Financial Conditions and Slow Recoveries

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Abstract

We argue that financial frictions and financial shocks can be an important factor behind the slow recoveries from the three most recent recessions. To illustrate this point, we augment a simple RBC model with a collateral constraint whose tightness is randomly disturbed by a shock that prescribes the general financial condition in the economy. We present evidence that such financial shock has become more persistent since the mid 1980s. We show that this can be an important contributor to the recent slow recoveries, and that a main mechanism may have to do with just-in-time-uses of capital and labor in the face of tight credit conditions during the recoveries. To assess the importance of such financial shock relative to other shocks in contributing to the slow recoveries, we enrich a New Keynesian model, which features various structural shocks and frictions widely considered in the literature, with the financial frictions and financial shocks studied in our parsimonious model. Our structural estimates of this comprehensive model indicate that financial shocks can play a dominant role in accounting for the slow recoveries, especially in employment growth rate.
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JEL classifications: E23, E32, E44, G01

Keywords: Collateral constraint; Financial shock; Slow recovery; Capital shortage; Extensive margin; Intensive margin

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1 Introduction

Recoveries from the three most recent recessions are slow, compared to the recoveries from other post-World War II recessions in the United States. This is illustrated in Figure 1, which plots the gross rates of output and employment growth from the NBER-dated trough in each of the past three (1991, 2001, and 2009) recessions and of the typical postwar recession prior to 1985 (taken to be the average of the recessions between 1967 and 1985) in the three years following the trough. As is evident from the figure, the three post-1985 recoveries are much slower and sluggish than the pre-1985 recovery, and the contrast is even more striking for employment (lower panel) than for output (upper panel).

![Figure 1. Pre- and Post 1985 recoveries](image)

Notes: The upper panel uses the Real GDP data from the Bureau of Economic Analysis. The lower panel uses the Civilian Employment data from the U.S. Bureau of Labor Statistics.
Figure 2 highlights such contrast between pre-1985 and post-1985 recoveries by displaying side-by-side across each sub-sample the average cumulative growth rate of output and of employment four (upper panel) and eight (lower panel) quarters into a recovery. As can be seen from the figure, the average output growth rate accumulated over four quarters following a trough is merely 3% in the post-1985 sample period, compared to more than 7% in the pre-1985 era (less than 7% versus more than 13% at an eight-quarter horizon); when we look at the average cumulative employment growth rate from the trough, we see an even more stark reduction across the two sub-sample periods: from 3% earlier to being slightly negative now at a four-quarter horizon (from 6% earlier to less than 1% now at an eight-quarter horizon).

![Figure 2. Cumulative growth after troughs](image)

It is worth noting that, in order to make a sensible comparison across recoveries from different business cycles, in both Figures 1 and 2, the data for each business cycle are indexed to the beginning of the recovery, that is, the trough. Indexing in this manner is useful not only because it helps isolate the comparison from the impact of potential long term factors, but also because the value of each indexed data point intuitively corresponds to the gross rate of growth in the underlying variable from the end of the relevant recession.
Several hypotheses have been proposed to explain the recent, slower recoveries. A just-
in-time-use-of-labor hypothesis (e.g., Schreft and Singh 2003; Hodgson, Schreft and Singh 2005) emphasizes the increased reliance of firms on adjustments along the intensive margin (versus the extensive margin) of labor inputs as a potential cause of the reduced speed of recoveries in employment growth rate. A recent empirical study (i.e., Panovska 2012) finds evidence in favor of this as opposed to some alternative hypotheses,¹ and suggests that changes in the relative importance of business-cycle shocks might have played a role in the increased importance of variations along the intensive margin of labor services.

The importance of various business-cycle shocks in accounting for the slower recoveries observed after 1985 is investigated by Gali, Smets and Wouters (2012) using a structural model, from which the intensive margin of labor adjustment is entirely abstracted away. Their main finding is that, for the recent three cyclical recoveries, the low growth rates of employment can be attributed entirely to the low growth rates of output, which are caused by relatively adverse shocks experienced during the recoveries. Of the eight shocks that they consider, investment-specific technology, risk premium, wage markup, and monetary policy shocks are shown to be the main factors behind the slow recoveries since 1985.

In another recent study, Jermann and Quadrini (2012) introduce a new type of business-cycle shocks into a structural framework with financial frictions in which firms’ ability to borrow is restrained by an enforcement (collateral) constraint. These “financial shocks” (labeled as such as they randomly disturb the value of firms’ collateral and thus firms’ ability to borrow) are shown to be a main driver of the three most recent recessions. The study fixes the intensive margin of labor services as in Gali, Smets and Wouters (2012), but does not take into account the labor supply shocks considered therein. And, most importantly for the purpose of motivating the present paper, it focuses on how adverse financial shocks may have contributed to the downturns in 1990-1991, 2001, and 2008-2009, but is salient about the potential implications of these shocks for the subsequent recoveries.

This paper studies such implications. Since the issue to be addressed here is why the pre-1985 recoveries were faster than the post-1985 recoveries, it is essential that we

examine both episodes rather than just the latter one that is the focus of Jermann and Quadrini (2012). Financial frictions and financial shocks are similarly introduced as in there, except that working capital loan is here modeled in a somewhat more conventional way, in that wage bills and purchases of investment goods must be paid at the beginning of each period, before production takes place and revenues are realized, whereas dividend and bond payments can be settled at the end of the period, after the realization of revenues. One of our main findings is that the financial shocks have become more persistent since 1985 and this is an important contributor to the slower recoveries during the post-1985 period.\(^2\) When we model working capital loan in the same way as in Jermann and Quadrini (2012), in that not only payments to workers and for purchases of investment goods but also payments to stockholders and bondholders must be made before production takes place and revenues are realized, our results become quantitatively less striking, though qualitatively similar.

We identify two channels through which a more persistent adverse financial shock originated in a recession can drag the subsequent recovery triggered by, say, a positive productivity shock. The first channel works in a relatively straight forward way: Since greater persistence of the financial shock implies that credit conditions are more likely to remain tight, at least in the early phase of the recovery, firms are more likely to continuously face difficulties in obtaining loans and thus be limited in their ability to increase labor and capital inputs to expand production. The growth rates of output and employment may recover more slowly as a result.

The second channel works through substitutions between adjustments along the two margins of labor and of capital inputs. This relates to another main finding of this paper: In the face of tight credit conditions, expanding along the extensive margins can be costly relative to expanding along the intensive margins. So in the early stage of the recovery, firms may rely more on increasing the utilization rate of capital and hours worked per employee, but less on growing employment and capital investment. The result is slow recovery in

\(^2\)Such a turning point may not come as a total surprise given the widespread financial innovations and deregulations since the 1980s, which have dramatically complicated the financial architecture, turning it into a somewhat shadow and opaque system with many loose links, and which have also made it hard for monetary policy (which itself has undergone a dramatic transformation in adapting to the changing financial world) to improve financial conditions during recessions and even recoveries. It is natural to take a longer time for an adverse financial shock to dissipate in such a more complex financial system. Corroborating evidence is also presented by Loutskina and Strahan (2009), Knotek and Terry (2009), and Hatzius, Hooper, Mishkin, Schoenholtz and Watson (2010).
employment growth rate accompanied by delayed investment and capital shortage in the early stage of the recovery phase. Whereas the substitution between the two margins of labor inputs echoes the just-in-time-use-of-labor hypothesis of Schreft and Singh (2003), Hodgson, Schreft and Singh (2005), and Panovska (2012), the substitution between the two margins of capital inputs finds its empirical support as is illustrated by Figure 3 that plots the cyclical components of capital utilization rate, investment, and capital stock for the US economy.\footnote{Starting with figure 3, in what follows, we will focus on cyclical components obtained by passing actual and simulated data through the Hodrick-Prescott filter with a smoothing parameter of 16000.}

As is seen from Figure 3, capital utilization rate usually increases as soon as a recovery begins, but investment rebounds only with a delay, while capital shortage can persist for an extended period into the recovery, and such contrasts are especially stark during the slower recoveries experienced in recent times when financial conditions are persistently tight.\footnote{The substitution between the two margins of capital inputs and its implication for capital shortage during a recovery have long been noted in the literature (e.g., Gertler and Hubbard 1989; Gertler and Gilchrist 1994; Kashyap et al. 1994). Several studies actually link such capital shortage to the persistently high long-term unemployment rate in Europe (e.g., Benassy 1999; Braumann 1997; Acemoglu 2001). When situated in this strand of the literature, a contribution of this paper is to demonstrate a transmission mechanism through which persistently tight financial conditions can intensify such substitution to cause severe capital shortage and slow recovery in employment and output growth from a recession.}

We use a parsimonious model with only financial and productivity shocks to help get a sense about the quantitative importance of the financial shocks and the relative contributions of the two channels. The time series for the shocks are here constructed following the same methodology used in the two-shock model of Jermann and Quadrini (2012) that however fixes the intensive margins of both labor and capital inputs. Since the shock series so constructed are independent of whether or not other shocks are also included in the model, their macroeconomic effects will not be overstated just because those other shocks are abstracted away from the model. Using the constructed series, we simulate the model, as well as its various versions in which the intensive margin of labor, or capital, or both labor and capital, is fixed. The simulation results show that the second channel is quantitatively important. Shutting down the intensive margins of capital and labor dramatically increases the employment growth rate following a trough for both the pre-1985 and post-1985 periods. However, the difference between the pre-1985 and the post-1985 employment growth rate still stands. It turns out that the first channel is crucial for explaining the difference between the pre-1985 and post-1985 employment growth rate.
Figure 3. Cyclical component (HP-16000), 1967-2012

Notes: The data for capital utilization come from the Board of Governors of the Federal Reserve System. The data for investment and capital stock come from the Flow of Funds Account and authors’ calculation. The shaded areas are NBER-dated recession bars.
When we divide the constructed series into two sub-series, one for the pre-1985 era and the other for the post-1985 episode, and use them to estimate two bivariate VAR processes respectively, we uncover statistically significant evidence that the financial shock has been more persistent after 1985 than it was before 1985. When we re-simulate the model, first using the VAR system estimated from the pre-1985 sub-series, and then using the VAR system estimated from the post-1985 sub-series, as a stochastic driving process, we find that recovery from a recession indeed is much slower in the latter case than in the former one, as is illustrated in Figure 4.

![4 quarter cumulative growth](image1)

![8 quarter cumulative growth](image2)

Figure 4. per-1985 and post-1985 recoveries

Taken together, we consider these results as providing strong evidence to suggest that not only the three recent episodes of considerable financial distress, namely, the S&L crisis in the late 1980s to the early 1990s, the dot-com bubble burst in the early 2000s, and especially most recently the more severe financial crisis that started in the mortgage markets in the late 2000s, contributed significantly to the downturns in 1990-1991, 2001, and 2008-2009, as demonstrated by Jermann and Quadrini (2012), but their adverse effects continued into the subsequent recoveries posting a major drag on employment and output growth.
In light of the findings by Gali, Smets and Wouters (2012), it is also fitting to assess the importance of financial shocks relative to other shocks in contributing to the recent slow recoveries. For this purpose, we enrich their framework, which is based on Gali, Smets and Wouters (2011) that features eight structural shocks and various frictions widely considered in the literature, with the financial frictions and financial shocks studied in our parsimonious model. Similar to their structural estimation approach, our richer model is also estimated with Bayesian maximum likelihood methods. Our main finding based on whole-sample estimates shows that financial shocks are the dominant contributor to the slow recoveries in employment growth rate, while for the slow recoveries in output growth rate technology shocks are also important. This main message remains unchanged when we re-estimate the model separately for two sub-sample periods. The sub-sample estimates assign some roles to wage-markup shocks for the slow recoveries in employment and output growth rates, and price-markup shocks for the slow recoveries in output growth rate, but these roles are much smaller than that played by financial shocks.

The rest of the paper is organized as follows. In section 2 we present our parsimonious model with only financial shocks and TFP shocks. In section 3 we calibrate the model using U.S. data from the first quarter of 1967 to the last quarter of 2012. In section 4 we demonstrate the quantitative results that show not only our model’s ability to generate the slower recoveries following the last three recessions, but also the quantitative significance of different factors of our model. In section 5 we provide a comprehensive model with nine shocks to check the quantitative contribution of financial shocks relative to other shocks to the slow recoveries following the post-1985 recessions. In section 6 we conclude.

2 The Parsimonious Model

We incorporate financial frictions and financial shocks as in Jermann and Quadrini (2012) into the framework of Burnside and Eichenbaum (1996). The later is a variant of Hansen’s (1985) indivisible labor model, modified to incorporate both variable capital-utilization rates and varying hours worked per worker. One feature of this model is that the units of effective labor input co-move positively with capital utilization because the two are complements in production. Financial frictions and financial shocks are similarly introduced as in Jermann and Quadrini (2012), except that working capital loan is here modeled in
a somewhat more conventional way, in that wage bills and purchases of investment goods
must be paid at the beginning of each period, before production takes place and revenues
are realized, whereas dividend and bond payments can be settled at the end of the period,
after the realization of revenues.

2.1 Environment

The model economy is populated by a continuum of homogeneous households. The total
number of households is normalized to 1. A representative household consists of a large
number of infinitely-lived individuals. To go to work, an individual incurs a fixed cost of ζ
hours. The time-\( t \) instantaneous utility function of such a person is given by \( \ln(c_t) + \theta \ln(T - ζ - h_t) \). Here, \( T \) denotes the individual’s time endowment, \( h_t \) denotes the total hours he
works, and \( c_t \) denotes time-\( t \) privately purchased consumption. The utility parameter \( \theta \) is
assumed to be positive. The time-\( t \) instantaneous utility of a person who does not go to
work is given by \( \ln(c_t) + \ln(T) \). At the end of period \( t - 1 \), a representative household
decides the number of its members that will go to work in period \( t \), \( n_t \). We will see that
this timing of labor supply is consistent with the firm’s timing of labor demand.

There is a continuum of firms, in the \([0, 1]\) interval. They have access to a common
production function

\[
y_t = z_t (k_t u_t)^\alpha (n_t h_t)^{1-\alpha}, \tag{1}
\]

where the capital share satisfies \( \alpha \in (0, 1) \), \( z_t \) represents the stochastic level of technology
common to all firms, \( k_t \) denotes the capital stock at the beginning of time \( t \), \( u_t \) represents
the capital utilization rate, and \( n_t \) denotes the number of individuals per household at
work at time \( t \). According to the production function, the production of output depends
on the total amount of effective capital, \( k_t u_t \), and the total effective hours of work, \( n_t h_t \). As
in Burnside and Eichenbaum (1996), equation (1) captures the notion that capital services
and labor input are complements in production.

We suppose that using capital more intensively increases the rate at which capital
depreciates. Specifically, we assume that the time-\( t \) depreciation rate of capital, \( \delta_t \), is given
by

\[
\delta_t = \delta u_t^\phi , \tag{2}
\]
where $0 < \bar{\delta} < 1$ and $\phi > 1$.\footnote{See Greenwood, Hercowitz and Huffman (1988) and Burnside and Eichenbaum (1996).} The stock of capital evolves according to

$$k_{t+1} = (1 - \delta_t)k_t + i_t,$$

where $i_t$ denotes gross investment at time $t$. Under these assumptions, firms can increase capital service without increasing capital stock.

Firms issue equity and debt. Debt, denoted by $b_t$, is preferred to equity because it has a tax advantage. Given the interest rate $r_t$, the effective gross interest rate for the firm is $R_t = 1 + r_t(1 - \tau)$, where $\tau$ represents the tax benefit. This tax advantage is financed by a lump-sum tax on households.

In addition to the intertemporal debt, $b_t$, firms raise funds with an intraperiod loan, $l_t$, to finance the total wage payment, $w_t n_t h_t$, and the investment, $i_t$, at the beginning of period $t$. That is, $l_t \geq w_t n_t h_t + k_{t+1} - (1 - \delta_t)k_t$, where $w_t$ is the wage rate. The intraperiod loan is repaid at the end of the period, and there is no interest. This specification is shared by most of the literature with working capital, but it is different from Jermann and Quadrini (2012), who also includes the dividend, $d_t$, and the intertemporal debt, $b_t$, in the working capital. We assume that $d_t$ and $b_t$ can be paid after the time-$t$ revenue is realized.

The ability to borrow (intra- and intertemporally) is bounded by the limited enforceability of debt contracts, as firms can default on their obligations. Following Kiyotaki and Moore (1997), we assume that the only asset available for liquidation is the physical capital, $k_{t+1}$. In other words, $k_{t+1}$ is the collateral for the debt $b_{t+1}/R_t$ and the intraperiod loan $l_t$. In the literature that follows Kiyotaki and Moore (1997), it is often assumed that only a proportion of the collateral can be liquidated to cover the debt. Their focus is often on the role of financial frictions in the propagation of TFP shocks. Here, following Jermann and Quadrini (2012), we model an exogenous financial shock. Specifically, we assume that at the moment of contracting a loan the liquidation value of physical capital is uncertain. With probability $\xi_t$, the lender can recover the full value $k_{t+1}$, but with probability $1 - \xi_t$, the recovery value is zero. As in Jermann and Quadrini (2012), we define the borrowing constraint by

$$\xi_t(k_{t+1} - b_{t+1}/(1 + r_t)) \geq l_t.$$  \hspace{1cm} (3)

Higher debt, either intertemporal or intratemporal, makes the borrowing constraint tighter. On the other hand, a higher stock of capital relaxes the enforcement constraint. The
probability $\xi_t$ is stochastic and depends on market conditions. Because this variable affects the tightness of the borrowing constraint, we refer to its stochastic innovations as "financial shocks." Notice that $\xi_t$ is common to all firms.

As shown, we have two sources of aggregate uncertainty: productivity, $z_t$, and financial, $\xi_t$. Since we do not have idiosyncratic shocks, we will concentrate on the symmetric equilibrium where all firms are alike. The exogenous shocks are assumed to follow a VAR process with one lag.

As in Jermann and Quadrini (2012), we use a quadratic cost function for the dividend adjustment

$$\varphi(d_t) = d_t + \kappa(d_t - \bar{d})^2,$$

where $\kappa \geq 0$, and $\bar{d}$ is the long-run dividend payout target (steady state). This form of adjustment cost captures the preferences of managers for dividend smoothing (Lintner 1956). The parameter $\kappa$ is key for determining the impact of financial shocks. When $\kappa > 0$, the adjustment of equity is costly. As a result, financial shocks will have non-negligible short-term effects on the production decision of firms.

To see more clearly how $\xi_t$ affects the financing and production decisions of firms, we rewrite the borrowing constraint (3), using the firm’s budget constraint

$$w_t n_t h_t + k_{t+1} - (1 - \delta_t) k_t + b_t + \varphi(d_t) = y_t + b_{t+1}/R_t. \quad (4)$$

That is,

$$\xi_t \left[ z_t (k_t u_t)^\alpha (n_t h_t)^{1-\alpha} + (1 - \delta_t) k_t - b_t - w_t n_t h_t - \varphi(d_t) \right] \geq l_t.$$

For demonstration purpose, we have assumed that $\tau = 0$. Since $k_t$, $n_t$, and $b_t$ are given at the beginning of period, the variables that are under the control of the firm are the hours worked per worker, $h_t$, the investment, $i_t$, the capital utilization, $u_t$, and the equity payout, $d_t$. The firm can vary these variables to satisfy a binding borrowing constraint. In contrast, in the model of Jermann and Quadrini (2012), the only variables that a firm can use to relax a tight borrowing constraint are the input of labor, $n_t$, and the equity payout, $d_t$.

If we start from a pre-shock state in which the borrowing constraint is binding and the firm wishes to keep the production plan unchanged, a negative financial shock (lower $\xi_t$) requires a reduction in the equity payout $d_t$. However, in our model, if the firm cannot reduce $d_t$, it has to cut $h_t$ and $i_t$, or increase $u_t$. Given that the labor service and the capital service are complements in production, a negative financial shock may induce the firm to
reduce both some labor service and some capital service. For labor service, it reduces \( h_t \). For capital service, it can reduce \( u_t \). Alternatively, to minimize this reduction and keep the current production plan, the firm can also reduce more of its investment, \( i_t \), at the expense of future production. But this later plan will lower the capital stock \( k_{t+1} \), and a persistent low profile of capital will keep the borrowing constraint persistently tight. The firm will find it difficult to hire more workers when the collateral is low and the financial conditions stay poor. With the same logic, even if the TFP recovers from a recession and the output increases, poor financial conditions may cause the employment to stay at a relatively low level for a longer period, associated with capital shortage.

### 2.2 Firm’s Problem

To capture the ideas that firms must make employment decisions conditional on their expectations on the future states of demand and technology, and that firms cannot adjust the number of employees instantly in response to the aggregate shocks, we let firms choose their \( n_t \) in the end of period \( t-1 \). The recursive formulation of the firm’s problem is as follows. Here we skip the subscript \( t \) in order to simplify the notations. The individual states are the capital stock, \( k \), the number of workers, \( n \), and the debt, \( b \). The aggregate states are TFP shock and financial shock, denoted by \( s \). The optimization problem is

\[
V(s; k, n, b) = \max_{h, u, n+1, k+1, b+1, d} \{ d + E m_{+1} V(s_{+1}, k_{+1}, n_{+1}, b_{+1}) \} \tag{5}
\]

subject to the budget constraint

\[
b + w n h + k_{+1} + \varphi(d) = (1 - \delta) k + z(ku)^\alpha (nh)^{1-\alpha} + b_{+1}/R, \tag{6}
\]

and the borrowing constraint

\[
\xi (k_{+1} - b_{+1}/(1 + r)) \geq w n h + k_{+1} - (1 - \delta) k. \tag{7}
\]

The function \( V(s; k, n, b) \) is the market value of the firm in terms of its cumulative dividend, and \( m_{+1} \) is the stochastic discount factor. The stochastic discount factor, the wage rate, and the interest rate are determined in the general equilibrium and are taken as given by an individual firm.

Denoting by \( \lambda \) and \( \mu \) the Lagrange multipliers associated with the budget constraint and the borrowing constraint, respectively, the first-order conditions for \( h, u, n_{+1}, k_{+1}, b_{+1} \) and \( d \) are
\[ (1 - \alpha) \frac{y}{nh} = (1 + \frac{\mu}{\lambda})w, \quad (8) \]
\[ \alpha \frac{y}{k} = (1 + \frac{\mu}{\lambda})\phi \delta, \quad (9) \]
\[ Em_{+1}\lambda_{+1} \left[ (1 - \alpha) \frac{y_{+1}}{n_{+1}} - (1 + \frac{\mu_{+1}}{\lambda_{+1}})w_{+1}h_{+1} \right] = 0, \quad (10) \]
\[ Em_{+1} \frac{\lambda_{+1}}{\lambda} [(1 - \delta_{+1}) + \alpha \frac{y_{+1}}{k_{+1}} + \frac{\mu_{+1}}{\lambda_{+1}}(1 - \delta_{+1})] + \frac{\mu}{\lambda} \xi = 1 + \frac{\mu}{\lambda}, \quad (11) \]
\[ REm_{+1} \frac{\lambda_{+1}}{\lambda} + \frac{\mu}{\lambda} \xi \frac{R}{1 + r} = 1, \quad (12) \]

where
\[ \lambda = \frac{1}{\varphi_d(d)} = \frac{1}{1 + 2\kappa(d - d)}, \quad (13) \]

First, we try to establish the relationship between \( \xi \) and the multiplier \( \mu \). It will be convenient to consider the special case in which the cost of equity payout is zero, that is, \( \kappa = 0 \). In this case, \( \lambda = \lambda_{+1} = 1 \) according to equation (13); equation (12) becomes \( REm_{+1} + \mu \xi \frac{R}{1 + r} = 1 \). Taking as given the aggregate prices \( R, r, \) and \( Em_{+1} \), it implies that there is a negative relationship between \( \xi \) and \( \mu \). In other words, a negative financial shock gives a higher \( \mu \), meaning that the borrowing constraint is tighter.

The optimality condition for hours worked per worker is expressed in equation (8). Equation (8) indicates that the marginal productivity of labor is equalized to the marginal cost. The marginal cost is the wage rate augmented by a wedge that depends on the tightness of the borrowing constraint, that is, \( \mu \). A tighter borrowing constraint increases the effective cost of labor and reduces its demand.

Equation (9) is the optimality condition for capacity utilization. The capacity utilization increases with TFP and decreases with capital stock, given that \( \phi > \alpha \). Moreover, a tighter borrowing constraint also reduces the capital utilization. That is because the firm would like to save some capital in order to relax the borrowing constraint in the next period. As a result, the capital utilization decreases immediately during a recession (with both negative TFP shock and negative financial shock). This mechanism is reinforced when \( \kappa > 0 \). In this case, it will be costly to readjust the dividend payment, and the change in \( \xi \) induces a larger movement in \( \mu \).
2.3 Household’s Problem and General Equilibrium

The representative household maximizes the expected lifetime utility

$$\max E_0 \sum_{t=0}^{\infty} \beta^t [\ln(c_t) + \theta n_t \ln(T - \zeta - h_t) + \theta (1 - n_t) \ln(T)],$$

(14)

where $\beta$ is the discount factor. Households are the owners of firms. In addition to equity shares, they hold non-contingent bonds issued by firms. The household’s budget constraint is

$$c_t + \frac{b_{t+1}}{1 + r_t} + x_{t+1} p_t + \Upsilon_t = w_t n_t h_t + b_t + x_t (d_t + p_t),$$

(15)

where $x_t$ is the equity shares, $p_t$ is the market price of shares, and $\Upsilon_t = B_{t+1}/[1 + r_t(1 - \tau)] - B_{t+1}/[1 + r_t]$ are the lump-sum taxes that finance the tax benefit of debt for firms. The first order conditions with respect to $h_t$, $n_{t+1}$, $b_{t+1}$, and $x_{t+1}$ are

$$\frac{w_t}{c_t} - \frac{\theta}{T - \zeta - h_t} = 0,$$

(16)

$$E_t \left[ \frac{w_{t+1} h_{t+1}}{c_{t+1}} + \theta \ln(T - \zeta - h_{t+1}) - \theta \ln(T) \right] = 0,$$

(17)

$$\beta (1 + r_t) E_t \frac{c_t}{c_{t+1}} = 1,$$

(18)

and

$$\beta E_t \frac{c_t}{c_{t+1}} \frac{d_{t+1} + p_{t+1}}{p_t} = 1.$$

(19)

Recall that $n_t$ is determined at the end of period $t-1$. In period $t$, the household makes a plan for $n_{t+1}$ according to its expectation on the future wage $w_{t+1}$ and the corresponding demand for $h_{t+1}$. Equations (18) and (19) determine the interest rate and share price, respectively.

Firm’s optimization is consistent with household’s optimization. Therefore, the stochastic discount factor is $m_{t+j} = \beta^i c_t/c_{t+j}$.

The definition of a general equilibrium is provided below. The aggregate states $s$ are the productivity $z$, the financial shock $\xi$, the aggregate capital $K$, and the aggregate bond $B$. The total share is normalized to 1.

**Definition 1** A recursive competitive equilibrium is defined as a set of functions for (i) household’s policies $c^h(s)$, $n^h_{t+1}(s)$, $h^h(s)$, and $b^h_{t+1}(s)$; (ii) firm’s policies $d(s; k, b)$, $h(s; k, b)$,
Now we can get better insight into why our model can explain capital shortage and slow recovery together. At the very beginning of a recovery, the financial conditions are still tight, i.e., $\xi_t$ is small. As TFP improves, a firm’s demand for labor input and capital services (capital utilization) increases, as shown in equations (8) and (9). Given that the borrowing constraint is binding, in order to increase output in response to a higher TFP a firm may increase the total wage payment $w_t n_t h_t$ at the cost of decreasing investment $i_t$, since the investment $i$ does not increase the current capital service. This reduction of $i_t$ causes a low level of capital stock in the future periods, which means a low level of collateral and a persistently tight borrowing constraint, until the financial conditions improve dramatically. As a consequence, employment can continue to decline even after the TFP and output have begun to increase. This mechanism shows that a slow improvement in financial conditions can cause a longer capital shortage, leading to a slow recovery.

3 Calibration

The parameters can be divided into two groups. The first group includes parameters that can be calibrated using steady state targets, some of which are typical in the business cycle literature. The second group includes parameters that cannot be calibrated using steady state targets. For these parameters, we use numerical methods. In order to solve the numerical model, we log-linearize the model, which is provided in Appendix A.

The period in the model is a quarter. The data we used for calibration are the empirical series from 1967:I to 2012:IV in the United States. For the series of capital, $k_{t+1}$, depreciation, $\delta_t$, and debt, $b_{t+1}/(1+r_t)$, we use end of period balance sheet data from the Flow of Funds Accounts. For the output, $y_t$, we use the GDP data from the Bureau of Economic Analysis. For $w_t$, $n_t$, $h_t$, and $u_t$, we use data from Federal Reserve Bank of St.
Louis. All the series are in real terms. A more detailed description of data is provided in Appendix B.

**Parameters set with steady state targets.** — We set $\beta = 0.9825$, $\tau = 0.35$, and $\alpha = 0.36$. The discount factor $\beta = 0.9825$ implies that the annual steady state return from holding shares is 7.12%, according to equation (19). The tax wedge $\tau = 0.35$ corresponds to the benefit of debt over equity if the marginal tax rate is 35%.$^6$ This parameter determines whether the borrowing constraint is binding. As we will see, with this value of $\tau$, the borrowing constraint is always binding in our simulation. We examine it by checking the Lagrange multiplier $\mu_t$. The Cobb-Douglas production function capital share $\alpha = 0.36$ is standard in the literature. The mean of productivity $\bar{z}$ is normalized to 1. The Utility parameter is set to $\theta = 2.5702$ so that the steady state employment rate is $n = 0.94$.

We set $T = 1369$ and $\zeta = 60$ following Burnside and Eichenbaum (1996), who examined the reasonable range of the value of $\zeta$ to be between 20 and 120 when $T = 1369$. The value of $\delta$ matters only in $\delta\omega$, so we only need the steady state value of the depreciation rate $\delta$. We take $\delta = 0.0250$, implying an annual depreciation rate of 10%, which is standard in the business cycle literature.

The value of $\bar{\xi}$ is chosen to have a steady state ratio of debt over GDP equal to 3. This is the average ratio over our sample period. In the steady state, $\lambda = 1$ is implied by equation (13). We use equation (18) to get the steady state value of interest rate $r$, $r = 1/\beta - 1$. Then, $R = 1 + (1/\beta - 1) (1 - \tau)$. Using the firm’s borrowing constraint (7), steady state condition $i = \delta k$, and the first order condition for $h$ (8) in the steady state, we get

$$\bar{\xi} \left( \frac{k - b}{y} \frac{1}{y + r} \right) = \frac{1 - \alpha}{1 + \mu} + \frac{\delta k}{y}. \quad (20)$$

Using the first order condition (11) for $k_{+1}$ in the steady state, we can express the steady state ratio $k/y$ by the following equation,

$$\frac{y}{k} = \frac{(1 + \mu) (1 - \beta (1 - \delta)) - \mu \bar{\xi}}{\beta \alpha}. \quad (21)$$

Using the first order condition (12) for $b_{+1}$ in the steady state, we get

$$R \beta (1 + \mu \bar{\xi}) = 1. \quad (22)$$

Using the equations (20), (21), (22), and the average ratio $b/y = 3$ over our sample period 1961:I-2012:IV, we solve for the parameters $\bar{\xi}$ together with the multiplier $\mu$.

$^6$We take a value of $\tau = 0.35$ following Jermann and Quadrini (2012).
The value of \( \phi \) is calibrated through the first order condition (9) for \( u \) in the steady state,

\[
\phi = \frac{\alpha}{(1 + \mu) \delta k/y}. \tag{23}
\]

<table>
<thead>
<tr>
<th>Description</th>
<th>Parameter/value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount factor</td>
<td>( \beta = 0.9825 )</td>
</tr>
<tr>
<td>Utility parameter</td>
<td>( \theta = 2.5702 )</td>
</tr>
<tr>
<td>Tax advantage</td>
<td>( \tau = 0.3500 )</td>
</tr>
<tr>
<td>Capital share</td>
<td>( \alpha = 0.3600 )</td>
</tr>
<tr>
<td>Depreciation</td>
<td>( \delta = 0.0250 )</td>
</tr>
<tr>
<td>Utilization para.</td>
<td>( \phi = 1.5935 )</td>
</tr>
<tr>
<td>Time endowment</td>
<td>( T = 1369 )</td>
</tr>
<tr>
<td>Fixed cost.</td>
<td>( \zeta = 60 )</td>
</tr>
<tr>
<td>Payout cost</td>
<td>( \kappa \delta = 0.0135 )</td>
</tr>
<tr>
<td>Ave. finan. con.</td>
<td>( \bar{\xi} = 0.1325 )</td>
</tr>
<tr>
<td>Stan. dev. of ( z_t )</td>
<td>( \sigma_z = 0.00668 )</td>
</tr>
<tr>
<td>Stan. dev. of ( \xi_t )</td>
<td>( \sigma_{\xi} = 0.02015 )</td>
</tr>
<tr>
<td>Matrix for shocks</td>
<td>( A = \begin{bmatrix} 0.853 &amp; 0.0018 \ 0.0309 &amp; 0.914 \end{bmatrix} )</td>
</tr>
</tbody>
</table>

Parameters set through numerical methods. — The parameters to be calibrated using numerical methods are those determining the stochastic process of shocks, and the cost of equity payout — the parameter \( \kappa \).

For the productivity variable \( z_t \) we use series estimated by Fernald (2012), which uses the same method as in Basu, Fernald, and Kimbal (2006). To construct the series for the financial variable \( \xi_t \), we follow a similar approach as in Jermann and Quadrini (2012). Using the borrowing constraint under the assumption that it is always binding, that is,

\[
\xi_t (k_{t+1} - b_{t+1}/(1 + r_t)) = w_t n_t h_t + i_t,
\]

we derive a similar "residual" \( \hat{\xi}_t \) and use the relevant data to construct the \( \hat{\xi}_t \) series (see appendix B for details).

After constructing the series for the productivity and the financial conditions over the period 1967:I-2012:IV, we estimate the autoregressive system

\[
\begin{pmatrix} \hat{z}_{t+1} \\ \hat{\xi}_{t+1} \end{pmatrix} = A \begin{pmatrix} \hat{z}_t \\ \hat{\xi}_t \end{pmatrix} + \begin{pmatrix} \epsilon_{z,t+1} \\ \epsilon_{\xi,t+1} \end{pmatrix}, \tag{24}
\]
where $\epsilon_{z,t+1}$ and $\epsilon_{\xi,t+1}$ are i.i.d. shock to TFP and financial condition, respectively. We assume that $\epsilon_{z,t}$ and $\epsilon_{\xi,t}$ have zero mean and standard deviation $\sigma_z$ and $\sigma_{\xi}$, respectively.

Finally, using the stochastic series for $\hat{z}_t$ and $\hat{\xi}_t$ generated by (24) and the parameters identified above, together with an initial guess of the value of $\kappa \tilde{d}$, we can simulate the model. The simulation generates a series of $d_t/y_t$. We check whether the standard deviation of $d_t/y_t$ generated by the simulated model equal to the counterpart in the data. If they are different, we vary the value of $\kappa \tilde{d}$ until they coincide. The set of parameters are provided in Table 1.

4 Quantitative Results

4.1 Financial shocks

Before we show the simulated results, we provide some sense of our financial shock. According to our model, our financial shocks should track the indicators of credit tightness. The Federal Reserve Board conducts a survey among senior loan officers of banks (Senior Loan Officer Opinion Survey on Bank Lending Practices) asking whether they have tightened the credit standards for commercial and industrial loans during the survey period. Based on the survey results, the Board constructs an index of credit tightness measured by the percentage of officers who tightened the standards. This index is a measure of the changes in the credit standard and it has a similar interpretation as the changes in the variable $\xi_t$ from our model. A proxy for the changes in $\xi_t$ is given by the innovation $\epsilon_{\xi,t}$. Therefore, in the model we can define the index of credit tightening as the negative of $\epsilon_{\xi,t}$.

![Figure 5. Financial shocks and survey indices](image)

Figure 5 plots the tightness indices constructed from the model and from the survey. To facilitate the comparison we have re-scaled the survey index by a factor of $1/1473$. The
survey of senior loan officers is available starting in the second quarter of 1990. As can be seen, our measure of credit tightness tracks reasonably well the survey index during the period when the survey index became available. In particular, we see sharp increases in both indices during the last three recessions.

4.2 Simulation with binding borrowing constraint

To study the dynamics of the model induced by the constructed series of shocks, we conduct the following simulation. Starting with initial values of \( \tilde{z}_{1967,1} \) and \( \hat{\xi}_{1967,1} \), we feed the innovation into the model and compute the responses of the variables of capital and labor. We have assumed that the borrowing constraint is always binding. Figure 6 reports the Lagrange multiplier for the borrowing constraint \( \mu_t \). The negative deviations of this variable from the steady state never exceed -100 percent, implying that the multiplier is always positive. Therefore, we can verify that the borrowing constraint is always binding during the simulation period.

![Figure 6. Lagrange multiplier \( \mu \)](image)

4.3 Results and the role of model elements

Our main purpose is to examine the model’s ability to generate the slower recoveries from the three most recent recessions, compared to the recoveries from other post-World War II recessions, and to identify the role of the model’s elements in generating the recent slow recoveries. More precisely, we should do two sub-sample estimations if we suspect a structural change in the financial shock process. But before doing that, we would like to check on the whole sample results if we do not eye ball a sharp change in the persistence
and volatility of financial shocks as shown in figure 5. Of course, potentially our whole sample estimates could be biased.

4.3.1 Benchmark model result

We simulate our benchmark model and use the model generated data series of output and employment to compute the average growth rate accumulated over 8 quarters following a trough for each variable in the pre-1985 sample period and the post-1985 sample period, respectively. The results are shown in Figure 7. The average output growth rate accumulated over 8 quarters following a trough is about 6% (1.8%) from our model, and about 6.5% (1.5%) in the data, in the pre-1985 (post-1985) sample period. Our model not only generates a sharp difference of the average output growth rate accumulated over 8 quarters following a trough between in the pre-1985 period and in the post-1985 period, but also match the model with the data in the magnitude of the average output growth rate accumulated over 8 quarters after a trough in both the pre-1985 and post-1985 periods.

Same for the employment, our model can generate a sharp contrast between the pre-1985 and the post-1985 average employment growth rate accumulated over 8 quarters following a trough. However, the average employment growth rate accumulated over 8 quarters following a trough generated from our model is systematically higher than that from the data in both the pre-1985 sample period and the post-1985 sample period. This could be due to ignoring the labor market frictions in our model. We will show how labor adjustment cost may refine our results in a later subsection.

Figure 7. Data versus benchmark model
4.3.2 The role of shocks to financial conditions

In order to see the role of financial shocks played in changing the recovery speed before and after 1985, we examine whether there is a structural break of the financial shocks in the middle of 1980s. Using the data before and after 1985 respectively, We run VAR for TFP and the financial condition variable recovered from the borrowing constraint. The autoregression matrix before 1985 is \[
\begin{pmatrix}
0.843 & 0.053 \\
-0.019 & 0.887
\end{pmatrix},
\] and after 1985 is \[
\begin{pmatrix}
0.862 & -0.003 \\
0.156 & 0.931
\end{pmatrix}.
\]

We observe that the financial conditions become more persistent after 1985. We verify our postulation by using the Chow test for parameter stability. Using the likelihood ratio test we reject the null hypothesis that the parameters are stable before and after 1985 with a p-value of 0.014.

It would be ideal if we can check this persistence change with the data from the survey mentioned above since the concept of our financial shock is close to that in the survey. That is the tightness of borrowing facing a firm. Unfortunately, the survey data cover only the recent three recessions. Alternatively, we do some check using data from the Chicago Fed, who constructed a National Financial Condition Index using a large range of indicators, including but not restricted to different types of interest rate spreads, exchange rates, and different survey data. The Senior Loan Officer Opinion Survey on Bank Lending Practices that we used above is one out of the hundreds of indicators included in the Chicago Fed financial index. So we use these data loosely to check the change of financial conditions.

We do the similar exercise here as we did for the financial condition variable recovered from our borrowing constraint. It shows that the autoregression matrix before 1985 is \[
\begin{pmatrix}
0.816 & 0.030 \\
0.127 & 0.811
\end{pmatrix},
\] and after 1985 is \[
\begin{pmatrix}
0.866 & -0.037 \\
0.158 & 0.854
\end{pmatrix}.
\]

Although the autoregression coefficient for the financial condition is relatively low in level compared to the financial variable recovered from our model, there is a significant increase in persistence after 1985, from 0.811 to 0.854.

The next question is how this change in persistence affects the behavior of recovery? In order to answer this question we do some counterfactuals. We use a random number generator to generate 3200 periods of TFP and Financial shocks according to the autoregression coefficients and standard deviations recovered from our model in periods before 1985 and after 1985, respectively. We feed these shocks into our model and obtain simulated series of employment and output. Then, we test the difference between the two simulated models.
using shock process before 1985 and after 1985 respectively. We pick 100 points that have the lowest level of output and mark them as troughs (a business cycle lasts about 32 periods or 8 years). We compute the 4 quarter (and 8 quarter) cumulative employment (and output) growth rates following these marked troughs. We find that the model generates a slower recovery of employment and output employing the after 1985 shock process than employing the before 1985 shock process, using a student-t test on the mean difference of the 4 quarter (and 8 quarter) cumulative employment (and output) growth rates. The results are reported in Table 2. The significant difference indicates that a slightly more persistent financial shock process can have important role in explaining the slower recovery after 1985.

<table>
<thead>
<tr>
<th></th>
<th>Pre- 1985</th>
<th>Post- 1985</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Employment 4 quarter</td>
<td>0.0223</td>
<td>0.0135</td>
<td>0.0089***</td>
</tr>
<tr>
<td>Employment 8 quarter</td>
<td>0.0337</td>
<td>0.0248</td>
<td>0.0089***</td>
</tr>
<tr>
<td>Output 4 quarter</td>
<td>0.0345</td>
<td>0.0203</td>
<td>0.0142***</td>
</tr>
<tr>
<td>Output 8 quarter</td>
<td>0.0491</td>
<td>0.0491</td>
<td>0.0162***</td>
</tr>
</tbody>
</table>

It is intuitive that a more persistent adverse financial shock originated in a recession can drag the subsequent recovery triggered by a positive productivity shock. This is because greater persistence of the financial shock implies that credit conditions are more likely to remain tight, at least in the early phase of the recovery, so firms are more likely to continuously face difficulties in obtaining loans and thus be limited in their ability to increase labor and capital inputs to expand production. The growth rates of output and employment may recover more slowly as a result. We call this channel the direct channel through which a more persistent adverse financial shock works to generate a slower recovery.

Next, we examine whether the model also embodies some transmission mechanism that can propagate a more persistent adverse financial shock originated in a recession to generate a slower recovery.

4.3.3 Transmission mechanism

The mechanism we check here is the substitutions between adjustments along the two margins of labor and of capital inputs. First, we construct a model that shuts down the
intensive margin of capital, in which the capital utilization rate is always 1. The production function becomes

\[ y_t = z_t k_t^\alpha (n_t h_t)^{1-\alpha}. \]

Second, we construct a model that shuts down the intensive margin of labor. Third, we construct a model that shuts down both the intensive margin of capital and the intensive margin of labor. Everything else is identical to our benchmark model. We re-calibrate the models using the same data and the same procedure as in our benchmark model. Then we compare the simulated outcomes from our benchmark model with those from the modified models. The results of these modified models are shown in figure 8.

![Figure 8. Employment recoveries - fixed intensive margin(s)](image)

We can read two messages from figure 8. First, all the three modified models generate much faster growth of employment during the early recoveries than the data do. Second, the modified models can still generate the sharp difference between the pre-1985 and the post-1985 average employment growth rate accumulated over 8 quarters from a trough. These messages confirm our hypothesis that allowing firms to choose both extensive and intensive margin of capital and labor delays the growth of employment during the early recoveries, but the driving force of the contrast of employment growth rate over 8 quarters accumulated from a trough before and after 1985 may be the financial shocks.

The reason that allowing firms to choose both extensive and intensive margin of labor delays the growth of employment during the early recoveries is two fold. First, since the firms make employment decision before the aggregate TFP and financial shocks are realized, but make the decision for hours per worker after the shocks are realized, the firms can use a provident plan of employment to cope with uncertainty. That is, the firms can use less workers if they can increase hours per worker after the shocks are realized.
than in the case where they cannot vary hours per worker. Second, since a negative TFP (financial) shock lowers the optimal (wage payable) number of hours per worker given a predetermined number of employed workers, if firms expect a low level of total working hours given the possible shocks, they can expect a lower wage if they use a relatively small number of workers but a relatively large number of hours per worker. This is because an additional worker incurs the household a fixed cost $\zeta$ and an additional hour of a working worker incurs the household disutility that is convex.

Incorporating capital utilization rate, our model weakens the dependence of the effective use of capital in production on the amount of capital stock. As a result, the marginal benefit of investment is lower than that in the case where capital utilization rate is shut down. Note that the marginal benefit of investment contains two parts in our model, the marginal product of capital and the marginal value from relaxing the borrowing constraint. This is due to the dual role of capital as both a factor in production and a collateral for borrowing. The second role of capital as a collateral for borrowing in our benchmark model is the same as in the modified model without capital utilization rate, but the first role of capital as a factor in production is different in the two models. If financial conditions are poor and firm’s borrowing constraint is tight, to respond to an increase in TFP, the firms would like to increase capital service by increasing capital utilization rate if this is allowed. Although they expect that the TFP shocks are persistent, they have less incentive to increase investment if they can increase capital utilization rate than in the case where capital utilization rate is fixed. This parsimonious decision on investment will, however, make the borrowing constraint persistently binding and leading to a low recovery.

4.3.4 Alternative financial constraint

In order to well understand the channel through which financial friction works, we construct a modified model with a different borrowing constraint,

$$\xi_t (k_{t+1} - b_{t+1}/(1 + r_t)) \geq y_t.$$ 

This alternative setup of the borrowing constraint is similar to that in Jermann and Quadrini (2012). As mentioned above, compared to this alternative setup of borrowing constraint, our setup of borrowing constraint rules out the possibility that the firm can vary it’s dividend payment to relax the borrowing constraint. This point will become
apparent if we substitute firm’s budget constraint into it’s borrowing constraint. In our benchmark model, the firm has to reduce its wage payment if it increases its current flow of capital, and vice versa. We named this modified model as alternative financial constraint. The result of this model is shown in figure 9. The alternative financial constraint generates much faster growth of employment during recoveries than the data do in both the periods before and after 1985. However, as we can see from the figure 9, the alternative model can still generate the sharp decline in the speed of recovery after 1985. This confirms our conjecture that a more persistent financial shock can be a driving force of the recent slow recoveries. The different setup of the financial constraint may reinforce or weaken the transmission mechanism via the second channel.

![Figure 9. Employment recoveries - alternative financial constraint](image)

4.3.5 Labor adjustment cost

Now we incorporate labor adjustment cost into our benchmark model and the various modified models analyzed above. Following the literature (Wen (2004)), we assume that a firm needs to pay \( \frac{\nu}{2} (n_t - n_{t-1})^2 k_t \) in order to adjust employment from \( n_{t-1} \) to \( n_t \). The budget constraint (4) becomes

\[
wt n_t b_t + k_{t+1} - (1 - \delta_t) k_t + b_t + \varphi(d_t) = y_t - \frac{\nu}{2} (n_t - n_{t-1})^2 k_t + b_{t+1}/R_t.
\] (25)

We choose a value of \( \nu, \nu = 0.025 \), such that in the benchmark model the model’s prediction of the average employment growth rate accumulated over 8 quarters following a trough in the pre-1985 period equals to that in the data.

The results of our model with labor adjustment cost is shown in Figure 10. The output growth rate is not affected much, while the employment growth rate accumulated over
8 quarters following a trough in the period post-1985 declines relative to our benchmark model and overshoots a little bit relative to the data. The results show that labor adjustment cost can systematically reduce the growth rate of employment, but it can not change the pattern of output growth nor be responsible for the sharp contrast between the pre-1985 and post-1985 employment growth rate.

![Figure 10. Data vs. benchmark model augmented with labor adjustment cost](image1)

We use the same target, the average employment growth rate accumulated over 8 quarters following a trough in the pre-1985 period, to recalibrate the various modified models analyzed above. The results of the models that fix the intensive margin(s) are shown in Figure 11. The figure shows that even with the added labor market frictions, it is unlikely for the modified models to generate sufficiently low speed of employment growth during the post-1985 recoveries.

![Figure 11. Fixed intensive margin(s) and labor adjustment cost](image2)

The result of the model with alternative financial constraint and labor adjustment cost is shown in Figure 12. This modified model generates an employment growth rate accumulated over 8 quarters into a post-1985 recovery that is 70% closer to the data, compared to...
the case absent any labor adjustment cost. However, compared to our benchmark model with labor adjustment cost, a much greater magnitude of labor adjustment cost is needed in this modified model in order to match the average employment growth rate accumulated over 8 quarters following a trough in the pre-1985 period.

Figure 12. Alternative financial constraint and labor adjustment cost

### 4.3.6 Investment adjustment cost

As another robustness check, we incorporate into our model investment adjustment cost so that,

\[ k_{t+1} = (1 - \delta_t)k_t + \varrho \left( \frac{i_t}{k_t} \right)^{\psi} k_t. \]

We calibrate the parameters, \( \varrho \) and \( \psi \), so that the steady state depreciation rate is \( \delta \) and the standard deviation of capital matches that in the data. It turns out that incorporating investment adjustment cost does not affect our main results, although it does affect the standard deviation of the cyclical component of capital stock.

### 5 Comprehensive Model

In this section we estimate a comprehensive model with financial and eight other shocks using the Bayesian dynamic stochastic general equilibrium approach. The purpose is to investigate the effects of financial shocks in explaining the observed employment patterns during cyclical recoveries. Based on the model of Gali, Smets and Wouters (2012), which includes eight shocks — productivity, investment, risk premium, wage mark-up, labor supply, price mark-up, government spending, and monetary policy, we also add in our model a financial shock, as in Jermann and Quadrini (2012) which however abstracts from
the labor supply shock (and involuntary unemployment). The estimation of our model uses nine empirical time series — GDP, investment, labor force, working hours, wage rate, federal fund rate, government spending, nominal prices, and debt purchases.

5.1 The Model
5.1.1 Household sector

There is a representative household with a continuum of members represented by a pair \((j, k) \in [0, 1] \times [0, 1]\), supplying specialized labor services. The first dimension, indexed by \(j \in [0, 1]\), indicates the type of labor service. The second dimension, indexed by \(k \in [0, 1]\), indicates the disutility from working. We denote the period utility of member \((j, k)\) by

\[
U(C_t(j, k), C_{t-1}, n_{j,k,t}) = \frac{[C_t(j, k) - \varpi C_{t-1}]^{1-\rho}}{1-\rho} - \theta_t(j, k) \Theta_t k^\delta,
\]

where \(C_t(j, k)\) is the consumption of member \((j, k)\), \(C_{t-1}\) is the household average consumption in period \(t - 1\). This setup incorporates the feature of habit formation. The parameter \(\varpi\) determines the degree of habit in consumption. The term \(\theta_t(j, k)\) is an indicator function, \(\theta_t(j, k) = 1\) if the individual is working and \(\theta_t(j, k) = 0\) if not working. For the sake of comparison with Gali, Smets and Wouters (2012), we do not model the intensive margin of labor in our comprehensive model.

A person with index \(k \in [0, 1]\) gets disutility \(\chi_t \Theta_t k^\xi\) from working, where \(\chi_t\) is an exogenous preference shifter and \(\Theta_t\) is an endogenous preference shifter. The exogenous preference shifter \(\chi_t\) is referred to as labor supply shock and assumed to follow an AR(1) process. The endogenous preference shifter \(\Theta_t\) is specified as \(\Theta_t = \Lambda_t / (C_t - \varpi C_{t-1})\), where \(\Lambda_t\) evolves according to the difference equation \(\Lambda_t = \Lambda_{t-1}^\zeta (C_t - \varpi C_{t-1})^\zeta\). Here \(\Lambda_t\) can be interpreted as a "smooth" trend for aggregate consumption. The parameter \(\zeta \in [0, 1]\) determines the importance of the endogenous preference shifter. The role of the endogenous preference shifter is to incorporate a short-term wealth effect. Note that, during a boom time when the aggregate consumption is higher than trend, the endogenous preference \(\Theta_t\) shifts down, and vice versa. The inverse of the parameter \(\varepsilon\) is the elasticity of labor supply.

We assume full insurance for household’s members so that \(C_t(j, k) = C_t\) for all \((j, k)\). We define \(l_{j,t}\) as the cut-off value of \(k\) below which the type \(j\) worker would choose to work, so \(l_{j,t}\) is also the proportion of type \(j\) workers that would participate in the labor
force given the current prevailing wage. We can integrate members’ utilities to get the household’s utility

$$\frac{[C_t(j,k) - \omega C_{t-1}]^{1-\sigma}}{1-\sigma} - \chi_t \Theta_t \int_0^1 \int_0^{l_{j,t}} k^\epsilon dk dj = \frac{[C_t - \omega C_{t-1}]^{1-\sigma}}{1-\sigma} - \chi_t \Theta_t \int_0^1 \frac{l_{j,t}^{1+\epsilon}}{1+\epsilon} dj.$$

A member $j$ is a monopolistic supplier of type $j$ labor. We assume that a member of type $j$ sets his price of type $j$ labor taken demand for labor as given. Further, we assume that member $j$ can change the posted wage only with probability $1 - \omega$ in a new period (Calvo’s price rigidity). While $l_{j,t}$ is the proportion of type $j$ workers the household would like to supply taken wage as given, the employed workers of type $j$, $n_{j,t}$, is determined by labor demand. The difference, $l_{j,t} - n_{j,t}$, is involuntary unemployment for type $j$ workers.

The specialized labor is supplied to the intermediate goods producer that integrates all the types of labor through

$$n_{i,t} = (\int_0^1 n_{j,i,t}^{1/\nu_t} dj)^{\nu_t},$$

where $n_{i,t}$ is firm $i$’s aggregate labor input, and $n_{j,i,t}$ is firm $i$’s type $j$ labor input. The stochastic variable $\nu_t$ captures wage markup shock and follows an AR(1) process. The demand for type $j$ labor (it will be derived from the firm $i$’s optimization problem) is

$$n_{j,t} = \left(\frac{w_{j,t}}{W_t}\right)^{\nu_t/1-\nu_t} n_t,$$

where $n_t$ is the aggregate labor demand, $w_{j,t}$ is the nominal wage rate set by type $j$ workers, and $W_t = \left(\int_0^1 w_{j,t}^{1/(1-\nu_t)} dj\right)^{1-\nu_t}$ is the aggregate nominal wage index.

The household’s period $t$ budget constraint is

$$\int_0^1 (w_{j,t}n_{j,t} + a_{j,t}) dj + b_t + d_t = \frac{b_{t+1}}{1 + r_t} + P_t C_t + T_t + \int_0^1 q_{j,t+1}^{\omega} a_{j,t+1} d\omega_{j,t+1} dj,$$

where $r_t$ is the nominal interest rate on bonds. The variable $b_{t+1}$ is the one-period nominal bond, $d_t$ is the equity payout, and $T_t$ denotes nominal lump-sum taxes. Households can buy state-contingent claims $a_{j,t+1}$ at the price $q_{j,t+1}^{\omega}$ to insure against wage shocks.

Household takes labor demand as given and maximizes its utility. Sticky wage is introduced like Calvo pricing setting: there is a probability $\omega$ that a household cannot change its wage. The wage choice of household $j$, which is allowed to post a new one in period $t$, is to solve the following problem:
\[
\max_{\{w_{j,t}, a_{j,t+1}, b_{t+1}\}} \sum_{s=0}^{\infty} (\beta \omega)^s \gamma_t \cdot U \left( C_{t-s-1}, C_{t+s}, \left( \frac{w_{j,t+s}}{W_{t+s}} \right)^{\nu_t} n_{t+s} \right) ,
\]
subject to its budget constraints (27). Here, \( \beta \) is the discount factor. The variable \( \gamma_t \) is stochastic and captures risk premium shock. The probability of not allowing a change in wage, \( \omega \), appears in the discount factor because only the periods preceding the resetting of a new wage are relevant for the choice of the wage in period \( t \). Since all the households that reset its wage choose the same \( w_{j,t} \), the aggregate wage index evolves according to
\[
W_t = \left[ \omega W_{t-1}^{1/(1-\nu_t)} + (1 - \omega) w_t^{1/(1-\nu_t)} \right]^{1-\nu_t}.
\]

Using household’s utility as a criterion, and taking as given current prevailing wage for its labor type, the household would find an individual of type \( j \) optimal to participate in the labor market in period \( t \) if and only if the disutility from working is smaller than the real wage in terms of util. So the labor force participation, \( l_{j,t} \), is determined by
\[
[ C_t - \omega C_{t-1} ]^{\sigma} \frac{w_{j,t}}{P_t} = \chi_t \Theta_t \chi_{j,t}^{1/\theta} . \tag{28}
\]

The first-order condition for \( b_{t+1} \) is
\[
1 = \beta (1 + r_t) E_t \left( \frac{\gamma_{t+1} U_{2,t+1}}{\gamma_t U_{2,t}} \right) \left( \frac{P_{t+1}}{P_t} \right) . \tag{29}
\]

Since the firm’s optimization is consistent with households’ optimization in equilibrium, the stochastic discount factor for firms is \( m_{t+1} = \beta \gamma_{t+1} U_{2,t+1}/(\gamma_t U_{2,t}) \).

### 5.1.2 Business sector

There is a continuum of firms indexed by \( i \in [0, 1] \), each producing an intermediate good \( x_i \). The intermediate good is used as an input in the final goods production, \( y_t = \int_0^1 x_{i,t}^{1/\eta_i} \, di \). The price markup \( \eta_i \) is stochastic, assumed following an AR(1) process.

A final goods producer maximizes its profit \( P_t y_t - \int_0^1 p_{i,t} x_{i,t} \, di \). The first order condition gives the inverse demand for intermediate good \( i \), \( p_{i,t} = P_t y_t^{(\eta_t-1)/\eta_t} x_{i,t}^{(1-\eta_t)/\eta_t} \), where \( p_t \) is the nominal price set by the producer of good \( i \), and \( P_t = \int_0^1 p_{i,t}^{1/(1-\eta_t)} \, di \) is the aggregate nominal price index.

Firm \( i \) produces its product using a Cobb-Douglas function, \( x_{i,t} = z_t (u_{i,t} k_{i,t})^{\theta} n_{i,t}^{1-\theta} \), where \( z_t \) is common to all intermediate firms and it follows an AR(1) process, \( k_{i,t} \) is the input of capital, \( u_{i,t} \) the capital utilization rate, and \( n_{i,t} = \int_0^1 n_{j,i,t}^{1/\theta} \, dj \) is the aggregation
of all types of the specialized labor inputs used by firm \( i \). The stochastic variable \( \nu_t \) affects the demand elasticity for the different types of labor.

Substituting the production into the inverse demand for the intermediate input, the price charged by firm \( i \) is:

\[
p_{i,t} = P_t y_t^{\eta_t} x_{i,t}^{1-\eta_t} = P_t D(k_{i,t}, u_{i,t}, n_{i,t}, s_t),
\]

where \( D(k_{i,t}, u_{i,t}, n_{i,t}, s_t) = y_t^{(\eta_t-1)/\eta_t} [z_t(u_{i,t}k_{i,t})^{\phi} n_{i,t}^{1-\phi}]^{1/(1-\eta_t)} \). To take into account the dependence on the aggregate production \( y_t \), we have included the vector of aggregate states, \( s_t \). The real revenue of firm \( i \) can also be expressed as a function of the production inputs and the aggregate states,

\[
p_{i,t}x_{i,t} = P_t y_t^{\eta_t} x_{i,t}^{1-\eta_t} = P_t F(k_{i,t}, u_{i,t}, n_{i,t}, s_t),
\]

where \( F(k_{i,t}, u_{i,t}, n_{i,t}, s_t) = y_t^{(\eta_t-1)/\eta_t} [z_t(u_{i,t}k_{i,t})^{\phi} n_{i,t}^{1-\phi}]^{1/(1-\eta_t)} \).

Physical capital is accumulated by firms. The law of motion of capital follows

\[
k_{t+1} = (1 - \delta)k_t + \varphi(i_{t-1}, i_t, \varsigma_t),
\]

where \( \varphi(i_{t-1}, i_t, \varsigma_t) = \varsigma_t[1 - \psi \left( \frac{i_t}{i_{t-1}} - 1 \right)^2]i_t \) incorporates the idea that adjusting investment is costly. The variable \( \varsigma_t \) is investment specific technology shock and assumed to follow an AR(1) process. In order to be comparable with Gali, Smets and Wouters (2012), here we assume that using capital more intensively also has a cost, and the utilization cost is \( \Psi(u_t)k_t \), where \( \Psi(u_t) = \varphi(u_t^{1+\phi} - 1)/(1 + \phi) \).

We use Rotemberg’s approach to introduce Sticky price by assuming a convex cost of adjusting the nominal price, in order to avoid price heterogeneity, which is a problem if we use Calvo’s approach. Given the nominal price \( p_{t-1} \) set in the previous period, the adjustment cost is \( G(p_{t-1}, p_t, s_t) = \frac{\varphi}{2} (\frac{p_t}{p_{t-1}} - 1)^2 y_t \).

As in the simpler model above, the financial structure is determined by two parameters \( \kappa \) and \( \tau \). The dividend payout costs \( \varphi(d_t) = d_t + \kappa(d_t - \bar{d})^2 \), and the effective gross rate paid by the firms is \( R_t = 1 + r_t(1 - \tau) \). If these two parameters are set to zero, then the model collapses to a New Keynesian model with complete market.

The individual state variables for the firm are the nominal price \( p_{t-1} \), the investment \( i_{t-1} \), the stock of capital, \( k \), and the debt, \( b \). Since all the firms make the same choices in equilibrium, from now on we omit the subscript \( i \). A firm’s optimization problem is
\[
V(s; k, n, b) = \max_{d, u, n+1, p, i, k+1, b+1} \{d + Em_{s+1}V(s_{s+1}; k_{k+1}, n_{n+1}, b_{b+1})\},
\]
subject to the budget constraint,
\[
Wn + PG(p_{-1}, p, s) + P\phi(d) + Pi = P[F(k, u, n, s) - \Psi(u)k] + \frac{b_{b+1}}{R} - b,
\]
the borrowing constraint,
\[
\xi(k_{k+1} - \frac{b_{b+1}}{P(1 + r)}) \geq \frac{Wn}{P} + i,
\]
the demand for the firm’s product,
\[
\frac{p}{P} = D(k, u, n; s),
\]
and the law of motion for capital, equation (31).

### 5.1.3 Public sector

The government faces the budget constraint
\[
P_tG_t + B_{t+1}\left(\frac{1}{R_t} - \frac{1}{1 + r_t}\right) = T_t,
\]
where \(G_t\) is real government purchases, which is assumed to be stochastic and follow an AR(1) process. Recall that \(r_t\) is the nominal interest rate and \(R_t = 1 + r_t(1 - \tau)\) is the effective gross interest rate paid by firms. The cost of the interest reduction is \(B_{t+1}[1/R_t - 1/(1 + r_t)]\). These total expenditures are financed with lump-sum taxes \(T_t\) paid by households. Government purchases follow the stochastic process
\[
\hat{G}_t = \rho_g\hat{G}_{t-1} + \sigma_g(\hat{\epsilon}_t - \hat{\epsilon}_{t-1}) + \epsilon_{g,t},
\]
where \(\epsilon_{g,t} \sim N(0, \sigma_G)\).

Monetary policy is assumed to target inflation and output growth deviations from the steady state,
\[
\hat{r}_t = \rho_r\hat{r}_{t-1} + (1 - \rho_r)\left[\nu_1(\hat{p}_t - \hat{p}_{t-1}) + (\nu_2 + \nu_3)\hat{y}_t - \nu_3\hat{y}_{t-1}\right] + \nu_t,
\]
where \(\rho_r, \nu_1, \nu_2, \nu_3\) are parameters, and \(\nu_t \sim N(0, \sigma_r)\).
5.2 Estimation

As a preliminary result, our initial estimation is based on the whole sample. Then we conduct two subsample-based estimations, taking into account seriously the possibility of a structural change in the middle of 1980s as indicated by our finding obtained in the parsimonious model.

A small number of the model parameters are pinned down using the standard calibration technique based on steady state targets. The remaining parameters are estimated using Bayesian methods similar to that described in Smets and Wouters (2007).

The period in the model is a quarter and the calibration targets for the few parameters are the same as those in the simpler model. We set discount parameter $\beta = 0.9825$, tax advantage parameter $\tau = 0.35$, capital share parameter $\alpha = 0.36$, depreciation parameter $\delta = 0.025$, and financial condition parameter $\bar{\xi} = 0.1325$. We set the average government purchases $\bar{G}$ to 0.18, which is chosen to have a steady state ratio of government purchases over output of 0.18.

We obtain other parameters by estimating the model using the nine empirical series from 1967:I to 2012:IV mentioned above. To generate artificial series, we solve the model numerically after log-linearizing around the steady state. We also check the multiplier for the borrowing constraint to make sure it is always binding in equilibrium. The complete set of equilibrium conditions is presented in Appendix C.

The choices of the prior distributions of the parameters governing the eight non-financial shock processes are similar to those used in Gali, Smets and Wouters (2012). The choices of the prior distributions of the parameters governing the financial shock process, $\rho_\xi$ and $\sigma_\xi$, and of the parameter governing the flexibility in equity payout, $\kappa$, are similar to those used in Jermenn and Quadrini (2012). We report the estimation results in Table 3, which lists the prior densities, the modes, and the standard deviations of the posteriors. As can be seen from the table, all of the estimations of the posteriors are statistically significant.

5.3 Results based on whole sample estimates

Our results based on whole sample estimates indicate a significant reduction in the speed of recovery from a recession trough, from the pre-1985 period to the post-1985 period. This is illustrated by the last row of Table 4.a and of Table 4.b, in terms of the cumulative
growth rate of output and of employment, respectively, eight quarters into a recovery. The difference between the two periods is significant at the 5 percent level for both output and employment growth rates.

Table 3. Parameterization

<table>
<thead>
<tr>
<th>Estimated parameters</th>
<th>Prior[mean, std]</th>
<th>Mode</th>
<th>Post_d</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elasticity of labor, $\varepsilon$</td>
<td>norm[2.00, 1.00]</td>
<td>0.091</td>
<td>0.0083</td>
</tr>
<tr>
<td>Utility parameter, $\sigma$</td>
<td>norm[1.20, 0.37]</td>
<td>1.263</td>
<td>0.0373</td>
</tr>
<tr>
<td>Habit in consumption, $\varpi$</td>
<td>beta[0.70, 0.10]</td>
<td>0.687</td>
<td>0.0066</td>
</tr>
<tr>
<td>Wage adjustment, $\omega$</td>
<td>beta[0.50, 0.15]</td>
<td>0.406</td>
<td>0.0121</td>
</tr>
<tr>
<td>endogenous preference shifter, $\zeta$</td>
<td>beta[0.10, 0.02]</td>
<td>0.088</td>
<td>0.0036</td>
</tr>
<tr>
<td>Investment adjustment cost, $\psi$</td>
<td>IGamma[0.20, 0.60]</td>
<td>0.050</td>
<td>0.0089</td>
</tr>
<tr>
<td>Price adjustment cost, $\rho$</td>
<td>IGamma[0.10, 0.30]</td>
<td>0.964</td>
<td>0.0411</td>
</tr>
<tr>
<td>Equity payout cost, $\kappa$</td>
<td>IGamma[0.20, 0.10]</td>
<td>0.104</td>
<td>0.0050</td>
</tr>
<tr>
<td>Capital utilization cost, $\phi$</td>
<td>beta[0.50, 0.15]</td>
<td>0.461</td>
<td>0.0304</td>
</tr>
<tr>
<td>Average price mark-up, $\bar{\eta}$</td>
<td>beta[1.20, 0.10]</td>
<td>1.730</td>
<td>0.0333</td>
</tr>
<tr>
<td>Average wage mark-up, $\bar{\nu}$</td>
<td>beta[1.20, 0.10]</td>
<td>1.327</td>
<td>0.0119</td>
</tr>
<tr>
<td>Monetary policy, $\nu_1$</td>
<td>norm[1.50, 0.25]</td>
<td>1.541</td>
<td>0.0159</td>
</tr>
<tr>
<td>Monetary policy, $\nu_2$</td>
<td>norm[0.12, 0.05]</td>
<td>0.065</td>
<td>0.0074</td>
</tr>
<tr>
<td>Monetary policy, $\nu_3$</td>
<td>norm[0.12, 0.05]</td>
<td>0.153</td>
<td>0.0085</td>
</tr>
<tr>
<td>Productivity shock persistence, $\rho_z$</td>
<td>Beta[0.50, 0.20]</td>
<td>0.971</td>
<td>0.0265</td>
</tr>
<tr>
<td>Labor supply shock persistence, $\rho_{\chi}$</td>
<td>Beta[0.50, 0.20]</td>
<td>0.908</td>
<td>0.0223</td>
</tr>
<tr>
<td>Financial shock persistence, $\rho_\lambda$</td>
<td>Beta[0.50, 0.20]</td>
<td>0.896</td>
<td>0.0131</td>
</tr>
<tr>
<td>Investment shock persistence, $\rho_\xi$</td>
<td>Beta[0.50, 0.20]</td>
<td>0.753</td>
<td>0.0214</td>
</tr>
<tr>
<td>Wage mark-up shock persistence, $\rho_{\bar{\eta}}$</td>
<td>Beta[0.50, 0.20]</td>
<td>0.383</td>
<td>0.0166</td>
</tr>
<tr>
<td>Price mark-up shock persistence, $\rho_{\bar{\nu}}$</td>
<td>Beta[0.50, 0.20]</td>
<td>0.930</td>
<td>0.0100</td>
</tr>
<tr>
<td>Intertemporal shock persistence, $\rho_{\gamma}$</td>
<td>Beta[0.50, 0.20]</td>
<td>0.348</td>
<td>0.0107</td>
</tr>
<tr>
<td>Government shock persistence, $\rho_g$</td>
<td>Beta[0.50, 0.20]</td>
<td>0.968</td>
<td>0.0251</td>
</tr>
<tr>
<td>Interaction prod-government, $\rho_{gz}$</td>
<td>Beta[0.50, 0.20]</td>
<td>0.319</td>
<td>0.0208</td>
</tr>
<tr>
<td>Monetary policy persistence, $\rho_r$</td>
<td>Beta[0.50, 0.20]</td>
<td>0.782</td>
<td>0.0094</td>
</tr>
<tr>
<td>Productivity shock volatility, $\sigma_z$</td>
<td>IGamma[0.001, 0.005]</td>
<td>0.0057</td>
<td>0.0003</td>
</tr>
<tr>
<td>Labor supply shock volatility, $\sigma_{\chi}$</td>
<td>IGamma[0.001, 0.05]</td>
<td>0.0342</td>
<td>0.0014</td>
</tr>
<tr>
<td>Financial shock volatility, $\sigma_\lambda$</td>
<td>IGamma[0.001, 0.05]</td>
<td>0.0178</td>
<td>0.0009</td>
</tr>
<tr>
<td>Investment shock volatility, $\sigma_{\xi}$</td>
<td>IGamma[0.001, 0.05]</td>
<td>0.0090</td>
<td>0.0007</td>
</tr>
<tr>
<td>Wage mark-up shock volatility, $\sigma_{\bar{\nu}}$</td>
<td>IGamma[0.001, 0.05]</td>
<td>0.2253</td>
<td>0.0088</td>
</tr>
<tr>
<td>Price mark-up shock volatility, $\sigma_{\bar{\eta}}$</td>
<td>IGamma[0.001, 0.05]</td>
<td>0.0063</td>
<td>0.0006</td>
</tr>
<tr>
<td>Intertemporal shock volatility, $\sigma_{\gamma}$</td>
<td>IGamma[0.001, 0.05]</td>
<td>0.0204</td>
<td>0.0021</td>
</tr>
<tr>
<td>Government shock volatility, $\sigma_g$</td>
<td>IGamma[0.001, 0.05]</td>
<td>0.0169</td>
<td>0.0010</td>
</tr>
<tr>
<td>Monetary policy shock volatility, $\sigma_r$</td>
<td>IGamma[0.001, 0.05]</td>
<td>0.0008</td>
<td>0.0001</td>
</tr>
</tbody>
</table>
The other rows of the tables illustrate the roles played by the nine structural shocks in accounting for the above difference between the two periods by reporting the average contributions of the shocks to the cumulative growth rate of output (Table 4.a) and of employment (Table 4.b) in a recovery before and after the year of 1985. Several observations can be made from these variance decomposition results.

<table>
<thead>
<tr>
<th>Shocks</th>
<th>Pre-1985</th>
<th>Post-1985</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>TFP shock</td>
<td>0.03481</td>
<td>0.01837</td>
<td>-0.01644**</td>
</tr>
<tr>
<td>Financial</td>
<td>0.02948</td>
<td>0.01454</td>
<td>-0.01495**</td>
</tr>
<tr>
<td>Investment</td>
<td>-0.00332</td>
<td>0.00299</td>
<td>0.00631*</td>
</tr>
<tr>
<td>Monetary</td>
<td>-0.00433</td>
<td>-0.00108</td>
<td>0.00326**</td>
</tr>
<tr>
<td>Gov. spending</td>
<td>-0.00559</td>
<td>-0.00884</td>
<td>-0.00325</td>
</tr>
<tr>
<td>Wage markup</td>
<td>-0.00770</td>
<td>-0.00838</td>
<td>-0.00068</td>
</tr>
<tr>
<td>Price markup</td>
<td>0.00390</td>
<td>0.000197</td>
<td>-0.00371**</td>
</tr>
<tr>
<td>Risk premium</td>
<td>0.00106</td>
<td>-0.00113</td>
<td>-0.00219</td>
</tr>
<tr>
<td>Labor supply</td>
<td>-0.00547</td>
<td>-0.00778</td>
<td>-0.00232</td>
</tr>
<tr>
<td>Initial state</td>
<td>-0.00111</td>
<td>-0.00012</td>
<td>0.00109</td>
</tr>
<tr>
<td>Total</td>
<td>0.04173</td>
<td>0.00886</td>
<td>-0.03287**</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Shocks</th>
<th>Pre-1985</th>
<th>Post-1985</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>TFP shock</td>
<td>0.00356</td>
<td>0.00039</td>
<td>-0.00317</td>
</tr>
<tr>
<td>Financial</td>
<td>0.03409</td>
<td>0.02026</td>
<td>-0.01384**</td>
</tr>
<tr>
<td>Investment</td>
<td>-0.00343</td>
<td>0.00221</td>
<td>0.00564**</td>
</tr>
<tr>
<td>Monetary</td>
<td>-0.00432</td>
<td>-0.00151</td>
<td>0.00281**</td>
</tr>
<tr>
<td>Gov. spending</td>
<td>-0.00584</td>
<td>-0.01088</td>
<td>-0.00504</td>
</tr>
<tr>
<td>Wage markup</td>
<td>-0.00758</td>
<td>-0.01050</td>
<td>-0.00292</td>
</tr>
<tr>
<td>Price markup</td>
<td>0.00197</td>
<td>0.00137</td>
<td>-0.00060</td>
</tr>
<tr>
<td>Risk premium</td>
<td>0.00131</td>
<td>0.00050</td>
<td>-0.00081</td>
</tr>
<tr>
<td>Labor supply</td>
<td>-0.00631</td>
<td>-0.00922</td>
<td>-0.00291</td>
</tr>
<tr>
<td>Initial state</td>
<td>0.00080</td>
<td>-0.00001</td>
<td>-0.00080</td>
</tr>
<tr>
<td>Total</td>
<td>0.01425</td>
<td>-0.00739</td>
<td>-0.02164**</td>
</tr>
</tbody>
</table>

First, the recent slower recovery in output growth is mainly due to reduced contributions by TFP and financial shocks, from the pre-1985 period to the post-1985 period. Both shocks make positive contributions in both subsample periods, but much less so in the more recent one. The difference is statistically significant at the 5 percent level for both shocks.

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Second, the slower recovery in employment growth after 1985 is almost entirely due to a reduced contribution by financial shocks. Although financial shocks have been the most important positive contributor across the entire sample period, it is much less so in the more recent episode. The difference is statistically significant at the 5 percent level.

Third, investment shocks make a negative contribution to output and employment recoveries in the pre-1985 episode, but a positive contribution to output and employment recoveries in the post-1985 era, though the effect in absolute value is an order of magnitude smaller than those of TFP and financial shocks. This reversal in the role of investment shocks is statistically significant at the 5 percent level. Nevertheless, the overtime improved role of investment shocks is much more than dominated by the overtime worsened role of finance shocks that limit how much of the investment opportunity can be materialized.

Fourth, though monetary policy shocks make a negative contribution to both output and employment recoveries in both subsample periods, the effect in absolute value, like that of investment shocks, is also an order of magnitude smaller than those of TFP and financial shocks. Further, there has been an improvement in the effect of monetary policy shocks on output and employment recoveries across the two subsample periods, and the difference is statistically significant at the 5 percent level. This implies that, hadn’t the effect of monetary policy shocks been improved overtime, the three most recent recoveries would have even been slightly slower than observed.

Fifth, price markup shocks make a positive contribution to output recovery in the pre-1985 period with the effect being an order of magnitude smaller than those of TFP and financial shocks, but an even much smaller positive contribution in the post-1985 period, and the difference is statistically significant only at the 10 percent level. As a result, the contribution of price markup shocks to the recent slower recovery in output growth is an order of magnitude smaller than the contributions of TFP and financial shocks. The role of price markup shocks is both economically and statistically insignificant in accounting for the recent slower recovery in employment growth.

Finally, none of the other four types of structural shocks plays an either economically or statistically significant role in accounting for either the recent slower output recovery or the recent slower employment recovery.
5.4 Results based on subsample estimates

The above result concerning the reduction in the contribution of financial shocks to cyclical recoveries from the pre-1985 era to the post-1985 episode can be consistent with our earlier finding about an increased persistence of financial shocks across the two periods. This is so, since a more persistent adverse financial shock originated in a recession can drag the subsequent recovery triggered by, say, a positive financial or productivity shock by offsetting part of the effect of the positive shock. To take this interpretation seriously, however, it is more appropriate to estimate the model separately for the two subsample periods. We conduct this exercise in this subsection.

The main results obtained in the previous subsection remain unchanged when we re-estimate the model separately for the two subsample periods. This is shown in Table 5. Based on the subsample estimates, price markup shocks play a somewhat bigger role in accounting for slow output recovery and a significantly greater role in accounting for slow employment recovery, although the effect remains an order of magnitude smaller than those of TFP and financial shocks as in the case based on whole sample estimates. The subsample estimates also assign a more significant role, both statistically and economically, to wage markup shocks in accounting for both slow output recovery and slow employment recovery, although the effect is only of the same order of magnitude as that of price markup shocks. Risk premium and labor supply shocks still do not show any statistically significant difference in their effects on recoveries before and after 1985.

<table>
<thead>
<tr>
<th>Shocks</th>
<th>Pre-1985</th>
<th>Post-1985</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>TFP shock</td>
<td>0.035529</td>
<td>0.019188</td>
<td>-0.016341**</td>
</tr>
<tr>
<td>Financial</td>
<td>0.036908</td>
<td>0.013620</td>
<td>-0.023287**</td>
</tr>
<tr>
<td>Investment</td>
<td>-0.010302</td>
<td>0.002625</td>
<td>0.012927**</td>
</tr>
<tr>
<td>Monetary</td>
<td>-0.006260</td>
<td>-0.000248</td>
<td>0.006012**</td>
</tr>
<tr>
<td>Gov. spending</td>
<td>-0.007262</td>
<td>-0.007635</td>
<td>-0.000372</td>
</tr>
<tr>
<td>Wage markup</td>
<td>-0.006980</td>
<td>-0.009504</td>
<td>-0.002524*</td>
</tr>
<tr>
<td>Price markup</td>
<td>0.004094</td>
<td>0.000006</td>
<td>-0.004087*</td>
</tr>
<tr>
<td>Risk premium</td>
<td>0.002423</td>
<td>-0.001304</td>
<td>-0.003727</td>
</tr>
<tr>
<td>Labor supply</td>
<td>-0.005197</td>
<td>-0.007552</td>
<td>-0.002355</td>
</tr>
<tr>
<td>Initial state</td>
<td>-0.001223</td>
<td>-0.000340</td>
<td>0.000883</td>
</tr>
<tr>
<td>Total</td>
<td>0.041729</td>
<td>0.008857</td>
<td>-0.032872</td>
</tr>
</tbody>
</table>

Table 5.a. 8 quarter cumulative growth of output: decomposition (sub-period)
Table 5.b. 8 quarter cumulative growth of employment: decomposition (sub-period)

<table>
<thead>
<tr>
<th>Shocks</th>
<th>Pre-1985</th>
<th>Post-1985</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>TFP shock</td>
<td>0.003842</td>
<td>0.002290</td>
<td>-0.001553</td>
</tr>
<tr>
<td>Financial</td>
<td>0.042095</td>
<td>0.019389</td>
<td>-0.022706**</td>
</tr>
<tr>
<td>Investment</td>
<td>-0.009946</td>
<td>0.002114</td>
<td>0.012060**</td>
</tr>
<tr>
<td>Monetary</td>
<td>-0.006045</td>
<td>-0.001127</td>
<td>0.004918**</td>
</tr>
<tr>
<td>Gov. spending</td>
<td>-0.007821</td>
<td>-0.009419</td>
<td>-0.001598</td>
</tr>
<tr>
<td>Wage markup</td>
<td>-0.007308</td>
<td>-0.012255</td>
<td>-0.004948*</td>
</tr>
<tr>
<td>Price markup</td>
<td>0.002198</td>
<td>0.000008</td>
<td>-0.002190</td>
</tr>
<tr>
<td>Risk premium</td>
<td>0.002460</td>
<td>0.000581</td>
<td>-0.001878</td>
</tr>
<tr>
<td>Labor supply</td>
<td>-0.005914</td>
<td>-0.008958</td>
<td>-0.003045</td>
</tr>
<tr>
<td>Initial state</td>
<td>0.000687</td>
<td>-0.000012</td>
<td>-0.000699</td>
</tr>
<tr>
<td>Total</td>
<td>0.014248</td>
<td>-0.007389</td>
<td>-0.021637**</td>
</tr>
</tbody>
</table>

6 Conclusion

Our model generates slow recoveries after the mid-1980s, but not before. This can be attributed to the increased persistence of poor financial conditions in the recoveries from the last three recessions. Our model accounts for capital shortage and slow recovery following a recession trough in a unified framework. This is achieved through two key mechanisms: (1) complementarity between capital and labor in production along with endogenous capital utilization rate and hours worked per employee; and (2) substitution between wage payment and investment in the face of a binding borrowing constraint.

7 Acknowledgement

We are grateful to John Matsusaka, Vincenzo Quadrini, Shouyong Shi, and participants at the 2013 Dynare Conference, the 2013 Shanghai Macroeconomic Workshop, and the 2013 China Meeting of the Econometric Society for helpful comments. Chen acknowledges financial support from the graduate innovation fund of Shanghai University of Finance and Economics. Huang acknowledges financial support from the Grey Fund at Vanderbilt University. Li acknowledges financial support from the Shanghai Pujiang Program and the National Science Foundation of China (71203127). Sun acknowledges financial support from the Shanghai Pujiang Program.
Appendix A

In appendix A, we log-linearize the parsimonious model. In order to simplify the notations, we define

\[ d_y = \frac{d}{y}, \quad b_y = \frac{b}{y}, \quad k_y = \frac{k}{y}, \quad c_y = \frac{c}{y}, \quad y_k = \frac{y}{k}, \quad i_y = \frac{i}{y}. \]

Here the variables without a subscript are steady state values, and the values with a hat is the log-deviations from the steady state value, for example, \( \hat{x}_t = \log x_t - \log x. \)

1. For the household side:

The log-linearization of the first order conditions:

\[ n_{t+1} : E_t \hat{w}_{t+1} = E_t \hat{c}_{t+1} \]

\[ h_t : \hat{w}_t - \hat{c}_t = \frac{h}{T - \zeta - h} \hat{h}_t \]

\[ b_{t+1} : E_t (\hat{c}_t - \hat{c}_{t+1} + \frac{R}{R - \tau} \hat{R}_t) = 0 \]

Budget constraint:

\[ c_y \hat{c}_t + b_y \left( \frac{b}{y} \hat{b}_{t+1} - \hat{R}_t \right) - d_y \hat{d}_t = whn/y(\hat{w}_t + \hat{n}_t + \hat{h}_t) + b_y \hat{b}_t \]

2. For the firm side:

Log-linearization of the first order conditions:

\[ d_t : \hat{\lambda}_t = -2k \delta \hat{d}_t \]

\[ h_t : \hat{y}_t - \hat{n}_t - \hat{h}_t = \frac{\mu}{1 + \mu} (\hat{\mu}_t - \hat{\lambda}_t) + \hat{w}_t \]

\[ u_t : \hat{y}_t - \hat{k}_t = \frac{\mu}{1 + \mu} (\hat{\mu}_t - \hat{\lambda}_t) + \hat{\delta}_t \]

\[ n_{t+1} : E_t (\hat{y}_{t+1} - \hat{n}_{t+1} - \hat{h}_{t+1}) = E_t \hat{w}_{t+1} + \frac{\mu}{1 + \mu} E_t (\hat{\mu}_{t+1} - \hat{\lambda}_{t+1}) \]

\[ k_{t+1} : \beta E_t [\alpha k (\hat{y}_{t+1} - \hat{k}_{t+1}) + \mu (1 - \delta)(\hat{\mu}_{t+1} - \hat{\lambda}_{t+1} - \frac{\delta}{1 - \delta} \hat{\delta}_{t+1}) + \beta \delta E_t \hat{\delta}_{t+1}] = -(1 + \mu - \mu \xi) E_t (\hat{c}_t - \hat{c}_{t+1} + \hat{\lambda}_{t+1} - \hat{\lambda}_t) - \mu \xi (\hat{\xi}_t + \hat{\mu}_t - \hat{\lambda}_t) + \mu (\hat{\mu}_t - \hat{\lambda}_t) \]
\[
\begin{align*}
    b_{t+1} & : \beta R E_t (\hat{c}_t - \hat{c}_{t+1} + \hat{\lambda}_{t+1} - \hat{\lambda}_t + \hat{R}_t) = \\
    & \quad - \frac{R}{R - \tau} (1 - \tau) \mu (\hat{\xi}_t + \hat{\mu}_t - \hat{\lambda}_t - \frac{\tau}{R - \tau} \hat{R}_t),
\end{align*}
\]

Law of motion for capital:
\[
\hat{\delta}_t = \phi \hat{u}_t
\]

Production function:
\[
\hat{y}_t = \hat{z}_t + \alpha (\hat{k}_t + \hat{u}_t) + (1 - \alpha) (\hat{n}_t + \hat{h}_t)
\]

Budget constraint:
\[
-\delta k_y \hat{\delta}_t + (1 - \delta) k_y \hat{k}_t + \hat{y}_t + \frac{b_y}{R} (\hat{h}_{t+1} - \hat{R}_t) = wnh/y (\hat{w}_t + \hat{n}_t + \hat{h}_t) + b_y \hat{b}_t + k_y \hat{k}_{t+1} + d_y \hat{d}_t
\]

Borrowing constraint:
\[
(wnh/y + \delta k_y) \hat{\xi}_t + \xi k_y \hat{k}_{t+1} - \xi \beta b_y (\hat{b}_{t+1} - \frac{R}{R - \tau} \hat{R}_t) = wnh/y (\hat{w}_t + \hat{n}_t + \hat{h}_t) + k_y \hat{k}_{t+1} - (1 - \delta) k_y (\hat{k}_t - \frac{\delta}{1 - \delta} \hat{\delta}_t)
\]

**Appendix B**

In appendix B we provide some details of the calibration and the sources of the data that we use to calibrate the parsimonious model.

In order to calibrate the parameters governing the processes for the aggregate shocks in the model, we first construct the series \(\hat{z}_t\) and \(\hat{\xi}_t\). For the productivity variable \(\hat{z}_t\) we use series estimated by Fernald (2012), which uses the same method as in Basu, Fernald, and Kimbal (2006). For the financial condition \(\hat{\xi}_t\), we get it from log-linearized financial constraint

\[
\hat{\xi}_t = \left[ \frac{wnh/y (\hat{w}_t + \hat{n}_t + \hat{h}_t) + i_y \hat{\mu}_t}{-\xi k_y \hat{k}_{t+1} + \xi \beta b_y \hat{b}_{t+1}} \right] (wnh/y + \delta k_y).
\]

Here, we define \(b_{t+1}^c = b_{t+1}/(1 + r_t)\). To construct the series of \(\hat{z}_t\) and \(\hat{\xi}_t\), we need to get the series of \(\hat{w}_t\), \(\hat{n}_t\), \(\hat{h}_t\), \(\hat{k}_t\), \(i_t\), \(\hat{y}_t\), and \(\hat{b}_t\).
The series for $k_t$ is constructed by using the law of motion for capital, $k_{t+1} = k_t - \text{depreciation}_t + \text{investment}_t$, and an initial value of capital. Following Jermann and Quadrini (2012), the data series $\text{depreciation}_t$ and $\text{investment}_t$ are taken from the Flow of Funds Accounts of the United States. The data series $\text{depreciation}_t$ includes two parts: depreciation in corporation (FA106300015.Q) and depreciation in noncorporation (FA116300005.Q). The data series $i_t = \text{investment}_t$ is the series FA145050005.Q. The debt series $b_t$ is also taken from the Flow of Funds Accounts of the United States. Its series number is FA144104005.Q. The GDP $y_t$ is taken from the Bureau of Economic Analysis Table 1.3.4.

For the wage series $w_t$, we use the business sector real compensation per hour from the Federal Reserve Bank of St. Louis. The data are available from their website: http://research.stlouisfed.org/fred2/series/RCPHBS. For the total hours $n_t h_t$, we use the Index of Aggregate Weekly Hours: Production and Nonsupervisory Employees: Total Private Industries (AWHI), Index 2002 = 100, Quarterly, Seasonally Adjusted. It is available in the website of the Federal Reserve Bank of St. Louis. The data are available from their website: http://research.stlouisfed.org/fred2/series/AWHI.

We take log of the variables $b_t^e, k_t, w_t, n_t h_t, y_t, i_t$ and then we detrend these log variables using Hodrick-Prescott filter with a smoothing parameter 16000 to get $\hat{b}_t^e, \hat{k}_t, \hat{w}_t, \hat{n}_t, \hat{h}_t, \hat{y}_t, \hat{i}_t$.

### Appendix C

In appendix C we log-linearize the comprehensive model.

Equation for wage set by household:

$$\hat{w}_t = \Phi \hat{P}_t + \Phi \hat{v}_t + \Phi (\hat{\chi}_t + \hat{\Lambda}_t) + \frac{\Phi}{\bar{v}} \hat{n}_t + \frac{\Phi}{\bar{v} - 1} \hat{W}_t + \beta \omega E_t \hat{w}_{t+1},$$

where $\Phi = \frac{1 - \beta \omega}{1 + \frac{1}{\bar{v} - 1} \epsilon}$, $\hat{W}_t = \omega \hat{W}_{t-1} + (1 - \omega)\hat{w}_t$, $\hat{\Lambda}_t = (1 - \zeta)\hat{\Lambda}_{t-1} + \zeta \hat{C}_t$, and $\hat{C}_t = \frac{\sigma}{1 - \omega} \left( \hat{C}_t - \omega \hat{C}_{t-1} \right)$.

Equation for labor participation:

$$\hat{W}_t - \hat{P}_t = \hat{\Lambda}_t + \frac{1}{\epsilon} \hat{l}_t + \hat{\chi}_t.$$
First order condition for bond on the household side:

\[ E_t(\gamma_{t+1} - \gamma_t + \tilde{C}_t - \tilde{C}_{t+1} + \hat{P}_t - \hat{P}_{t+1} + \frac{R}{R-\tau} \hat{R}_t) = 0. \]

Budget constraint of the household:

\[ \frac{w_n}{y} (\tilde{w}_t + \tilde{n}_t - \hat{P}_t) + d_y \tilde{d}_t - G_y \tilde{G}_t - \frac{b_y}{R} (b_{t+1} - \hat{R}_t - \hat{P}_t) + b_y (b_t - \hat{P}_t) - C_y \tilde{C}_t. \]

Let \( \lambda, Q, \mu \) be the Lagrange multiplier of budget constraint, financial constraint and law of motion of capital on the firm side, respectively. Then, the first order condition for \( u_t, P_t, k_{t+1}, i_t, b_{t+1} \) are (after log-linearized):

\[ \hat{k}_t + (1 + \phi) \hat{u}_t = \hat{n}_t + \hat{W}_t - \hat{P}_t + \frac{\bar{\mu}}{1 + \bar{\mu}} (\bar{\mu}_t + \kappa \hat{d}_t), \]

where \( \kappa = 2\bar{\kappa} \) and \( \bar{\mu} = \frac{\bar{\mu}^{-1}}{\bar{\xi}}; \)

\[ \rho \hat{\pi}_t - \beta \rho E_t \hat{\pi}_{t+1} = \frac{1}{1 - \bar{\eta}} (\hat{\eta}_t - \hat{\eta}_t - (1 + \phi) \hat{u}_t - \hat{k}_t), \]

where \( \hat{\pi}_t = \hat{P}_t - \hat{P}_{t-1}; \)

\[ (1 + \bar{\mu}) \hat{Q}_t = (1 + \bar{\mu} - \bar{\mu} \bar{\xi}) E_t (\gamma_{t+1} - \gamma_t + \tilde{C}_t - \tilde{C}_{t+1}) + \beta (1 - \delta) (1 + \bar{\mu}) E_t \hat{Q}_{t+1} + \bar{\mu} \bar{\xi} (\hat{\mu}_t + \hat{\xi}_t) + \beta \partial E_t (\phi \hat{u}_{t+1} - \kappa \hat{d}_{t+1}); \]

\[ (1 + \bar{\mu}) (\hat{Q}_t + \hat{\kappa}_t) - \psi (1 + \bar{\mu}) (\hat{i}_t - \hat{i}_{t-1}) + \beta \psi (1 + \bar{\mu}) E_t (\hat{i}_{t+1} - \hat{i}_t) = -\kappa \hat{d}_t + \bar{\mu} \hat{\mu}_t; \]

\[ \beta R E_t (\gamma_{t+1} - \gamma_t + \tilde{C}_t - \tilde{C}_{t+1} + \hat{P}_t - \hat{P}_{t+1} + \kappa \hat{d}_t - \kappa \hat{d}_{t+1}) + \bar{\mu} \bar{\xi} \frac{1 - \tau}{R - \tau} R (\hat{\mu}_t + \hat{\xi}_t + \kappa \hat{d}_t - \frac{\tau}{R - \tau} \hat{R}_t) = 0; \]

The law of motion for capital:

\[ \hat{k}_{t+1} = (1 - \delta) \hat{k}_t + \delta (\hat{\kappa}_t + \hat{i}_t). \]

Product Function:
\[ \hat{y}_t = \hat{z}_t + (1 - \theta)\hat{n}_t + \theta(\hat{k}_t + \hat{u}_t). \]

Budget constraint of firm:

\[ \hat{y}_t - \partial k_y \hat{u}_t + \frac{b_y}{R} (\hat{b}_{t+1} - \hat{R}_t - \hat{P}_t) - b_y (\hat{b}_t - \hat{P}_t) - \frac{w_n}{y} (\hat{w}_t + \hat{n}_t - \hat{P}_t) - d_y \hat{d}_t - i_y \hat{i}_t. \]

Financial constraint of firm:

\[ \hat{x}_t + \frac{k_y}{k_y - b_y} \hat{k}_{t+1} - \frac{b_y}{k_y - b_y} (\hat{b}_{t+1} - \frac{R}{R - \tau} \hat{R}_t - \hat{P}_t) \]
\[ = \frac{w_n}{y + \delta k_y} (\hat{W}_t + \hat{n}_t - \hat{P}_t) + \frac{\delta k_y}{y + \delta k_y} \hat{i}_t. \]

Fiscal Policy:

\[ \hat{G}_t = \rho_g \hat{G}_{t-1} + \rho_{gz} (\hat{z}_t - \hat{z}_{t-1}) + \epsilon_{g,t}. \]

Monetary Policy:

\[ \hat{r}_t = \rho_r \hat{r}_{t-1} + (1 - \rho_r) [\nu_1 (\hat{p}_t - \hat{p}_{t-1}) + (\nu_2 + \nu_3) \hat{y}_t - \nu_3 \hat{y}_{t-1}] + \nu_t. \]

All other shocks are assumed to follow AR(1) process:

\[ \hat{x}_t = \theta_x \hat{x}_{t-1} + \epsilon_{x,t}, \text{ for } x = z, \gamma, \xi, \xi, v, \eta, \chi. \]
References


