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Social Welfare Functions

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Abstract. This article provides an introduction to the use of social welfare functions for the comparative evaluation of social alternatives. Three main approaches are considered: Bergson–Samuelson social welfare functions, Arrovian social welfare functions, and Sen's social welfare functionals. How the measurability and comparability of utility can be modelled and how limitations on the types of utility comparisons that are possible restricts the kinds of social welfare functions that can be considered is also discussed. Extensive social choice theory is used to deal with heterogeneous opinions about how to make utility comparisons.

Keywords. Arrovian social welfare function, Bergson–Samuelson social welfare function, social welfare functional, welfare economics

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1. Introduction

A number of different methods are used in welfare economics and social choice theory to comparatively evaluate social alternatives using a social preference. In this chapter, I provide an introduction to social welfare function approaches to making such comparisons. With a social welfare function, social preferences depend on individual well-beings. These well-beings are expressed either in terms of preferences or utilities.

There are three main kinds of social welfare functions. A *Bergson–Samuelson social welfare function* (Bergson, 1938; Samuelson, 1947) assigns a welfare number to each alternative and then uses these numbers to determine a social preference ordering of them. In its individualistic form, a Bergson–Samuelson social welfare function is constructed in two stages: the individual utilities obtained with an alternative are first determined and then these utilities are aggregated into the value of social welfare. The function that aggregates the utilities provides a *social welfare ordering* of vectors of individual utilities. With an *Arrovian social welfare function* (Arrow, 1963), social preferences for the alternatives are determined as a function of the individual preferences for them. It can be regarded as being a procedure for identifying a Bergson–Samuelson social preference for each configuration of individual preferences. A limitation of the Arrovian approach is that it only uses information about individual preferences to determine a social preference. A *social welfare functional* (Sen, 1970a, 1974) generalizes an Arrovian social welfare function by letting the social preference depend on individual utilities, not just individual preferences.

Consequentialist approaches to social evaluation compare and evaluate alternatives exclusively in terms of the outcomes asosciated with each alternative. *Welfarism* is a form of consequentialism in which only welfare consequences matter. Individualistic Bergson–Samuelson social welfare functions are welfarist. Although social welfare functionals need not be welfarist, the existing literature has supposed that they are, and I shall do the same.

The limited ability to make intrapersonal and interpersonal utility comparisons constrains the kinds of social preferences that can be considered. The Arrovian framework is particularly constrained because it eschews interpersonal comparisons entirely and only takes acount of utility level comparisons intrapersonally. A major focus of this chapter is to describe how the measurability and comparability of utility can be modelled and how limitations on the types of utility comparisons that are possible restricts the kinds of social welfare functions that can be considered. Possible bases for making interpersonal comparisons will also be discussed as different bases have different implications about what types of utility comparisons are meaningful.

The *Pareto quasiordering* is a social preference on the set of social alternatives that only ranks alternatives when there is unanimity about how this should be done. With heterogeneous individual preferences, Pareto rankings may be quite incomplete. Incomplete social preferences can result in no clear-cut decision if these preferences are used to guide public policy. Policy paralysis is avoided if a social preference is able to compare any pair of alternatives, as has typically been assumed. Standard welfare criteria, such as utilitarianism and leximin, provide complete social rankings. Requiring social preferences to be complete may be overdemanding, so I allow some social incompleteness to remain provided that Pareto comparisons are respected. Incomplete social preferences arise quite naturally when they are based on utility comparisons made by different individuals who do not agree on how to make them. Heterogeneneous utility comparisons are formalized here using an *extensive social welfare functional* (Roberts, 1995; Ooghe and Lauwers, 2005), which is a generalization of a social welfare functional.

2. Preliminaries

The objective is to socially rank a set A of *social alternatives*. This social ranking is meant to have normative significance. To say that one alternative is socially better than a second is to make a moral claim. Moreover, this social ranking must be positively related to how well off the individuals in society are with the alternatives being compared. The alternatives in A can be given different interpretations. For example, they may be allocations for an economy, complete histories of the world, or anything that affects the well-being of individuals. It is assumed that there are at least three alternatives. If A is a set allocations of divisible goods, then it typically contains an infinite number of alternatives.

The social ranking is determined for a particular *society*. The number of individuals in this society is a fixed number n with $n \geq 2$. The individuals are numbered from 1 to n and $S = \{1, \dots, n\}$ is used to denote this society.

A *binary relation* B compares pairs of objects in some set O. The statement that x stands in the relation B to y is written as xBy. The relation B is (i) *reflexive* if xBx for all $x \in O$, (ii) *complete* if xBy or yBx for all distinct $x, y \in O$, (iii) *transitive* if xBy and yBz imply xBz for all $x, y, z \in O$, and

(iv) *symmetric* if xBy implies yBx for all distinct $x, y \in O$. The relation B is a *quasiordering* if it is reflexive and transitive. It is an *ordering* if it is a complete quasiordering and it is an *equivalence relation* if it is a symmetric quasiordering. Thus, an ordering is a quasiordering, but not all quasiorderings are orderings. Three relations of interest may be derived from B. The *asymmetric factor* B^A is defined by setting xB^Ay if and only if $[xBy$ and $\neg(yBx)]$, where $\neg$ means "not". The *symmetric factor* B^S is defined by setting xB^Sy if and only if $[xBy$ and $yBx]$. Finally, the *noncomparability factor* B^N is defined by setting xB^Ay if and only if $[\neg(xBy)$ and $\neg(yBx)]$. If B is a quasiordering, then its symmetric factor B^S is an equivalence relation.

Both individuals and society rank the alternatives in A. These rankings are described by a binary relation, R for society and R_i for individual i. These relations can be interpreted in various ways, such as in terms of preference or betterness. For concreteness, I shall use the preference interpretation. More precisely, these relations are interpreted as *weak preference* relations. Thus, for example, xRy means that society weakly prefers x to y. The corresponding asymmetric, symmetric, and noncomparability factors of R (resp. R_i) are denoted by P, I, and N (resp. P_i, I_i, and N_i), respectively. The asymmetric factor of R or R_i corresponds to having a strict preference. Similarly, the symmetric factor indicates indifference, whereas the noncomparabilty factor identifies which alternatives cannot be compared in terms of preference.

It shall henceforth be assumed that individual preferences are orderings. Thus, for any individual i and any pair of alternatives x and y, either i strictly prefers x to y, strictly prefers y to x, or is indiffferent between them; i never regards x and y as being noncomparable (i.e., $\neg(xN_iy)$). It is sometimes the case that requiring social preferences to compare any pair of alternatives is overly demanding, so I shall asssume that a social preference may be either an ordering or a quasiordering.

The set of all possible orderings (resp. quasiorderings) of A is denoted by $\mathcal{R}$ (resp. $\mathcal{Q}$). A *profile* of individual preferences is a list $\mathbf{R} = (R_1, \ldots, R_n)$ of one preference for each individual. By assumption $\mathbf{R} \in \mathcal{R}^n$, the n-fold Cartesian product of $\mathbf{R}$.

The most familiar example of a social preference quasiordering is provided by the Pareto quasiordering. For a given profile $\mathbf{R}$, the *Pareto quasiordering* $R^P_{\mathbf{R}}$ of A is defined by setting

$$xR^P_{\mathbf{R}}y \leftrightarrow xR_iy \text{ for all } i \in S. \tag{1}$$

In words, x is weakly Pareto preferred to y if and only if everybody weakly prefers x to y. It is readily confirmed that (i) x is strictly Pareto preferred to y ($xP^P_{\mathbf{R}}y$) if everybody weakly prefers x to y and there exists at least one person who strictly prefers x to y, (ii) x is Pareto indifferent to y ($xI^P_{\mathbf{R}}y$) if and only if everybody is indifferent between them, and (iii) x is Pareto noncomparable to y ($xN^P_{\mathbf{R}}y$) if and only if someone strictly prefers x to y and somebody else strictly prefers y to x. Thus, the Pareto quasiordering identifies the pairs of alternatives on which there is a consensus preference.

Individuals may also have utilities. Utility is a numerical measure of an individual's well-being. The nature of well-being is the subject of continuing controversy. There are three main bases for determining well-being considered in the literature: (1) preferences or desire fulfillment, (2) mental states such as happiness or satisfaction, and (3) goods that are of intrinsic value to individuals (the objective list approach). The kinds of intrapersonal and interpersonal utility comparisons that are possible may depend on which account of well-being is adopted.[1]

Individual i's *utility function* is $U_i \colon A \to \mathbb{R}$.[2] The number $U_i(x)$ is the utility u_i that i obtains from x. It is supposed that i's utility function *represents* i's preferences in the sense that for all $x, y \in A$,

$$U_i(x) \geq U_i(y) \leftrightarrow xR_iy. \tag{2}$$

In other words, more utility is obtained with more preferred alternatives and the same utility is obtained with alternatives that are indifferent to each other. I let $\mathcal{U}$ denote the set of all possible utility functions and $\mathcal{U}^n$ denote the set of all profiles of individual utility functions. A profile of utility functions is written as $U = (U_1, \ldots, U_n)$. Thus, $U(x) = (U_1(x), \ldots, U_n(x))$.

The expression in (2) does not indicate whether utility or preference is the primitive concept. It could be that preference is defined from utility using (2) or, alternatively, that (2) is used to define utility from preference. In the latter case, utility only has meaning as a representation of preference. As a consequence, if U_i represents R_i, then so does any increasing transform of U_i, such as the function V_i which cubes each utility; that is, the function

[1] For detailed discussions of these alternative approaches to measuring well-being, see Adler (2012), Hausman and McPherson (1996), and Mongin and d'Aspremont (1998). Fleurbaey and Hammond (2004) provide a useful discussion of how different conceptions of well-being are used to make interpersonal utility comparisons.

[2] $\mathbb{R}$ (resp. $\mathbb{R}_+$, $\mathbb{R}_{++}$) is the set of all (resp. all nonnegative, all positive) real numbers. The n-fold Cartesian products of these sets are the Euclidean n-space $\mathbb{R}^n$ and its nonnegative and positive orthants $\mathbb{R}^n_+$ and $\mathbb{R}^n_{++}$.

V_i defined by setting $V_i(x) = [U_i(x)]^3$ for all $x \in A$.[3] In this case, the only meaningful information conveyed by two utility numbers is that one of them is bigger or that they are both the same. In other words, the utility function preserves the order of preference, but has no other meaning. It is therefore meaningful to say that i has either more or the same utility with x than with y. However, it is not meaningful to say how much more utility i has with x compared with y, nor is it possible to make any interpersonal comparisons of utility. In contrast, if utility is the primitive concept, it may be possible to make some kinds of interpersonal utility comparisons as well or to make some additional intrapersonal utility comparisons. This is an issue that I shall consider in more detail in the next section.

It is also possible to define a Pareto quasiordering R_U^P for a profile of utility functions U. Formally, R_U^P is defined by setting, for all $x, y \in A$,

$$xR_U^P y \leftrightarrow U_i(x) \geq U_i(y) \text{ for all } i \in S. \tag{3}$$

For all $x, y \in A$, the equivalence in (3) can be written more compactly as

$$xR_U^P y \leftrightarrow U(x) \geq U(y).^4 \tag{4}$$

The interpretation of R_U^P and its factors parallel those given above for the preference based Pareto quasiordering.

3. Meaningful Utility Comparisons

The limited ability to make intrapersonal and interpersonal comparisons of utility restricts the kinds of social rankings of the alternatives in A that are possible and the kinds of statements about utility that are meaningful. For example, if it is not possible to make any interpersonal utility comparisons, then one cannot rank alternatives by the sum of utilities obtained with them, as in classical utilitarianism, or to rank them by the utility obtained by the worst-off individual, as in some egalitarian theories.

Because the ability to measure and compare the well-beings of individuals is limited, distinct profiles of utility functions may contain the same usable information, and so cannot be distinguished. For example, if preference is

[3] A function $h\colon \mathbb{R} \to \mathbb{R}$ is an *increasing transform* if $h(t) \geq h(\bar{t})$ if and only if $t \geq \bar{t}$ for all $t, \bar{t} \in \mathbb{R}$.

[4] For two vectors $a, b \in \mathbb{R}^n$, (i) $a \geq b \leftrightarrow a_i \geq b_i$ for all $i \in S$, (ii) $a \gg b \leftrightarrow a_i > b_i$ for all $i \in S$, and (iii) $a > b \leftrightarrow a \geq b$ and $a \neq b$.

used to define utility as in (2), then the profiles $U = (U_1, \ldots, U_n)$ and $V = (V_1, \ldots, V_n)$ are informationally equivalent if V_i is an increasing transform of U_i for all $i \in S$ because the only information available about the well-being of any individual i is i's preference R_i and both U_i and V_i represent this preference.

More generally, two profiles of utility functions are *informationally equivalent* if they contain the same usable information about the individual well-beings. Information equivalence is formally modelled by an equivalence relation $\sim$ on the set of all possible profiles of utility functions $\mathcal{U}^n$, with $U{\sim}V$ denoting that U is informationally equivalent to V. This relation can be used to partition $\mathcal{U}^n$ into *information sets*. Two profiles of utility functions are in the same information set if and only if they are informationally equivalent to each other. Information sets formalize the ability to discriminate between different profiles of utility functions. Information sets are denoted by $\mathbf{U}$, $\mathbf{V}$, etc. It is important for $\sim$ to be an equivalence relation. Reflexivity of $\sim$ simply reflects the fact that a profile U cannot be distinguished from itself. Symmetry ensures that if U is informationally equivalent to V then V is also informationally equivalent to U. Transitivity implies that if T and U are informationally equivalent and so are U and V, then T and V cannot be distinguished either.

The most common approach in the literature to defining the relation $\sim$ is based on the way that physical quantities such as length, weight, and temperature are modelled in formal measurement theory (see Krantz, Luce, Suppes, and Tversky, 1971). In this approach, a statement about some quantity is meaningful if it is valid for all allowable transforms of the scale being used to measure it. For example, a statement about length measured in meters is only meaningful if it is also true if distance is measured by any positive multiple of the distance in meters. The set of scale transforms that needs to be considered is determined by the empirical procedure used in the measurement exercise (e.g., using a ruler to measure length).[5]

Applied to utility theory, this approach to measurement begins by specifying an allowable set of transforms that may be applied to profiles of utility functions and declares two profiles to be informationally equivalent if one can be obtained from the other by such a transform.[6] In other words, the infor-

[5]For a discussion of the empirical foundations of measurement theory and of the empirical procedure underlying the measurement of utility in expected utility theory, see Weymark (2005).

[6]The modelling of interpersonal utility comparisons in terms of transforms of utility

mational equivalence relation $\sim$ is not foundational in this approach; rather, it is determined by applying this procedure. The transforms specify the precision with which an individual's utility can be measured (the *measurability* of utility) and the extent to which the kinds of utility comparisons that can be made intrapersonally can also be made interpersonally (the *comparability* of utility). If any utility comparisons that can be made intrapersonally can also be made interpersonally, then utility is *fully comparable* for that degree of measurability.

Formally, an *invariance transform* is a list $\phi = (\phi_1, \ldots, \phi_n)$ of n functions $\phi_i \colon \mathbb{R} \to \mathbb{R}$, $i \in S$, that when applied to a profile of utility functions U results in an informationally equivalent profile. The vector-valued function ϕ is applied component-wise to U. That is, $\phi \circ U = (\phi_1 \circ U_1, \ldots, \phi_n \circ U_n)$, where $\circ$ denotes function composition. The degree to which utility is measurable intrapersonally and comparable interpersonally is formalized by specifying a set Φ of such transforms and defining the profiles of utility functions U and V to be informationally equivalent if and only if V can be obtained from U using some invariance transform $\phi \in \Phi$. That is, $U \sim V$ if and only if there exists a $\phi \in \Phi$ such that $V = \phi \circ U$. In order for the relation $\sim$ generated this way to be an equivalence relation, not any set Φ of invariance transforms can be employed; it must be an algebraic group.[7]

Invariance transforms are used to determine what kinds of statements about utilities are meaningful. To be meaningful, a statement obtained using the profile of utility functions U must also be valid when U is subjected to any transform in the set of invariance transforms Φ being considered. For example, the statement that i has more utility with x than j has with y with the profile U (i.e., $U_i(x) > U_j(y)$) is only meaningful if for any $\phi \in \Phi$, $\phi_i(U_i(x)) > \phi_j(U_j(y))$. The profiles U and $\phi \circ U$ are informationally equivalent, so any statement which is not true for both of these profiles has no meaning. An important implication of this observation is that any statement that is meaningful with one set of invariance transforms Φ is also valid for any smaller (in terms of set inclusion) set Φ'. The fewer the transforms that

functions was developed by Sen (1974), d'Aspremont and Gevers (1977), and Roberts (1980), among others. Detailed expositions of this approach may be found in Sen (1977), Boadway and Bruce (1984), d'Aspremont (1985), d'Aspremont and Gevers (2002), Bossert and Weymark (2004), and Fleurbaey and Hammond (2004).

[7]The set of functions Φ is an *algebraic group* if it (i) includes the identity transform that maps a profile to itself, (ii) contains the inverse of each transform in this set, and (iii) is closed under function composition (i.e., $\bar{\phi} \circ \phi$ is in Φ if both ϕ and $\bar{\phi}$ are in Φ).

utilities must be subjected to, the more meaningful information that can be obtained from any profile of utility functions.

I have focused on the use of invariance transforms to identify what profiles of utility functions are informationally equivalent and to identify what kinds of statements about utility are meaningful because this is the most common and most developed way of modelling these issues. An alternative approach proceeds by directly specifying what kinds of statements about utility are meaningful (e.g., utility levels are interpersonally comparable) and then uses this specification to determine the information equivalence relation $\sim$. For an introduction to this approach, see Bossert and Weymark (2004).

4. Alternative Measurability and Comparability Assumptions

The sets of invariance transforms that have been considered in the literature encapsulate different assumptions about the measurability and comparability of utility. The following examples describe the main alternatives.

Ordinal Measurability (OM). $\phi \in \Phi^{\mathrm{OM}}$ if and only if ϕ_i is an increasing transform for all $i \in S$.

In this case, individual utility is measured on an ordinal scale and no interpersonal utility comparisons are possible. It is meaningful to compare levels of utility intrapersonally because $U_i(x) \geq U_i(y)$ if and only $\phi_i(U_i(x)) \geq \phi_i(U_i(y))$ for any increasing transform ϕ_i. Levels of utility are not interpersonally comparable because the transforms for different people can be chosen independently. For example, if $U_1(x) > U_2(y)$, this inequality can be reversed by applying the identity transform to person 1's utility function and by adding a constant whose value is $U_1(y) - U_2(y) + 1$ to the utility assigned to each alternative by U_2. No other kinds of utility comparisons are possible either intrapersonally or interpersonally, such as comparisons of utility differences or ratios, because any such comparison for a given profile can be undone by a suitable choice of increasing transforms.

In effect, with OM, the only meaningful statements about utility that can be obtained from the profile U are those that can be made with the corresponding profile $\mathbf{R}$ of preference orderings. This conclusion is immediate if utility is defined from preference using (2). However, the same conclusion follows if utility functions are the primitives of the model, but utility comparisons are only meaningful if they are preserved by any independently chosen increasing transforms of the individual utility functions.

Ordinal Measurability and Full Comparability (OFC). $\phi \in \Phi^{\text{OFC}}$ if and only if for all $i \in S$, $\phi_i = \phi_0$ for some increasing transform ϕ_0.

With this set of invariance transforms, utility is measured on a common ordinal scale. Because any transform ϕ permitted by OFC is also permitted by OM, any statement about utility that is meaningful with OM is also meaningful with OFC. In particular, intrapersonal comparisons of levels of utility are meaningful. Now, in addition, utility levels are also meaningful interpersonally because the ordering of utilities interpersonally is preserved if the individual utility functions are subjected to a common increasing transform. No other kinds of utility comparisons are meaningful with OFC.

Cardinal Measurability (CM). $\phi \in \Phi^{\text{CM}}$ if and only if there exist numbers $a_1, \ldots, a_n$ and positive numbers $b_1, \ldots, b_n$ such that $\phi_i(t) = a_i + b_i t$ for all $i \in S$.

With CM, individual utility is measured on a cardinal scale, by which it is meant that a statement about individual utility is meaningful if it is valid for some set of increasing affine transforms of this person's utility function.[8] The scaling factor b_i determines the size of the unit that utility is measured in and a_i determines its origin. Because the individual transforms can be chosen independently, CM does not permit any interpersonal utility comparisons. Because an increasing affine transform is an increasing transform, levels of utility are intrapersonally comparable. Furthermore, intrapersonal comparisons of utility differences are now meaningful. This follows because the statement $U_i(w) - U_i(x) \geq U_i(y) - U_i(z)$ holds if and only if $[a_i + b_i U_i(w)] - [a_i + b_i U_i(x)] \geq [a_i + b_i U_i(y)] - [a_i + b_i U_i(z)]$ when $b_i > 0$.[9]

Cardinal Measurability and Unit Comparability (CUC). $\phi \in \Phi^{\text{CUC}}$ if and only if there exist numbers $a_1, \ldots, a_n$ and a positive number b such that $\phi_i(t) = a_i + bt$ for all $i \in S$.

As with CM, intrapersonal comparisons of utilty levels and differences are meaningful. In addition, utility difference comparisons are meaningful

[8] A function $h\colon \mathbb{R} \to \mathbb{R}$ is an *increasing affine transform* if and only if $h(t) = a + bt$ for all $t \in \mathbb{R}$, where a is any number and b is any positive number.

[9] To show this equivalence, first delete the a_i terms on each side of the latter inequality because they sum to 0 and then divide both sides of the resulting inequality by the positive number b_i.

interpersonally. This follows because $U_i(w) - U_i(x) \geq U_j(y) - U_j(z)$ if and only if $[a_i + bU_i(w)] - [a_i + bU_i(x)] \geq [a_j + bU_i(y)] - [a_j + bU_i(z)]$ when $b > 0$.[10]

Cardinal Measurability and Full Comparability (CFC). $\phi \in \Phi^{\text{CFC}}$ if and only if there exists a number a and a positive number b such that $\phi_i(t) = a + bt$ for all $i \in S$.

With CFC, only increasing affine transforms that are the same for everybody are considered. These transforms are also used in OM, OFC, CM, and CUC, so any statement about utility that is meaningful with them is also meaningful with CFC. In particular, with CFC, utility levels and differences are both intrapersonally and interpersonally comparable.

Ratio-Scale Measurability (RSM). $\phi \in \Phi^{\text{RS}}$ if and only if there exist positive numbers $b_1, \ldots, b_n$ such that $\phi_i(t) = b_i t$ for all $i \in S$.

With RSM, each individual's utility is measured on a ratio scale, just like height and distance. Each of the transforms used in the definition of RSM is an increasing similarity transform.[11] As with CM, utility levels and differences are comparable interpersonally. Even though each individual's transform can be chosen independently, they all map 0 back to itself. As a consequence, it is meaningful to say that an individual has a utility of 0 and that this origin for utility is the same for everybody. This utility value could correspond to the value assigned to any alternative in which an individual is indfferent between living or dying (see Blackorby and Donaldson, 1984; Blackorby, Bossert, and Donaldson, 2005). Because the utility origin has meaning, it is also possible to compare ratios of utilities intrapersonally and interpersonally. This follows because the inequality $U_i(w)/U_i(x) \geq U_j(y)/U_j(z)$ is unaffected if i's and j's utilities are subject to similarity transforms (i and j need not be distinct).

Ratio-Scale Measurability and Full Comparability (RSF). $\phi \in \Phi^{\text{RSF}}$ if and only if there exists a positive number b such that $\phi_i(t) = bt$ for all $i \in S$.

[10] As in the previous footnote, the origin terms on each side of the latter inequality cancel even though a_i need not equal a_j. Because the unit term b is the same and positive on both sides of the resulting inequality, it is then possible to divide both sides of it by b. This would not be possible if the unit terms of the affine transforms were different for i and j.

[11] A *similarity transform* is an affine transform in which the origin term is 0.

With RSF, each individual's utility is measured on a common ratio scale. Any transform in this set is also a member of each of the other sets of invariance transforms previously considered, so any utility statement that is meaningful with them is also meaningful with RSF. Morevover, a ratio of the form $U_i(x)/U_j(y)$ is now meaningful because it is invariant to a common similarity transform of i's and j's utility functions.

Each of the examples of sets of invariance transforms discussed above has the property that the transforms which can be applied to any one individual's utility function can also be applied to anybody else's. This is not always a reasonable assumption. In the next section, I shall discuss a proposal advanced by Harsanyi (1977) in which a member of society makes intrapersonal and interpersonal utility comparisons by a process of empathetic identification. One would expect that he would be able to make meaningful statements about his own utility that he could not make about somebody else. It is possible to model this kind of asymmetry using sets of invariance transforms that differ across individuals, but this is an issue that has received very little attention to date.[12]

5. Possible Bases for Making Utility Comparisons

The preceding discussion has shown how limits on the ability to make utility can be modelled, but it has not considered the basis on which such comparisons are made. Harsanyi (1955) has described one such basis. He has a mental state account of well-being; for him, utility is a measure of satisfaction.[13] According to Harsanyi, an individual's utility is determined by a function, common to all individuals, of an alternative and the causal variables that determine the tastes and other subjective factors that influence how much satisfaction is obtained with this alternative. This view provides a *causal variables* account of well-being. Formally, for all $x, y \in A$,

$$U_i(x) = v(x; c_i), \tag{5}$$

where c_i denotes the values of the causal variables for person i. The causal variables provide a comprehensive description of everything that affects an individual's well-being other than the alternative being evaluated itself, such

[12] For an example of this approach, see Khmelnitskaya and Weymark (2000).

[13] Harsanyi's view that utility measures satisfaction is discussed in Weymark (1991).

as his biological features and life history. With this causal variables interpretation of utility, two individuals are equally well off with the same alternative if they have the same values of the causal variables. This *similarity postulate* is an *a priori* nonempirical claim. Harsanyi regards interpersonal utility comparisons as being empirical statements that rely on this postulate for their validity.[14]

With this account of well-being, an individual makes interpersonal utility comparisons through a process of empathetic identification. Individual k determines how well off i is with alternative x by imagining how well off he would be himself with this alternative and with i's causal varibles; that is, k determines the value of $U_i(x)$ using (5). In effect, all interpersonal utility comparisons are reduced to intrapersonal comparisons. Morover, this approach can accommodate taste changes. If i's tastes change, this is reflected in a change in his causal variables, with the consequence that the utility obtained with x as given by (5) can differ before and after the taste change. If the functional form of v in (5) and the values of each individual's causal variables are commonly known, then everybody would make the same utility comparisons. However, as Harsanyi emphasizes, this information is only imprecisely known, so this is not the case in practice.

An alternative basis for making utility comparisons is provided by the concept of an extended preference. A *social situation* is a pair (x, i) consisting of being individual i complete with i's characteristics, both objective and subjective, when the alternative is x. Through a process of empathetic identification, each individual k in society is thought of as forming an *extended preference* $\tilde{R}_k$ of $A \times S$. Thus, for example, $(x, i)\tilde{R}_k(y, j)$ indicates that k weakly prefers to be i when the alternative is x than to be j when the alternative is y. The idea of putting oneself in the shoes of others in order to compare the well-beings of different individuals may be found in Adam Smith (1759) and for this reason when k makes such comparisons, following Smith I call him a *spectator*. This idea has been formalized by Harsanyi (1977) and Kolm (1997).

Spectator k may also have a utility function $\tilde{U}$ for extended alternatives, with

$$\tilde{U}_k(x, i) \geq \tilde{U}_k(y, j) \leftrightarrow (x, i)\tilde{R}_k(y, j) \tag{6}$$

[14]Kolm (1997) offers a similar causal variables account of well-being using his concept of a *fundamental preference or utility*.

for all $(x, i), (y, j) \in A \times S$. For all $x \in A$ and all $i, k \in S$, by setting

$$U_i^k(x) = \tilde{U}_k(x, i), \tag{7}$$

the inequality in (6) may be rewritten as

$$U_i^k(x) \geq U_j^k(y) \tag{8}$$

for all $(x, i), (y, j) \in A \times S$. Thus, for each $i \in S$, k is attributing the utility function U_i^k to i and making intrapersonal and interpersonal utility comparisons using these functions.

If k's extended preference $\tilde{R}_k$ is the primitive in (6), which is typically what is assumed when extended preferences are used, then it can be represented by any increasing transform of $\tilde{U}_k$. This is equivalent to saying that the utility measurability-comparability assumption is Ordinal Measurability and Full Comparability (OFC). As we have seen, with OFC it is not possible to make any utility difference or ratio comparisons.

The use of an extended preference as a foundation for making interpersonal utility comparisons is widely regarded as being problematic. For example, this approach raises troublesome questions about personal identity, such as whether it is meaningful for k to identify himself with somebody else (see, e.g., Adler, 2012). Attempts to circumvent this problem by adopting a causal variables account of *preferences* analogous to the causal variables account of utility described above (i.e., by regarding any individual i's preferences on A as being determined by the values of the causal variables using a function that is common to all individfuals) is also problematic. For example, the meaning of a comparison involving individuals with different values of the causal variables is unclear.[15]

These problems do not arise if utility, rather than preference, is the primitive in (6). Then, to say that k weakly prefers to be i with x than j with y means that, in k's estimation, i is better off in x than j is in y. This way of determining extended preferences is used by Adler (2012). Harsanyi (1977) is not completely clear on this issue, but it appears that he does the same using his causal variables account of *utility*. With utility as the primitive concept, one is not *a priori* restricted to only making comparisons of utility levels.[16]

[15]For a critique of the foundations of extended preferences, see Broome (1993).

[16]Other proposals for making utility comparisons have been suggested, including ones

6. Bergson–Samuelson Social Welfare Functions

Late 18th and early 19th century economists such as Francis Edgeworth, Alfred Marshall, and Arthur Pigou evaluated alternative social policies using the sum of utilities criterion proposed by the classical utilitarians.[17] This approach fell into disrepute following Lionel Robbins' influential critique of the scientific basis for the interpersonal utility comparisions used by the utilitarians (see Robbins, 1932). Robbins did not dispute that interpersonal utility comparisons are routinely made; what he argued was that they are normative and cannot be determined objectively. One response to Robbins' critique was to limit welfare evaluations to the comparisons of alternatives that can be ranked by the Pareto quasiordering. However, in practice, this approach made it difficult to endorse any major shift in public policy because it would typically benefit some individuals at the expense of others.

Bergson (1938), and later Samuelson (1947), took a different tack. For them, value judgments are involved in making interpersonal utility comparisons, but that does not mean that welfare economics must restrict attention to Paretian comparisons. Rather, "[i]t is a legitimate exercise of economic analysis to examine the consequences of various value judgments" (Samuelson, 1947, p. 220) such as those involved in making interpersonal utility comparisons.

Bergson and Samuelson evaluated different social alternatives using what is now known as a *Bergson–Samuelson social welfare function*. This is a function $W\colon A \to \mathbb{R}$ that assigns a number to each social alternative in A. Bergson and Samuelson were interested in deriving necessary conditions for a welfare optimum when the alternatives are allocations of economic goods (how much of each good is consumed or produced by each individual or firm), but there is no necessity for restricting alternatives in this way if non-economic alternatives are relevant for the welfare evaluation. The function W is used to determine a social preference R by setting

$$xRy \leftrightarrow W(x) \geq W(y) \tag{9}$$

for all $x, y \in A$. As Samuelson recognized, only the ordinal properties of W are relevant for ranking social alternatives as any increasing transform of

that regard interpersonal utility comparisons as being inherently normative. For extended discussions of a number of these approaches, see Adler (2012) and Fleurbaey and Hammond (2004).

[17] For an illuminating discussion of the history of welfare economics, see Mandler (1999).

W would result in the same social preference. Of course, one could instead start with a preference ordering R as in Samuelson (1981), in which case a function W that satisfies (9) is a Bergson–Samuelson social welfare function representation of R.

The function W makes no explicit use of utility information. In order to derive their welfare optimality conditions, Bergson and Samuelson employed an *individualistic Bergson–Samuelson social welfare function.* The welfare function W is individualistic if, for a given profile of utility functions $U \in \mathcal{U}^n$, W can be written as

$$W(x) = W^*(U(x)) = W^*(U_1(x), \dots, U_n(x)) \tag{10}$$

for all $x \in A$.[18] The function $W^*\colon \mathbb{R}^n \to \mathbb{R}$ is usually assumed to be increasing in each person's utility. By combining (9) and (10), for a given given $U \in \mathcal{U}^n$, we obtain

$$xRy \leftrightarrow W^*(U(x)) \geq W^*(U(y)) \tag{11}$$

for all $x, y \in A$.

In (10), W is obtained by composing the profile of utility functions U with the function W^*. The argument of W^* is an n-vector of individual utilities $u = (u_1, \dots, u_n)$. Such a function is called a *social welfare function* (without any qualifying adjective). Any approach to social evaluation that uses a social welfare function is necessarily welfarist because it only takes account of the individual utilities obtained with each alternative. A *social welfare ordering* is an ordering of vectors of individual utilities. Corresponding to W^* is the social welfare ordering R^* of $\mathbb{R}^n$ given by

$$uR^*v \leftrightarrow W^*(u) \geq W^*(v) \tag{12}$$

for all $u, v \in \mathbb{R}^n$.

In order to socially rank the alternatives in A, only the social welfare ordering R^* is needed. That is, by combining (11) and (12), it follows that for all $x, y \in A$,

$$xRy \leftrightarrow U(x)R^*U(y). \tag{13}$$

In other words, in order to determine how x and y are socially ranked, one first computes the vectors of individual utilities $U(x)$ and $U(y)$ associated

[18] In their versions of (10), Bergson and Samuelson suppose that an individual's utility function only depends on what he consumes and produces.

with them and then compares these vectors using the social welfare ordering R^*. For this reason, R^* is often taken as the primitive of the analysis. When this is the case, W^* is a representation of R^*.

In (11), an individualistic Bergson–Samuelson social welfare function W generates a social ordering R_U of A for each profile $U \in \mathcal{U}^n$ given the social welfare function W^*. If W^* is increasing in each person's utility, then R^*_U agrees with the Pareto quasiordering R^P_U introduced in Section 2 on any pair of alternatives that R^P_U regards as being comparable. That is, R^*_U extends R^P_U.[19] The use of a Bergson–Samuelson social welfare function provides a way comparing all alternatives while at the same time respecting the consensus ranking that is captured by the Pareto quasiordering.

7. Informationally Invariant Social Welfare Orderings

An individualistic Bergson–Samuelson social welfare function is defined for a fixed profile of utility functions U. The social ranking of A must be unaffected if U is replaced by an informationally equivalent (according to $\sim$) profile of utility functions V.[20] For all $U \in \mathcal{U}^n$, letting R_U denote the social ranking associated with the profile U, this invariance restriction can be stated formally as follows.

Information Invariance. For all $U, V \in \mathcal{U}^n$, $R_U = R_V$ if $U \sim V$.

To satisfy this restriction, if U is replaced by the informationally equivalent profile V, then the functional form of W^* may have to be adjusted so as to preserve the social ranking of the alternatives in (11). For example, suppose that $n = 2$ and that W^* is defined by setting $W^*_U(u) = u_1 + u_2$ for all $u \in \mathbb{R}^2$ when the profile is U. Now suppose that the profile V is defined by setting $V_1 = 2U_1$ and $V_2 = U_2$. If W^*_U is replaced by W^*_V defined by setting $W^*_V(u) = u_1/2 + u_2$ for all $u \in \mathbb{R}^2$, then $W(x) = W^*_U(U_1(x), U_2(x)) = W^*_V(V_1(x), V_2(x))$ for all $x \in A$ and, hence, $R_U = R_V$.

[19] For binary relations B and $\bar{B}$ on O, $\bar{B}$ *extends* B if for all $x, y \in O$, (i) $xBy \rightarrow x\bar{B}y$ and (ii) $xB^Ay \rightarrow x\bar{B}^Ay$.

[20] In my account of an individualistic Bergson–Samuelson social welfare function, it is not assumed that the individual utility functions are ordinally measurable and interpersonally noncomparable. In this regard, I follow Bergson (1938) and Samuelson (1947). However, in some of their later writings, for example in Samuelson (1981), and in much of the literature on Bergson–Samuelson social welfare functions, this assumption is made. For an exegesis of what Bergson and Samuelson say on this issue, see Fleurbaey and Mongin (2005).

In order to satisfy Information Invariance, it is not always the case that the functional form of W^* needs to be modified when one profile is replaced by one that is informationally equivalently to it. This observation plays a fundamental role in the literature on social welfare functionals discussed in Sections 10 and 11. For example, if in the preceding example, V_2 is instead given by $V_2 = 5+2U_2$, then with $W^*_U = W^*_V = W^*$ where $W^*(u) = u_1+u_2$ for all $u \in \mathbb{R}^2$, $W_V(x) = W^*(V_1(x), V_2(x)) = 5+2W^*(U_1(x), U_2(x)) = 5+2W_U(x)$ for all $x \in A$. Thus, the Bergson–Samuelson social welfare functions W_U and W_V for these two profiles are ordinally equivalent, and so generate the same ranking of the alternatives.[21]

For a welfarist, it is an unsatisfactory feature of the Bergson–Samuelson approach that the functional form of the social welfare function W^* may depend on the profile of utility functions being considered. For that reason, it is more natural to begin with a fixed social welfare function W^* and to then use (11) to determine the social ranking R_U for each profile $U \in \mathcal{U}^n$. In order to satisfy Information Invariance, it is therefore necessary to place restrictions on the functional form of W^*. These restrictions depend on the structure of the information sets generated by informational equivalence relation $\sim$. But, as we have seen, only the ordinal properties of a (Bergson–Samuelson) social welfare function are relevant for determining its underlying ordering. So, it is in fact only necessary to place restrictions on the social welfare ordering R^* that ensure that the social ranking of the alternatives in (13) is invariant when U is replaced by an informationally equivalent profile.

When the information sets of $\sim$ are identified using a set of invariance transforms Φ, as in Section 3, in order for Information Invariance to be satisfied, the social welfare ordering R^* must satisfy Φ *Information Invariance.*

Φ Information Invariance. For all $u, v \in \mathbb{R}^n$ and all $\phi \in \Phi$, $uR^*v \leftrightarrow \phi(u)R^*\phi(v)$.

If R^* satisfies this invariance property, then from (13), it follows that $xRy \leftrightarrow U(x)R^*U(y) \leftrightarrow \phi(U(x))R^*\phi(U(y))$ for all $x, y \in A$ if $V = \phi(U)$ for some $\phi \in \Phi$, as required by Information Invariance.

As an illustration of the application of Φ Information Invariance, consider the set of transforms Φ^{OM} for Ordinal Measurability (OM). With OM, ϕ_i can

[21] Note that while Information Invariance requires the social ranking of the alternatives in A to be the same for informationally equivalent profiles of utility functions, the corresponding Bergson-Samuelson social welfare functions only need to be ordinally equivalent.

be any increasing transform. Thus, $u_i \geq v_i \leftrightarrow \phi_i(u_i) \geq \phi_i(v_i)$ for all $i \in S$. That is, for each individual i, the utility $\phi_i(u_i)$ is at least as large as the utility $\phi_i(v_i)$ if and only if the utility u_i is at least as large as the utility v_i. Intrapersonal comparisons of utility levels are the only meaningful statements about utility with OM, and so the social welfare ordering R^* of $\phi(u)$ and $\phi(v)$ must be the same as that of u and v.[22]

8. Examples of Social Welfare Orderings

I now present some of the most familiar social welfare orderings found in the literature and discuss what kinds of utility comparisons are needed in order to employ them.[23]

The *utilitarian* social welfare ordering is defined by setting

$$uR^*v \leftrightarrow \sum_{i=1}^{n} u_i \geq \sum_{i=1}^{n} v_i \tag{14}$$

for all $u, v \in \mathbb{R}^n$. With this criterion, utility vectors are ranked by the sum of their utilities. The RHS of (14) may be equivalently written as $\sum_{i=1}^{n}(u_i - v_i) \geq 0$. So, in order to use the utilitarian social welfare ordering, one must be able to make intrapersonal and interpersonal comparisons of utility differences. However, it is not necesssary to compare utility levels interpersonally because adding person-specific constants to the utilities does not affect the sum of the utility differences. Hence, Φ Information Invariance must be satisfied for a set of invariance transforms Φ that is contained in Φ^{CUC}, the set of transforms for Cardinal Measurability and Unit Comparability.

For $u \in \mathbb{R}^n$, let $u_{()} = (u_{(1)}, \ldots, u_{(n)})$ be a permutation of u such that $u_{(i)} \geq u_{(i+1)}$ for all $i = 1, \ldots, n-1$; that is, the permutation rank orders the utilities in a nonincreasing order. The *leximin* social welfare ordering is defined by setting, for all $u, v \in \mathbb{R}^n$,

$$\begin{aligned} uR^*v \leftrightarrow u \text{ is a permutation of } v \text{ or there exists a } j \in S \text{ such that} \\ u_{(i)} = v_{(i)} \text{ for all } i > j \text{ and } u_{(j)} > v_{(j)}. \end{aligned} \tag{15}$$

[22]For geometrical illustrations of this invariance property for different sets of invariance tranforms, see Blackorby, Donaldson, and Weymark (1984).

[23]Further examples may be found in Boadway and Bruce (1984) and Bossert and Weymark (2004).

In (15), the utility vector u is socially preferred to the utility vector v if the worst-off individual in u is better off than the worst-off individual in v. If they are equally well off, the social preference is determined by comparing the utilities of the individuals who are second-worst-off in u and v, and so on. There is social indifference if and only if $u_{()} = v_{()}$. Leximin compares utility vectors by comparing the utility levels of individuals who occupy the same rank in the utility hierarchy, and so requires that these levels be intrapersonally and interpersonally comparable. No other kinds of utility comparsons are needed, so Φ Information Invariance must be satisfied for a set of invariance transforms Φ that is contained in Φ^{OFC}, the set of transforms for Ordinal Measurability and Full Comparability.

A social welfare ordering is a *single-parameter Gini* social welfare ordering if there exists a $\delta \geq 1$ such that for all $u, v \in \mathbb{R}^n$,

$$uR^*v \leftrightarrow \sum_{i=1}^{n} \left[i^{\delta} - (i-1)^{\delta}\right] u_{(i)} \geq \sum_{i=1}^{n} \left[i^{\delta} - (i-1)^{\delta}\right] v_{(i)}. \tag{16}$$

This class of social welfare orderings was introduced by Donaldson and Weymark (1980). A different social welfare ordering is obtained for each value of the parameter δ. In (16), utility vectors are compared using a weighted sum of rank-ordered utilities. When $\delta > 1$, lower ranks receive higher weights.[24] The parameter δ is a measure of inequality aversion, with higher values of δ corresponding to more social aversion to inequality in the distribution of utilities. When $\delta = 2$, for each $i \in S$, the ith weight is the ith odd number $2i - 1$. These are the weights used in the social welfare ordering underlying the Gini index of inequality. Comparisons of utility levels are needed to rank order utilities and comparisons of utility differences are needed to compare weighted sums of utilities. Hence, Φ Information Invariance must be satisfied for a set of invariance transforms Φ that is contained in Φ^{CFC}, the set of transforms for Cardinal Measurability and Full Comparability.

Another parameterized class of social welfare orderings is the class underlying the Atkinson (1970) inequality indices. In this example, only nonnegative utilities are considered. A social welfare ordering is an *Atkinson* social

[24]When $\delta = 1$, the weights are all equal to 1 and (16) reduces to (14), the utilitarian social welfare ordering.

welfare ordering if there exists a $\gamma \leq 1$ such that for all $u, v \in \mathbb{R}^n_+$,

$$uR^*v \leftrightarrow \begin{cases} \left[\frac{1}{n}\sum_{i=1}^n u_i^\gamma\right]^{1/\gamma} \geq \left[\frac{1}{n}\sum_{i=1}^n v_i^\gamma\right]^{1/\gamma}, & \text{if } \gamma \leq 1 \text{ and } \gamma \neq 0; \\ \prod_{i=1}^n u_i^{1/n} \geq \prod_{i=1}^n v_i^{1/n}, & \text{if } \gamma = 0. \end{cases} \tag{17}$$

Adler (2012) advocates using an Atkinson social welfare ordering for morally evaluating decisions with significant social consequences. As with the single-parmeter Ginis, γ is an inequality aversion parameter, but now the parameter value is inversely related to the degree of inequality aversion.[25] Because the population size is fixed, for $\gamma \neq 0$, utility vectors are being compared by first raising them all to the power γ and then summing these transformed utilities. The first step introduces inequality aversion (except when $\gamma = 1$) into what would otherwise be a utilitarian sum. When $\gamma = 0$, the criterion employed in (17) is equivalent to comparing utility vectors using the product of the individual utilities. Utilities must be measured on a common ratio scale for (17) to be meaningful. Hence, Φ Information Invariance must be satisfied for a set of invariance transforms Φ that is contained in Φ^{RSF}, the set of transforms for Ratio-Scale Measurability and Full Comparability.

All of the preceding examples treat individuals symmetrically. As a final example, I consider a class of social welfare orderings that do not. A social welfare ordering is a *serial dictatorship* if there exists a permutation $\pi\colon S \to S$ of the individuals such that for all $u, v \in \mathbb{R}^n$,

$$\begin{aligned} uR^*v \leftrightarrow u \text{ is a permutation of } v \text{ or there exists a } j \in S \text{ such that} \\ u_{\pi(i)} = v_{\pi(i)} \text{ for all } i < j \text{ and } u_{\pi(j)} > v_{\pi(j)}. \end{aligned} \tag{18}$$

In (18), $\pi(1)$ dictates when he is not equally well off in u and v, $\pi(2)$ dictates when $\pi(1)$ is equally well off in u and v but $\pi(2)$ is not, and so on. To employ this social welfare ordering, it is only necessary to make intrapersonal comparisons of utility levels. Hence, Φ Information Invariance must be satisfied for a set of invariance transforms Φ that is contained in Φ^{OM}, the set of transforms for Ordinal Measurability, which is the least demanding information assumption.

Leximin and serially dictatorial social welfare orderings are not continuous and cannot be represented by a social welfare function. In each of the other examples, a social welfare function W representing the social welfare

[25] When $\gamma = 1$, (17) is the utilitarian order (14).

ordering can be obtained by setting $W(u)$ equal to the term on the LHS of the inequality used in its definition. For example, the utilitarian social welfare function is defined by setting $W(u) = \sum_{i=1}^{n} u_i$ for all $u \in \mathbb{R}^n$.

9. Arrovian Social Welfare Functions

With an individualistic Bergson–Samuelson social welfare function, a social preference on the set of social alternatives A is determined for a given profile of utility functions. If an informationally equivalent profile is used instead, the new social preference must coincide with the original one. In my formulation of a Bergson–Samuelson social welfare function, it is not assumed that utility is ordinally measurable and interpersonally noncomparable. However, as previously noted, the literature on Bergson–Samuelson welfare economics often makes this assumption. With this assumption, the social preference is in effect determined by a given profile of individual preference orderings on A. This is often described as saying that the tastes or values of the individuals in society are fixed. What if these tastes change? Little (1952, p. 423) summarizes the Bergson–Samuelson approach to taste change quite succiently:

> If tastes change, we may expect a new ordering of all the conceivable states; but we do not require that the difference between the new and the old ordering should bear any particular relation to the changes of taste which have occurred. We have, so to speak, a new world and a new order; and we do not demand correspondence between the change in the world and the change in the order.

Arrow in his *Social Choice and Individual Values* (Arrow, 1963) has a different view; he requires the social preferences associated with different tastes to satisfy a consistency condition, his Independence of Irrelevant Alernatives condition (defined below). Arrow regards the problem of collective decision-making as being one of preference aggregation; the objective is to determine a social ranking of the social alternatives based on the preferences of the individuals in society. This procedure for aggregating preferences is formalized by an *Arrovian social welfare function*, which is a function $f\colon \mathcal{D} \to \mathcal{B}$ that assigns a social binary preference relation $R_{\mathbf{R}} = f(\mathbf{R})$ on A to each profile $\mathbf{R}$ in some domain $\mathcal{D} \subseteq \mathcal{R}^n$ of profiles of individual preference orderings on

A, where $\mathcal{B}$ is the set of all binary relations on A. It is usually assumed that social preferences are orderings and that the domain is $\mathcal{R}^n$, the set of all conceivable profiles of individual preference orderings, but it is useful not to build these assumptions into the very definition of an Arrovian social welfare function so as to allow for alternative possibilities. Arrow (1963, p. 23) describes his approach as follows: "In effect, the social welfare function described here is a method of choosing which social welfare function of the Bergson type will be applicable . . .".[26]

Arrow fundamentally changed the way that collective decision-making is analyzed. Rather than starting with a particular method of making collective decisions, such as majority rule, he instead sought to identify which preference aggregation procedures satisfy a number of *a priori* desirable properties that are formalized as axioms. Arrow thereby founded the field of axiomatic social choice theory. Unfortunately, as Arrow has shown in his famous impossibility theorem, the properties that he thought a social welfare function should satisfy are incompatible.[27]

Arrow requires social preferences to be orderings, which is his *collective rationality* assumption.

Social Ordering. For each preference profile $\mathbf{R} \in \mathcal{D}$, the corresponding social preference $R_{\mathbf{R}}$ is an ordering.

Arrow envisages the aggregation procedure as being designed before the individual preferences are known and so wants it to be able to deal with any conceivable preference profile.

Unrestricted Domain. The domain $\mathcal{D}$ is the set of all profiles of individual preference orderings $\mathcal{R}^n$.

As in Bergson–Samuelson welfare economics, Arrows wants a social preference to be consistent with the Pareto quasiordering. He formulates this

[26]Samuelson (1967) has argued that the fundamental difference between Bergson–Samuelson and Arrovian social welfare functions is that the former are defined for a single profile, whereas the latter have a multi-profile domain. This claim led to the development of a number of single preference profile Arrovian impossibility theorems. See Fleurbaey and Mongin (2005). Fleurbaey and Mongin convincingly argue that the essential difference between Bergson and Samuelson on the one hand and Arrow on the other is whether there is any consistency condition for taste changes, not whether multiple profiles are being considered.

[27]My discussion of Arrow's Theorem is based on the version that appears in the second edition of his book. For a particularly lucid presentation of this theorem, see Sen (1970a).

condition in a weak form, only requiring unanimous strict preferences to be respected.

Weak Pareto. For each preference profile $\mathbf{R} \in \mathcal{D}$ and every pair of alternatives $x, y \in A$, if xP_iy for all $i \in S$, then $xP_{\mathbf{R}}y$.

Arrow's independence condition places restrictions on how the social preferences for different preference profiles are related. It requires the social preference on any pair of alternatives x and y to only depend on the individual preferences for them. In other words, the way that any individual ranks any pair of alternatives that includes at least one alternative different from x or y is irrelevant for socially ranking x and y.[28]

Independence of Irrelevant Alternatives. For all preference profiles $\mathbf{R}, \mathbf{R}' \in \mathcal{D}$ and every pair of alternatives $x, y \in A$, if R_i and R'_i coincide on $\{x, y\}$ for all $i \in S$, then so do $R_{\mathbf{R}}$ and $R_{\mathbf{R}'}$.[29]

A *dictator* is an individual for whom the social preference always coincides with this person's preference on a pair of alternatives whenever he is not indifferent between them. Only when he is indifferent may other individuals' preferences be taken into account. This definition does not preclude respecting the dictator's preferences even when he is indifferent. Arrow's final axiom requires that nobody should be given this much power.

Nondictatorship. There is no dictator.

Arrow's Theorem shows that these five axioms are inconsistent if there are at least three alternatives and the number of individuals is finite and greater than 1.[30] To faciliate the comparison of this theorem with the axiomatizations of utilitarianism and leximin in Section 11, I state it in the form of a possibility theorem.

Theorem 1 (Arrow's Theorem). *If n is finite with $n \geq 2$ and $|A| \geq 3$, then a social welfare function is dictatorial if it satisfies Social Ordering, Unrestricted Domain, Weak Pareto, and Independence of Irrelevant Alternatives.*

[28]It is sometimes suggested that this axiom is what prevents taking account of intensities of preference or interpersonal utility comparisons, but, in fact, they are ruled out of consideration by the very definition of an Arrovian social welfare function.

[29]Two binary relations B and B' *coincide* on $\{x, y\}$ if $xBy \leftrightarrow xB'y$ and $yBx \leftrightarrow yB'x$.

[30]Majority rule satisfies Arrow's axioms if there are only two alternatives.

Each of the Arrow axioms has received a fair amount of criticism, but none more than Independence of Irrelevant Alternatives. It is possible to relax this axiom without abandoning it altogether as Little (1952) does in the text quoted above. Fleurbaey, Suzumura, and Tadenuma (2005) have suggested a weakening of Arrow's independence condition that requires the social preference on a pair of alternatives to coincide for two profiles if the indifference curves containing these alternatives coincide in both profiles for each individual. This relaxation of Arrow's independence condition is consistent with Arrow's other nondomain axioms when natural economic structure is placed on the set of alternatives and on the individual preferences. Moreover, with this weaker independence condition, as shown by Fleurbaey and Maniquet (2011), it is also possible to satisfy fairness norms of the kind considered in the literature on the fair allocation of economic goods.

Alternatively, one may retain the full force of Arrow's independence condition, but rather than demanding the social welfare function to determine a social ordering for each preference profile one may instead suppose that it is only required to determine a quasiordering that extends the Pareto quasiordering.[31] Not only is this possible, it is also possible to treat individuals impartially in the sense that permuting their preferences has no affect on the social preference. Unfortunately, as Weymark (1984) has shown, there is only one social welfare function that satisfies all of these desiderata, namely, the social welfare function whose social preferences are always given by the Pareto quasiordering. So relaxing Arrow's collective rationality assumption in this way fails in the objective of extending the Pareto quasiordering in a nontrivial way if individuals are treated impartially.

10. Social Welfare Functionals and Welfarism

The nihilism of Arrow's Theorem led Sen (1970a, 1974) to criticize the informational basis used by Arrow to determine a social preference. The very formulation of an Arrovian social welfare function as a mechanism for aggregating preferences precludes the use of any utility information that cannot be obtained from individual preferences on the set of social alternatives. In particular, it precludes the use of interpersonal utility comparisons. As a consequence, prominent social choice procedures such as utilitarianism or

[31] Recall that this means that this quasiordering agrees with the Pareto quasiordering whenever the latter ranks pairs of alternatives, but it may also make futher comparisons.

leximin are ruled out of consideration from the outset because they cannot be expressed using an Arrovian social welfare function. The informational poverty of Arrow's framework, not just his axioms, plays a fundamental role in precipitating his impossibility theorem.[32]

In order to overcome the limitations imposed by Arrow's informational assumption, Sen has proposed a more general framework for determining social preferences. He models the collective choice problem using a *social welfare functional*, which is a function $F\colon \mathcal{D} \to \mathcal{B}$ that assigns a social preference $R_U = F(U)$ on A to each profile of utility functions U in some domain $\mathcal{D} \subseteq \mathcal{U}^n$.[33] If social preferences are required to be orderings, then a social welfare functional specifies how the choice of a Bergson–Samuelson social welfare function (more precisely, the ordering underlying such a function) depends on the profile of individual utility functions being considered. Restrictions on the ability to make utility comparisons are formalized by requiring informationally equivalent profiles to be mapped by F into the same social preference; that is, by requiring F to satisfy Information Invariance. If utility is ordinally measurable and interpersonally noncomparable, this requirement is equivalent to using only the individual preferences to determine the social preference. Hence, the Arrovian framework is a special case of the one proposed by Sen.[34] Futhermore, ranking social alternatives using any of the nondictatorial social welfare orderings considered in Section 8 are not ruled out of consideration *a priori*, as they are with an Arrovian social welfare function. Rather, which of them can be used depends on the kinds of utility comparisons that are possible.

With richer utility information, there are fewer profiles of utility functions informationally equivalent to a given profile, and so fewer profiles must be assigned the same social preference by F. As a consequence, there are also fewer informationally imposed restrictions on the social welfare functional and, hence, more ways of mapping the individual utility functions into a

[32] Sen (1970a) has shown that a version of Arrow's Theorem holds even if there is cardinal utility information available for each individual provided that these utilities are interpersonally noncomparable.

[33] As with my definition of an Arrovian social welfare function, I have permitted a social preference to be any binary relation on A. Sen requires social preferences to be orderings.

[34] Strictly speaking, this is only true if all of the individual preference orderings have utility representations, which is a necessarily the case if there are a finite number of alternatives. In applications with an infinite number of alternatives, it is often natural to restrict attention to preferences that are representable.

social preference are possible.

The use of a social welfare functional does not presuppose that alternatives are socially ranked only on the basis of the utilities associated with them. The social welfare functional is welfarist however if it satisfies versions of four of the Arrow axioms reformulated in terms of utilities.

The collective rationality condition takes one of two forms depending on whether one requires social preferences to be orderings or merely requires them to be quasiorderings.

Social Ordering. For each profile of utility functions $U \in \mathcal{D}$, the corresponding social preference R_U is an ordering.

Social Quasiordering. For each profile of utility functions $U \in \mathcal{D}$, the corresponding social preference R_U is a quasiordering.

As in Arrow's Theorem, the domain is assumed to be unrestricted, but now this is an assumption about a domain of profiles of utility functions.

Unrestricted Domain. The domain $\mathcal{D}$ is the set of all profiles of individual utility functions $\mathcal{U}^n$.[35]

The Pareto axiom requires two alternatives to be socially indifferent if everybody is indifferent between them.

Pareto Indifference. For each profile of utility functions $U \in \mathcal{D}$ and every pair of alternatives $x, y \in A$, if $U(x) = U(y)$, then xI_Uy.

The independence condition requires the social preference for any two alternatives to be independent of the utilities obtained with any other alternatives.

Independence of Irrelevant Alternatives. For all profiles of utility functions $U, V \in \mathcal{D}$ and every pair of alternatives $x, y \in A$, if $U(x) = V(x)$ and $U(y) = V(y)$, then R_U and R_V coincide on $\{x, y\}$.

This axiom is equivalent to Arrow's axiom if utilities are ordinally measurable and interpersonally noncomparable and the social preference is invariant to any independently chosen increasing transforms of the individual utility functions.

[35] If there is a natural origin for utility and only nonnegative or positive utilities are possible, the domain restriction can be modified accordingly.

With a *welfarist social welfare functional*, the only information that can be used to social rank two alternatives x and y are the vectors of individual utilities associated with them. The physical descriptions of the alternatives and the profile used to generate these utilities are irrelevant for a welfarist. These restrictions on the social welfare functional are formalized in the following neutrality condition.

Strong Neutrality. For all profiles of utility functions $U, V \in \mathcal{D}$ and any four alternatives $w, x, y, z \in A$, if $U(w) = V(y)$ and $U(x) = V(z)$, then $wR_U x \leftrightarrow yR_V z$ and $xR_U w \leftrightarrow zR_V y$.

In the Bergson–Samuelson approach to welfare economics, we have seen from (13) that the social ranking of any pair of alternatives can be determined by comparing how the corresponding vectors of individual utilities are ranked according to a social welfare ordering R^*. The restriction that R^* is an ordering can be relaxed by simply requiring it to be a binary relation on $\mathbb{R}^n$, the set of possible vectors of utilities. With this modification, a welfarist analogue of the procedure used in (13) for a social welfare functional requires the equivalence in (13) to hold for all profiles of utility functions using a *profile-independent* social preference relation R^*. Formally, for all $U \in \mathcal{U}^n$ and all $x, y \in A$, there exists a social welfare binary relation R^* on A such that

$$xR_U y \leftrightarrow U(x)R^*U(y) \text{ and } yR_U x \leftrightarrow U(y)R^*U(x).^{36} \tag{19}$$

Here, it is supposed that R^* is either an ordering or a quasiordering (so as to allow for social incompleteness). In the latter case, R^* is a *social welfare quasiordering.*

The relationship between the axioms introduced in this section and (19) are summarized in the following theorem.

Theorem 2 (Welfarism Theorem). *If n is finite with $n \geq 2$, $|A| \geq 3$, and a social welfare functional F satisfies Unrestricted Domain and Social Ordering* (*resp. Social Quasiordering*), *then the following statements are equivalent:*

(i) *F satisfies Pareto Indifference and Independence of Irrelevant Alternatives;*

(ii) *F satisfies Strong Neutrality; and*

[36] If R^* is an ordering, the second equivalence is redundant.

(iii) *there exists a social welfare ordering (resp. quasiordering) R^* for which* (19) *holds.*[37]

Theorem 2 identifies exactly what properties a social welfare functional must satisfy if it is to be welfarist. Moreover, it shows that when these properties hold, then all of the information needed to determine the social preference on any pair of alternatives $x, y \in A$ for any profile of utility functions $U \in \mathcal{U}^n$ is provided by the single social welfare ordering or quasiordering R^* on $\mathbb{R}^n$. Specifically, by applying R^* to the utility vectors $U(x)$ and $U(y)$, the social ranking of x and y can be obtained using (19). Being able to determine social preferences in this way is informationally much less demanding than doing so by directly employing the social welfare functional F.

11. Axiomatic Characterizations of Social Welfare Functions

Social welfare functionals have been used to axiomatically characterize many standard welfarist social choice procedures.[38] These axiomatizations either explicitly or implicitly make use of restrictions on the measurability or comparability of utility. As illustrative examples of the many characterization theorems that have been established, here I present axiomatizations of utilitarianism and leximin. Because both utilitarianism and leximin are welfarist and completely order all alternatives, rather than defining the axioms directly on a social welfare functional F, for simplicity I appeal to Theorem 2 and state the axioms on the corresponding social welfare ordering R^*.[39] Restrictions on the kinds of utility comparisons that are possible are formalized

[37] The equivalences in Theorem 2 are obtained by combining Pareto Indifference versions of equivalences established by d'Aspremont and Gevers (1977) and Hammond (1979), as in Bossert and Weymark (2004). These results are for social welfare orderings, but their proofs also establish Theorem 2 for social welfare quasiorderings. Profile-dependent versions of Strong Neutrality and (19) can be defined by setting $U = V$ in the definition of Strong Neutrality and by replacing R^* by a profile-dependent relation R^*_U in (19). Blackorby, Donaldson, and Weymark (1990) have shown that these modified versions of (ii) and (iii) are equivalent to Pareto Indifference. In effect, adding Independence of Irrelevant Alternatives to Pareto Indifference forces the profile-dependent relations R^*_U to all be the same.

[38] See Sen (1977), Blackorby, Donaldson, and Weymark (1984), Boadway and Bruce (1984), d'Aspremont (1985), d'Aspremont and Gevers (2002), and Bossert and Weymark (2004) for surveys of these results.

[39] It is straightfoward to state the social welfare functional analogues of the social welfare ordering axioms used in this section.

by requiring R^* to satisfy Φ Information Invariance for the relevant class of invariance transforms Φ.

Two versions of the Pareto Principle for R^* are considered. The first regards an increase in each person's utility to be a social improvement. It is a utility analogue of Arrow's Weak Pareto axiom. The second requires R^* to be reflexive and regards an increase in anybody's utility as being a social improvement provided nobody is made worse off.

Weak Pareto. For all vectors of individual utilities $u, v \in \mathbb{R}^n$, if $U(x) \gg U(y)$, then uP^*v.

Strong Pareto. For all vectors of individual utilities $u, v \in \mathbb{R}^n$, (i) if $U(x) = U(y)$, then uI^*v and (ii) if $U(x) > U(y)$, then uP^*v.

Impartiality is modelled by requiring R^* to be symmetric. In other words, permuting who has which utility is a matter of social indifference.

Anonymity. For any vector of individual utilities $u \in \mathbb{R}^n$ and any permutation $\pi\colon S \to S$, $uI^*(u_{\pi(1)}, \ldots, u_{\pi(n)})$.[40]

One way of modelling inequality aversion is provided by an equity axiom introduced by Hammond (1976). This axiom applies to comparisons of two utility vectors u and v in which the utilities of all but two individuals are fixed and in which the worst-off of the two concerned individuals is the same in both u and v. It requires u to be socially preferred to v if the worst-off (resp. better-off) of the two concerned individuals is made better (resp. worse) off if v is replaced with u.

Hammond Equity. For all individuals $i, j \in S$ and all vectors of individual utilities $u, v \in \mathbb{R}^n$, if $u_k = v_k$ for all $k \neq i, j$ and $v_j > u_j > u_i > v_i$, then uP^*v.

Hammond Equity requires utility levels to be interpersonally comparable. In this axiom, the sums of the utilities in u and v need not be the same. Indeed, it is not necessary for utility differences to be interpersonally comparable in order to apply Hammond Equity.

The following characterization of utilitarianism has been established by d'Aspremont and Gevers (1977).

[40]If $j = \pi(i)$, then j obtains u_i after the utilities in u have been permuted.

Theorem 3 (d'Aspremont and Gevers' Theorem). *A social welfare ordering R^* on $\mathbb{R}^n$ is the utilitarian social welfare function* (14) *if and only if R^* satisfies Weak Pareto, Anonymity, and Φ^{CUC} Information Invariance.*[41]

Utilitarianism is only meaningful if it is possible to compare utility differences. In the d'Aspremont–Gevers Theorem, it is not only supposed that such comparisons are possible, it is also supposed that no other kinds of utility comparisons can be made except for intrapersonal level comparisons. If further kinds of utility comparisons are possible, then additional social welfare orderings satisfy d'Aspremont and Gevers' other two axioms. For example, the Atkinson social welfare orderings (17) satisfy Weak Pareto and Anonymity, but require utility to be ratio-scale measurable and fully comparable.

Hammond (1976) has established the following characterization of the leximin social welfare ordering.

Theorem 4 (Hammond's Theorem). *A social welfare ordering R^* on $\mathbb{R}^n$ is the leximin social welfare function* (15) *if and only if R^* satisfies Strong Pareto, Anonymity, and Hammond Equity.*

Unlike with the d'Aspremont–Gevers Theorem, Hammond's Theorem does not make use of a specific set of information invariance transforms. Provided that utility levels are interpersonally comparable (so as to be able to use Hammond Equity), Hammond's characterization of leximin holds no matter what other kinds of utility comparisons are possible.

If utilities are cardinally measurable and fully comparable, then it is possible to use either leximin or utilitarianism. With this informational assumption, Deschamps and Gevers (1978) have identified conditions that are necessary and sufficient for a social welfare ordering to be one of these two rules.[42]

12. Extensive Social Choice

In Section 5, I described how utility comparisons can be modelled using extended preferences. Recall that each individual $k \in S$ in his role as a

[41] The utilitarian social welfare ordering also satisfies Strong Pareto. Strong Pareto is implied by the axioms used in Theorem 3.

[42] Strictly speaking, their version of utilitarianism only requires that if $\sum_{i=1}^n u_i > \sum_{i=1}^n v_i$, then uP^*v.

spectator has a profile of utility functions $U^k = (U^k_1, \ldots, U^k_n)$, where U^k_i is the utility function attributed to person i by spectator k. Different spectators may not agree on how to make utility comparisons. Roberts (1995) has proposed a generalization of a social welfare functional for determining how to social rank the alternatives in A that takes account of the differing ways in which spectators make utility comparisons. Formally, an *extensive social welfare functional* is a function $E\colon \mathcal{D} \to \mathcal{B}$, where now $\mathcal{D} \subseteq \mathcal{U}^{n^n}$. A element of $\mathcal{D}$ is an list $\mathfrak{U} = (U^1, \ldots, U^n)$ consisting of the profiles of utility functions for each spectator. Thus, $R_{\mathfrak{U}} = E(\mathfrak{U})$ is the social ranking of A when the spectators have the n profiles of utility functions given by $\mathfrak{U}$.[43]

A welfarist compares alternatives exclusively in terms of their utility consequences. For each alternative $x \in A$ and each profile $\mathfrak{U} \in \mathcal{D}$, there are now n^n of these utilities given by $(U^1_1(x), \ldots, U^1_n(x), \ldots, U^n_1(x), \ldots, U^n_n(x))$. A *welfarist extensive social welfare functional* determines the social preference for the alternatives in A for a given profile $\mathfrak{U} \in \mathcal{D}$ using a single social welfare ordering or quasiordering on the set $\mathbb{R}^{n^n}$ of all such utility vectors using the procedure that is the natural analogue of (19). For example, *extensive utilitarianism* socially ranks alternatives by the sums of the corresponding n^n utilities.[44]

In this framework, one needs to specify how utilities are measured and compared for each spectator separately, as well what kinds of utility comparisons are possible between spectators. Ooghe and Lauwers (2005) have exhaustively considered the main combinations that are possible, and have used this framework to axiomatically characterize a wide range of social decision procedures that are welfarist.

The construction of an extensive social welfare functional is particularly simple if it is a two-stage aggregator. With a *two-stege aggregator*, each spectator $k \in S$ uses a social welfare functional F^k to determine what Harsanyi (1977) calls his *moral preference* on A in the first stage and then the social preference is set equal to the Pareto quasiordering of these n moral preferences in the second stage.[45] For example, when $n = 2$, if person 1

[43] Alternatively, one can assume that the input to an extensive social welfare functional is a list of the profiles of utility functions for m social planners. With this interpretation, a social welfare functional is an extensive social welfare functional for one planner.

[44] A version of the Welfarism Theorem for extensive social choice can be established by reinterpreting each each of the functions U^k_i as the utility function of a different individual and than applying Theorem 2. See Ooghe and Lauwers (2005).

[45] A number of the extensive social welfare functionals axiomatized in Ooghe and Lauwers

is a utilitarian and person 2 uses the Atkinson social welfare function with $\gamma = 0$, then x is socially weakly preferred to y if and only if both the sum and product of the utilities is no smaller with x than with y. Because two-stage aggregators use a Pareto quasiordering in the second stage, in general, social preferences are incomplete.

If every spectator uses the same social welfare functional F in the first stage of a two-stage aggregator, then the restrictions imposed by their limited abilities to make utility comparisons are determined by pooling their information sets. For example, consider the profile $\mathfrak{U} = (U^1, \ldots, U^n)$ and let $\mathbf{U}^k$ denote the profiles of individual utility functions that are informationally equivalent to U^k for spectator k. In order to respect spectator k's limited ability to make utility comparisons, F must assign the same social preference to all profiles in $\mathbf{U}^k$. But this must be true for all spectators, so the same social preference must be assigned to all profiles in $\cup_{k=1}^{n}\mathbf{U}^k$. Adler (2012) models social decision-making in this way.

13. Other Issues

The social welfare function approaches discussed here have been extended in a number of directions. In this concluding section, I briefly consider three of them.

With the exception of the discussion of fairness norms, no special structure has been placed on the set of alternatives. When outcomes are uncertain, there are natural restrictions on the set of alternatives and on preferences or utility functions. For example, alternatives may be a set of lotteries on a finite set of outcomes and preferences may satisfy the axioms of expected utility theory. Harsanyi (1955) has exploited this special structure to provide decision-theoretic foundations for the utilitarian social welfare function.[46]

Many social decisions affect the size and composition of the population. Variable populations raise a number of difficult life and death issues (see Broome, 2004). Fixed-population social welfare functionals have been generalized by Blackorby and Donaldson (1984) to deal with variable populations. Blackorby, Bossert, and Donaldson (2005) explore their use in some detail.

To grapple with environmental concerns such as those raised by global

(2005) are two-stage aggregators.

[46]There is considerable controversy concerning whether Harsanyi may legitimately intepret his formal theorems in this way. See Sen (1976) and Weymark (1991).

warming, it is necessary to consider future generations of individuals stretching into the indefinite future. Social welfare functions with a finite number of individuals cannot deal with this intergenerational structure. Unfortunately, constructing social welfare functions for an infinite population that respects Pareto rankings and treats individuals impartially is problematic. See, for example, Zame (2007) and Lauwers (2012).

Social welfare functions that are welfarist or, at least, respect some form of the Pareto Principle cannot capture what many regard as being important considerations for a normative social evaluation. For example, Sen (1970b) has shown that an Arrovian social welfare function that satisfies Weak Pareto is inconsistent with some natural assignments of rights for some profiles of preferences. Sen (1979) has also stressed the importance of the reasons why individuals have the preferences or utilities they do (e.g., whether they result from sadistic motivations). Taking these reasons into account may well be incompatible with the Pareto Principle.[47] So, in spite of their many virtues, the limitations of using social welfare functions should also be borne in mind.

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[47] See Blackorby, Donaldson, and Weymark (1990) for a discussion of this incompatability.

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