young agent's problem is now:

$$max_{H,y_{tt},x_t} \beta(c_{t+1} + \gamma H_{t+1}) - \theta^t v(x_t + y_t) \tag{60}$$

$$s.t.\ (1),\ (2) \text{ and } (68).$$

The first order condition that x has to satisfy is now:

$$\beta R \Omega = v'(x + y) \tag{61}$$

Claim 13. We can now show that the policy choice $r = \theta - 1$ and $\Omega = \frac{1}{R}$ achieves the first best outcome.

To show this claim note that for this policy choice, the steady state solution (17) coincides with the planner's first order condition (23) and the first order condition (70) coincides with the planner's first order condition (22).

Subsidizing mortgages can also work. Young agents finance the housing they buy from the old by taking mortgages at the rate $R^* < R$ and use their perishable good income to buy government bonds that pay the interest rate R. The consumption of the perishable good at old age is now:

$$c_{t+1} = R\theta x_t - Rp_t^* H_t + H_{t+1} E(p_{t+1}) - \overline{T}_{t+1} = R\theta x_t - Rp_t^* H_t + H_{t+1}\left(qg_t p_t + (1-q)I\right) - \overline{T}_{t+1} \tag{62}$$

The first order condition with respect to H_t is now:

$$Rp_t^* = (1-\delta)\left(qg_t p_t + (1-q)I + \gamma\right) \tag{63}$$

Substituting this in (7) leads to:

$$f'(y) = \left(\frac{\theta}{\psi}\right)^t \frac{R}{R^* p_t} \tag{64}$$

Note that (73) can be written as: $R^* p_t \psi^t f'(y) = \theta^t R$. The right hand side, $\theta^t R$, is the wage in the perishable good industry (in terms of next period consumption). The left hand side, $R^* p_t \psi^t f'(y)$, is the value of the marginal product in the housing sector and this must equal the wage rate.

In the steady state the price is given by (16) and y is determined by:

$$f'(y) = \frac{R(r+\delta)}{R^*(1-\delta)\gamma} \tag{65}$$

We can now show the following Claim.

Claim 14. The policy choice $R = 1$ and $R^* = \frac{\delta}{\theta+\delta-1}$ achieves the first best.

To show this Claim substitute the policy choice in (74) to get the first order condition for the planner's problem (23). The choice $R = 1$ insures that condition (22) is also satisfied.

Appendix C: Population growth

I now show that Claim 10 holds if we allow for population growth. Let:

$$\frac{N_t}{N_{t-1}} = 1 + n \tag{66}$$

where N_t is the size of the generation born at t and n is the rate of population growth. Lemmas 1 and 2 use only the first order conditions of the individual agent and therefore they hold also for this case.

I assume that $\psi(1+n) > 1 - \delta - 1$ and modify Lemma 3 as follows.

Lemma 6. *When labor input in the housing sector is constant over time and is given by $y_t = y$ for all t, and the evolution of housing stock satisfies (2) then eventually (when t is sufficiently large), housing stock is given by:*

$$H_t = \psi^t H \tag{67}$$

where

$$H = \frac{(1-\delta)f(y)}{\psi(1+n) + \delta - 1} \tag{68}$$

Proof. The clearing of the housing market requires:

$$N_t H_t = N_{t-1}(1-\delta)\left(H_{t-1} + \psi^{t-1} f(y_{t-1})\right) \tag{69}$$

When $y_t = y$ we get:

$$\frac{H_t}{H_{t-1}} = \frac{1-\delta}{1+n}\left(1 + \psi^{t-1}\frac{f(y)}{H_{t-1}}\right) \tag{70}$$

Substituting (45) in (48) leads to (46). □

The proof of Claim 10 should be modified as follows. In the long run when y_t is close to $\overline{y}$, the stock of houses grows at the rate

$$\frac{p_{t+1}N_{t+1}H_{t+1}}{p_t N_t H_t} = \frac{p_{t+1}(1+n)\psi^{t+1}H}{p_t \psi^t H} = g(1+n)\psi \tag{71}$$

The aggregate amount of the perishable good grows at the rate

$$\frac{N_t \theta^t x}{N_{t-1}\theta^{t-1}x} = (1+n)\theta \tag{72}$$

When the bubble lasts for a long time, there will not be sufficient funds to buy houses if $g\psi > \theta$ which implies that (49) is greater than (50). The proof of Claim 10 goes through. It says that bubble are not possible if $R > \theta(1-\delta)q/\psi$.

Thus here the rate of population growth does not play a role in the condition that allows for bubbles. It does not play a role because higher population growth affects the output in both sectors symmetrically and is therefore not relevant for the affordability issue.

This is different from the analysis of pure bubbles in Tirole (1985). In his model sufficient growth allows for bubbles and population growth is relevant because it contributes to growth.

Optimal steady state policy

The constraint to the planner's problem (21) is now (46) instead of (19). After substituting $\psi = \theta$, the first order conditions for this problem are:

$$v'(L) = \beta \tag{73}$$

$$f'(y) = \frac{\theta(1+n)+\delta-1}{(1-\delta)\gamma} \tag{74}$$

With two policy instruments the first best can be achieved with the policy choice: $r = \theta(1+n) - 1$ and $\Omega = \frac{1}{\beta}$. Alternatively, we can set $R = 1$ and $R^* = \frac{\delta}{\theta(1+n)+\delta-1}$.

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