Vanderbilt University Department of Economics Working Paper Series
Nashville, TN 37235-1819

Vanderbilt University Department of Economics Working Papers 18-00006

REAL INTEREST POLICY AND THE HOUSING CYCLE

Bejamin Eden
Vanderbilt University

Abstract

I use a model of rational bubbles to account for housing cycles and to discuss the effects of government loans and its real interest policy on the possibility of cycles. Cycles occur when the government is willing to lend to the young generation. Cycles do not occur if the government does not lend and the interest rate is sufficiently high. The level of interest required to discourage cycles (in the no lending case) is high when the rate of technological change in the non-housing sector is high relative to the rate of technological change in the housing sector.

Citation: Bejamin Eden, (2018) "REAL INTEREST POLICY AND THE HOUSING CYCLE", *Vanderbilt University Department of Economics Working Papers*, VUECON-18-00006.
Contact: Bejamin Eden - ben.eden@vanderbilt.edu.
Submitted: June 11, 2018. **Published:** August 24, 2018.
URL:http://www.accessecon.com/Pubs/VUECON/VUECON-18-00006.pdf

REAL INTEREST POLICY AND THE HOUSING CYCLE

Benjamin Eden

May 2018

I use a model of rational bubbles to account for housing cycles and to discuss the effects of government loans and its real interest policy on the possibility of cycles. Cycles occur when the government is willing to lend to the young generation. Cycles do not occur if the government does not lend and the interest rate is sufficiently high. The level of interest required to discourage cycles (in the no lending case) is high when the rate of technological change in the non-housing sector is high relative to the rate of technological change in the housing sector.

Key Words: Housing-cycles, Interest Rate, Bubbles, Government loans.
JEL Codes: E32, E60

1. INTRODUCTION

In his Nobel lecture Shiller says that: "Home prices do indeed go through years of price increases and then years of price decreases. So, the random walk model of home price behavior is just not even close to being true for home prices". (Shiller [2014, page 1502]).

In an earlier paper Shiller (2007) observed that housing prices in the US rose 86% in real terms between 1996-2006. During this period real rent has been extremely stable and increased by only 4% during the period and labor cost experienced a slight decline. To explain these observations he proposes “a psychological theory that represents the boom as taking place because of a feedback mechanism or social epidemics that encourages a view of housing as an important investment opportunity”.

There is also no consensus about the role of government in bubbles. Gali (2014), for example, argues against the common view that tighter monetary policy in the form of higher interest rates may help dis-inflate bubbles.

Here I explore the ability of the standard theory of rational bubbles to account for the housing cycle and discuss some policy issues. I use an overlapping generations economy with two assets: Government bonds and houses. Here, the real interest rate on government bonds is a policy choice: The real price of a bond is one unit of consumption and it promises R units of consumption in the next period where R is the announced gross real interest rate. I distinguish between two cases. In the first, the government is willing to lend and borrow at the announced interest rate. In the second, the government borrows but does not lend and holding a negative amount of government bonds is not allowed.

Cycles are possible in the first case, when borrowing from the government is allowed. In the second case, cycles are not possible if the interest rate is sufficiently high. The affordability issue discussed in Scheinkman (1980) and Tirole (1985) plays a key role. When borrowing is not allowed and the interest rate is sufficiently high, the young will not be able to buy the housing stock, if the bubble lasts for a long time. The level of the interest rate is relevant because when the interest rate is high, the growth in the value of the housing stock is higher than the growth in the non-housing sector and therefore eventually the young will not be able to buy the housing stock. Since rational bubbles require market clearing even when the bubble lasts for a long time, a high interest rate will rule out rational bubbles in the no borrowing case.

The young in the model allocate their labor between the production of a perishable good and houses. Houses yield services in addition to their use as a store of value. I allow differences in the rate of technological change between the housing industry and the perishable good industry. This is required to explain the downward trend in the fraction of workers that are employed in construction and the upper trend in housing prices. It also seems realistic. For example, in many industries robots play an increasing role but this did not happen in construction.

The level of interest that is required to prevent the formation of bubbles (in the no borrowing case) depends on the rate of technological progress in the perishable good industry relative to the rate of technological change in the housing sector. When the rate of technological change in the perishable good industry is relatively high, the rate of growth of this sector is high and the affordability issue does not arise when the interest rate is low and the value of the housing stock grows at a relatively low rate. Thus, when the non-housing sector grows at a relatively fast rate, a high interest is required for ruling out bubbles. Surprisingly, the rate of population growth does not play a role in determining the level of interest rate that is required to discourage the formation of rational bubbles. This is different from bubbles in single good overlapping generations economies, where population growth may "solve" the affordability problem.

A high interest rate may eliminate bubbles but does it improve welfare? I compare welfare across deterministic steady states and show that allowing two interest rates, one on government bonds and a lower one on mortgages, is enough to achieve the first best. The interest rates that maximize steady state welfare may not be sufficiently high to eliminate bubbles and this pose a dilemma for the policy maker.

Section 2 is about related literature. Section 3 provides some stylized facts. Section 4 is the model with no restrictions on borrowing. Section 5 provides steady state welfare analysis. Section 6 is about deterministic non-steady state equilibria and section 7 is about cycles. Section 8 assumes no government loans. In section 9 the government supplies mortgages that require down payment. Section 10 is about population growth. Section 11 allows for deviations from strict rationality and section 12 concludes.

2. THE AFFORDABILITY ISSUE AND RELATED LITERATURE

I start by elaborating on the affordability issue. Tirole (1985) argues that in an overlapping generations model, a bubble can emerge if there is sufficient growth in the economy. Otherwise, if the long run interest rate is positive, the asset bubble - which must grow at the interest rate - eventually becomes so big that young generation cannot buy the asset.

Although the model here is different from Tirole's model, the question of whether the young can afford the housing stock (the "affordability" question) plays a central role also in this paper.[1]

I argue here that the government can play a role in "solving" the "affordability problem" and as a result bubbles can emerge even if there is no growth in the economy. There are many ways in which government policies can "solve" the "affordability problem". For example, the government can announce that in states of the world in which

[1] Here there are two goods while in Tirole's model there is one. Here the interest rate is a policy choice

the young cannot afford the housing stock it will collect lump sum taxes from the owner of houses (the old in my model) and transfer them to the young.[2] Here the government "solves" the problem when it supplies loans to the young.

The loans to the young are financed by the loan payments that the government receives from the old and by lump sum taxes. In the model, a crash in housing prices shifts labor to the non-housing sector and as a result the output in the non-housing sector and the demand for government bonds increase. This increase in the demand for government bonds leads to lower taxes. Indeed, a crash is not associated with an increase in unemployment and a loss of welfare. Here I abstract from important frictions and focus on the conditions under which bubbles can arise rather than the consequences of a crash.

Population growth is an important source of growth and in single good models it may "solve" the affordability problem. Here population growth is not relevant for affordability because it affects the long run growth in both sectors in a symmetric way. Here relative technological change is relevant. In the long run the quantity of housing grows at the rate of technological change in the housing sector. If the rest of the economy grows at a faster rate, the young may be able to afford the housing stock even when housing prices increase over time.

The affordability problem is thus less "severe" (imposes less restrictions on bubble creation) when the rate of technological change in the rest of the economy is high relative to the rate of technological change in the housing sector. It is more "severe" when the interest rate is high because the long run rate of growth in housing prices increases with the interest rate. A policy maker who wants to impose restrictions on bubble creation may look for a policy that makes the affordability problem more "severe" so that

[2] The expectations that the government will step in when housing prices are too high, is not unfounded. Something close to that happened in Israel. In 2011 there were big demonstrations by young people who could not afford housing. To make the point that housing is not affordable, young people erected tents in parks in the middle of Tel Aviv and lived there for the entire summer. This turned out to be quite effective. The finance minister was elected on the promise to solve the housing problem. His proposed solution includes subsidies to the young who do not have apartments and taxes on those who have three or more apartments.

bubbles will be ruled out on the ground that the young will not be able to afford the housing stock if the bubble lasts for a long time. I argue that such a policy exists.

This is different from Gali (2014) who argues that monetary policy cannot affect bubbles. An increase in the interest rate will be matched by an increase in the rate of return on the bubble. In Gali's model there are no government bonds. There are only private bonds and in equilibrium the holding of private bonds is zero. The equilibrium definition (on page 732) in Gali's model does not require that the budget constraint of the young (unnumbered equation on page 728) must be satisfied. I thus think that Gali reaches his conclusion because he ignores the affordability problem.

Here there are government bonds and it makes a big difference if the young can borrow from the government or not. When borrowing is allowed, the young can always buy the housing owned by the old by taking loans from the government and as in Gali (2014), high interest rate will not discourage bubble formation. It is also true in my model (as in Gali's model) that when borrowing is not restricted housing prices will rise faster when the interest rate is high. But when borrowing from the government is not allowed a path of fast rising housing prices may not be possible because of the affordability problem discussed above: Eventually the young may not be able to buy the housing stock. It follows that since high interest leads to fast rate of growth in housing prices, high interest rate can rule out rational bubbles in the no borrowing case.

A policy maker who wants to discourage rational bubbles must therefore restrict lending and choose a sufficiently high interest rate. The interest rate that he chooses must be high when the rate of technological change in the rest of the economy is high relative to the rate in the housing sector.

Other papers assume that monetary policy can affect bubbles but argue against it. Bernanke and Gertler (2001, 1999) advocate monetary policy reaction to changes in asset prices that affect the central bank’s forecast of inflation. But once the predictive content for inflation has been accounted for, there should be no additional response of monetary policy to asset price fluctuations. Gilchrist and Leahy (2002) summarize the literature on monetary policy and asset prices. They also do not find a case for including asset prices in monetary policy rules.

Using the terminology of Allen, Morris and Postlewaite (1993) the model here is about "strong bubbles" in which the lack of fundamentals is common knowledge as in Samuelson (1958) and Tirole (1985). There is a vast literature that asks under what conditions "strong bubbles" can exist. See Scheinkman (1977, 1988), Brock (1979, 1982), Wallace (1980), Tirole (1985), Azariadis (1993) and Santos and Woodford (1997). It seems that "strong bubbles" may arise in economies that are "close" to dynamically inefficient OG (overlapping generations) economies.[3] For early models that allow pops of "strong bubbles" see Blanchard and Watson (1982) and Weil (1987).

Recently there is a growing literature on the possibility of "strong bubbles" in economies with financial constraints. See for example, Caballero and Krishnamurthy (2006), Kocherlakota (2009), Farhi and Tirole (2012), Basco (2014) and Miao and Wang (forthcoming). Woodford (1990) notes that some economies populated by infinitely-lived-liquidity-constrained agents are similar to an OG economy. For an excellent review of this literature, see Martin and Ventura (2018).

Part of the literature surveyed by Martin and Ventura assumes a small open economy and allows the young to borrow at the world interest rate. Bubbles may be used as a collateral and therefore they relax the borrowing constraint. The assumption of a "small open economy" does not completely solve the affordability problem. If the bubble does not pop for a long time (which is indeed a small probability event in most models)

---

[3] In OG models the economy is dynamically inefficient whenever a planner can improve the terms in which the young can save. In the steady state the planner can promise a rate of return that is equal to the rate of population growth by taking goods from the young and transferring them to the old. A planner can improve matters when the rate of population growth is higher than the rate that the young can get in equilibrium that has no bubbles.

then it becomes "big" in the sense that the world's young generation will not be able to buy it.

Here the government intermediates between generations. In the baseline model, the government is willing to lend and borrow at a given interest rate. There is thus, no financial friction and therefore a bubble in the housing market does not relax financial constraints.[4]

In most of the literature on bubbles the interest rate is endogenous. Here the government determines the interest rate as is typically assumed in the policy debate.

<u>More on the policy debate</u>

In his famous presidential address, Friedman (1968) argues that monetary policy cannot peg (a) the interest rate and (b) the rate of unemployment. Friedman starts by arguing that increasing the money supply will reduce interest rate in the short run but not in the long run. He then argue (on page 7) that

"Paradoxically, the monetary authority could assure low nominal rates of interest-but to do so it would have to start out in what seems like the opposite direction, by engaging in a deflationary monetary policy. Similarly, it could assure high nominal interest rates by engaging in an inflationary policy and accepting a temporary movement in interest rates in the opposite direction."

He then cite Wicksell for the concept of the "natural" interest rate and argue that

" It will require not merely deflation but more and more rapid deflation to hold the market rate above the initial "natural" rate."

---

[4] The relationship of the government lending and borrowing policy to the bubble literature is not straightforward. When the government supplies bonds at a zero interest rate, these bonds are like money that is the classical example of a bubble. But are government bonds a bubble if they promise strictly positive (or a strictly negative) interest? If they are defined as a bubble than the question of whether bubbles can exist is trivial because the government can always sell bonds at a high enough interest. From a policy point of view, the question of whether government bonds are a bubble is not relevant. The relevant question is whether the government's intermediation improves welfare. For a discussion on this topic see Eden (2011).

Based on Friedman's analysis many have concluded that in the long run both the real interest rate and the rate of unemployment must equal their respective "natural" levels. This is not true. Friedman argues that the long run levels of these variables cannot be affected by monetary policy defined as changes in the money supply that are the result of open market operations. But the level of both can be affected by other policies. The long run level of unemployment can be affected by unemployment insurance, for example. And it is possible to peg the real interest rate by the following policy. The government announces that it will lend and borrow at a given interest rate and that it will use taxes to make the difference between the revenue from selling bonds and the payment that occurs when retiring bonds. Usually pegging a relative price is a "bad" policy. The real interest may be different because it affects bubble formation.

To get a better sense of the current policy debate, I now turn to discuss a thoughtful paper by Neel Kashkari the president of the Minneapolis Fed.[5] Kashkari initial remarks are about the importance of financial stability.

" In 1977, Congress gave the Fed its dual mandate: stable prices and maximum employment. However, we can't ignore the implicit role the Fed also has to try to achieve financial stability. After all, when Congress first created the Fed in 1913, it did so in response to financial crises that repeatedly hammered the U.S. economy in the late 1800s and in the panic of 1907. The Board of Governors and 12 regional Federal Reserve Banks were specifically created with the goal of promoting financial stability. Price stability and maximum employment came almost 70 years later."

And

"Achieving financial stability is hard—really hard. Human societies are prone to mass delusion and to bubbles; history has numerous examples, from the tulip bubble in Holland in the 1600s to the stock market bubble in the 1920s to the housing bubble in the 2000s. Future generations are exceptionally good at repeating past mistakes. Even if we focus just on the Fed's official dual mandate, financial crises can cause very high unemployment and low inflation or even deflation. My perspective is that whether it is officially acknowledged or not, whether we want the responsibility or not, the Fed has an important role to try to ensure financial stability."

---

[5] Published May 17, 2017 https://www.minneapolisfed.org/news-and-events/messages/monetary-policy-and-bubbles

Kashkari assumes mass delusion while here I focus on rational bubbles. The analysis of rational bubbles may serve as a useful benchmark. For example, he argues for increasing down payment requirements noting that:

"Going into the financial crisis, people were putting little to nothing down with those infamous no-doc loans. Those loans were bundled into mortgage-backed securities, which were then bundled into collateralized debt obligations, and then banks bought them with yet more borrowed money. It was leverage on top of leverage with little equity supporting it all."

Here I show that having some down payment requirement can prevent the formation of rational bubbles. The argument is as follows. When the bubble lasts for a long time the young may not have enough funds to buy the housing stock and this cannot occur in equilibrium where markets are cleared. When the down payment requirement is low the probability that the bubble will last until the young generation will not be able to afford the housing stock is low. In reality agents are not completely rational and may ignore small probability events. When we increase the down payment requirement the probability that the young will not be able to buy the entire housing stock goes up and the chance that this possibility will be ignored goes down. A policy maker who thinks that agents are not fully rational may use this argument to increase the down payment requirement.

Kashkari distinguishes between two related policy issues: (a) should the Fed try to burst bubbles and (b) should it attempt to prevent the formation of future bubbles. He answers the first question in the negative because it is difficult to identify a bubble and even if the Fed can identify it the interest rate is not a good instrument to deal with it. Regarding the second question, he says the following.

Current estimates are that the neutral real rate (net of inflation) is currently around zero or perhaps slightly negative. Could it be that such low rates make bubbles more likely to form and, if so, what should we do about it? The truth is we don't have a good answer to this question. If inflation is low and there is slack in the labor market, how high should we raise rates to reduce the chances of bubbles forming? We don't have a good economic theory to analyze this scenario and offer policy guidance. It is a question that needs more research. Until we have such a theory that we have confidence in, I believe we should continue to focus on our dual mandate goals to set monetary policy and then keep our eyes open for potential bubbles and respond as best we can. The cost of keeping rates high to reduce the chances for future bubbles would be higher unemployment and a risk of unanchoring inflation expectations to the downside. Those are large economic costs.

Here, high interest does not lead to less employment. High interest actually increases employment, when the government lends money or when the "no borrowing constraint" is not binding. In the model the young work in the first period and consume only in the second. Therefore, an increase in the interest rate is equivalent to an increase in the real wage.[6]

3. STYLIZED FACTS

Figures 1 describes post world war data about housing prices and stock prices.[7] The solid lines are the logs of the real price (right scale). The dotted lines are the rates of change in the real price in the last year.

The real price of housing exhibits cycles. Housing prices decreased by about 7% from the peak of 1971 to the trough in 1974. They decrease by 11% from the peak of 1978 to the trough of 1982. And they decrease by roughly 40% from the peak of 2005 to the trough of 2011.

The rate of change in housing price can also be described by the use of business cycle language. The annual rate of change peaked around 1970 at 2%. This means that on average, someone who bought a house in 1969 experienced a 2% increase in the price of his house after a year. The rate of change peaked in 1977. On average, someone who bought a house in 1976 experienced a 7% increase in the price of his house after a year. It

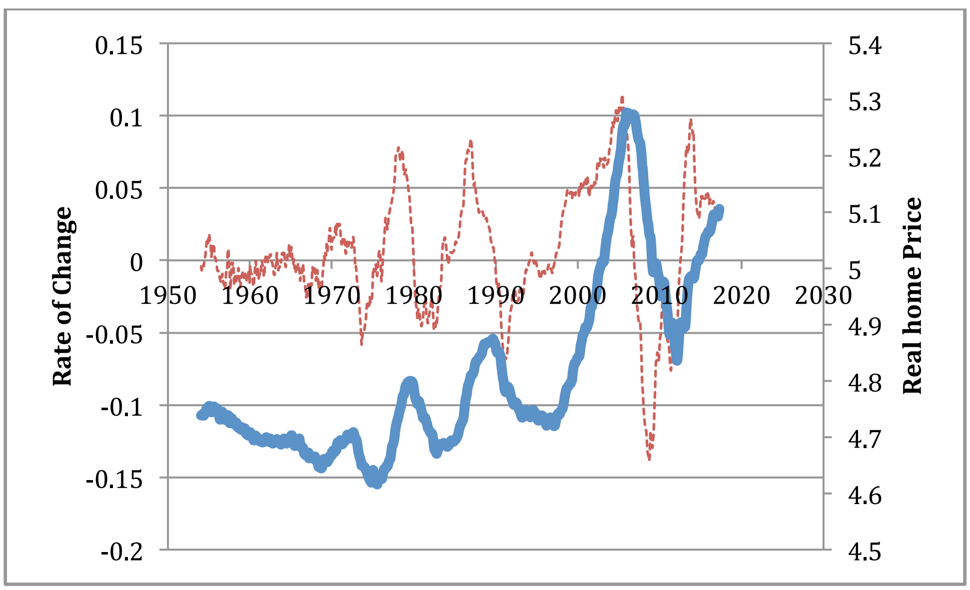
[6] In reality, an increase in the interest rate is good for savings, including savings for retirement. Currently savings is subsidized (by tax deferral for 401Ks) so raising the interest rate and eliminating the 401Ks maybe a good idea.

[7] The data about real housing price and real stock prices is from Robert Shiller's web page. The employment share was calculated using data from the St. Louis Fed web site.

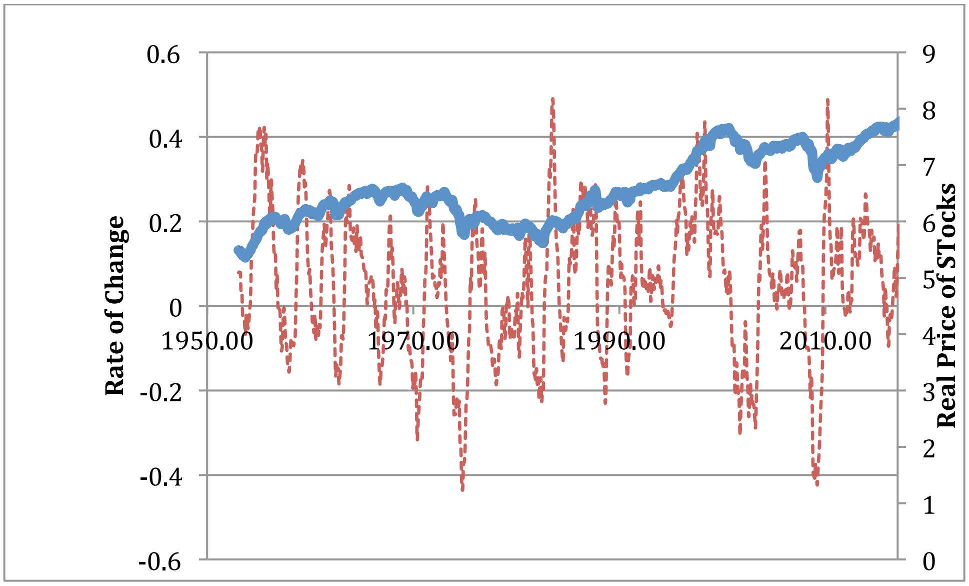
also peaked in 1988 at the same level of 7%, at 2005 at the level of 11% and at 2013 at the level of 9%.

The rate of return on housing fluctuates much less than the rate of return on stocks. It is in the range of 11 to -14 percent while the rate of return on stocks is in the range of 48 to - 42 percent. The correlation between the rate of change in housing real price and its one-year lag is 0.74. This says that the rate of change is likely to be high if it was high a year ago. The same correlation for stocks is 0.08. This says that for stocks the rate of change does not depend on the last year rate of change.

Although houses and stocks are both assets, Figure 1 suggests that their price behavior is very different and maybe we should not attempt to capture both behaviors in the same model.



A: Real Home price (in logs, right axis) and the rate of price change

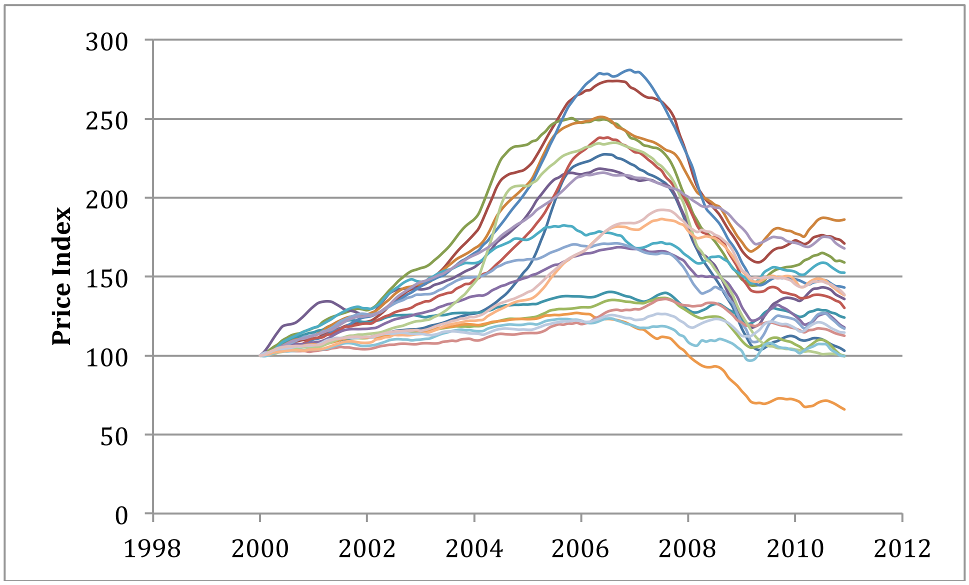


B: Real Stock Price (in logs, right axis) and their rate of change

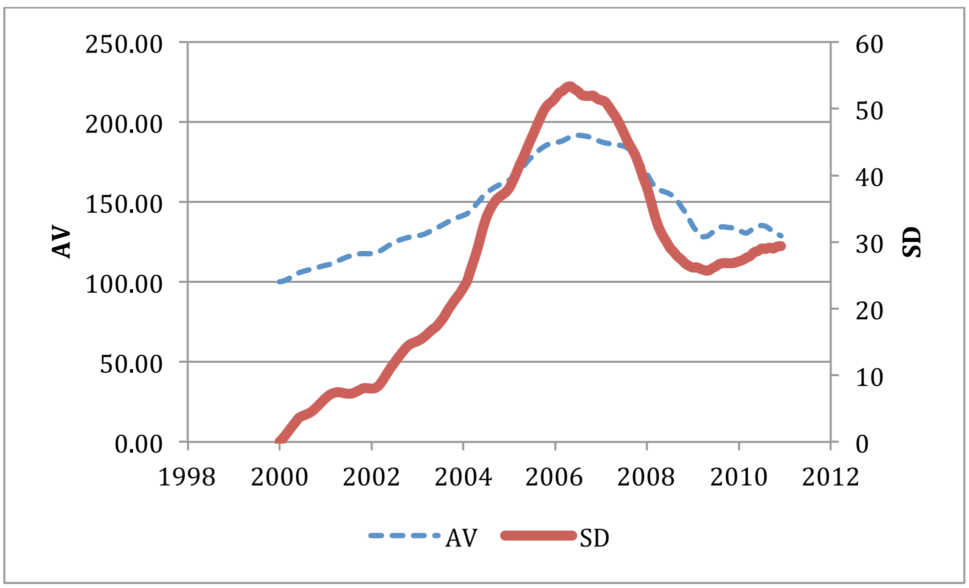
Figure 1: Real Housing Prices and Real Stock Prices. Rate of change are calculated as the percentage change from the same month in the last year (12 months lag).

Figure 2 focuses on the last housing cycle. Figure 2A describes the Case-Shiller index of housing prices in 20 major metropolitan areas across the US. The index is set at 100 in the year 2000 for all the 20 metropolitan areas. It then increased for all observations reaching a level above 250 in 2006 in some cases (273 for Los Angeles CA and 278 for Miami FL). By Dec. 2010 the indices were much lower (170 for Los Angeles and 143 for Miami). As can be seen from Figure 2B the cross sectional standard deviation of the Case-Shiller price indices fell by 50% during the period 2006-2009 from 52 to 26.

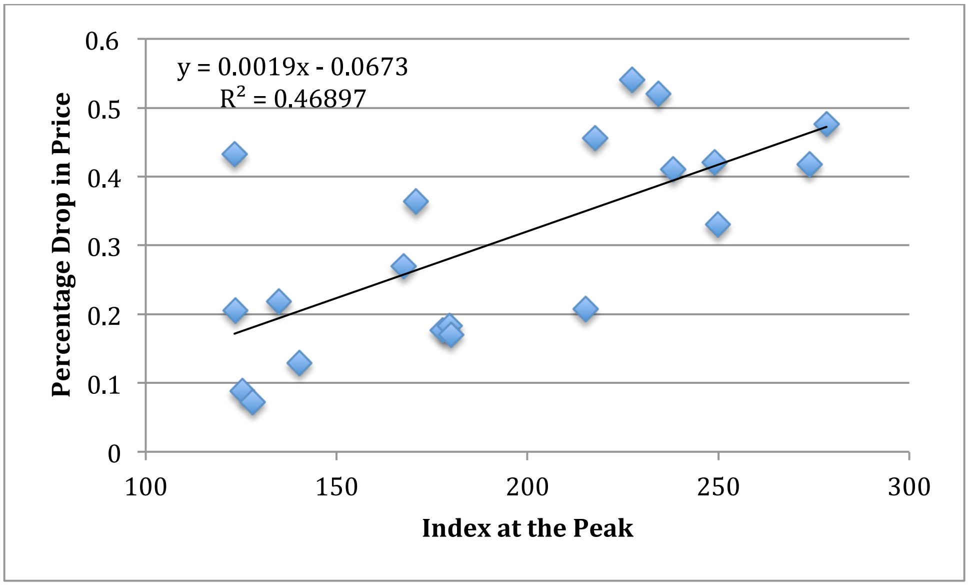
The fall in the standard deviation of the indices suggests that the fall in prices (from the peak to the trough) was larger for cities that experienced a large increase in price (from 2000 to 2006). Figure 2C plots the rate of decrease in price from the peak to the trough against the price index at the peak. It shows that on average the cities that experienced a large increase in prices also experienced a large fall in prices.



A. Price Indices for 20 metropolitan areas (1/1/2000=100)



B. The Cross Sectional Standard Deviation and the Mean of the Price Indices



C. The percentage drop in price from the peak of July 2006 to the trough of April 2009. The regression equation and the R squared are in the upper left corner of the graph.

Figure 2: The Case-Shiller Price Indices for 20 Metropolitan areas

Figure 3 is a plot of the employment share in the housing sector and real housing price.[8] The correlation between the two is close to zero. But this near zero correlation is the result of two opposite forces. The correlation between the trends is negative: The share of employment in the housing sector exhibits a negative trend while the real housing price exhibits a positive trend. At the cycle frequency the correlation is positive. For example, employment share has been growing from 4.2% in 1992 to 5.6% in 2007. During this period housing prices grew by 66%. The negative trend in the employment share that occurs in spite of the positive trend in the real price suggests a non-neutral technological change that reduces the marginal product of labor. The increase in housing price during the boom is strong enough to increase the employment share in spite of the technological changes that do not favor labor.

[8] The data about real housing price is from Robert Shiller's web page. The employment share was calculated using data from the St. Louis Fed web site.

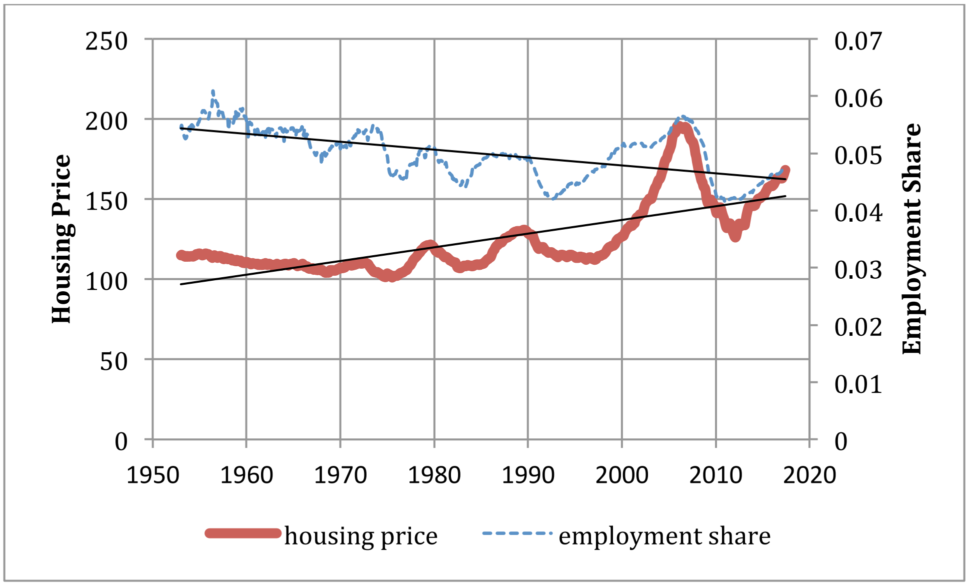
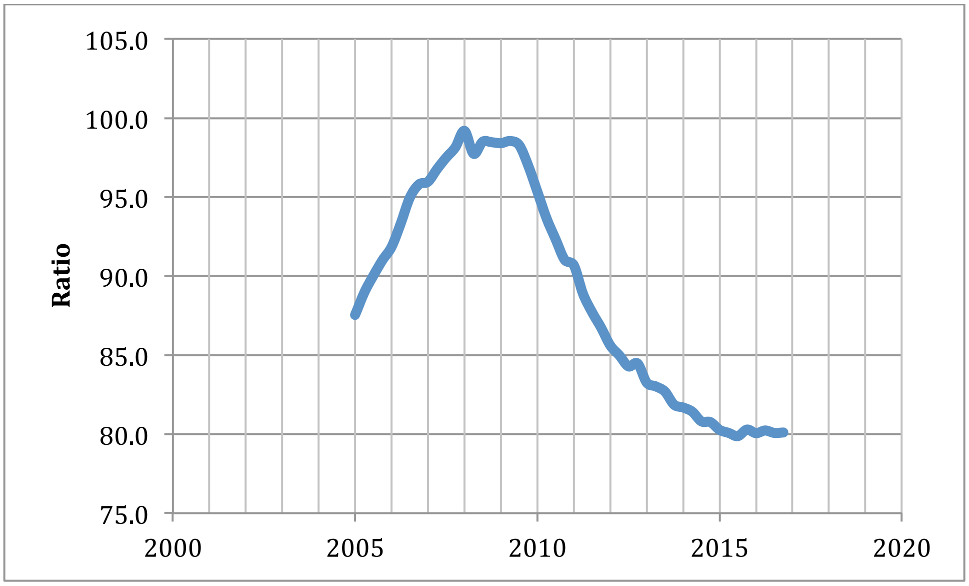


Figure 3: Employment in construction as a share of total non-farm employment and the real price of housing. Both series are seasonally adjusted. Trends lines are added.

Figure 4 is about household debt. The ratio of household debt to GDP reached a peak of close to 100% in 2008 and then declined to about 80% by 2015. Who did the household borrow from? It seems that Foreigners plaid an important role in buying mortgage-backed securities. But eventually the US government bought much of these securities. This brings the question of how to model Government implicit loan guarantees. Here I assume that the government gave the loans and implicitly promised to bailout the representative agent in the case of a crash.[9]

[9] The role of government in mortgages is large. See for example, Frame and Wall (2002a, 2002b). The US congressional budget office (CBO) (2001) reports that at the end of 2000 Fannie Mae and Freddie Mac held or guaranteed 39% of all residential mortgages and 71% of all fixed-rate conforming mortgages. By one estimate, these Government Sponsored Enterprises (GSEs) backed half of the mortgages on the eve of the crisis and after the crisis many criticize these GSEs for providing mortgages with no sufficient guarantees and low down payments. During the wave of default in 2007 the Federal government stepped in. Fannie Mae and Freddie Mac were placed under the conservatorship of the Federal Housing Finance Agency (FHFA) in September 2008. The Fed also purchased toxic mortgage backed securities in its quantity easing policies.



D. Household debt to GDP

I now turn to the model.

4. THE MODEL

I assume an overlapping generations economy. There is one agent per generation. He lives for two periods: produces in the first and consumes in the second. There are two goods: A perishable good and houses. I use x to denote the amount of time allocated to the production of the perishable good and y to denote the amount of time allocated to the production of houses. The endowment of time (labor input) of the young is $\bar{L}$ and the total time devoted to both activities cannot exceed the endowment:

(1) $$L_t = x_t + y_t \leq \bar{L}$$

The agent born at time t gets θ^t units of the perishable good for each unit of labor, where $\theta \geq 1$ is a productivity parameter. At time t, y_t units of labor in the housing sector yields $\psi^t f(y_t)$ units of housing where $\psi > 0$ is a productivity parameter, $f(y)$ has a maximum at $\bar{y} < \bar{L}$ and is differentiable with $f'(y) > 0$ and $f''(y) < 0$ for

$y \le \bar{y}$. These assumptions impose a limit on the amount of housing that can be produced in each period.

Houses are homogeneous and the quantity of houses is measured by say, square feet. The productivity parameter ψ may be different from θ and because the quantity of land is fixed, it may be less than unity. For example, in Manhattan NY, the cost of adding say 10,000 square foot of living space may grow over time because of the need to build taller buildings.[10]

In addition to producing houses, the young can also buy houses from the old. A young agent who buys H_t units of housing and devotes y_t units of labor for housing production will have at his old age:

(2) $$H_{t+1} = (1-\delta)\left(H_t + \psi^t f(y_t)\right)$$

units of housing, where $0 < \delta < 1$ is the depreciation rate. Thus, here new houses depreciate before they get used.

The utility function of the agent born at t is[11]:

(3) $$\beta\left(c_{t+1} + \gamma H_{t+1}\right) - \theta^t v(x_t + y_t)$$

where c is the expected consumption of the perishable good, $\beta > 0$, $\gamma > 0$ are parameters and $v(L = x + y)$ is a strictly convex and strictly monotone cost function. Unlike an Inada type condition, I assume $\lim_{L \to \bar{L}} v'(L) < \infty$. With some abuse of notation I denote the upper bound of the derivatives by $v'(\bar{L}) < \infty$. Note that the cost of labor grows over time at the rate of productivity growth in the non-housing sector reflecting the growth of productivity in leisure activities.

The government lends and borrows at the gross real rate $R = 1 + r$. There is no money and the payment is in terms of the perishable good.

---

[10] In the 19 century you could still build a house on a vacant piece of land. Now you have to take out an old building and build a higher building in its place. The net gain in square footage may be small in spite of the high labor input.

[11] The results will not change much if we assume $\beta\left(c_{t+1} + U(H_{t+1})\right) - \theta^t v(x_t + y_t)$ where U is monotone and strictly concave.

The price of housing in terms of the perishable good evolves according to:

(4) $$p_{t+1} = \{g_t p_t \text{ with probability } q \text{ and } I \text{ otherwise}\}$$

Thus, the price of houses grows at the rate g_t with probability q and it "crashes" to the price I with probability $1-q$. I assume that g_t may change over time but q and I are constants. This is of course not the only way to model bubbles. See Blanchard and Watson (1982). Here I look at a special case that is consistent with the stylized facts.

The sequence of events within the period is as follows. First the old get the housing services which is a fraction γ of the stock of houses they own. Then the price of houses is announced and the young make labor choices (choosing x and y). At the end of the period the old sell their houses for the perishable good and settle debt with the government (the government pays interest and principle on its bonds and collects lump sum taxes).

After the completion of trade, the young agent has $\theta^t x_t - p_t H_t$ units of government bonds and $H_t + \psi^t f(y_t)$ units of housing. When old he will get $R(\theta^t x_t - p_t H_t)$ from the government in exchange for his bonds and $p_{t+1} H_{t+1}$ units in exchange for his houses.

Lump sum tax is contingent on the state: It is T^1_{t+1} in the non-crash state and T^2_{t+1} in the crash state. I assume: $R(\theta^t x_t - p_t H_t) + (1-\delta)\left(H_t + \psi^t f(y_t)\right) g p_t - T^1_{t+1} \geq 0$ and $R(\theta^t x_t - p_t H_t) + (1-\delta)\left(H_t + \psi^t f(y_t)\right) I - T^2_{t+1} \geq 0$. This assumption guarantees that bankruptcy does not occur. For example, $T^2_{t+1} = R(\theta^t x_t - p_t H_t) < 0$ is equivalent to debt forgiveness or bailout.

The expected consumption of the perishable good is:

(5) $$\begin{aligned} c_{t+1} &= R(\theta^t x_t - p_t H_t) + H_{t+1} E(p_{t+1}) - \overline{T}_{t+1} \\ &= R(\theta^t x_t - p_t H_t) + (1-\delta)\left(H_t + \psi^t f(y_t)\right)\left(q g_t p_t + (1-q) I\right) - \overline{T}_{t+1} \end{aligned}$$

where the second equality uses (2) and (4) and $\overline{T}_{t+1} = q T^1_{t+1} + (1-q) T^2_{t+1}$ is the expected lump sum tax.

The representative young agent chooses (H_t, y_t, x_t) by solving the following problem.

(6) $$\max_{H_t, y_t, x_t} \beta(c_{t+1} + \gamma H_{t+1}) - \theta^t v(x_t + y_t) \text{ s.t. (1), (2) and (5).}$$

An interior solution to this problem must satisfy the following first order conditions:

(7) $$\theta^t \beta R = \beta(1-\delta)\psi^t f'(y_t)\left(qg_t p_t + (1-q)I + \gamma\right) = \theta^t v'(x_t + y_t)$$

(8) $$Rp_t = (1-\delta)\left(qg_t p_t + (1-q)I + \gamma\right)$$

Condition (7) says that the expected real return (the expected real wage) from an additional unit of x must equal to the expected real return from an additional unit of y and to the marginal cost. Condition (8) requires that the rate of return on housing is equal to the interest rate. The left hand side of (8) is the cost of housing in terms of next period's consumption of the perishable good. The right hand side of (8) is the expected consumption from buying an additional housing unit: After depreciation the agent will have $(1-\delta)$ units that will yield $(1-\delta)\gamma$ units of services and then will be sold at the expected price $(qgp + (1-q)I)$.

Market clearing

Under (8) the demand for housing is infinitely elastic and therefore when (8) is satisfied the housing market clears. The old want to consume the revenue from selling their houses, plus whatever was left after settling the account with the government. The demand for the perishable good at time t is therefore: $p_t H_t + RB_{t-1} - T_t$, where B_{t-1} is the amount of bonds that were purchased by the current old at time $t-1$ and T_t is the realization of the lump sum tax. The clearing of the perishable good market requires:

(9) $$p_t H_t + RB_{t-1} - T_t = \theta^t x_t$$

The demand for government bonds by the young is:

(10) $$B_t = \theta^t x_t - p_t H_t$$

The government debt to the old is: RB_{t-1}. A lump sum tax on the old makes the difference:

$$T_t = RB_{t-1} - B_t \tag{11}$$

Using (10) and (11) we write the left hand side of (9) as:

$RB_{t-1} + p_t H_t - T_t = p_t H_t + B_t = \theta^t x_t$.

Thus the government policy insures the clearing of the perishable good market.

Here there are no frictions and the economy adjusts painlessly to a crash in housing prices. After a crash the supply of labor to the housing sector drops because of the drop in housing price. Labor moves immediately to the perishable good sector. Since the interest rate does not change total labor supply does not change and therefore the production of the perishable good increases. According with (10), the young buy more government bonds because they produce more of the perishable good and spend less on housing. According with (11) the tax on the old goes down. Since only the old consume the perishable good, the perishable good consumption of the old goes up. The services that the old get from housing do not depend on the price of houses and therefore the total consumption of the old goes up. This is of-course an unrealistic view of a crisis. In a real crisis labor will not move immediately from the housing sector to the perishable good sector and as a result there will be some unemployment. Here the focus is on the condition that allows for cycle in housing market and not on the consequences of a crash.

Equilibrium

A housing cycle is the period between two consecutive crashes. The length of the cycle $\tilde{\Omega}$ is a random variable and with small probability it may last for a long time. I consider a cycle that starts at time τ. The price and the housing stock at the time of the crash are $p_\tau = I$ and H_τ. Equilibrium for a cycle that starts at time τ requires that for each realization Ω of $\tilde{\Omega}$ the sequence of non-negative magnitudes $\{p_t, g_t, y_t, x_t, H_t\}_{t=\tau}^{\Omega+\tau}$ satisfies (1), (2), (7), (8) and $p_{t+1} = g_t p_t$.

I start with the following Claim.

Claim 1: In equilibrium with an interior solution, the production of housing is given by the solution to:

(12)
$$f'(y_t) = \left(\frac{\theta}{\psi}\right)^t \frac{1}{p_t}$$

Proof: Substituting (8) in (7) leads to (12). □

Note that the right hand side of (12) increases over time when $\frac{\theta}{\psi} > \frac{p_t}{p_{t-1}}$. We can therefore state the following.

Corollary: The amount of labor allocated to the housing sector (y) decreases between time $t-1$ and time t if $\frac{\theta}{\psi} > \frac{p_t}{p_{t-1}}$. It increases if this inequality is reversed and does not change if $\frac{\theta}{\psi} = \frac{p_t}{p_{t-1}}$.

To get the intuition note that when prices do not change and $\psi < \theta$ labor moves to the non-housing sector with relatively high productivity growth. In the data (Figure 3) the long run trend in housing prices is positive and the long run trend in the employment share of the housing sector is negative. This is consistent with $\psi < \theta$. In what follows I assume: $\psi \le \theta$.

Claim 2: When $v'(\bar{y}) < R\beta$, there exists a unique interior solution $\left(y(p) > 0, x(R,p) > 0\right)$ to (7) and (8).

Proof: Let $y(p_t)$ denote the solution to (12). We can now solve for x by using

(13) $$\beta R = v'\left(x + y(p)\right)$$

Equation (13) has a unique and strictly positive solution because $v'\left(y(p)\right) < v'(\bar{y}) < R\beta$. Figure 5 illustrates. In the Figure, the total labor supply $L(R) = x + y$ is the solution to (13) and $y(p)$ is the solution to (12). The amount of labor devoted to the production of the perishable good is: $x(R,p) = L(R) - y(p)$. □

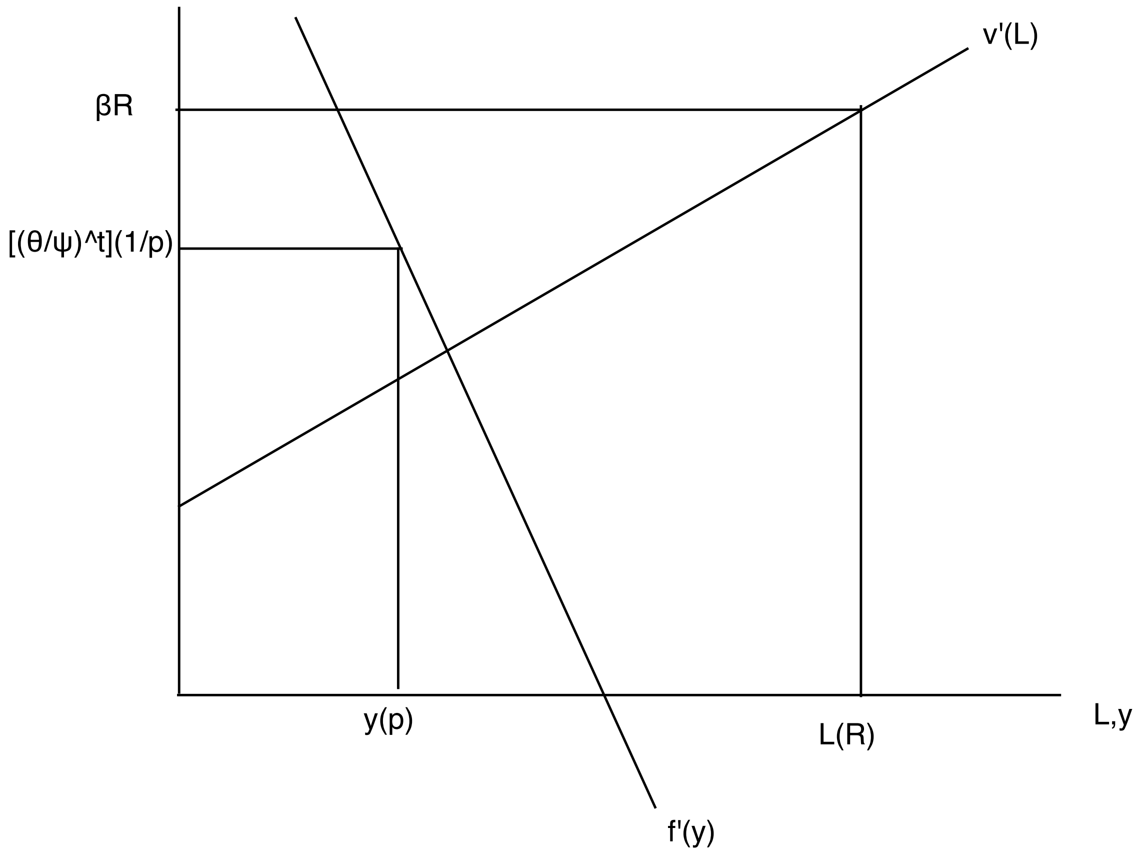


Figure 5: The solution to (12) and (13)

Note that a policy of high interest rate is associated with higher employment. Here consumption occurs with a one period delay and therefore the real interest rate affects the real wage. This delay occurs in the cash in advance model and other models. In reality even long term interest rate affect the real wage because part of earnings goes to savings for retirement. This is different from the point of view expressed by Kashkari in his 2017 paper and it is different from the IS-LM model. Here investment in housing is not sensitive to the interest rate and an increase in the interest rate leads to an increase in income and consumption by the same amount. The result that the real interest does not affect the level of employment holds also in a more standard overlapping generations model in which investment is sensitive to the interest rate and consumption occurs in both periods. See Eden (2011).

I now turn to a deterministic equilibria.

5. DETERMINISTIC STEADY STATE EQUILIBRIUM

A deterministic steady state equilibrium is equilibrium with $q=1$, $g_t = g$, and $y_t = y$ for all t.

Claim 3: A deterministic steady state equilibrium exists if and only if $\theta = \psi$.

A deterministic steady state does not exist when $\psi \neq \theta$, because labor keeps moving to the sector that is becoming relatively more productive.

Proof: I substitute $q=1$, $g_t = g$ in (8) to get:

(14) $$p = \frac{(1-\delta)\gamma}{R-(1-\delta)g}$$

This says that the price does not change over time and therefore $g=1$. Substituting $y_t = y$ in (12) leads to:

(15) $$g = \frac{p_{t+1}}{p_t} = \frac{\theta}{\psi}$$

This says that $g \neq 1$ when $\theta \neq \psi$. Therefore a deterministic steady state cannot exist when $\psi \neq \theta$.

When $\theta = \psi$ a steady state with stable prices exists. In this special case, $g=1$ and

(16) $$p = \frac{(1-\delta)\gamma}{r+\delta}$$

where $r = R-1$ is the interest rate. □

To characterize the steady state, I substitute (16) in (12). This leads to:

(17) $$f'(y) = \frac{r+\delta}{(1-\delta)\gamma} = J(r,\delta,\gamma)$$

Figure 6 illustrates the solution $y(r,\delta,\gamma)$ to (17). It can be shown that $y(r,\delta,\gamma)$ is decreasing in r and in δ and is increasing in γ.

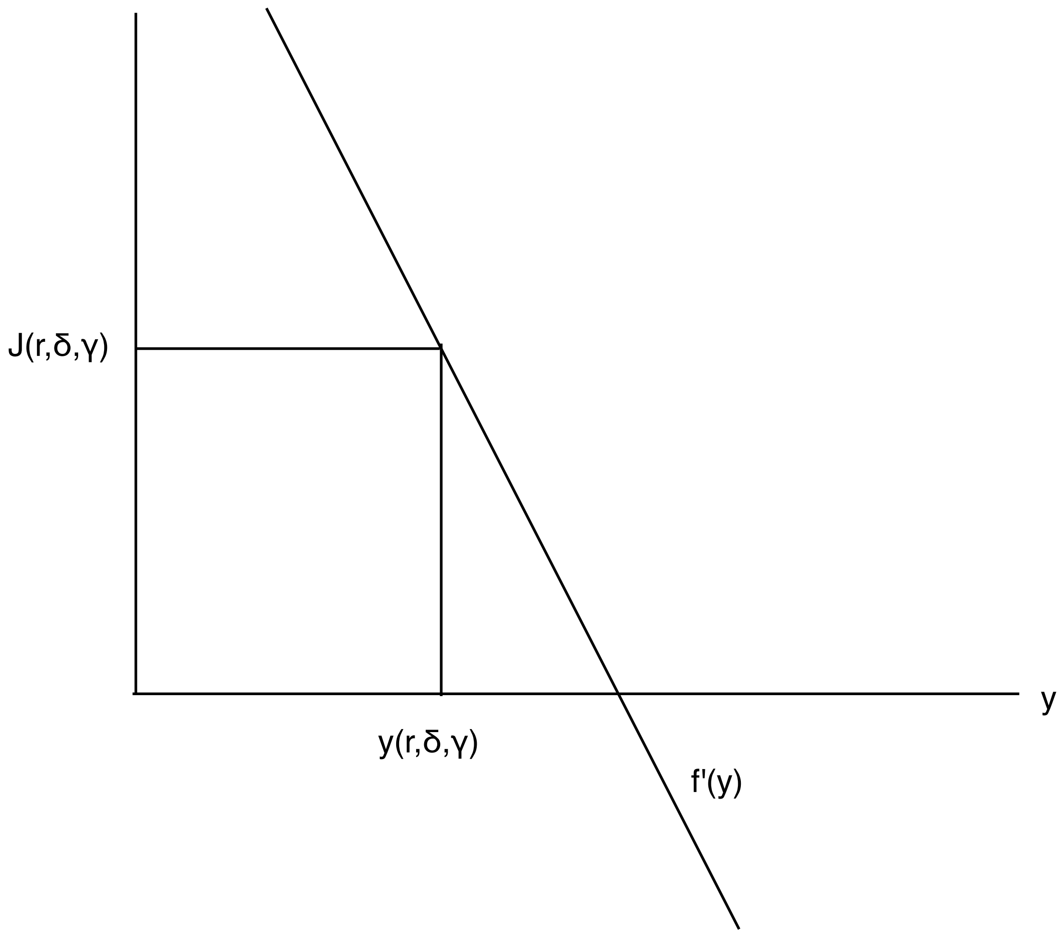


Figure 6: The solution to (17)

In the steady state,

(18) $$H_t = \theta^t H$$

Substituting (18) in (2) leads to $\theta^{t+1} H = (1-\delta)\left(\theta^t H + \theta^t f(y)\right)$ and

(19) $$H = \frac{(1-\delta)f(y)}{\theta - 1 + \delta}$$

The policy that maximizes steady state welfare

The steady state utility of an individual born at t is:

(20) $$\theta^t\left(\beta(x+\gamma H)-v(x+y)\right)$$

Therefore a planner that wants to maximize steady state welfare will solve the following problem:

(21) $$\max_{x,y,H} \beta(x+\gamma H)-v(x+y) \text{ s.t. (19).}$$

Substituting the constraint in the objective function leads to:

(22) $$\max_{x,y} \beta\left(x+\frac{\gamma(1-\delta)}{\theta+\delta-1}f(y)\right)-v(x+y)$$

The first order conditions for an interior solution to (22) are:

(23) $$v'(x+y)=\beta=\beta\frac{\gamma(1-\delta)}{\theta+\delta-1}f'(y)$$

The second inequality in (23) implies:

(24) $$f'(y)=\frac{\theta+\delta-1}{\gamma(1-\delta)}$$

Claim 4: (a) When $\psi=\theta=1$, the steady state allocation solves the planner's problem (21) when $r=0$; (b) When $\psi=\theta>1$, the steady state allocation does not solve the planner's problem (21) regardless of the choice of r.

Proof: The first order condition (7) requires $v'(x+y)=\beta R$. To satisfy the first equality in (23) we must have $R=1$ and $r=0$. Substituting $r=0$ in the steady state solution (17) leads to: $f'(y)=\frac{\delta}{\gamma(1-\delta)}$ which coincides with (24) only when $\theta=1$. □

This result is not surprising. The policy maker needs to determine two magnitudes: Total labor supply $(L=x+y)$ and the allocation of labor to the housing sector (y). He therefore needs an additional policy instrument.

Adding a policy instrument

Achieving the first best requires preferential treatment to the housing sector. One way of doing it is to impose a tax on output in the perishable good sector so that the young agent gets $\Omega = 1 - \rho$ units for each unit produced, where ρ is the per unit tax. The expected consumption of the perishable good is now:

(5')
$$c_{t+1} = R(\theta^t \Omega x_t - p_t H_t) + H_{t+1} E(p_{t+1}) - T_{t+1}$$
$$= R(\theta^t \Omega x_t - p_t H_t) + (1-\delta)\left(H_t + \psi^t f(y_t)\right)\left(q g_t p_t + (1-q)I\right) - T_{t+1}$$

and the young agent's problem is:

(6')
$$\max_{H_t, y_t, x_t} \beta(c_{t+1} + \gamma H_{t+1}) - \theta^t v(x_t + y_t) \text{ s.t. (1) and (5').}$$

The first order condition with respect to x is now:

(7')
$$\beta R \Omega = v'(L)$$

Claim 5: The policy choice $r = \theta - 1$ and $\Omega = \frac{1}{R}$ achieves the first best outcome.

To show this claim note that for this policy choice, the steady state solution (17) coincides with the planner's first order condition (24) and the first order condition (7') coincides with the planner's first order condition $v'(L) = \beta$.

Subsidizing mortgages can also work. Young agents finance the housing they buy from the old by taking mortgages at the rate $R^* < R$ and use their perishable good income to buy government bonds that pay a higher interest R. The consumption of the perishable good at old age is now:

(5'')
$$c_{t+1} = R\theta^t x_t - R^* p_t H_t + H_{t+1} E(p_{t+1}) - T_{t+1}$$
$$= R\theta^t x_t - R^* p_t H_t + (1-\delta)\left(H_t + \psi^t f(y_t)\right)\left(q g_t p_t + (1-q)I\right) - T_{t+1}$$

The first order condition with respect to H_t is now:

(8')
$$R^* p_t = (1-\delta)\left(q g_t p_t + (1-q)I + \gamma\right)$$

Substituting this in (7) leads to:

(12')
$$f'(y_t)=\left(\frac{\theta}{\psi}\right)^t \frac{R}{R^* p_t}$$

Note that (12') can be written as: $R^* p_t \psi^t f'(y_t)=\theta^t R$. The right hand side, $\theta^t R$, is the wage in the perishable good industry, where the wage is paid in the next period. By (8'), the value of a unit of housing in terms of next period's perishable good is: $R^* p_t$. The left hand side, $R^* p_t \psi^t f'(y_t)$, is the value of the marginal product and this must equal the wage rate.

In the steady state (14) and (15) hold. When $\psi=\theta$ and $g=1$ the price is given by (16) and y is determined by:

(20')
$$f'(y)=\frac{R(r+\delta)}{R^*(1-\delta)\gamma}$$

We can now choose $R=1$ to get an efficient labor supply and $R^*=\dfrac{\delta}{\theta+\delta-1}$ to get the efficient labor input in the housing sector. Note that $R^*<1$ when $\theta>1$. The interest on mortgages is thus subsidized to achieve the first best. We have thus shown:

<u>Claim 6</u>: The policy choice $R=1$ and $R^*=\dfrac{\delta}{\theta+\delta-1}$ achieves the first best.

6. DETERMINISTIC EQUILIBRIUM

A deterministic equilibrium assumes $q=1$ but allows y and g to change over time. I now show the following Claim.

<u>Claim 7</u>: (a) There is a deterministic equilibrium in which the price does not change over time and is equal to (16); (b) In a deterministic equilibrium if $g_t>1$ then $g_{t+1}>g_t$ and $\lim_{t\to\infty} g_t=\dfrac{R}{1-\delta}$.

Thus if housing prices are increasing they must be increasing at an increasing rate and the rate of change converges to $R/_{1-\delta}$.

Proof: I substitute $q=1$ in (8) to get:

$$g_t = \frac{R}{1-\delta} - \frac{\gamma}{p_t} \qquad (25)$$

Substituting (16) in (25) leads to $g=1$. Thus there exists a deterministic equilibrium with stable price equal to (16). In this equilibrium y will change over time.

To show (b) note that if $g_t > 1$ then $p_{t+1} > p_t$ and $g_{t+1} = \frac{R}{1-\delta} - \frac{\gamma}{p_{t+1}} > g_t$. Since prices are increasing in this equilibrium the right hand side of (25) converges to $R/_{1-\delta}$. □

To get the intuition I consider now the special case: $\delta = 0$ and $\gamma > 0$. In this case (25) implies: $g_t + (\gamma / p_t) = R$. The left hand side is the gross return from a unit of housing. It has a capital gains component and a dividend component. When the price of housing increases, the dividend component of the return decreases and the capital gains component increases. In the limit, when the price of housing is high, the capital gains component equals the interest rate.

Note that according to (12), $f'(y)$ grows at the rate $\frac{\theta}{\psi g_t}$. In the long run g_t converges to $R/(1-\delta)$ and $f'(y)$ grows at the rate $\frac{\theta(1-\delta)}{\psi R}$. In a deterministic equilibrium, the long run trend in the share of employment in the housing sector is therefore negative, if $\frac{\theta(1-\delta)}{\psi R} > 1$ and $\theta(1-\delta) > \psi R$.

7. CYCLES

I now allow $q < 1$ and use (8) to get:

(26) $$g_t = g(p_t, I) = \frac{R}{(1-\delta)q} - \frac{(1-q)I+\gamma}{qp_t}$$

To write (26) in more familiar terms, note that when $\delta = 0$ we can write (18) as: $qg_t + (1-q)(I/p_t) + (\gamma/p_t) = R$. The first two terms on the left are the expected capital gains. The last term on the left is the dividend yield. The equation thus says that the total expected rate of return on holding houses is equal the interest rate. With depreciation the total expected rate of return is: $(1-\delta)\left(qg_t + (1-q)(I/p_t) + (\gamma/p_t)\right)$ and (26) requires that it will equal the interest rate.

Immediately after the crash, $p = I$, and the (gross) rate of change is:

(27) $$g(I,I) = \frac{R}{(1-\delta)q} - \frac{1-q}{q} - \frac{\gamma}{qI}$$

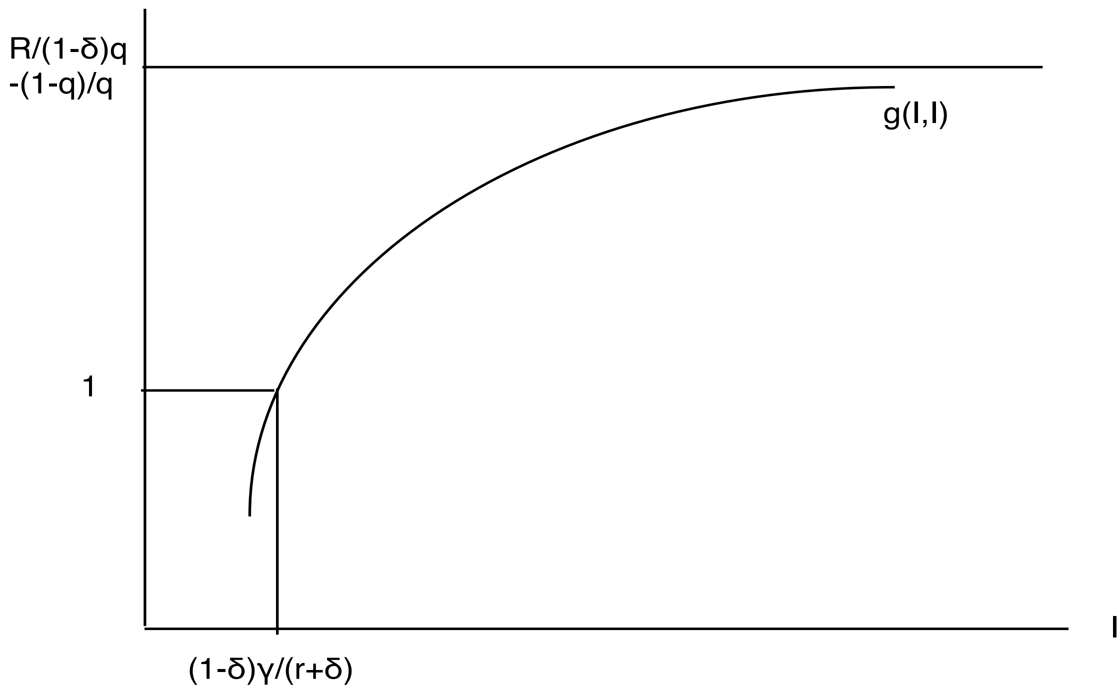
We may get price stability if the price after the crash satisfies: $g(I,I) = \frac{R}{(1-\delta)q} - \frac{1-q}{q} - \frac{\gamma}{qI} = 1$. This leads to $I = \frac{(1-\delta)\gamma}{r+\delta}$ which is the same as (16). Note that if the price drops to (16) it will remain constant but the amount of labor devoted to housing will change if $\psi \neq \theta$.

Since $g(p,I)$ is an increasing function in both arguments, $g(I,I)$ is an increasing function. It follows that when housing prices immediately after the crash are greater than one, they will increase over time at an increasing rate. (The rate of housing price change increases over time because $g(p,I)$ is increasing in its first argument). When $g(I,I) < 1$ housing prices decrease over time until they jump up. I focus here on the case in which housing price are either stable or increasing.

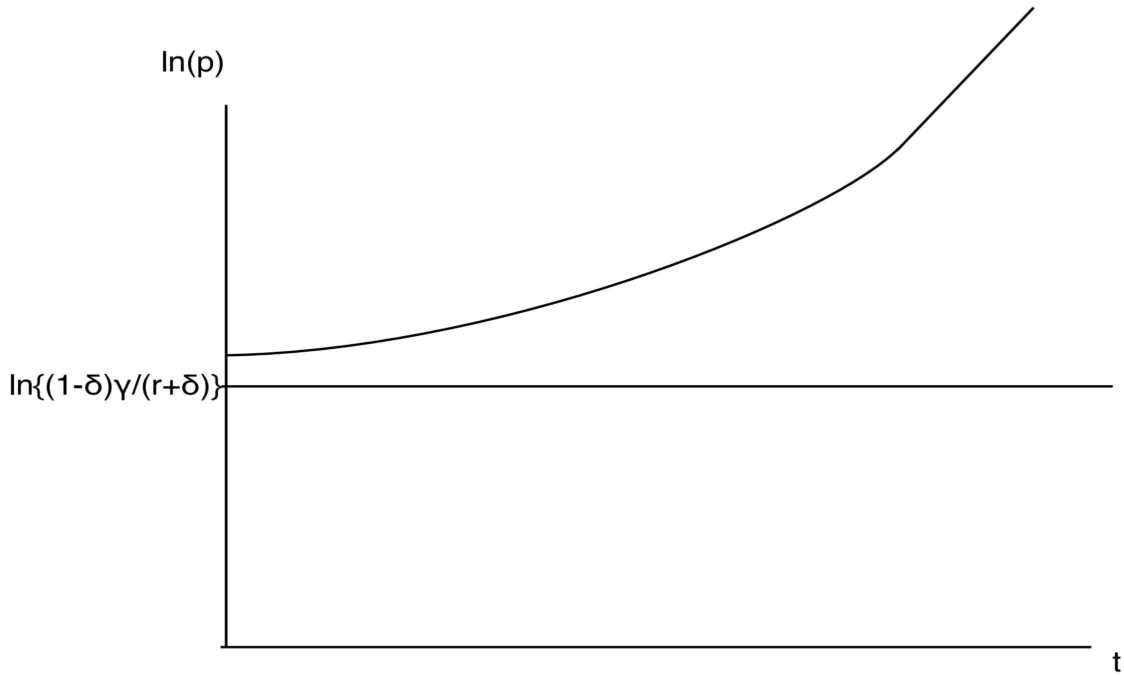
We have thus shown the following claim.

<u>Claim 8</u>: (a) if the price after the crash is greater than the deterministic steady state level (16), then housing price will rise at an increasing rate, until the next crush; (b) if the price after the crash is equal to (16) then housing prices will remain stable; (c) housing price growth rate is less than $\frac{R}{(1-\delta)q}$ and converges to this level when the cycle lasts for a long time.

Figure 7 illustrates.



A. The rate of change in housing price immediately after the crash as a function of the price immediately after the crash (I)

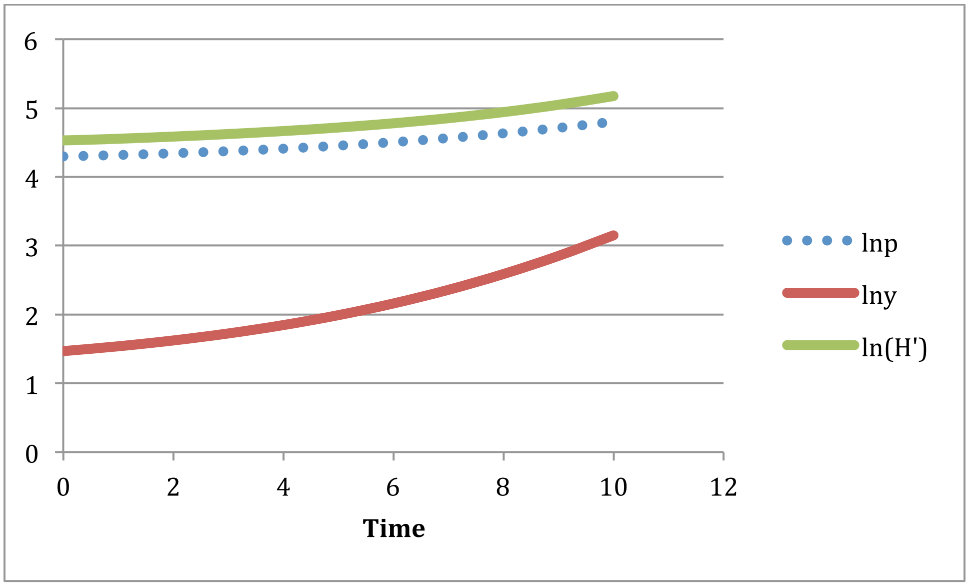


B. The evolution of housing prices after a crash

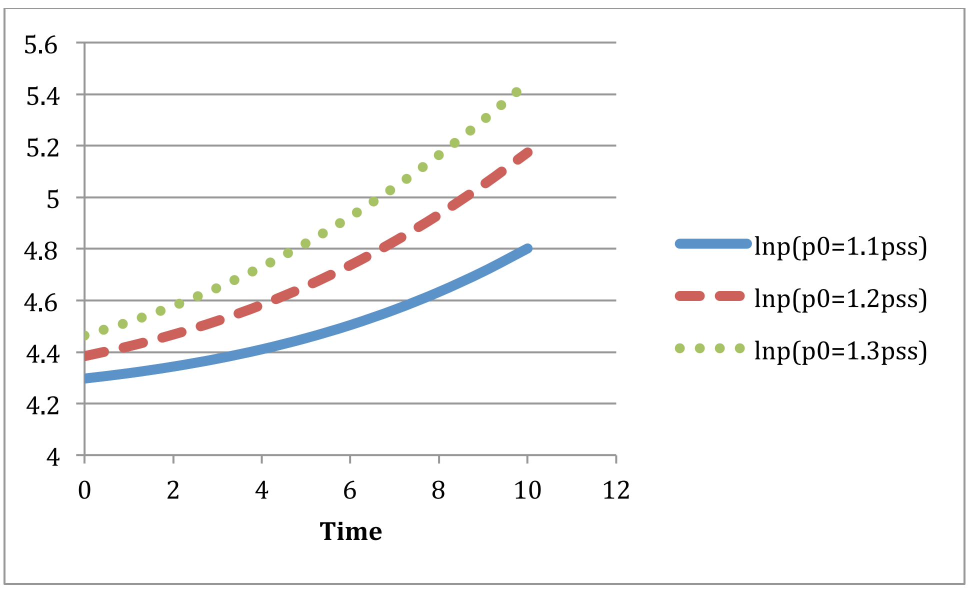
Figure 7: The evolution of housing prices depends critically on the price immediately after the crash.

I now turn to a numerical example that assumes: $f(y)=y^{0.7}$ when $y \le \bar{y}$ and $f(y) \le \bar{y}^{0.7}$ when $y > \bar{y}$. I also assume that $\bar{y}$ is large and $\psi=\theta=1$, $\gamma=0.05$ and $q=0.9$, $n=0$ and $R^*=R$.

Figure 8A illustrates what happens to housing price, the amount of labor in construction and next period's housing stock (H') in response to a shock of 10% in the housing price. All magnitude increases in response to the increase in the price and the rate of increase increases over time. Figure 8B compares 3 different shocks: An increase in the price by 10% above the steady state level, an increase by 20% and an increase by 30%. As we can see the variance of log prices increases over time. Assuming that different cities experienced different shocks, this may explain why the cross sectional variance of the index of housing prices increases during the boom and decreases after the crash as in Figure 2.



A. The log of Price (lnp), housing stock (lnH') and labor employed in construction (lny) when the price at time zero is 10% higher than the steady state level. Housing stock at time zero is at the steady state level. The number 5 was added to the logs to get positive magnitudes that look better on the graph. For example lnp is the log of price plus 5.



B. The log of Price when the price at time zero is 10% higher than the steady state level [lnp(p0=1.1pss)], 20% higher [lnp(p0=1.2pss)] and 30% higher [lnp(p0=1.3pss)].

Figure 8: Starting from the steady state there is a shock to the price of housing. $f(y)=y^{0.7}, \psi=\theta=1$, $\gamma=0.05$ and $q=0.9$.

8. NO BORROWING

I now consider the case in which borrowing from the government is not possible and the young agent must satisfy the following no borrowing constraint:

(28) $$\theta^t x_t \geq p_t H_t$$

I show that in this case, rational bubbles can be ruled out if the interest rate is sufficiently high. The intuition is as follows. When the interest rate is high housing prices increases at a high rate and eventually the young generation will not be able to buy the housing stock.

I start with three Lemmas. The first shows that Claim 1 holds even when borrowing is not allowed.

Lemma 1: The production of housing must satisfy (12) when the solution to the young agent's problem satisfies $(y>0, x>0, y+x<\bar{L})$ and (28) is imposed.

Proof: Since (1) is not binding the agent can increase X by p units and buy a unit of housing. The amount of labor required for doing it is p/θ^t units and the cost of doing it (the Pain) is $(p/\theta^t)\theta^t v'(x+y) = pv'(x+y)$. The benefit (the Gain) is: $\beta(1-\delta)\left(qg_t p_t + (1-q)I + \gamma\right)$. Since at the optimum $Pain \geq Gain$ we must have: $\beta(1-\delta)\left(qg_t p_t + (1-q)I + \gamma\right) \leq pv'(x+y)$. The agent can also reduce housing by a unit and reduce labor input by p/θ^t units. Therefore, at the optimum we must have: $\beta(1-\delta)\left(qg_t p_t + (1-q)I + \gamma\right) \geq pv'(x+y)$. It follows that the solution must satisfy:

(29) $$\beta(1-\delta)\left(qg_t p_t + (1-q)I + \gamma\right) = pv'(x+y)$$

Since we assume an interior solution $(y>0, x>0, y+x<\bar{L})$, the second equality in (7) must hold. That is:

(30) $$\beta(1-\delta)\psi^t f'(y_t)\left(qg_t p_t + (1-q)I + \gamma\right) = \theta^t v'(x_t + y_t)$$

Substituting (29) in (30) leads to (12). □

I now show that the lower bound of the rate of housing price change is increasing in the interest rate.

Lemma 2: Under the no borrowing constraint (28) the rate of price growth in the boom phase must satisfy:

(31) $$g_t \geq \frac{R}{(1-\delta)q} - \frac{(1-q)I+\gamma}{qp_t} \quad \text{with equality if } B_t > 0$$

Proof: When borrowing is not allowed and the housing stock is strictly positive, the young agent can sell houses and buy bonds. If he sells a unit of housing and invests in government bonds he will get Rp_t utils in the next period. But he will loose $(1-\delta)\left(qg_tp_t+(1-q)I+\gamma\right)$ utils. Since in equilibrium he is willing to hold the existing housing stock, we must have:

(8") $$Rp_t \leq (1-\delta)\left(qg_tp_t+(1-q)I+\gamma\right) \quad \text{with equality if } B_t > 0$$

This leads to (31). □

The third Lemma is about the evolution of the housing stock when y is constant over time.

Lemma 3: When labor input in the housing sector is constant over time and is given by $y_t = y$ for all t, and the evolution of housing stock satisfies (2) then eventually (when t is sufficiently large), housing stock is given by

(32) $$H_t = \psi^t H$$

where $H = \dfrac{(1-\delta)f(y)}{\psi+\delta-1}$.

Proof: Substituting (32) in (2) leads to:

(33) $$\psi^{t+1}H = (1-\delta)\left(\psi^t H + \psi^t f(y)\right)$$

Thus if (32) holds at time t it will also hold at time $t+\tau$ for all $\tau > 0$. Suppose now that

(34) $$H_t > \frac{\psi^t f(y)(1-\delta)}{\psi - 1 + \delta} = \psi^t H$$

Then,

(35) $$H_t \psi > (1-\delta)\left(H_t + \psi^t f(y)\right) = H_{t+1}$$

which says that housing stock grows at a rate that is lower than ψ . Eventually, when τ is large enough, $H_{t+\tau}$ will converge to $\psi^{t+\tau} H$ as illustrated by Figure 9. A similar argument can be made when the inequality in (34) is reversed. □

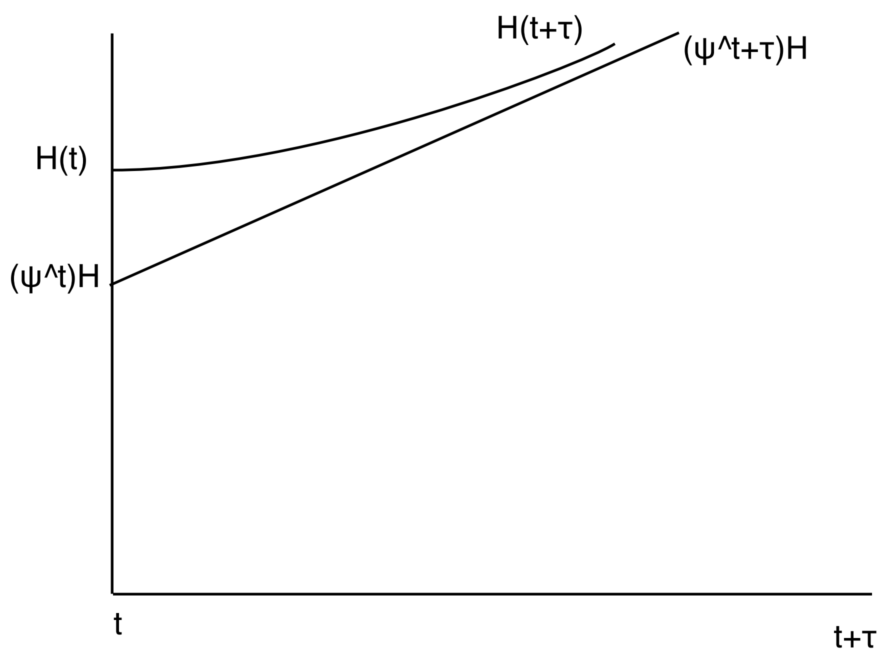


Figure 9: The convergence to the steady state

I now show that bubbles can be ruled out when the interest rate is sufficiently high.

<u>Claim 9</u>: Bubbles are not possible if $R > \dfrac{\theta(1-\delta)q}{\psi}$.

Proof: If the bubble lasts for a long time, $p_t \to \infty$ and the right hand side of (31) goes to $\frac{R}{(1-\delta)q}$. Under the condition in the claim, $\frac{R}{(1-\delta)q} > \frac{\theta}{\psi}$ and therefore in the limit $g > \frac{\theta}{\psi}$ and $g\psi > \theta$. By Lemma 1, eventually labor input will be close to $\bar{y}$, and by Lemma 3 the stock of houses in the long run will grow at a rate close to ψ and the value of houses will grow at the rate close to $g\psi > \theta$. Therefore total spending on houses grow at a rate that is higher than θ and eventually the no borrowing constraint (28) will be binding. □

Note that to prevent bubbles the interest rate should be higher the higher is the ratio of the rate of technological change between the two sectors. At a given interest rate bubbles are more likely to emerge the higher is the ratio of the technological change parameters.

9. SUBSIDIZED MORTGAGES

I now assume that the government supplies subsidized mortgages but no other loans. Subsidized mortgages were mention before as an additional policy instrument that may be used to achieve the first best. Here I add a down payment requirement.

As before, the interest on mortgages is $R^* \le R$. Unlike the previous case, here mortgages require a down payment of a fraction λ of the price of the house, where $0 < \lambda < 1$. The down payment requirement is:

(28*) $$\lambda p_t H_t \le \theta^t x_t$$

As before we assume that the government transfers policy rules out bankruptcies.

Expected consumption is now:

(5*) $$c_{t+1} = R(\theta^t x_t - \lambda p_t H_t) - (1-\lambda)R^* p_t H_t + (1-\delta)\left(H_t + \psi^t f(y_t)\right)\left(q g_t p_t + (1-q)I\right) - T_{t+1}$$

And the problem of the young agent is:

(6*) $\max_{H_t, y_t, x_t} \beta(c_{t+1} + \gamma H_{t+1}) - \theta^t v(x_t + y_t)$ s.t. (1), (5*) and (28*).

I assume that the solution to (6*) satisfies $(H_t > 0, y_t > 0, x_t > 0, y_t + x_t < \bar{L})$.

Lemma 1*: (a) $v'(L) \geq \beta R$ with equality when (28*) is not binding.

(b) The amount of labor supplied to the housing sector must satisfy

(36) $$f'(y_t) = \left(\frac{\theta}{\psi}\right)^t \frac{v'(L_t)}{p_t\left(\lambda v'(L_t) + \beta(1-\lambda)R^*\right)}$$

(c) When (28*) is not binding, $v'(L) = \beta R$ and

(37) $$f'(y_t) = \left(\frac{\theta}{\psi}\right)^t \frac{R}{R^{**} p_t},$$

where $R^{**} = (1-\lambda)R^* + \lambda R$.

Proof: To show (a) note that the young agent can increase work by a unit and get θ^t units of the perishable good and invest it in government bonds. This yields $\theta^t R$ in the next period. The cost of doing that is $\theta^t v'(L)$ and since at the optimum deviations cannot improve matters we must have: $\theta^t v'(L) \geq \beta \theta^t R$ and this leads to $v'(L) \geq \beta R$. When (28*) is not binding the young can reduce labor by a unit and reduce his holding of government bonds by θ^t units. Since deviations from the optimal solution do not increase welfare we must have $v'(L) \geq \beta R$ and $v'(L) \leq \beta R$ which leads to $v'(L) = \beta R$.

I now turn to show (c) which assumes that (28*) is not binding. In this case, $\theta^t R$ is the wage in the perishable good sector (paid in the next period). Since (28*) is not binding the young agent is taking a mortgage and $R^{**} p_t$ is the value of a unit of houses in terms of next period's perishable good: If you buy an additional unit of housing you will pay next period $R^{**} p_t$ units of the perishable good. The value of the marginal product must equal the wage: $R^{**} p_t \psi^t f'(y_t) = \theta^t R$. This leads to (37).

When (28*) is binding, the implicit wage (or the value of time) is: $\theta^t v'(L_t)$. The implicit price of a house is calculated as follows. Since (1) is not binding the agent can increase X by λp units and take a mortgage of $(1-\lambda)p$ to buy a unit of housing. The amount of

labor required for doing it is $\frac{\lambda p}{\theta^t}$ units and the labor cost is $\frac{\lambda p}{\theta^t}\theta^t v'(L) = \lambda p v'(L)$. The mortgage cost in terms of future perishable good is $(1-\lambda)pR^*$. The utility cost of buying an additional unit of housing is therefore: $\lambda p v'(L) + \beta(1-\lambda)pR^*$. At the optimum the value of the marginal product must equal the wage:
$\left(\lambda p v'(L) + \beta(1-\lambda)pR^*\right)\psi^t f'(y) = \theta^t v'(L)$. This leads to (36). □

I now assume equilibrium with $H_t > 0$ and show the following Claim.

Lemma 2*: (a) the rate of price growth in the boom phase must satisfy:

(38) $$g_t \geq \frac{R^{**}}{(1-\delta)q} - \frac{(1-q)I+\gamma}{qp_t}$$

Proof: The young agent can reduce his holding of houses by one unit. If he does it, his down payment will go down by λp_t and he can reduce his labor input by:

$\frac{\lambda p_t}{\theta^t}\theta^t v'(L) = \lambda p_t v'(L)$. In addition, his mortgage payment will go down by $(1-\lambda)p_t R^*$ units. The gains from reducing the amount of housing that he buys by a unit are therefore: $\lambda p_t v'(L) + \beta(1-\lambda)p_t R^*$. A unit of housing yields $(1-\delta)\left(qg_t p_t + (1-q)I + \gamma\right)$ utils. Since at the optimum he does not choose to do it we must have:

(40) $\lambda p_t v'(L) + \beta(1-\lambda)p_t R^* \leq \beta(1-\delta)\left(qg_t p_t + (1-q)I + \gamma\right)$

This leads to:

$$g_t \geq \frac{\left(\lambda v'(L) + \beta(1-\lambda)R^*\right)}{\beta(1-\delta)q} - \frac{(1-q)I+\gamma}{qp_t} \geq \frac{R^{**}}{(1-\delta)q} - \frac{(1-q)I+\gamma}{qp_t}$$

The second inequality follows from the claim $v'(L) \geq \beta R$ in Lemma 1*. □

Note that Lemma 3 holds also in this case. I now show the following Claim.

Claim 9*: Bubbles are not possible if $R^{**} > \frac{\theta(1-\delta)q}{\psi}$

Proof: If the bubble lasts for a long time, $p_t \to \infty$ and the right hand side of (38) goes to $\frac{R^{**}}{(1-\delta)q}$. Under the condition in the claim, $\frac{R^{**}}{(1-\delta)q} > \frac{\theta}{\psi}$ and therefore in the limit $g > \frac{\theta}{\psi}$ and $g\psi > \theta$. Since $v'(\bar{L}) < \infty$, Lemma 1* implies that eventually labor input will be close to $\bar{y}$, and by Lemma 3 the stock of houses in the long run will grow at a rate ψ and the value of houses will grow at the rate $g\psi > \theta$. Therefore total spending on houses grow at a rate that is higher than θ and eventually (28*) will be violated. □

Note that there is a difference between $\lambda = 0$ and $\lambda > 0$. When $\lambda = 0$ there is no constraint on borrowing and cycles may occur. When $\lambda > 0$ the affordability issue emerges when the bubble lasts for a long time and bubbles can be ruled out when the interest rate is sufficiently large. There is not much difference between $\lambda = 1$ in which the government does not supply any loans and $0 < \lambda < 1$. The only difference is in the computation of the relevant interest rate.

10. POPULATION GROWTH

I now show that Claim 9 holds if we allow for population growth. Let:

(39) $$\frac{N_t}{N_{t-1}} = 1 + n$$

where N_t is the size of the generation born at t and n is the rate of population growth. Lemmas 1 and 2 use only the first order conditions of the individual agent and therefore they hold also for this case.

Lemma 3 should be modified as follows.

<u>Lemma 3'</u>: When labor input in the housing sector is constant over time and is given by $y_t = y$ for all t, and the evolution of housing stock satisfies (2) then eventually (when t is sufficiently large), housing stock is given by:

$$H_t = \psi^t H \tag{40}$$

where

$$H = \frac{(1-\delta)f(y)}{\psi(1+n)+\delta-1} \tag{41}$$

<u>Proof</u>: The clearing of the housing market requires:

$$N_t H_t = N_{t-1}(1-\delta)\left(H_{t-1} + \psi^{t-1} f(y_{t-1})\right) \tag{42}$$

When $y_t = y$ we get:

$$\frac{H_t}{H_{t-1}} = \frac{1-\delta}{1+n}\left(1+\psi^{t-1}\frac{f(y)}{H_{t-1}}\right) \tag{43}$$

Substituting (43) in (40) leads to (41). □

The proof of Claim 9 should be modified as follows. In the long run when y_t is close to $\bar{y}$, the stock of houses grows at the rate

$$\frac{p_{t+1}N_{t+1}H_{t+1}}{p_t N_t H_t} = \frac{p_{t+1}(1+n)\psi^{t+1}H}{p_t \psi^t H} = g(1+n)\psi \tag{44}$$

The aggregate amount of the perishable good grows at the rate

$$\frac{N_t \theta^t x}{N_{t-1}\theta^{t-1}x} = (1+n)\theta \tag{45}$$

When the bubble lasts for a long time, there will not be sufficient funds to buy houses if $g\psi > \theta$ which implies that (44) is greater than (45). The proof of Claim 9 goes through. It says that bubble are not possible if $R > \theta(1-\delta)q/\psi$.

Thus here the rate of population growth does not play a role in the condition that allows for bubbles. It does not play a role because higher population growth affects the output in both sectors symmetrically and is therefore not relevant for the affordability issue.

This is different from the analysis of pure bubbles in Tirole (1985). In his model sufficient growth allows for bubbles and population growth is relevant because it contributes to growth.

Optimal steady state policy

The constraint to the planner's problem (21) is now (41) instead of (19). After substituting $\psi = \theta$, the first order conditions for this problem are:

(46) $$v'(L) = \beta$$

(47) $$f'(y) = \frac{\theta(1+n)+\delta-1}{(1-\delta)\gamma}$$

With two policy instruments the first best can be achieved with the policy choice: $r = \theta(1+n)-1$ and $\Omega = \frac{1}{R}$. Alternatively, we can set $R = 1$ and $R^* = \dfrac{\delta}{\theta(1+n)+\delta-1}$.

11. DEVIATIONS FROM STRICT RATIONALITY

Kahneman and Tversky (1979) argued that people edit prospects before evaluating them and that "A particularly important form of simplification involves the discarding of extremely unlikely outcomes". Bubbles are more likely if we discard extremely unlikely outcome.

The proof of Claim 9 argues that under certain conditions, bubbles are not possible because eventually the young may not have sufficient purchasing power to buy the stock of houses. The likelihood of the insufficient funds event may depend on the interest rate. Since the right hand side of (31) is decreasing in R, this will occur when (31) holds with equality. In this case, the higher R is, the higher is the rate of price increase and the shorter is the time it takes to get to the insufficient funds point.[12]

[12] In detail, let τ denote the number of no pop periods that it takes to get to the insufficient fund point. Since the probability of "no pop in the next τ periods", $(1-q)^{\tau}$, is decreasing in τ and since τ is

It is thus possible, that an increase in R increases the probability that the young will not be able to buy the housing stock and therefore when R is sufficiently high this event will not be ignored. Once this event is not ignored agents will see that the only equilibrium possible is the constant price equilibrium.

12. CONCLUDING REMARKS

Housing cycles can emerge when the government provides loans and pursue a policy that prevents bankruptcies. They can also emerge when the government does not provide loans but the interest rate on its bonds is low and the rate of technological change in the housing sector is low relative to the rate of change in the perishable good sector. The intuition is as follows. When the interest rate is low housing prices and the value of the housing stock increase at a low rate. If on top of that the rate of technological change in the perishable good sector is relatively high the young will be able to buy the housing stock even when the boom lasts for a long time and even when the government does not provide loans. These results hold with some modification when the government supplies mortgages that require down payments.

It seems that the conditions prior to the collapse of housing prices in 2007-2008 were conducive for bubble formation. The relative rate of technological change in the non-housing sector was high because of advancement in information technology and robotics. The interest was low and the involvement of the government in mortgages was large. Furthermore, during the wave of default in 2007 the government stepped in suggesting that expectations of some form of bailout in the case of a crisis were not unfounded.

---

decreasing in R, it follows that the probability of getting to the insufficient funds point is increasing in R.

The model can account for puzzling observations under the assumption of rationality. It can account for a period in which prices are increasing while production cost and rent are relatively stable. When the government provides loans, we get an equilibrium in which the price of housing rise but labor cost and rent are stable: the cost of labor is constant and is given by $v'(L) = \beta R$ and rent is γ . This can account for the observation in Shiller (2007) that were cited in the introduction.

Our model can also account for some of the observations made in Figures 1 and 2. In the model, the rate of growth of housing prices increases during the boom phase. This is roughly the case in Figure 1. The model allows for the case in which employment in the housing sector increases during the boom but exhibits downward trend. In the model, the percentage difference between housing prices in different cities will increase in the boom phase if they do not start from the same initial conditions. This will occur if there is a difference in the level I across cities. The cities that start the cycle with a modest overpricing (the price after the crash is above but close to the deterministic steady state level) will experience a modest cycle while the cities that start the cycle with a substantial overpricing will experience a more dramatic cycle.

In the model the fall in prices does not take time. In the data it does. It seems that the average time that a house is "on the market" is counter-cyclical. This suggests downward price rigidity that is not in our model. Indeed, Shiller (2007) note that home sellers tend to hold out for high prices when prices are falling and there was a 17% decline in the volume of US existing home sales since the peak in volume sales in 2005.

I also attempted to compare welfare across steady states. A deterministic steady state exists only when the rate of technological change is the same across the two sectors. The optimal policy in this case requires two instruments. One possibility is to set the interest on government sponsored mortgages at a level that is lower than the interest on other types of government's loans. Another possibility is to exempt housing from an income tax that is imposed on the other sector. The second alternative does not encourage the formation of bubbles.

REFERENCES

Allen, Franklin & Morris S. & Postlewaite A., 1993. "**Finite Bubbles with Short Sale Constraints and Asymmetric Information**," Journal of Economic Theory, Elsevier, vol. 61(2), pages 206-229, December.

Bernanke Ben S. & Mark Gertler, 2001. "**Should Central Banks Respond to Movements in Asset Prices?**," American Economic Review, American Economic Association, vol. 91(2), pages 253-257, May.

_______ 1999. "**Monetary policy and asset price volatility**," Economic Review, Federal Reserve Bank of Kansas City, issue Q IV, pages 17-51.

Blanchard Olivier J. "Speculative Bubbles, Crashes and Rational Expectations", Economics Letters 3 (1979) 387-389.

_______ and Mark W. Watson "Bubbles, Rational Expectations and Financial Markets" NBER WP #945, July 1982. Published in P. Wachtel (ed.) Crisis in the Economic and Financial Structure: Bubbles, Bursts, and Shocks, Lexington Press, Lexington, MA.

Craig Burnside & Martin Eichenbaum & Sergio Rebelo, 2016. "**Understanding Booms and Busts in Housing Markets**," Journal of Political Economy, University of Chicago Press, vol. 124(4), pages 1088-1147.

Dupor, Bill, 2005. "**Stabilizing non-fundamental asset price movements under discretion and limited information**," Journal of Monetary Economics, Elsevier, vol. 52(4), pages 727-747, May.

Eden, Benjamin, 2011. "**Intergenerational Intermediation and Altruistic Preferences**," Vanderbilt University Department of Economics Working Papers 1108, (2011) Vanderbilt University Department of Economics.

Frame, Scott and Larry Wall (2002a): "Financing Housing through Government-Sponsored Enterprises" Economic Review, Federal Reserve Bank of Atlanta, First Quarter 2002.

______ (2002b): "Fannie Mae's and Freddie Mac's Voluntary Initiatives: Lessons from Banking". Economic Review, Federal Reserve Bank of Atlanta, First Quarter 2002.

Friedman Milton., "The role of Monetary Policy", The American Economic Review Volume 58, March 1968 Number 1.

Gilchrist, Simon & Leahy, John V., 2002. "**Monetary policy and asset prices**," Journal of Monetary Economics, Elsevier, vol. 49(1), pages 75-97, January.

Gali Jordi., "Monetary Policy and Rational Asset Price Bubbles" American Economic Review, 2014, 104(3):721-752.

Kahneman Daniel and Amos Tversky "Prospect Theory: An Analysis of Decision under Risk Econometrica Vol. 47, No. 2 (Mar., 1979), pp. 263-292

Kocherlakota, Narayana, 2009. "Bursting Bubbles: Consequences and Cures" Paper presented at the Macroeconomic and Policy Challenges Following Financial Meltdown Conference. Hosted by the IMF. Washington, DC- April 3, 2009.

Martin Alberto & Jaume Ventura, 2018. "**The Macroeconomics of Rational Bubbles: A User's Guide**," NBER Working Papers 24234, National Bureau of Economic Research, Inc.

Miao Jianjun and Pengfei Wang, "Asset Bubbles and Credit Constraints". forthcoming in The American Economic Review.

Samuelson, Paul A., 1958: "An Exact Consumption-Loan Model of Interest with or without the Social Contrivance of Money," Journal of Political Economy, 66 (6), 467–82.

Santos Manuel S. and Michael Woodford., "Rational Asset Pricing Bubbles" Econometrica, Vol. 65, No. 1 (Jan. 1997), pp. 19-57.

Robert J. Shiller, 2007. "**Understanding Recent Trends in House Prices and Home Ownership**," NBER Working Papers 13553, National Bureau of Economic Research, Inc. Published in Proceedings - Economic Policy Symposium - Jackson Hole, Federal Reserve Bank of Kansas City, pages 89-123.

_______ "Speculative Asset Prices" American Economic Review 2014, 104**(**6**)**: 1486–1517.

Scheinkman J.: "Note on Asset Trading in an Overlapping Generations Model," mimeo, 1980.

Tirole Jean., "Asset Bubbles and Overlapping Generations" Econometrica, Vol. 53, No. 6 (Nov., 1985), pp. 1499-1528.

Woodford, Michael, 1990. "**Public Debt as Private Liquidity**," American Economic Review, American Economic Association, vol. 80(2), pages 382-88, May.