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Price dispersion and efficiency

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Abstract

The paper distinguishes between rigid price and flexible price versions of the Prescott (1975) "hotels" model. I focus on two dynamic models that allow for storage: The Bental and Eden (1993) model of all year round goods and the more recent Deneckere and Peck (2012) model of seasonal goods. The formulation follows the standard competitive analysis tradition with non-standard definition of markets: The set of markets that open depends on the state of demand. I show that when prices are flexible, the equilibrium outcome is efficient if the probability of becoming active depends on the aggregate state but not on the buyer's type. If prices are rigid the equilibrium outcome is in general not efficient except for the case in which there are no costs for delaying trade. The cost of delay is also relevant for price dispersion: Lower cost of delays leads to lower price dispersion.

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PRICE DISPERSION AND EFFICIENCY

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July 2013

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Keywords: Price dispersion, demand uncertainty, efficiency, sequential trade, inventories, costs of delaying trade, price rigidity.

JEL Codes: D41, D83, D84

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1. INTRODUCTION

Recently Deneckere and Peck (DP, 2012) analyzed a version of the Prescott (1975) model and argue that (a) the outcome in the one period case is inefficient (Proposition 2) and (b) the outcome in their multi-period version of the model is efficient. Their first result is different from the results in Eden (1990, 2009). The difference between the results is in the efficiency criteria used. In Eden's Uncertain and Sequential Trade (UST) model, sellers must make some irreversible (selling) decisions before they know the state of nature (demand). These informational constraints are not present in the DP definition of feasible allocation and therefore their Proposition 2 says that a planner who knows the state (after the choice of output but before the allocation of the output to buyers) can always improve matters.

The source of the disagreement about the relevant efficiency criterion may be traced to the difference between the original Prescott model and the UST model. In Prescott's original model, prices are set in advance and we may assume that the state of demand becomes known before actual transactions take place. After learning the state of demand, sellers want to change prices but cannot. In this environment, it makes sense to compare the equilibrium outcome with the solution to the problem of a planner who knows the state, as in Dana (1998, 1999) and DP.[2] In the UST model trade is sequential and price dispersion arises as a result of informational constraint rather than price rigidity. The relevant comparison is therefore with the solution to the problem of a planner who faces informational constraints that are similar to the constraints faced by the sellers in the model.

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[2] See also Bryan (1980) and Deneckere, Marvel, and Peck (1996). For other type of models that deals with pricing in the presence of demand uncertainty, see for example, Harris and Raviv (1981), Lazear (1986) and Pashigian (1988).

The second main result in the DP paper says that the equilibrium outcome in a multi-period setting is efficient even from the point of view of a planner who knows the state before the beginning of actual trade. I argue that this result critically depends on the assumption that the cost of delaying trade (the cost of postponing transactions) is not important (their assumption 4).

To study the effect of the cost of delaying trade on efficiency and price dispersion, I consider two dynamic versions of the Prescott models that allow for storage: a general equilibrium version of the UST model in Bental and Eden (BE, 1993) and a version of the DP model. These models complement each other. The BE model is about all year round goods while the DP model is about seasonal goods. I show that if the probability of becoming active depends on the aggregate state but not on the buyer's type, then the UST equilibrium outcome is efficient from the point of view of a planner who has the same information as the sellers in the model, but in general it is not efficient from the point of view of a planner that does not face informational constraint.

The cost of trade delays is also relevant for price dispersion. In both models a reduction in the cost of delays leads to a reduction in price dispersion. In the BE model price dispersion vanishes when there is no cost for delaying trade. Somewhat surprisingly, in the DP model price dispersion does not vanish when delaying is costless.

Using the distinction between rigid and flexible price versions of the Prescott model we can summarize the main results of the paper as follows: (a) When the costs of delaying trade are important, the equilibrium outcome in models that assume price flexibility may be efficient while the equilibrium outcome in models that assume price rigidity is not efficient, (b) Price rigidity does not impose welfare loss if there is no cost of delays and (c) Price dispersion increases with the cost of delays. Although some of the results are new, I see the main contribution of the paper in providing a unified simple framework for comparing alternative and sometimes quite complex versions of the Prescott model. The new results are the general equilibrium and welfare analysis in the BE model and the case in which the cost of delaying trade is important in the DP model. I also provide a unified framework in which the two models (BE and DP) can be obtained as special cases.

2. THE MODEL

I start with a single period economy. This relatively simple case introduces the UST approach to modeling trade and allows for a comparison with the DP approach. DP stress the price posting interpretation of the model. Formally, the UST approach adopts the price taking assumption and it is easier to think of sellers as choosing quantities rather than prices. But unlike the traditional approach here the set of markets that open depends on the realization of demand and as a result selling itself is a random event. The markets are for the same good and each market is characterized by the probability that it will open and the price in which trade will occur when it opens. We can therefore think of sellers as allocating goods across markets by posting prices. Thus the distinction between choosing prices and choosing quantities is not important in this model. I assume that buyers have a quasi-linear utility function that gives rise to a downward sloping demand curve. This is different from DP who assume that active buyers try to maximize the surplus from buying one unit. I later argue that this difference is also not important for the main results. Unlike the analysis in Eden (2009) here I consider the problem of a planner who can choose quantities rather than the problem of a policy-maker that can choose taxes and tariffs.

I consider an economy with two dates $(t = 0,1)$ and two goods: X and Y with lower case letters denoting quantities. There are Z possible aggregate states of nature. State s occurs with probability π_s.

There are many identical sellers\producers. The number of sellers is known and is normalized to 1. Sellers are risk neutral and derive utility from Y only. Sellers can produce X at the per-unit cost of λ units of Y. Unlike sellers, buyers are heterogeneous. There are J types of buyers. The number of type j (potential) buyers is n_j. All buyers are endowed with a large quantity of Y. In aggregate state s, the utility function that a fraction ϕ_{js} of type j buyers realize is: $u_{js}(x,y)=U_j(x)+y$, where $U_j(x)$ is strictly monotone, strictly concave and differentiable. To simplify, I assume that $U'_j(0)=\infty$. The remaining $(1-\phi_{js})n_j$ buyers realize the utility function $u_{js}(x,y)=y$ and are not active. The random utility of a type j buyer in aggregate state s is thus:

(1) $$u_{js}(x,y)=U_j(x)+y \text{ with probability } \phi_{js} \text{ and } u_{js}(x,y)=y \text{ otherwise.}$$

An active type j buyer demands $d_j(p)$ units of X at the price p where the individual demand function is defined by:

(2) $$d_j(p)=\arg\max_{x\geq 0} U_j(x)-px.$$

An interior solution to (2) must satisfy the following first order condition:

(3) $$U_j'(x)=p$$

Production (capacity choice) occurs at $t=0$. After production choice is made, buyers realize a utility function and active buyers form a line. I treat all active buyers symmetrically and assume that any segment taken from this line accurately represents the type composition of buyers who want to consume: In state s, $\sum_i \phi_{is}n_i$ buyers want to consume and the fraction of type j buyers in any segment of the line is:

$\vartheta_{js} = \frac{\phi_{js} n_j}{\sum_i \phi_{is} n_i}$. After the line is formed, active buyers arrive at the market place one by one according to their place in the line and buy at the cheapest available offer. The sequential trade does not take real time (and occurs in meta time).

Price dispersion is necessary:

In search models that assume no uncertainty about aggregate demand it is difficult to get price dispersion. The opposite is true in a UST environment in which buyers see all available offers. To elaborate, I consider a game in which the sellers must choose production and prices before they know the realization of aggregate demand. A single price P is a symmetric Nash equilibrium strategy if (a) all sellers choose to post this price and produce a strictly positive amount and (b) deviation from this strategy cannot increase expected profits.

Claim 1: There is no single price symmetric Nash equilibrium strategy.

To show this Claim note that at a single price P there are some states in which the market does not clear. We can therefore have one of the following three cases:

(a) In some states there is excess supply.

(b) In some states there is excess demand.

(c) There is excess demand in some states and excess supply in some other states.

In the excess supply case (a), $P > \lambda$ is not a symmetric Nash equilibrium strategy (or equilibrium strategy for short) because the individual seller can do better by reducing his price by an arbitrarily small amount and sell with probability 1. In this case also $P \le \lambda$ is not an equilibrium strategy because the individual seller can do better by not producing.

In the excess demand case (b), any $P < \infty$ is not an equilibrium strategy because the seller can do better by increasing his price to $U'(0) = \infty$ which is the price that unsatisfied buyers are willing to pay. This argument also applies to case (c).

Equilibrium with price dispersion:

Since there is no single price equilibrium I look for equilibrium with price dispersion. I start from the following case.

Assumption 1: The probability of becoming active depends only on the aggregate state and not on the buyer's type: $\phi_{js} = \phi_{1s} = \phi_s$ for all j.

I choose indices such that demand is increasing in the state: $0 = \phi_0 < \phi_1 < ... < \phi_Z = 1$. In state s, the number of active buyers is $N_s = \phi_s N$ where $N = \sum_j n_j$ is the number of potential buyers. Under assumption 1, the fraction of type j buyers in any segment of the line, $\vartheta_j = \frac{\phi_s n_j}{\sum_i \phi_s n_i} = \frac{n_j}{N}$, is independent of s.

Trade occurs in a sequence of Walrasian markets described as follows. The minimum number of buyers that will arrive is $\phi_1 N = \min_s\{\phi_s N\}$ and these buyers buy in the first market. The demand in the first market (at the price p) is: $D_1(p) = \phi_1 \sum_j n_j d_j(p)$ units. If $s > 1$, there are $N_s - N_1$ buyers who could not make a buy in the first market. The minimum number of unsatisfied buyers if $s > 1$, is $(\phi_2 - \phi_1)N = \min_{s>1}\{(\phi_s - \phi_1)N\}$ and this is the number of buyers who will buy in the second market. The demand of this second batch of buyers is: $D_2(p) = (\phi_2 - \phi_1)\sum_j n_j d_j(p)$ units. In general, if batch i arrives, its demand at the price p is: $D_i(p) = (\phi_i - \phi_{i-1})\sum_j n_j d_j(p)$ and this is the potential demand in market i. The probability that batch i arrives and market i opens is: $q_i = \sum_{s=i}^{Z} \pi_s$.

The seller is a "conditional price-taker" and behaves as if he can sell any amount at the price P_i if market i opens. The expected revenue from supplying a unit to market i is $q_i P_i$. In equilibrium expected profits are zero and prices satisfy: $q_i P_i = \lambda$. I now define equilibrium as follows.

A UST equilibrium is a vector of prices $(P_1,...,P_Z)$ and a vector of supplies $(x_1,...,x_Z)$ such that: (a) $P_i = \lambda / q_i$ and (b) $x_i = D_i(P_i)$.

<u>A "weak" planner</u>:

In equilibrium a type j buyer who arrives in batch i consumes $d_j(\lambda / q_i)$ units. To evaluate this outcome I assume a planner that can choose the amount x_{ji} that will be delivered to a type j agent that arrive in batch i. I call this planner "weak" because like the sellers in the model (and unlike the "strong" planner that will be introduce shortly) he must make choices before he knows the realization of demand. The "weak" planner solves the following problem.

(4) $$\max_{x_{ji}} \sum_{i=1}^{Z} q_i(\phi_i - \phi_{i-1}) \sum_{j=1}^{J} n_j U_j(x_{ji}) - \lambda \sum_{i=1}^{Z} \sum_{j=1}^{J} (\phi_i - \phi_{i-1}) n_j x_{ji}$$

The first order conditions to this problem are:

(5) $$q_i U_j'(x_{ji}) = \lambda$$

Since in equilibrium $P_i = \lambda / q_i$, a type j agent that arrive in batch i will choose to consume x_{ji} such that $U_j'(x_{ji}) = P_i = \lambda / q_i$ and therefore the equilibrium outcome satisfies (5). We have thus shown the following claim.

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<u>Claim 2</u>: The UST equilibrium outcome is a solution to the "weak" planner's problem (4).

Prices may appear rigid but they are not:

Posted prices may appear rigid because they do not respond to the realization of demand (the state). Nevertheless, prices are flexible in the sense that the seller's plan is time consistent and he has no incentive to change prices during trade. To show this claim, note that the probability of state $s \geq i$ given that market i open is:

$$\Pr ob(\phi = \phi_s \mid \phi \geq \phi_i) = \frac{\Pr ob(\phi = \phi_s \bigcup \phi \geq \phi_i)}{\Pr ob(\phi \geq \phi_i)} = \frac{\pi_s}{q_i}$$

The expected revenue from supplying to market $j \geq i$ when market i opens is therefore:

(6) $$P_j \sum_{s=j}^{Z} \frac{\pi_s}{q_i} = P_j \frac{q_j}{q_i} = P_i \quad \text{for all } j \geq i .$$

The second equality in (6) follows from the equilibrium condition: $q_j P_j = q_i P_i$. The equality in (6) implies that after updating of the probabilities all higher index markets have the same expected revenues and therefore the seller has no incentive to change the allocation of the remaining unsold goods across markets. In this sense, prices are perfectly flexible.

A "strong" planner:

A planner that knows the state before any decision is made (before $t = 0$) will produce exactly the amount that he plans to deliver. In state s, the planner will choose to deliver x_{js} units to type j by solving the following problem.

(7) $$\max_{x_{js}} \sum_j \phi_s n_j U_j(x_{js}) - \lambda \sum_j \phi_s n_j x_{js}$$

The first order conditions for this problem are:

(8) $$U_j'(x_{js}) = \lambda$$

Clearly the UST outcome characterized by (5) is not a solution to the "strong" planner's problem.

A "semi-strong" planner:

I now consider the case in which the planner knows the state after the capacity, k, is chosen but before trade occurs (that is, at $0 < t < 1$). Here I use x_{js} to denote the amount allocated to a type j agent in state s. Under the informational assumption for the "semi-strong" planner, the allocation $(k; x_{11},...,x_{1Z};...;x_{J1},...,x_{JZ})$ is feasible if it satisfies the following condition:

(9) $$\sum_j \phi_s n_j x_{js} = k \quad \text{for all } s$$

This is the definition of feasible allocation in DP (See their Definition 1). With this notion of feasible allocations we can write the problem of the "semi-strong" planner as follows.

(10) $$\max_k \sum_s \pi_s V_s(k)$$
$$\text{where } V_s(k) = \max_{x_{js}} \sum_j \phi_s n_j U(x_{js}) - \lambda k \quad \text{s.t.} \quad \sum_j \phi_s n_j x_{js} = k$$

Thus, $V_s(k)$ is the maximum welfare (sum of utilities) that the planner can achieve in state s when capacity is k. The first order conditions for (10) are:

(11) $$U_j'(x_{js}) = U_1'(x_{1s}) \text{ for all } j \text{ and } s$$

(12) $$\sum_s \pi_s U_j'(x_{js}) = \lambda \text{ for all } j.$$

Unlike the weak planner, here the allocation does not depend on the order of arrival (the batch) and unlike the strong planner here only the expected marginal utility is equal to λ (and not the marginal utility in each state). Since the “semi-planner” has better information than the sellers in the model he can improve on the UST equilibrium outcome.

As I said in the introduction the comparison with the semi-strong planner is reasonable if prices are set in advance (at $t = 0$) and sellers observe the state before actual trade occurs. In this case sellers would like to change their prices at the time of trade but cannot do so and a semi-strong planner who does not use rigid prices can improve matters. However, in the UST model, sellers observe only the amount sold at each stage (or the number of the hypothetical markets that were opened) and therefore the weak planner is the appropriate bench-mark.

I read Proposition 2 in DP as saying that a “semi-strong” planner can always improve matters.

Can the government improve matters? Under Assumption 1, the government can improve matters if it has informational advantage over the sellers in the economy. Once we relax assumption 1 the government can improve matters even if it has no informational advantage but can discriminate by type. To get the intuition, assume that type 1 is active only in states in which aggregate demand is low. Then the weak planner will give to a type 1 buyer that arrives in the first batch more than his competitive allocation (the amount he buys at the price P_1) because the fact that type 1 is active is a signal that aggregate demand is low and the chance of hitting the capacity constraint is low. I elaborate in the Appendix. In general, when we relax Assumption 1, the definition of batches is endogenous and so is the probability that market i will open. But prices are still given by $P_i = \lambda / q_i$.

3. STORAGE

Bental and Eden (BE, 1993) extended the UST model to the case in which the economy lasts forever and storage is possible. To do welfare analysis, I use here a general equilibrium version of their model.

As in the single period case there are $J+1$ types of agents (a seller and J types of buyers). Each agent gets a large endowment of Y each period. The demand of each of the active buyer does not change over time and is given by (2). As before the probability of becoming active does not depend on the type. The number of active buyers is iid. Sellers can store goods but buyers cannot (and in equilibrium they do not have an incentive to do so). The seller uses the discount factor $0<\beta<1$ to evaluate future revenues. The discount may also capture storage costs and depreciation.

<u>The "weak" planner's problem:</u>

Each period the "weak" planner chooses the amount x_{ji} that will be delivered to a type j agent that arrives in batch i. Goods that were allocated to batches that did not arrive are not delivered and are carried as inventories to the next period. The planner can also choose to hold Γ units of purely speculative inventories that will be stored regardless of the state. Thus, in state i, $\Gamma + \sum_{s=i+1}^{Z}(\phi_s - \phi_{s-1})\sum_{j=1}^{J} n_j x_{js}$ units will not be delivered and will be carried to the next period as inventories. I use L to denote current production and I to denote the beginning of period inventories.

When $\Gamma = 0$, the amount of inventories in state s is:

(13) $$I^s = \sum_{i=s+1}^{Z}\sum_{j=1}^{J}(\phi_i - \phi_{i-1}) n_j x_{ji}$$

The maximum amount that will be carried as inventories is $I^{\max} = I^1$. The value of inventories is a function, $V(I)$, from the beginning of period inventories $0 \le I \le I^{\max}$ to the real line (R_+) defined by the following Bellman equation:

$$(14)\; V(I) = \max_{L, x_{ji}, \Gamma \ge 0} \sum_{i=1}^{Z} q_i(\phi_i - \phi_{i-1}) \sum_{j=1}^{J} n_j U_j(x_{ji}) - \lambda L + \beta \sum_{i=1}^{Z} \pi_i V\left(\sum_{s=i+1}^{Z} (\phi_s - \phi_{s-1}) \sum_{j=1}^{J} n_j x_{js} + \Gamma \right)$$

$$\text{s.t.} \quad \Gamma + \sum_{i=1}^{Z} \sum_{j=1}^{J} (\phi_i - \phi_{i-1}) n_j x_{ji} = L + I\,.$$

I now show the following Claim.

Claim 3: The solution to the planner's problem (14) is characterized by $L > 0$, $\Gamma = 0$ and the following first order condition:

$$(15) \qquad U_j'(x_{ji}) = \beta\lambda + \frac{\lambda(1-\beta)}{q_i}$$

Proof: Since $U_j'(0) = \infty$, the amounts supplied are strictly positive ($x_{ji} > 0$). Total supply in each period is: $k = \sum_{i=1}^{Z} \sum_{j=1}^{J} (\phi_i - \phi_{i-1}) n_j x_{ji} > I^{\max}$ because the supply to the first market $\phi_1 \sum_{j=1}^{J} n_j x_{j1}$ is strictly positive. Since $I \le I^{\max}$, production $L = k - I$ is strictly positive.

Since $L > 0$ we must have $\Gamma = 0$. To see this claim, note that when $\Gamma > 0$ the seller can do better by cutting purely speculative inventories and current production by a unit and increasing production in the next period by a unit.

We can therefore write the first order conditions and the envelope condition as follows:

$$(16) \qquad q_i U_j'(x_{ji}) + \beta \sum_{k=1}^{i-1} \pi_k V'\left(\sum_{s=k+1}^{Z} (\phi_s - \phi_{s-1}) \sum_{j=1}^{J} n_j x_{js} \right) = \lambda$$

(17) $$V'(I) = \lambda$$

Substituting (17) in (16) leads to:

(18) $$q_i U_j'(x_{ji}) + (1 - q_i)\beta\lambda = \lambda$$

The first order condition (15) follows from (18). □

Note that a strictly positive amount of production is required to keep total supply at the level k and that inventories are always in the range $[0, I^{\max}]$. Thus (14) is well defined. Furthermore, optimal production fluctuates with inventories: the larger the amount of beginning of period inventories the lower is the amount produced. Here a unit increase in the beginning of period inventories reduces production by a unit. In a more general setting in which the marginal cost is increasing it will reduce production by less than a unit.

<u>UST equilibrium</u>

Prices in a typical period are given by $(P_1, ..., P_Z)$. With some abuse of notation, I describe the seller's problem by the following Bellman's equation:

(19) $$V(I) = \max_{x_i, \Gamma, L \geq 0} \sum_{i=1}^{Z} q_i P_i x_i - \lambda L + \beta \sum_{i=1}^{Z} \pi_i V\left(\Gamma + \sum_{s=i+1}^{Z} x_s\right)$$

$$\text{s.t. } \Gamma + \sum_{i=1}^{Z} x_i = L + I$$

Here x_i is the amount the seller allocates to market i and as before the range of $V(I)$ is $0 \leq I \leq I^{\max}$. A solution to (19) with $L, x_i > 0$ must satisfy $\Gamma = 0$ and the following first order and envelope conditions:

(20) $$q_i P_i + \beta(1-q_i)V'\left(\sum_{s=i+1}^{Z} x_s\right) = \lambda$$

(21) $$V'(I) = \lambda$$

Substituting (21) into (20) yields:

(22) $$q_i P_i + (1-q_i)\beta\lambda = \lambda$$

A UST equilibrium is a vector $(P_1,...,P_Z;x_1,...,x_Z)$ that satisfies (22) and the following market clearing conditions:

(23) $$\sum_j (\phi_i - \phi_{i-1}) n_j d_j(P_i) = x_i$$

Equilibrium prices (22) can be written as:

(24) $$P_i = \beta\lambda + \frac{\lambda(1-\beta)}{q_i}$$

Let $x_{ji} = d_j(P_i)$ denotes the amount bought by a type j buyer who arrives in batch i. Then (24) and (3) imply:

(25) $$U_j'(x_{ji}) = P_i = \beta\lambda + \frac{\lambda(1-\beta)}{q_i}$$

We can now show the following Claim.

Claim 4: The equilibrium outcome is a solution to the planner's problem (14).

This claim follows from the observation that (25) is the same as (15).

Price dispersion:

Prices in (24) are a weighted average between λ and λ/q_i. A higher β reduces the mean and the dispersion measures of the price distribution because all prices get closer to the lowest price λ. When $\beta \to 1$, (24) implies that all prices are approximately equal to λ and price dispersion vanishes. Thus discounting is required to get price dispersion in equilibrium.

In general, price dispersion requires some costs for delaying trade. This cost maybe due to discounting of future profits, storage costs or depreciation.

The "strong" planner's problem:

A planner who knows the state will produce exactly the amount that he plans to deliver and will not carry inventories. In state s, the planner will choose to deliver x_{js} units to type j by solving the problem (7). Clearly the UST outcome characterized by (25) is not a solution to the "strong" planner's problem. But when $\beta \to 1$, the "strong" planner cannot improve matters by much. To see this claim note that when $\beta \to 1$, the UST allocation of X is close to the "strong" planner's choice and the benefits from economizing on inventories are small because the maximum amount of inventories held is finite.

4. SEASONAL GOODS

Deneckere and Peck (DP, 2012) consider the case in which the good is offered for sale for a limited length of time called the "sale season". Goods that their use depends on the weather (like "summer clothes") may serve as an example. Here I analyze this case in a relatively simple economy in which the season is divided into two periods (sub-periods) and all potential buyers are identical (there is one type). Unlike the DP model, buyers have a downward sloping demand curve. As in the DP model, the seller chooses capacity at the beginning of the sale season and cannot change it during the season. The problem has some common elements with the storage model in the previous section but there are some important differences.

The economy lasts for 3 periods. Production decisions are made at $t = 0$. Trade occurs at $t = 1$ and $t = 2$. I start with the case in which the number of active buyers $\tilde{N}$ can take 2 possible realizations: $1 = N_1 < N_2$. As before the probability of state s ($\tilde{N} = N_s$) is π_s.

In the first trading period ($t = 1$) there are 2 hypothetical UST markets. Market 1 opens with probability 1 at the price P_1. Market 2 opens with probability π_2 and if it opens trade in this market occurs at the price P_2. If market 2 does not open (with probability $\pi_1 = 1 - \pi_2$) the seller can sell the unsold goods in the second period Walrasian market at the price $p \le P_1$.[3]

The utility of an active buyer is given by:

(26) $$U(C + \delta c) + y$$

where C is the amount bought in the first period ($t = 1$), c is the amount bought in the second period and $0 < \delta \le 1$ is a parameter that reflects the cost of delay. The delay cost may result from the shortening of the length of time that the buyer uses the good: A consumer who buys a short sleeve shirt at the beginning of the summer gets more use out of it than a consumer who buys it towards the end of the summer. The function U is strictly monotone, strictly concave and differentiable with $U'(0) = \infty$.

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[3] There is no real distinction between the UST and the Walrasian markets. In both cases a market opens only if there is both supply and demand. The UST second market may not open if there is no demand. The second period Walrasian market may not open if there is no supply. And each market that open is cleared. But in the first period there is a sequence of Walrasian markets and price dispersion. In the second period there is at most one market and one price. So I hope this language will help in keeping the two trading periods separated without creating confusion.

A buyer who buys at $t=1$ in the first market will take into account the possibility that a Walrasian market will open at $t=2$ and he will be able to buy at a cheaper price. He solves the following problem.

(27) $$\max_{C_1} \pi_1\left(\max_c U(C_1+\delta c)-P_1C_1-pc\right)+\pi_2\left(U(C_1)-P_1C_1\right)$$

Here C_1 is the amount he buys at $t=1$ in the first UST market (at the price P_1) and c is the amount he buys at $t=2$ in the Walrasian market (at the price p) if the state of demand is low (state 1).

A buyer who buys at $t=1$ in market 2 knows that the state of demand is high and the Walrasian market will not open in the next period because inventories will not be carried to the next period. He therefore solves the following problem.

(28) $$\max_{C_2} U(C_2)-P_2C_2$$

The first order conditions that an interior solution to (27) must satisfy are:

(29) $$\delta U'(C_1+\delta c)=p$$

(30) $$\pi_1 U'(C_1+\delta c)+\pi_2 U'(C_1)=P_1$$

The first order condition for an interior solution to (28) is:

(31) $$U'(C_2)=P_2$$

The seller chooses the amount allocated to each of the hypothetical markets at $t=1$ (x_i) by solving the following problem[4].

(32) $$\max_{x_i} \pi_1(P_1x_1+px_2)+\pi_2(P_1x_1+P_2x_2)-\lambda(x_1+x_2)$$

Note that the revenue per unit allocated to the first market is P_1. The revenue per unit allocated to the second market is random: It is equal to p in the low demand state and P_2 in the high demand state.

The first order conditions for the seller's problem are:

(33) $$P_1=\lambda$$

(34) $$\pi_2P_2+\pi_1p=\lambda$$

Condition (34) is similar to (22). The revenues are the quoted price in case the market opens and the value of inventories in case it does not open. The left hand side of (34) is therefore the expected revenues that must equal the cost.

Equilibrium is a vector $(C_1,C_2,c,x_1,x_2,P_1,P_2,p)$ that satisfies the buyers' first order conditions (29)-(31), the seller first order conditions (33),(34), the inequalities $p\le P_1<P_2$, and the following market clearing conditions:

(35) $$C_1=x_1$$

(36) $$(N_2-1)C_2=x_2$$

(37) $$c=x_2$$

---

[4] A more general formulation may allow for pure speculations. Let x_3 denote the amount that the seller does not plan to sell in the first period regardless of the number of markets open. Then we can write the seller's problem as:

$$\max_{x_i} \pi_1(P_1x_1+px_2)+\pi_2\left(P_1x_1+P_2x_2\right)+px_3-\lambda(x_1+x_2+x_3)$$

It will be shown that in equilibrium $p<P_1$, and therefore the seller chooses $x_3=0$.

Note that also here markets that open are cleared.

Solving for the equilibrium vector:

Substituting the market clearing conditions in the buyers' first order conditions (29)-(31) and using $\Delta = N_2 - 1$, leads to:

(38) $$\delta U'(x_1 + \delta x_2) = p$$

(39) $$\pi_1 U'(x_1 + \delta x_2) + \pi_2 U'(x_1) = P_1$$

(40) $$U'(\Delta^{-1} x_2) = P_2$$

Substituting (38) and (40) in (34) leads to:

(41) $$\pi_2 U'(\Delta^{-1} x_2) + \pi_1 \delta U'(x_1 + \delta x_2) = \lambda$$

Substituting (39) in (33) leads to:

(42) $$\pi_1 U'(x_1 + \delta x_2) + \pi_2 U'(x_1) = \lambda$$

We now have 2 equations (41)-(42) with 2 unknowns (x_1, x_2).

Claim 5: There exists a unique equilibrium.

Proof: I start by showing that there exists a unique solution $(\hat{x}_1, \hat{x}_2)$ to (41) and (42). For this purpose note that the slope of the locus of points that solve (41) is:

(43) $$\frac{dx_2}{dx_1} = -\frac{\pi_1 \delta U''(x_1 + \delta x_2)}{\pi_2 U''\big((\Delta)^{-1} x_2\big)(\Delta)^{-1} + \pi_1 \delta^2 U''(x_1 + \delta x_2)} > -\frac{1}{\delta}$$

The slope of the locus of points that solve (42) is:

(44) $$\frac{dx_2}{dx_1} = -\frac{\pi_2 U''(x_1) + \pi_1 U''(x_1 + \delta x_2)}{\pi_1 U''(x_1 + \delta x_2)\delta} < -\frac{1}{\delta}$$

In Figure 1 the locus labeled AA is the solutions to (41) and the locus labeled BB is the solutions to (42). When $x_1 = 0$, the amount x_2 that solves (41) is finite and therefore the locus AA intersects the vertical axis. But since $U'(0) = \infty$, the locus BB does not intersect the vertical axis. Therefore there exists a unique solution to (41) and (42) illustrated by Figure 1.

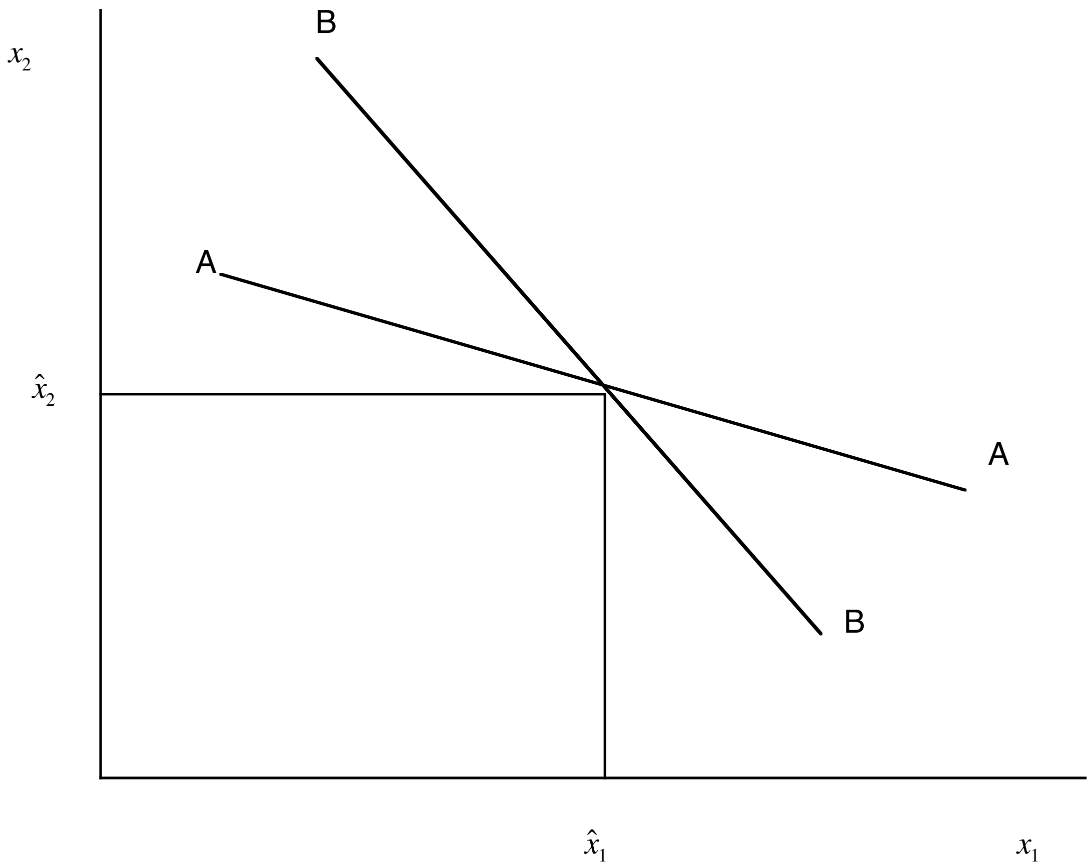

Figure 1: The solution to (41) and (42)

We still need to show that $p \le P_1 < P_2$. To show this note that since $U'(x_1) > U'(x_1 + \delta x_2)$, (42) implies: $U'(x_1) > \lambda$ and $U'(x_1 + \delta x_2) < \lambda$. It follows that $\delta U'(x_1 + \delta x_2) = p < \lambda$. Since $p < \lambda$, (34) implies $P_2 > \lambda = P_1$. □

The case of no cost of delay:

The special case in which $\delta = 1$ provides useful intuition. In this case, (41) and (42) imply:

(45) $$U'(x_1) = U'(\Delta^{-1} x_2)$$

(46) $$\pi_1 U'(x_1 + x_2) + \pi_2 U'(x_1) = \lambda$$

Since (45) implies $x_2 = \Delta x_1$ we can write (46) as:

(47) $$\pi_1 U'\big(x_1(1+\Delta)\big) + \pi_2 U'(x_1) = \lambda$$

Since $U'(0) = \infty$, there exists a unique solution $\hat{x}_1$ to (47). Since $U'\big(x_1(1+\Delta)\big) < U'(x_1)$, (47) implies $U'(\hat{x}_1) > \lambda$. This, (45) and (40) imply that $P_2 = U'(\hat{x}_1) = U'(\Delta^{-1}\hat{x}_2) > \lambda$.

The equilibrium when $\delta = 1$ can be described as follows. The first batch of buyers buys at the price $P_1 = \lambda$ a quantity that is equal to their demand at the higher price P_2. They buy less than their demand at the price P_1 because there is a chance that they will be able to buy more next period at the price $p < \lambda$. In state 2 the second batch arrives and buy the quantity allocated to the second market (x_2) at $t = 1$. In state 1, this quantity is bought by the first batch at $t = 2$.

Comparative statics:

I now assume that the absolute risk aversion measure is not too low and satisfies:

$$-\frac{U''(x_1+\delta x_2)}{U'(x_1+\delta x_2)} < \delta^{-2} \tag{48}$$

Claim 6: Under (48) an increase in δ leads to: (a) an increase in x_2 and a decrease in x_1, (b) a decrease in P_2 and (c) an increase in p.

To show (a), note that under (48), an increase in δ will shift the AA curve in Figure 1 up and to the right and the BB curve to the left and down. Part (b) follows from (40) and the increase in x_2. Part (c) follows from (34) and the fact that P_2 went down.

Note that since a reduction in δ increases P_2, an increase in the cost of delay increases price dispersion at $t=1$.

The Weak planner's problem:

The weak planner who has to choose production and the quantity he gives to the first batch before he knows whether the second batch will arrive or not. He therefore solves the following problem.

$$\max_{x_1,x_2} \pi_1 U(x_1+\delta x_2) + \pi_2\left(U(x_1)+\Delta U(\Delta^{-1}x_2)\right) - \lambda(x_1+x_2) \tag{49}$$

The first order conditions for this problem are (41) and (42). Thus,

Claim 7: The equilibrium outcome solves the weak planner's problem (49).

The Semi-strong planner's problem:

The semi-strong planner has to choose capacity $(x_1 + x_2)$ before he knows the state but he chooses the amount that he gives to the first batch of buyers at $t = 1$ after he knows the state. Since the semi-strong planner will give the entire capacity to the first batch in state 1, we can write his problem as follows.

(50) $$\max_{x_i} \pi_1 U(x_1 + x_2) + \pi_2 \left(U(x_1) + \Delta U(\Delta^{-1} x_2)\right) - \lambda(x_1 + x_2)$$

This problem is the same as the "weak" planner's problem (49) only if $\delta = 1$. Since the semi-strong planner has an informational advantage over the weak planner and the weak planner mimics the equilibrium outcome, this observation leads to the following Claim.

Claim 8: When $\delta < 1$, the semi-strong planner can improve on the equilibrium outcome but when $\delta = 1$ he cannot.

Claim 8 reiterates the importance of the cost of delay. The intuition is in the value of the informational advantage that the semi-strong planner has over the weak planner (and the sellers in the model). When there is no cost of delay the value of the information about the state is zero because the weak planner can distribute a limited amount at $t = 1$ and once he learns about the state, at $t = 2$ he can deliver the rest making sure that each buyer gets an equal amount (or more generally an amount that will equate the marginal utility across active buyers) regardless of the order of arrival.

Note that unlike the inventories model in the previous section, here there is price dispersion (at $t = 1$) even when $\delta = 1$ and there is no cost of delay.

Note also that buyers in the first batch do not buy the amount demanded at the first market price because they know that there is the possibility of buying at a bargain prices next period. This speculative behavior is similar to the endogenous rationing in DP.

In general, the language here is different from the language in DP. I am using the language of general competitive analysis with a non-standard definition of markets, while they use game theory. But I think that the model here is essentially the same as a 2 periods version of their model. To make the connection between the two models let us think of the utility function (26) as describing the preferences of a household that consists of many infinitesimal buyers. The head of the household assigns a reservation price to each member and instructs him to maximize the expected surplus from buying at most one (infinitesimal) unit. The highest reservation price $U'(0)$ is assigned to the member indexed 0 and in general the reservation price $U'(x)$ is assigned to the member indexed x. With this in mind we can get endogenous rationing in the sense described by DP. The members with indices less than $\hat{x}_1$ buy in the first market while those with higher indices are "endogenously rationed" or to use the language of competitive equilibrium, adopt speculative behavior.

5. THE MANY STATES CASE

I now assume that the number of active buyers $\tilde{N}$ may take $Z \geq 2$ possible realizations. To simplify notation I assume $N_s - N_{s-1} = 1$ so there is one buyer per batch. As before, the probability that exactly s markets will open in the first period is π_s and the probability that market i will open is $q_i = \sum_{s \geq i} \pi_s$.

The price in the first period hypothetical UST market i is P_i. A single Walrasian market will open in the second period at the price p_s if the number of markets open in the first period is $s < Z$. A buyer who arrives in the first period market i will buy C_i units and will make plans to buy in the next period c_s^i units if $i \le s < Z$. He will choose these quantities by solving the following problem.

(51) $$\max_{C_i} - P_i C_i + (1/q_i)\pi_Z U(C_i) + (1/q_i)\sum_{s=i}^{Z-1} \pi_s \left(\max_{c_s^i} U(C_i + \delta c_s^i) - p_s c_s^i \right)$$

For notational purposes I use $c_Z^i = 0$ and write the first order conditions for (51) as:

(52) $$\delta U'(C_i + \delta c_s^i) = p_s \quad \text{for } i \le s < Z$$

(53) $$\sum_{s=i}^{Z} \pi_s U'(C_i + \delta c_s^i) = q_i P_i$$

Condition (52) says that the buyer will choose c_s^i in the second period to equate the marginal utility with the second period price. To interpret (53) note that $(1/q_i)\pi_s$ is the probability that state s will occur given that $s \ge i$. Since the buyer in market i use these conditional probabilities, (53) says that the expected marginal utility for a buyer in market i must equal the price in market i (after dividing both sides of [53] by q_i).

Sellers will sell in market i, if the price in market i is greater than the value of inventories:

(54) $$P_i \ge (1/q_i) \sum_{s=i+1}^{Z-1} \pi_s p_s \quad \text{for all } i$$

Under (54) the seller chooses the supply to market i by solving the following problem.

(55) $$\max_{x_i} - \lambda x_i + q_i P_i x_i + x_i \sum_{s=1}^{i-1} \pi_s p_s$$

To interpret (55) note, that the seller pays the cost λ regardless of whether he sells the good or not. With probability q_i he sells the good and gets P_i per unit. If $s < i$, he does not sell the good in the first period but will sell it next period at the price p_s. The first order condition for an interior solution to the seller's problem (55) is:

(56) $$q_i P_i + \sum_{s=1}^{i-1} \pi_s p_s = \lambda$$

This condition says that the expected revenue equal the cost. The expected revenue calculations take into account the value of inventories in case market i does not open. In this respect it is similar to (22). But in (22) the value of inventories is the value from reducing production next period. Here there is no production in the second period and it will be shown that the value is less than the cost of production.

To define equilibrium I use $c^i = (c_i^i, ..., c_{Z-1}^i)$ to denote the plan (second period's purchases) of a buyer who arrive in market i.

Equilibrium is a vector $(C_1, ..., C_Z; c^1, ..., c^{Z-1}; x_1, ..., x_Z; P_1, ..., P_Z; p_1, ..., p_{Z-1})$ that satisfies (54), $P_1 < P_2, ... < P_Z$, the first order conditions (52), (53), (56) and the following market clearing conditions:

(57) $$C_i = x_i$$

(58) $$\sum_{j=1}^{s} c_s^j = \sum_{j=s+1}^{Z} x_j$$

Condition (57) says that the first period market i must clear if it opens. Condition (58) says that the second period market must clear if it opens (that is if $s < Z$).

<u>The Weak planner's problem</u>

I use x_i to denote the amount that the weak planner distributes to batch i in the first period (if it arrives and carry as inventories if batch i does not arrive) and c_s^i to denote the amount that he will distribute in the second period in state s to buyers that arrive in batch $i \le s$. The weak planner chooses these quantities by solving the following problem.

(59) $$\max_{x_j, c_s^j} -\lambda \sum_{j=1}^{Z} x_j + \sum_{s=1}^{Z} \pi_s \sum_{j=1}^{s} U\left(x_j + \delta c_s^j\right) \quad \text{s.t. (58).}$$

The first term in (59) is the cost of production. The second term is the expected sum of the utilities from the consumption of X. The first order conditions for an interior solution to this problem are:

(60) $$U'(x_j + \delta c_s^j) = U'(x_1 + \delta c_s^1) \quad \text{for all } j \le s$$

(61) $$\sum_{s=i}^{Z} \pi_s U'(x_i + \delta c_s^i) + \sum_{s=1}^{i-1} \pi_s \delta U'(x_1 + \delta c_s^1) = \lambda$$

Condition (60) says that the marginal utility after distributing c_s^j in the second period, must be the same across all active buyers. Condition (61) says that the expected marginal utility from consuming x_i must equal the cost of production. The first term in the left hand side is the weighted sum of the marginal utilities when batch i arrives. The second term on the left hand side is the weighted sum of the marginal utilities when batch i does not arrive. In this case the marginal utility is $\delta U'\left(x_1 + \delta c_s^1\right)$ because the good will be distributed in the second period and will "depreciate" by that time. The first order condition (61) can also be described as a no arbitrage condition. If the left hand side of (61) is greater than the right hand side, a planner could do better by increasing production by a unit and allocating the unit to batch i if it arrives and to batch 1 if batch i does not arrive.

We can now show the following Claim.

Claim 9: (a) There exists a unique equilibrium, (b) the equilibrium outcome is a solution to the "weak" planner's problem, (c) prices in the second period's Walrasian market are increasing with the state ($p_1 < ... < p_{Z-1}$), (d) the first UST market price is λ (and therefore $\lambda = P_1 < ... < P_Z$) and (e) $p_i < P_i$.

The intuition for (c) is straightforward: In higher states, more stuff is sold and less inventories are carried to the second period. The intuition for (d) is in the arbitrage condition: Market 1 opens with certainty and therefore if $P_1 > \lambda$ the supply will be infinite which is not consistent with market clearing (and if $P_1 < \lambda$, the supply is zero which is also not consistent with market clearing). The intuition for (e) is that when $p_i \geq P_i$ a seller who observes that market i opens will refuse to sell because he can sell it to batch $i+1$ at a higher price if it arrives and sell it in the second period at no loss if batch $i+1$ does not arrive. Since $p_i < P_i$, the price in the second period must be less than the highest transaction price in the first period.

Proof: The first order conditions (60) and (61) must hold in equilibrium. To show this claim, note that substituting (52) and (53) in (56) leads to (61). And (52) insures that (60) holds.

I now compute the equilibrium vector from the solution to the planner's problem. Using the planner's allocation to the first batch $(x_1; c_1^1, ..., c_{Z-1}^1)$, we can compute the second period prices:

(62) $$p_s = \delta U'(x_1 + \delta c_s^1)$$

When more batches arrive there is less to distribute in the second period and therefore the marginal utility $\delta U'(x_1 + \delta c_s^1)$ is increasing in s and therefore: $p_1 < ... < p_{Z-1}$.

We can use the planner's allocation to batch i, $(x_i; c_i^i, ..., c_{Z-1}^i)$, to compute the price in market i:

(63) $$P_i = (1/q_i) \sum_{s=i}^{Z} \pi_s U'(x_i + \delta c_s^i)$$

To show that $P_i < P_{i+1}$ note that the solution to the planner's problem must satisfy the following condition.

(64) $$(1/q_i) \sum_{s=i}^{Z} \pi_s U'(x_i + \delta c_s^i) = (1/q_i) \sum_{s=i+1}^{Z} \pi_s U'(x_{i+1} + \delta c_s^{i+1}) + (1/q_i) \pi_i \delta U'(x_i + \delta c_i^i)$$

To interpret (64) consider the point of view of a planner who observes that batch i arrives but does not know yet if batch $i+1$ will arrive. The left hand side of (64) is the expected loss from reducing the amount given to batch i in the first period by a unit. The right hand side is the expected gain from supplying a unit to batch $i+1$ if it arrives and supplying the unit to batch i if batch $i+1$ does not arrive. Condition (64) thus says that at the optimum the expected loss from transferring a unit from x_i to x_{i+1} must equal the expected gain so that a small deviation from the optimal plan does not reduce welfare. From (64) we get:

(65) $$(1/q_i) \sum_{s=i+1}^{Z} \pi_s U'(x_i + \delta c_s^i) \le (1/q_i) \sum_{s=i+1}^{Z} \pi_s U'(x_{i+1} + \delta c_s^{i+1})$$

Since $q_{i+1} < q_i$, (65) implies:

(66) $$(1/q_i)\sum_{s=i}^{Z}\pi_s U'(x_i+\delta c_s^i) < (1/q_{i+1})\sum_{s=i+1}^{Z}\pi_s U'(x_{i+1}+\delta c_s^{i+1})$$

The inequality $P_i < P_{i+1}$ follows from substituting (63) in (66).

Note that (61) and (63) imply:

(67) $$P_1 = \sum_{s=1}^{Z}\pi_s U'\left(x_1+\delta c_s^1\right) = \lambda$$

To show that (54) is satisfied note that (62) and (63) imply:

(68) $$P_i = (1/q_i)\sum_{s=i}^{Z}\pi_s U'(x_i+\delta c_s^i) \geq (1/q_i)\sum_{s=i+1}^{Z-1}\pi_s \delta U'(x_i+\delta c_s^i) = (1/q_i)\sum_{s=i+1}^{Z-1}\pi_s p_s .$$

To show (e) note that a seller who observe that market i opens must be indifferent between selling a unit in market i to transferring it to x_{i+1}. This leads to the following arbitrage condition.

(69) $$P_i = p_i(1/q_i)\pi_i + P_{i+1}(1/q_i)\sum_{s>i}\pi_s$$

Part (e) follows from the observation that P_i is a weighted average of p_i and P_{i+1} and $P_{i+1} > P_i$. This completes the proof.

6. A UNIFIED FRAMEWORK

Many goods have both a seasonal and all year round aspects. For example, short sleeve shirt is typically used in the summer that it was bought but can also be stored and used in the next summer. We may capture both aspects by combining the two models (BE and DP).

For this purpose, I assume an economy that lasts for infinitely many periods, and each period is divided into two sub-periods. In the first sub period demand is not known and trade is done in the UST hypothetical markets. The trade in the first sub-period reveals the state and in the second sub-period there is a single Walrasian market with a single price. As in the previous models prices do not change over time and I therefore drop the time index.

As in the DP model, the buyer's problem is described by (51).

The value of a unit carried as inventories to the next period is $\beta\lambda$ (because the seller can cut next period's production by a unit and save the unit's cost). Therefore, the seller will supply to the second sub-period market only if:

(70) $$p_s \geq \beta\lambda$$

Assuming that (70) holds, the seller will choose the amount supplied to the first sub-period UST market i (x_i) by solving the problem (55). If market i opens he will sell at the price P_i. If market i does not open in the first sub-period, he will sell in the second sub-period $x_i - I_s^i$ units at the price p_s and carry I_s^i units as inventories to the next period. If the second sub-period Walrasian market opens he will choose the amount of inventories by solving the following problem.

(71) $$\max_{0 \leq I_s^i \leq x_i} p_s(x_i - I_s^i) + \beta\lambda I_s^i$$

The first order conditions to this problem requires:

(72) $$I_s^i = 0 \text{ if } p_s > \beta\lambda \text{ and } 0 \leq I_s^i \leq x_i \text{ if } p_s = \beta\lambda$$

The amount of inventories carried to the next period in state s is $I_s = \sum_{i>s} I_s^i$. Given (72) the aggregate amount of inventories must satisfy the following condition.

(73) $$I_s = 0 \text{ if } p_s > \beta\lambda \text{ and } 0 \le I_s \le \sum_{i=s+1}^{Z} x_i \text{ if } p_s = \beta\lambda$$

To allow for inventories I modify the equilibrium definition in the previous section as follows.

Equilibrium is a vector $(P_1,...,P_Z; p_1,...,p_{Z-1}; x_1,...,x_Z; C_1,...,C_Z; c^1,...,c^{Z-1}; I_1,...,I_{Z-1})$ that satisfies (a) the incentive to supply conditions (54), (70) and $P_1 < ... < P_Z$; (b) the first order conditions (52), (53), (56), (73); and (c) the market clearing conditions (57) and

(58') $$I_s + \sum_{i=1}^{s} c_s^i = \sum_{i=s+1}^{Z} x_i$$

Note that the BE model is a special case that assumes $\delta = 0$, while the DP is a special case that assumes $\beta = 0$.

The weak planner's problem for the unified model

The planner's problem for the unified model environment is:

(74) $$\max_{x_j, c_s^j, I_s \ge 0} - \lambda \sum_{j=1}^{Z} x_j + \sum_{s=1}^{Z} \pi_s \left(\beta\lambda I_s + \sum_{j=1}^{s} U(x_j + \delta c_s^j) \right) \quad \text{s.t. (58')}$$

The first order conditions for this problem are (61) and

(75) $$\delta U'(x_j + \delta c_s^j) = \delta U'(x_1 + \delta c_s^1) \ge \beta\lambda \quad \text{with equality if } I_s > 0$$

We can now modify Claim 9 as follows.

Claim 9’: (a) There exists a unique equilibrium, (b) the equilibrium outcome is a solution to the “weak” planner’s problem, (c) prices in the second period’s Walrasian market are increasing with the state ($\beta\lambda \le p_1 \le ... \le p_{Z-1}$), (d) the first UST market price is λ and (e) $p_i < P_i$.

The proof is similar to the proof of Claim 9.

7. CONCLUDING REMARKS

This paper attempts to understand price dispersion and efficiency in versions of the Prescott (1975) model. In the paper I focus on flexible price (UST) version of the model and use three different benchmarks or planner’s problems to judge the equilibrium outcome. The three planners are distinguished by the information they have when making decisions. The “weak” planner has the same information as the sellers in the model. The “strong” planner knows the state before making any decisions and the “semi-strong” planner knows the state only after capacity decisions are made but before the arrival of buyers. In all the versions studied the UST outcome is a solution to the “weak” planner’s problem under the assumption that the probability of becoming active depends on the aggregate state but not on the buyer’s type. And in general, the “semi-strong” and the “strong” planners can improve matters. An exception is the case in which there are no costs for delaying trade. In this case the “semi-strong” planner cannot improve matters and in the general equilibrium version of the BE model even the “strong” planner cannot improve matters. The “strong” planner can still improve matters in the DP model because of the finite horizon assumption.

The paper provides a unified framework for Prescott types models. The formulation of equilibrium is the same whether we assume rigid or flexible prices. But the formulation of the relevant planner's problem or the definition of feasible allocation is different. In rigid price versions the planner faces the constraint that he cannot distribute more than the output produced. In flexible price versions he faces an additional constraint that he must choose the allocation to each batch of buyers before he knows whether more batches will arrive or not. The "semi-strong" planner is therefore relevant for the rigid price versions while the "weak" planner is relevant for the flexible price versions.

The intuition for the above results is as follows. Prices are rigid when at the time of trade sellers would like to change them but cannot. Therefore, in rigid price versions, a planner who has the same information as the sellers in the model but does not use rigid prices can in general improve matters. This is not surprising. It is also not surprising that in the flexible price versions (UST) a planner that has the same information as the sellers in the model cannot in general improve matters. What maybe surprising are the exceptions to the rules.

An exception to the first rule occurs when the costs of delays are not important. In this case the value of early information about the state is close to zero and price rigidity does not matter much. In the BE model with approximately no discounting, sellers will set a single price equal to the marginal cost and will not "regret" this choice even if information about the state becomes available before the beginning of trade. Similarly, when discounting is not important, the optimal policy of a "weak" planner in the BE model is to keep inventories at the beginning of the period at some target level and distribute to each active buyer a quantity that does not depend on the state (and equates the marginal utility with the marginal cost of production). Information about the state can be used to eliminate inventories but since there is no discounting the value of doing it is small.

When there are no costs of delays, a planner in the DP model, will not value information about the state because he can distribute relatively small amounts in the first period, learn about the state and then distribute the rest making sure that the marginal utility of all buyers is the same regardless of the order of arrival. Buyers in the model do what the planner wants them to do. They engage in speculative behavior and buy a relatively small amount in the first period. Then once the state is revealed in the second period they buy the amount that was not sold in the first period at the market-clearing price.

An exception to the second rule occurs when the probability of becoming active is not the same across types. In this case the fact that a buyer of a certain type is active is a signal about the state. Sellers do not use this signal because they cannot price discriminate and therefore a planner that has the same information as the sellers in the model (but is allowed to discriminate by type) can improve matters even in the flexible price version of the model.

The efficiency results can also be stated in terms of the following policy implications. If prices are flexible and the government (or the central bank) has no informational advantage over the sellers in the model there is in general no room for government intervention. An exception to this standard general rule is the case in which the government can discriminate in a way that sellers cannot. (The governments can use tariffs for example, to discriminate among residents of different countries, see Eden [2009]). There is room for government intervention if prices are rigid. These conclusions critically depend on the source of the uncertainty about demand. Here the source is in taste shocks and there is nothing much that the government can do about fluctuations in the desire to consume. In this respect taste shocks are similar to productivity shocks. But the government should attempt to minimize shocks to demand that arise as a result of shocks to government spending, money supply and bubbles. See Eden (1994, 2012).

The flexible price version of the model suggests that price dispersion is not evidence of market failure. It is required to cope with aggregate demand uncertainty in an efficient way.

The positive implications of the theory are about the relationship between price dispersion, demand uncertainty and the cost of delays. In Eden (2013) I analyze the case in which demand is uniformly distributed and show that price dispersion increases with demand uncertainty and with the cost of delays.

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APPENDIX: RELAXING ASSUMPTION 1

I now relax Assumption 1 for the single period case. I use $N_s = \sum_j \phi_{js} n_j$ for the number of active buyers and start with the following special case.

Assumption 2: $U_j(x) = U(x)$, $d_j(p) = d(p)$ for all j and $N_1 < N_2 < ... < N_S$.

Here the type composition of the buyers who arrive in each batch (ϑ_{js}) depends on the state. Because all types have the same demand function, the value of ϑ_{js} is not relevant for computing the demand of each batch and for defining equilibrium. But as we shall see it is relevant for the social planner.

The algorithm for computing the number of buyers in each batch is similar to what we had in the previous case. The minimum number of (active) buyers is: $\Delta_1 = N_1$. The first batch of Δ_1 buyers arrives with certainty. After buyers in this first batch complete trade and go away there are two possibilities. If $s = 1$ trade ends. If $s > 1$, there are $N_s - N_1$ unsatisfied buyers. The minimum number of unsatisfied buyers if $s > 1$ is: $\Delta_2 = \min_s\{N_s - N_1\} = N_2 - N_1$ and this is the number of buyers in batch 2. The probability that $s > 1$ is $q_2 = 1 - \pi_1$ and this is the probability that batch 2 will arrive. Proceeding in this way we define q_s and Δ_s for all $s = 1,...,Z$. As before, it is convenient to think of a sequence of Walrasian markets, where batch i buys in market i and the seller supplies x_i units to market i.

A UST equilibrium is a vector of prices $(P_1,...,P_Z)$ and a vector of supplies $(x_1,...,x_Z)$ such that: (a) $P_i = \lambda/q_i$ and (b) $x_i = (N_i - N_{i-1})d(P_i) = \Delta_i d(P_i)$.

<u>The "weak" planner's problem</u>:

I assume that the "weak" planner can choose the amount x_{ji} that he will give to a type j buyer who arrives in batch i but does not observe the fraction of type j buyers that arrive in each batch (ϑ_{js}) and like the sellers in the model, must therefore make allocation decisions before he observes the state. The planner's problem is:

(A1) $$\max_{x_{ji}} \sum_{j=1}^{J}\sum_{s=1}^{Z}\pi_s\vartheta_{js}\sum_{i=1}^{s}(N_i - N_{i-1})U(x_{ji}) - \lambda \max_s\left(\sum_{j=1}^{J}\sum_{i=1}^{Z}\vartheta_{js}(N_i - N_{i-1})x_{ji}\right)$$

To understand the first term in the objective function note that $\vartheta_{js}\sum_{i=1}^{s}(N_i - N_{i-1})U(x_{ji})$ is the total utility that the planner gets from type j buyers in state s. The second term is the production cost of implementing the plan. To simplify, I assume that the maximum amount distributed occurs in state Z:
$Z = \arg\max_s\left(\sum_{j=1}^{J}\sum_{i=1}^{Z}\vartheta_{js}(N_i - N_{i-1})x_{ji}\right)$.

Note that x_{ji} matters only when $s \geq i$ and market i opens. We can therefore find the first order condition to the problem in (A1) by taking the derivative of $(N_i - N_{i-1})U(x_{ji})\sum_{s=i}^{Z}\pi_s\vartheta_{js} - \lambda\vartheta_{jZ}(N_i - N_{i-1})x_{ji}$. The first order condition is:

(A2) $$U'(x_{ji}) = \frac{\lambda\vartheta_{jZ}}{\sum_{s=i}^{Z}\pi_s\vartheta_{js}}$$

This is different from (5), implying that the UST outcome is not a solution to the "weak" planner's problem.

Note that under assumption 1, $\vartheta_{js} = \vartheta_j$ for all j and (A2) is the same as (5). The difference between (A2) and (5) arises when $\vartheta_{jZ} \neq \vartheta_{js}$. In this case, the planner will give type j more relative to the UST outcome, when $\vartheta_{jZ} < \vartheta_{js}$ for all s. In the extreme case when $\vartheta_{jZ} = 0$ he will satiate type j agents because this type arrives only in state in which there is excess capacity.

<u>The general case</u>:

I now relax assumption 2. As before buyers arrive in batches but here the size of each batch is endogenous and depends on the prices: $P_1 \le P_2 \le ... \le P_Z$. Roughly speaking, the size of the first batch is the minimum demand at the price P_1. Market 2 opens if there are some buyers who wanted to buy in the first market but could not. In general, market i opens if there is residual demand after transactions in market $i-1$ are complete. The size of batch i is the minimum residual demand.

The definition of equilibrium is essentially a choice of indices: $(y_1,...,y_Z)$. Demand in state s at the price P_1 is: $\sum_j \phi_{js} n_j d_j(P_1)$. I choose indices such that state y_1 is the state of minimum demand at the price P_1: $y_1 = \arg\min_s \left\{ \sum_j \phi_{js} n_j d_j(P_1) \right\}$. State 2 is the minimum demand at the price P_2 out of the states $s \ne y_1$: $y_2 = \arg\min_s \sum_j \phi_{js} n_j d_j(P_i)$ s.t. $s \ne y_1$. And in general: $y_i = \arg\min_s \sum_j \phi_{js} n_j d_j(P_i)$ s.t. $s \ne y_k$ for all $k < i$.

With the above choice of indices, we may describe equilibrium in the following way. The seller puts a price tag of P_i on x_i units and then remains passive. He knows that the lowest priced x_1 units will be sold first with certainty. Then if there is additional demand the x_2 units with the price tag P_2 will be sold and so on. I assume that the seller does not use the type composition of batch i to update the probabilities of the states. We may therefore think of the seller as having many outlets and since trade does not take real time he cannot get aggregate statistics on the type composition during trade.

Since the definition of equilibrium describes this choice of indices it is simpler to write some subscripts in parenthesis. I use $\pi(s)$ instead of π_s for the probability that state s occurs and $\phi_j(s)$ instead of ϕ_{js} for the fraction of type j buyers that are active in state s.

A UST equilibrium is a vector of distinct Z integers $(y_1,...,y_Z)$ and a vector of real numbers $(P_1,...,P_Z;x_1,...,x_Z;\Pi_1,...,\Pi_Z;q_1,...,q_Z;\Phi_{11},...,\Phi_{J1};\Phi_{12},...,\Phi_{J2};...;\Phi_{1Z},...,\Phi_{JZ})$ such that:

(a) $1 \le y_i \le Z$ for all i,

(b) $\sum_j \phi_j(y_i) n_j d_j(P_i) < \sum_j \phi_j(y_k) n_j d_j(P_i)$ for all $i < k \le Z$,

(c) $\Phi_{ji} = \phi_j(y_i)$,

(d) $\Pi_i = \pi(y_i)$,

(e) $q_i = \sum_{s \ge i} \Pi_s$,

(f) $x_i = \sum_j \Phi_{ji} n_j d_j(P_i) - \sum_{s=1}^{i-1} x_s$ and

(g) $P_i = \dfrac{\lambda}{q_i}$.

Part (a) implies that y is a one to one mapping from $(1,...,Z)$ to $(1,...,Z)$.
Part (b) requires that at the price P_i demand in state y_i is less than demand in state y_k for all $k > i$. Suppose for example that $y_1 = 6$ and $y_2 = 3$. Then demand at the price P_1 is lowest in state 6 and demand at the price P_2 is lower in state 3 then in all states $s \ne 6$. The fraction of type j buyers who are active in state 6 is denoted by $\Phi_{j1} = \phi_j(y_1)$ and the fraction of type j buyers who are active in state 3 is denoted by: $\Phi_{j2} = \phi_j(y_2)$. The probability that state 6 occurs is denoted by $\Pi_1 = \pi(y_1)$ and the probability that state 3 occurs is denoted by $\Pi_2 = \pi(y_2)$. Thus Π_1 is the probability that exactly one batch will arrive and Π_2 is the probability that exactly two batches will arrive. The probability $q_i = \sum_{s \ge i} \Pi_s$ is the probability that more than i batches will arrive or the probability that market i opens. Part (f) is a market clearing condition: After transactions in market $i-1$ are complete, the minimum residual demand at the price P_i is $\sum_j \Phi_{ji} n_j d_j(P_i) - \sum_{s=1}^{i-1} x_s$ and this must equal the supply to

market i. Part (g) requires that the expected revenue per unit is the same across markets.

<u>The "weak" planner's problem</u>:

I assume that the "weak" planner can observe the aggregate amount distributed and the type of each buyer but not the type composition of the buyers. The "weak" planner chooses Z quantities $(x_1,...,x_Z)$ and Z allocation rules. The first allocation rule is applied to the distribution of the first batch of x_1 units. The second allocation rule is applied to the distribution of the second batch of x_2 units and so on. In detail, the planner distributes x_{j1} units to type j buyers that arrive until the first x_1 units are distributed. He then use the second allocation rule and distributes x_{j2} units to type j buyers that arrive until the second batch of x_2 units are distributed and in general he uses the allocation rule x_{ji} after $\sum_{s=1}^{i-1} x_s$ units were already distributed to distribute the next x_i units.

We may say that buyers who arrive after $\sum_{s=1}^{i-1} x_s$ units were distributed and before $\sum_{s=1}^{i} x_s$ units were distributed, arrive in batch i and x_{ji} is the amount allocated to a type j agent who arrives in batch i.

The choice of x_i and x_{ji} determine the probability that x_i will satisfy the additional demand. If for example x_1 is large and x_{ji} are small, the probability that more buyers will arrive after the distribution of x_1 units is small. Therefore the probabilities of delivery depends on the choice of x_i and x_{ji}.

We may therefore write the "weak" planner's problem in the following way.

Choose Z distinct integers $(y_1,...,y_Z)$ and a vector of real numbers
$(x_1,...,x_Z;\Pi_1,...,\Pi_Z;q_1,...,q_Z;$
$\Phi_{11},...,\Phi_{J1};\Phi_{12},...,\Phi_{J2};...;\Phi_{1Z},...,\Phi_{JZ};x_{11},...,x_{J1};x_{12},...,x_{J2};...;x_{1Z},...,x_{JZ})$

such that:

(a) $1 \le y_i \le Z$ for all i,

(b) $\sum_j \phi_j(y_i) n_j x_{ji} < \sum_j \phi_j(y_k) n_j x_{ji}$ for all $i < k \le Z$

(c) $\Phi_{ji} = \phi_j(y_i)$,

(d) $\Pi_i = \pi(y_i)$,

(e) $x_i = \sum_{j=1}^{J} n_j \Phi_{js} x_{ji} - \sum_{j=1}^{J} n_j \Phi_{ji-1} x_{ji-1} > 0$ And

(f) x_{ji} solve the following problem:

$$\max_{x_{js}} \sum_{j=1}^{J}\sum_{s=1}^{Z} \Pi_s \sum_{i=1}^{s} \Phi_{js} n_j U_j(x_{ji}) - \lambda \max_s \left(\sum_{j=1}^{J}\sum_{i=1}^{Z} n_j \Phi_{js} x_{ji} \right)$$

To solve for the planner's first order condition I assume as before, that the planner wants to distribute the maximum amount in state Z: $Z = \arg\max_s \left(\sum_{j=1}^{J}\sum_{i=1}^{Z} n_j \Phi_{js} x_{ji} \right)$.

Since the planner knows ϑ_{js} he can compute for each state s, the number of buyers served in batch i, $(N_{is} - N_{i-1s})$ and the number of type j buyers served in batch i, $\vartheta_{js}(N_{is} - N_{i-1s})$. In detail, the equations:

$N_{1s} \sum_{j=1}^{J} \vartheta_{js} x_{j1} = x_1$ and $(N_{is} - N_{i-1s}) \sum_{j=1}^{J} \vartheta_{js} x_{ji} = x_i$ lead to: $N_{1s} = x_1 \left(\sum_{j=1}^{J} \vartheta_{js} x_{j1} \right)^{-1}$ and

$N_{is} - N_{i-1s} = x_i \left(\sum_{j=1}^{J} \vartheta_{js} x_{ji} \right)^{-1}$.

The planner will choose the amount allocated to a type j buyers who arrive in batch i by maximizing:

(A3) $$\max_{x_{ji}} U_j(x_{ji}) \sum_{s=i}^{Z} \Pi_s \vartheta_{js} (N_{is} - N_{i-1s}) - \lambda \vartheta_{jZ} (N_{iZ} - N_{i-1Z}) x_{ji}$$

The first order condition for this problem is:

(A4) $$U_j'(x_{ji})\Delta_{ji} = \lambda\vartheta_{jZ}(N_{iZ} - N_{i-1Z}),$$

where $\Delta_{ji} = \sum_{s=i}^{Z} \Pi_s \vartheta_{js}(N_{is} - N_{i-1s})$ is the expected number of buyers served in market i

. The interpretation of this first order condition is as follows. The expected marginal utility from increasing the allocation to type j in market i by one unit: $U_j'(x_{ji})\Delta_{ji}$. The cost of doing it is: $\lambda\vartheta_{jZ}(N_{iZ} - N_{i-1Z})$, because only in state Z we hit the capacity constraint. Therefore, (A4) says that the marginal benefits equal the marginal cost.

Rearranging (A4) leads to:

(A5) $$U_j'(x_{ji}) = \frac{\lambda\vartheta_{jZ}(N_{iZ} - N_{i-1Z})}{\sum_{s=i}^{Z} \Pi_s \vartheta_{js}(N_{is} - N_{i-1s})}$$

This is different from the UST allocation rule (5) implying that in general, the UST outcome is not a solution to the “weak” planner’s problem (and of course not to the “strong” planner’s problem).