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Abstract

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DOES A LOW INTEREST RATE SUPPORT PRIVATE BUBBLES?

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I examine the argument that a low interest rate policy can lead to “overvalued” private assets or privately created bubbles (private bubbles). Using the standard approach to bubbles, I find that a policy of a low real interest rate may support private bubbles but a policy of a low nominal interest rate may actually reduce the importance of private bubbles. I then attempt a less conventional way of modeling bubbles focusing on the supply of private bubbles. The paper uses results from the Friedman rule literature, the fiscal approach to the price level and the literature on rational bubbles.

Key Words: Interest Rate Policy, The Friedman Rule, The Fiscal Approach to Bubbles.

JEL codes: E31, E32, E42, E52.

1. INTRODUCTION

The recent financial crisis has led to a renewed interest in bubbles and their effect on the economy. One of the open questions is what kind of policies can reduce the likelihood of privately created bubble assets. For example, it is often argued that the low interest rate policy that preceded the great recession of 2008 has led to the housing bubble. Will a high interest rate policy reduce the likelihood of privately created bubbles?

To discuss this type of questions we need an economy that can support bubbles. This is not a trivial issue. Roughly speaking, the question is whether a Walrasian auctioneer can announce prices that are not strongly correlated with fundamentals but nevertheless clear markets. In their well-known working paper, Blanchard and Watson (1982) answer this question in the positive. Santos and Woodford (1997) show that it is more difficult to get bubbles when we get closer to an economy with infinitely lived agents (IL economy) because in such economies individuals will not want to hold the accumulated bubble wealth when it becomes large. See Jovanovic (2007) for a discussion in the context of durable goods that are used as a store of value.

Santos and Woodford (1997) argue that rational bubbles cannot exist under fairly general assumptions. But this does not imply that government policy cannot create bubbles. Here I use the logic of the fiscal approach to the price level, to show that government policy can leads to bubbles even in IL economies. But in IL economies the policy-maker cannot choose the real interest rate. I therefore focus on overlapping generations (OG) economies. Unlike Samuelson (1958), here private monies may compete with government's money and bubble assets may pop.

The paper has 5 additional sections. Section 2 uses Friedman (1969) and the fiscal approach to argue that the government can create bubbles even in an IL economy. Section 3 assumes a fully indexed OG economy in which the government can choose the steady state real interest rate. It is shown that a real interest rate that is higher than the rate of population growth (assumed to be zero here) can eliminate privately created bubbles if the government can commit to a no-bailout policy: A policy of not bailing out agents who cannot pay their taxes because they cannot sell their private bubble for money. Section 4 illustrates the difference between an indexed and a non-indexed economy. It is shown that in a non-indexed economy bubbles can exists even if the nominal interest rate is high. Section 5 uses non-indexed productive economies to discuss policy tradeoffs and to model the supply of bubbles. Section 6 provides summary and further discussion about policy issues.

Sections 2-4 are somewhere between an extended literature review and an original contribution. As I read the literature it focuses on the question of the existence of private bubbles while I focus on the effect of changes in interest rate policy. The reader may choose to read section 5 first and then read sections 2-4 to gain a broader perspective.

2. AN INFINITELY LIVED REPRESENTATIVE AGENT ECONOMY AND THE FISCAL APPROACH TO BUBBLES

Sargent and Wallace (1981), derives a “budget constraint” for the government by substituting market-clearing conditions in the individuals’ budget constraint. This budget constraint requires equality between the present value of the primary deficit and the present value of seigniorage revenues. The idea of a “government budget constraint” was used to determine the price level in an economy in which money does not yield “liquidity services” and does not appear as an argument in the utility function. See the literature on the fiscal approach to the price level pioneered by Leeper (1991), Sims (1994), Woodford (1995), Dupor (2000) and Cochrane (2001).

The fiscal approach raises a question about the definition of a bubble. To illustrate, I start with an endowment economy populated by infinitely lived identical agents (IL economy). There is a single good (called “corn”) and the representative agent gets d units of corn per period. In addition to his endowment of corn the agent has at $t=0$, M_0 dollars. The government announces a tax of τ units of corn per period, payable in dollars but does not use the tax revenues to buy goods. It either burns the dollars it gets or stores them.

The representative agent expects that the dollar price of corn at time t will be P_t and solves the following problem.

(2.1)
$$\max_{C_t, M_t} \sum_{t=1}^{\infty} \beta^t U(C_t)$$
$$\text{s.t. } P_t C_t + M_t = P_t(d-\tau) + M_{t-1} \quad \text{and } M_0 \text{ is given.}$$

The notation used are standard: C_t is corn consumption, M_t is nominal balances, P_t is the dollar price of corn (the price level), $0 < \beta = \dfrac{1}{1+\rho} < 1$ is the discount factor and U is strictly monotone and strictly concave period utility function.

Equilibrium is a sequence $\{M_t, C_t, P_t\}_{t=1}^{\infty}$ such that (a) given the sequence $\{P_t\}$, the sequence $\{M_t, C_t\}$ solves (2.1) and (b) $C_t = d$ for all t.

A steady-state equilibrium is equilibrium in which
$\dfrac{P_{t-1}}{P_t} = 1 + r_m$, $m_t = \dfrac{M_t}{P_t} = m$ for all $t \geq 1$.

Thus, in a steady-state equilibrium the gross real rate of return on money $1 + r_m = \dfrac{P_{t-1}}{P_t}$ and the holdings of real balances do not change over time. To solve for a steady-state equilibrium I define $P_0 = (1+r_m)^{-1} P_1$ and write the representative agent's problem in real terms.[1]

(2.1') $$\max_{C_t, m_t} \sum_{t=1}^{\infty} \beta^t U(C_t)$$

$$\text{s.t. } C_t + m_t = d - \tau + m_{t-1}(1+r_m) \text{ and } m_0 \text{ is given.}$$

Note that in the steady state: $C = d - \tau + mr_m$. This and the market clearing condition $C = d$, implies a "government budget constraint": $mr_m = \tau$. And it leads to the following Claim.

<u>Claim 1</u>: There exists a unique steady-state equilibrium in which $r_m = \rho$ and $\rho m = \tau$.

To show this Claim, note that when $r_m = \rho$, an agent that start with $m = \tau/\rho$ units of real balances can consume his endowment and use the capital gains (or "interest") on his asset to pay the tax. When the rate of return is $r_m = \rho$ this feasible plan is also optimal. To show uniqueness note that at any other "interest rate" the agent will not choose to consume his endowment.

Once we solve for the "real magnitudes" we can compute the nominal magnitudes as follows. The price level at $t = 0$ is determined by: $m = \tau/\rho = \dfrac{M_0}{P_0}$. The price level at $t = 1$ is $P_1 = P_0/(1+\rho)$ and in general $P_t = P_{t-1}/(1+\rho)$. Thus we have deflation and the value of the bubble asset appreciate at the rate of the subjective interest rate.

The above analysis provides a non standard interpretation to Friedman (1969) and the "money in the utility function" approach. It is often argued that agents hold money in these models because money appears as an argument in the utility function. This is

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[1] To get the budget constraint in real terms divide both sides of the constraint in (1) by P_t and then use: $\dfrac{M_{t-1}}{P_t} = \dfrac{M_{t-1}}{P_{t-1}} \dfrac{P_{t-1}}{P_t} = m_{t-1}(1+r_m)$.

correct only when $r_m < \rho$. At the Friedman rule when $r_m = \rho$, agents will hold money even when they do not derive utility from it.

To elaborate, consider for example the case in which the representative agent derives utility from holding real balances up to the satiation level of $\bar{m}$ and additional balances do not affect his utility. In this case, if $\tau/\rho > \bar{m}$, the agent will hold $\tau/\rho - \bar{m}$ units because holding these additional units provide a “hedge” against his tax obligations. Consider another example in which some agents derive utility from money and some do not. Assume further that the agents who derive utility from holding money are satiated when $m \geq \bar{m}$. Then if $\tau/\rho > \bar{m}$, all agents will hold $m = \tau/\rho$ regardless of whether or not they derive utility from holding money. We may therefore say that the reason for holding money at the Friedman rule, is that it provides a “hedge” against taxes and not because it provides utility.

In terms of the bubble literature there is a distinction between “dividends” and the price of an asset. Blanchard and Watson (1982) define “dividends” as direct returns that may take pecuniary or non-pecuniary forms. The question is whether the ability to use the asset to pay taxes (related to the “legal tender” characteristic of money) is a form of non-pecuniary returns. This is not the way Friedman uses the concept: According to Friedman (1969), money yields non-pecuniary returns at the margin only when $r_m < \rho$. Thus the ability to sell the asset and use it to make payments is not considered a form of non-pecuniary return.

2.1 PRIVATE BUBBLES

I now assume that in addition to money there is a Rock like mount Rushmore. Everyone think that the Rock looks good and want to have a piece of it. But the Rock does not yield services and does not pay dividends. (I thus assume that it is illegal or impossible to charge a fee for visiting the Rock).[2] The representative agent's budget constraint is:

(2.2) $$P_t C_t + M_t + q_t a_t = P_t(d-\tau) + M_{t-1} + q_t a_{t-1},$$

where a_t is the fraction of the Rock after trade in the asset market and q_t is the dollar price of the Rock. Let $S_t = q_t a_t$ denote the post-trade dollar value of Rock shares. Then $q_t a_{t-1} = S_{t-1}(1+i_t)$, where $1+i_t = \frac{q_t}{q_{t-1}}$ is the gross nominal rate of return on Rock shares.

Using this notation I write (2.2) as:

(2.3) $$P_t C_t + M_t + S_t = P_t(d-\tau) + M_{t-1} + S_{t-1}(1+i_t)$$

Using $1+r_t = (1+i_t)(1+r_{mt})$ for the gross real rate of return on Rock shares and $s_t = \frac{S_t}{P_t}$ for their real value, I write (2.3) as:

(2.4) $$C_t + m_t + s_t = d - \tau + m_{t-1}(1+r_{mt}) + s_{t-1}(1+r_t)$$

The problem of the representative agent can thus be written as:

(2.5) $$\max_{C_t, m_t, s_t \geq 0} \sum_{t=1}^{\infty} \beta^t U(C_t) \quad \text{s.t. (2.4) and given } (m_0, s_0)$$

Equilibrium is a vector (τ, m_0, s_0) and a sequence $\{r_t, r_{mt}, C_t, m_t, s_t\}_{t=1}^{\infty}$ such that

[2] This is somewhat similar to Jovanovic (2007) who assumes that the bubble asset is potentially useful but this potential is never realized. For example, oil in the ground that will never be used. Here there is no potential usage for the Rock.

(a) $r_t = r_{mt}$, (b) given (τ, m_0, s_0) and the sequence $\{r_t, r_{mt}\}_{t=1}^{\infty}$ the sequence $\{C_t, m_t, s_t\}_{t=1}^{\infty}$ solves (2.5) and (c) $C_t = d$.

It is straightforward to show that there exists a steady-state equilibrium with $m_t = m_0$, $s_t = s_0$, $r_{mt} = r_t = \rho$ for all t and $m_0 + s_0 = \tau/\rho$. There are also non steady state equilibria in which $r_{mt} = r_t = \rho$ and $m_t + s_t = \tau/\rho$ for all t . But the equilibrium is unique if we impose the constraint that taxes can be paid in terms of money only and there will be no bailout for agents who cannot sell their Rock shares. I will call this policy a no-bailout policy for short.

To show this claim I define a no-bailout equilibrium as an equilibrium that satisfies the added constraint:

(2.6) $$m_t = m_{t-1}(1 + r_{mt}) - \tau \geq 0 .$$

Claim 2: Rock shares must be worthless when (2.6) is imposed.

To show the Claim for the steady state, note that in the steady state (2.6) implies $r_m m \geq \tau$. Substituting this and $C_t = d$ in (2.4) yields $s \leq s(1 + r)$ which cannot hold when $r = \rho$. I now turn to show the Claim for any (non steady state) equilibrium.

Proof: To show this Claim note that when $r_t = r_{mt}$, we can write (2.4) as:

(2.7) $$C_t + z_t = d - \tau + z_{t-1}(1 + r_t) \quad \text{and} \quad z_t = m_t + s_t$$

Substituting the market clearing condition $C_t = d$ in (2.7) leads to:

(2.8) $$z_t = z_{t-1}(1 + r_t) - \tau$$

In equilibrium the Euler condition must hold and therefore $r_t = r = \rho > 0$. I now introduce the following Lemma.

Lemma: In equilibrium $z_t = z = \tau/\rho$ for all t.

Proof: if $\tau > \rho z_0$, then

$z_1 = z_0(1+\rho) - \tau < z_0$, $z_2 = z_1(1+\rho) - \tau < z_1$ and in general, $z_t < z_{t-1}$. This cannot be in equilibrium because eventually z_t will be negative (the agent will not be able to consume $C_t = d$ and pay his taxes). When $\tau < \rho z_0$, the agent will accumulate assets and this cannot be optimal because he can increase consumption at some date without violating his budget constraint (that is, z_t that is increasing over time violates the transversality condition). □

Using the Lemma we can now see that under (2.6),

(2.9) $m_t = m_{t-1}(1+r) - \tau = m_{t-1} + r(m_{t-1} - z) \le m_{t-1}$ with strict inequality if $z > m_{t-1}$.

Suppose now that $s_0 > 0$, then $s_1 = (1+\rho)s_0 > s_0$ and in general, $s_t > s_{t-1} > 0$. Therefore $z - m_t > 0$ and is growing over time. This and (2.9) imply that (2.6) will eventually be violated. It follows that equilibrium with the restriction (2.6) requires $s_t = 0$ for all t. □

Can the government commit to a no-bailout policy? To examine this question, consider the hypothetical case in which agents run out of money at time t^*. Then the government must announce a change in policy at t^*. It may (a) accept tax payments in terms of Rock shares, or (b) buy Rock shares with newly printed money. Under both alternatives, Rock shares may be valued. In general committing to a no-bailout policy is difficult. Kocherlakota (2010) for example, assume that bailouts are inevitable because government will rescue firms whose collapse may cause systemic failure.

The result that in the absence of a no-bail out policy, the Rock may have value seems different from the general conclusion in the literature. Azariadis (1993, page 474) summarizes the literature and argue that bubbles occur only if two preconditions are satisfied: the lifespan of the asset must be potentially infinite and the underlying fundamental equilibrium must be dynamically inefficient. See Wallace (1980) and Tirole (1985). Here the underlying fundamental equilibrium is efficient but nevertheless the Rock looks like a bubble. Again, it may be argued that Rock shares have non-pecuniary returns because the government will bail out agents that cannot sell their Rock shares for money. This is a matter of definition. Here I will refer to money as a government created bubble and to the Rock as a privately created bubble and for most of the paper I will just use the term "bubble" for both.

To better understand the difference between bubbles here and bubbles in the literature, we may reexamine the intuitive argument made by Blanchard and Watson (1982, page 10). They claim that: "The only reason to hold an asset whose price is above its fundamental value is to resell it at some time and to realize the expected capital gain. But if all agents intend to sell in finite time, nobody will be holding the asset thereafter, and this cannot be an equilibrium…Therefore, with rationality and infinitely lived agents, bubbles cannot emerge". Here agents use the capital gains to pay taxes. The bubble survives because the government is willing to hold it forever (or to burn the tax proceeds as in Friedman [1969]).

The IL model was used here to illustrate the possibility of a government-supported bubbles and to discuss the relationship between the literature on bubbles, the fiscal approach and the Friedman rule. But this model is not suitable to answer the question in the title because here the real rate of return on money holdings must be ρ. I therefore turn now to an overlapping generations model in which the policy-maker can choose the real interest rate.

3. AN OVERLAPPING GENERATIONS INDEXED ECONOMY

I assume a special case of the OG model in Diamond (1965). Agents live for two periods and each gets an endowment of x units of corn in the first period of his life. The agent can sow corn and enjoy the next period harvest: If he sows k units he will harvest $F(k)$ units where F is a standard production function ($F'>0, F''<0$). Under autarky the representative agent chooses consumption (C_1, C_2) subject to the constraint:

$$C_2 = F(x - C_1) \tag{3.1}$$

The choice under autarky is point A in Figure 1.

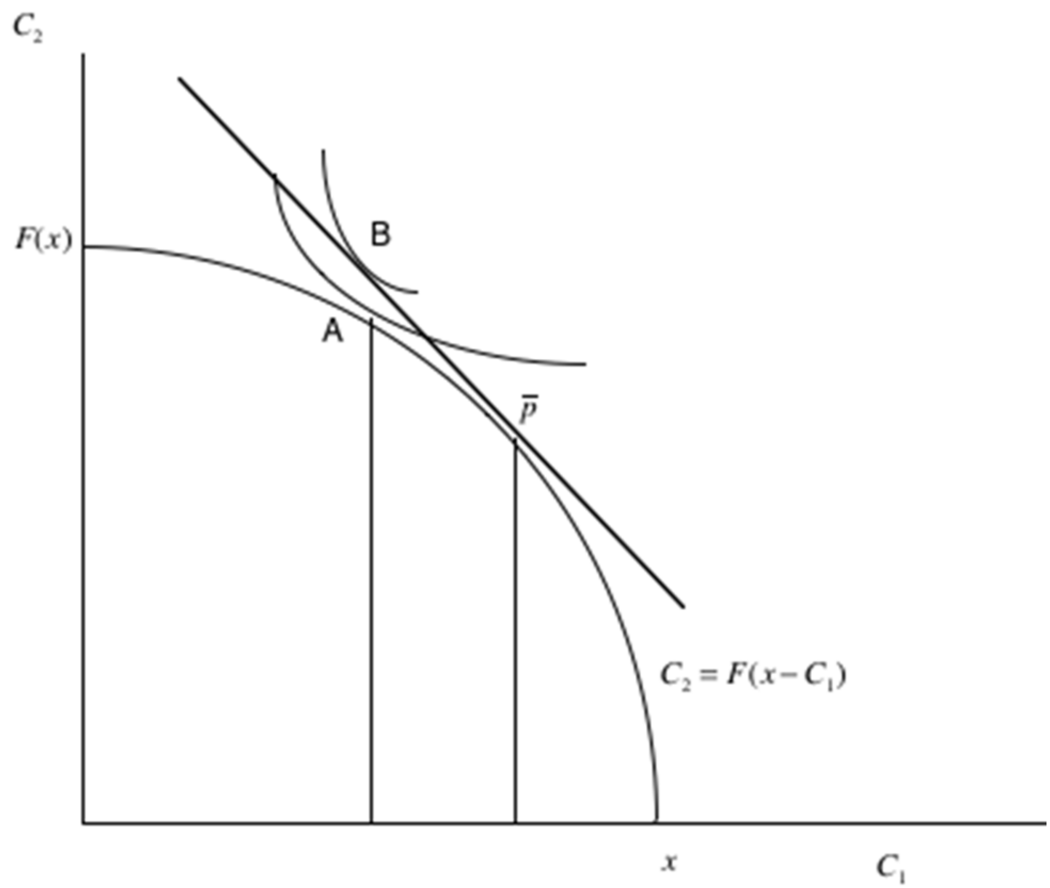

Figure 1: Autarky and the Planner's solution

A planner who wants to maximize welfare in the steady state face the constraint:

(3.2) $$C_1 + C_2 = x + F(k) - k$$

where now C_1 is the amount he gives to the young and C_2 is the amount he gives to the old. The planner will choose to sow $\bar{k}$ units of corn where $F'(\bar{k}) = 1$ because this choice maximizes the total resources available for distribution (the right hand side of [3.2]). Optimal production is at point $\bar{p}$ in Figure 1 and optimal consumption is at point B. Note that the planner can improve welfare for all generations only if under autarky $k^A > \bar{k}$ and $F'(k^A) < 1$. In this case a planner that takes control at $t = 0$, can give the old generation at $t = 0$, $k^A - \bar{k}$ units and then give the basket B to all future generations. When the production under autarky is to the right of $\bar{p}$ a Pareto improvement is not possible because the planner can get to the optimal steady state only if the current old generations are willing to give up $\bar{k} - k^A > 0$ units.

Money that promises zero real rate of return can be used to implement the planner's solution, when the autarkic point A is to the left of $\bar{p}$. Trade in privately created bubbles (Rock shares) with a stable price can achieve the same outcome. This may not be desirable for reasons that will be discussed shortly. For now I consider the question of whether a high interest rate can eliminate privately created bubbles.

I assume that the representative agent's utility function is $U(C_1) + \beta U(C_2)$, where U is strictly monotone and strictly concave and $\beta > 0$ is a discount factor. In the previous section, the price level changed over time and the real rate of return on money was approximately equal to the rate of deflation. Here I assume that the government pays interest on money explicitly and the price level remains constant at the level of unity. This case is somewhat simpler but the main results do not depend on whether interest is paid implicitly or explicitly.

The government promises to pay a gross real interest rate $R = 1 + r$ on money and impose a tax of τ units of corn on the old. (Note that here I drop the sub m from the real rate of return on money). I start from the case in which money is the only bubble asset and the representative agent chooses capital, real balances and consumption (k, m, C_1, C_2) out of the budget constraints:

(3.3) $$k + m + C_1 = x \text{ ; } C_2 = F(k) + Rm - \tau \text{; and } (k, m, C_1, C_2) \geq 0$$

The agent's problem is thus:

(3.4) $$\max_{k,m,C_t \geq 0} U(C_1) + \beta U(C_2) \quad \text{s.t. (3.3).}$$

The first order conditions that an interior solution to this problem must satisfy are:

(3.5) $$F'(k) = R = \frac{U'(C_1)}{\beta U'(C_2)}$$

Equilibrium is a vector $(k, m, C_1, C_2, R, \tau)$ that satisfies (3.2), (3.3) and (3.5). Substituting (3.3) in (3.2) leads to: $\tau = rm$. Thus in equilibrium the government budget is balanced.

Figure 2 describes equilibrium for the case $R > 1$. Production is at $\hat{p}$ which is to the right of the optimal production ($\overline{p}$). Consumption is at the point $\hat{C}$. Note that to satisfy the resource constraint (3.2), the consumption choice $\hat{C}$ must be on a line that goes through the production point $\hat{p}$ and has a slope of unity. As can be seen the choice of $R > 1$ is feasible (but not optimal).

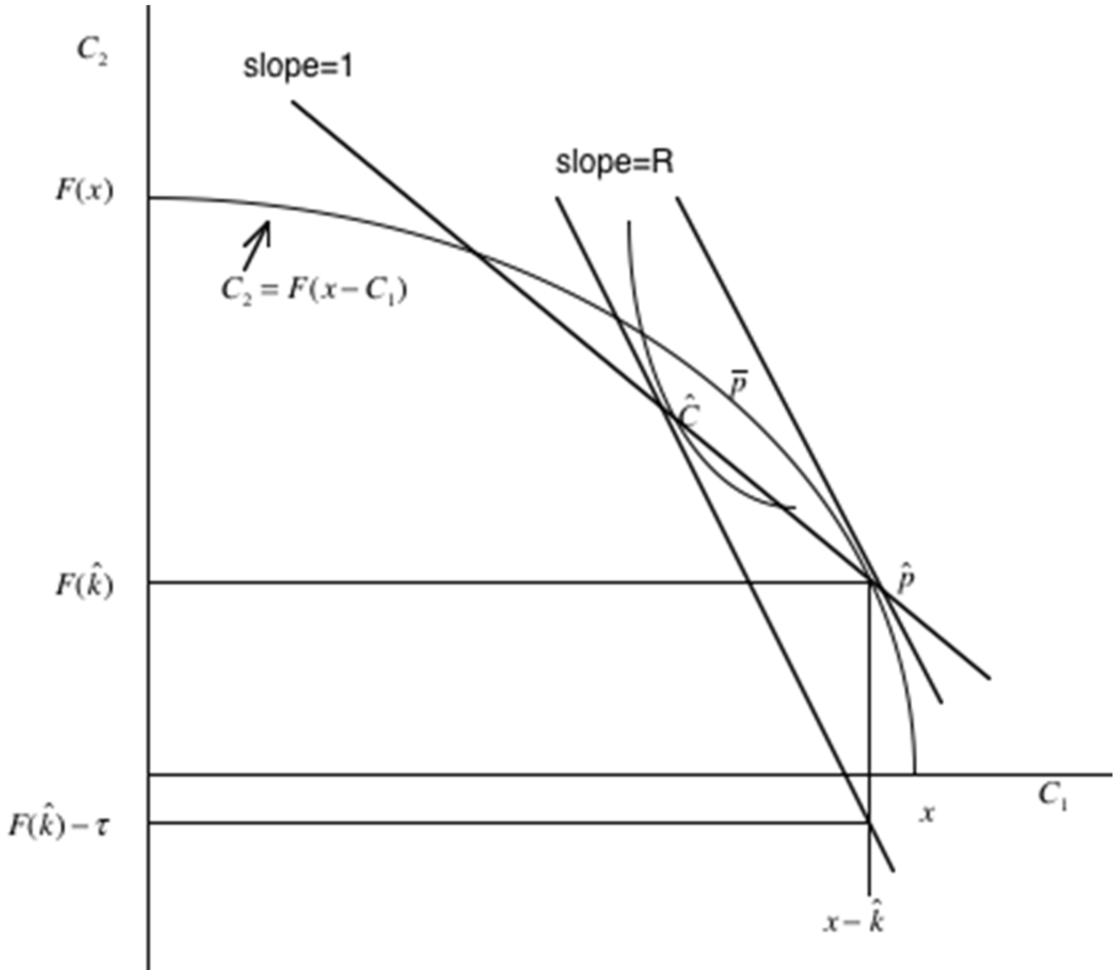

Figure 2: A steady state equilibrium with $R > 1$

Note also that when $R > 1$, $\hat{m} = x - \hat{k} - \hat{C}_1 > 0$. Thus money has value in this case even when the underlying fundamental economy may be efficient.

Figure 3 describes equilibrium for the case $R < 1$ and $\tau = rm < 0$. In this case we have a transfer rather than a tax.

<u>Existence</u>: Assuming indifference curves that are convex to the origin, equilibrium always exist. There is always a policy choice $R = F'(k^A)$ and $\tau = 0$ that supports the autarkic outcome. A more interesting question is about the range of interest rates that can be supported in equilibrium and can be chosen by the policy-maker.

Figure 4 shows that a given interest rate R is consistent with equilibrium if the income expansion path (IEP) that holds the slope of the budget lines constant at the level of R is to the left of the production point $\hat{p}$. This is the case because for each choice of R, real balances m and $\tau = rm$ can be varied. Note that the equilibrium is unique if the expansion path is strictly increasing.

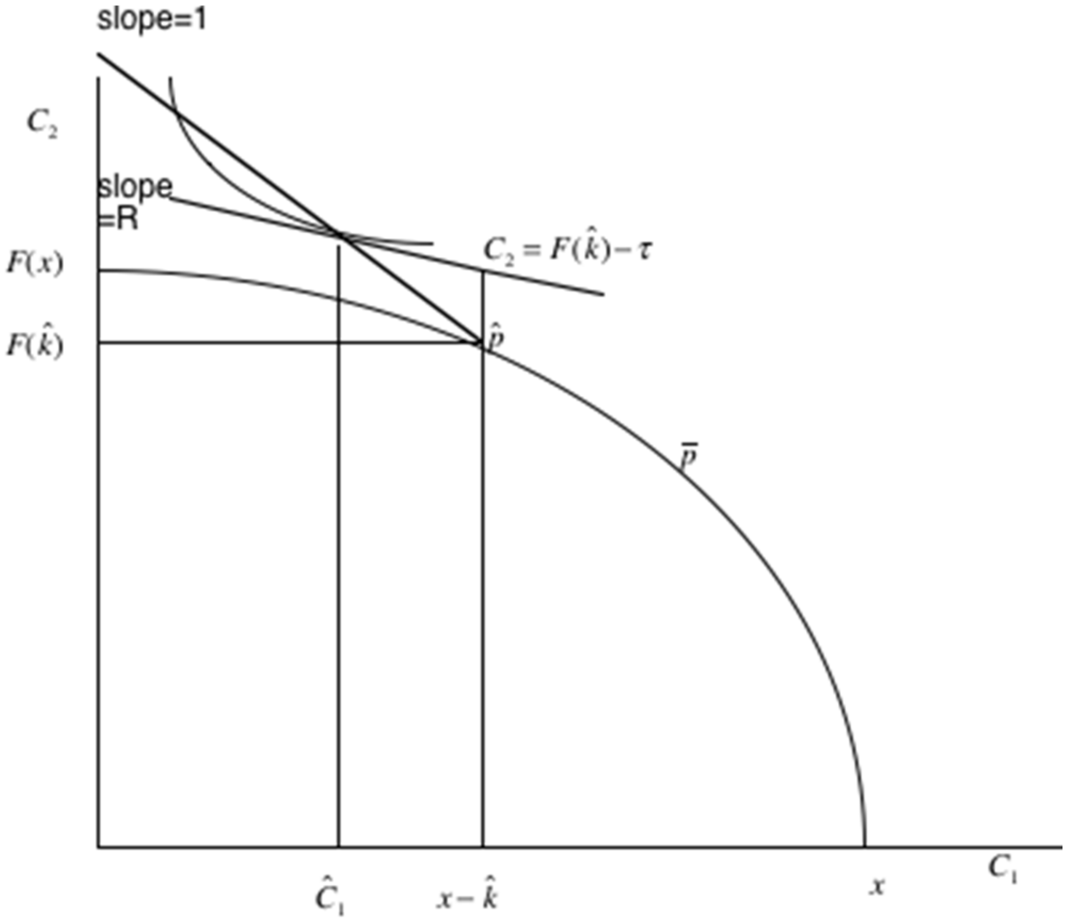

Figure 3: A steady state equilibrium with $R < 1$

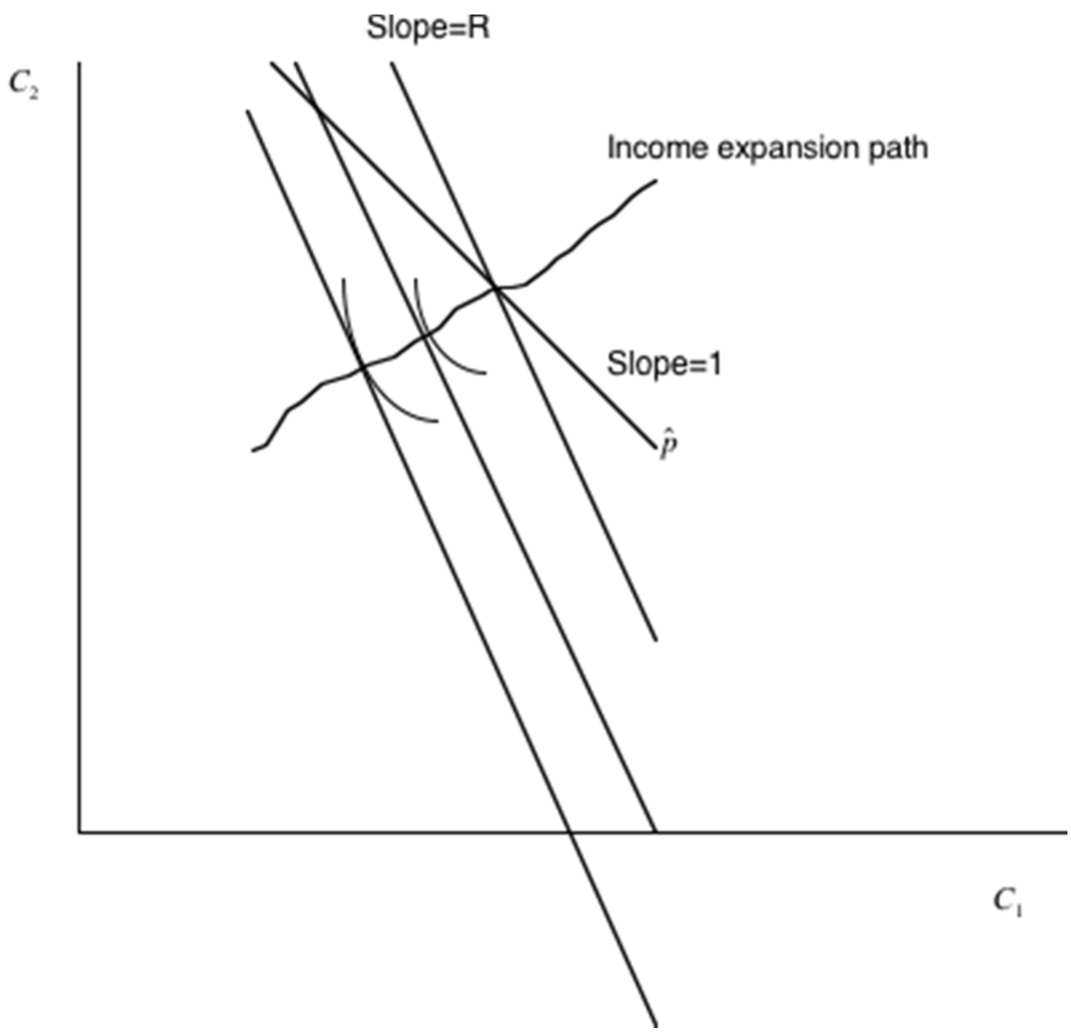

Figure 4: Equilibrium exists if the income expansion path is to the left of $\hat{p}$.

To characterize the range of interest rates that are consistent with equilibrium, I assume that both goods are normal and all income expansion paths are strictly increasing and show that equilibrium exists if R is sufficiently high.

Claim 3: Equilibrium exists if and only if $R \geq R^A = F'(k^A)$.

I start by showing that if equilibrium exists for R' then it must exist for $R > R'$. To show this Claim I use Figure 5 to show that Income Expansion paths that are strictly increasing do not cross. Let $IEP(R)$ denote the Income Expansion Path for the gross interest R. Suppose now that $IEP(R)$ cross $IEP(R'<R)$ and $IEP(R')$ passes through points A and B in the Figure. Then a line with a slope of R' that goes through point B intersects $IEP(R)$. This leads to a contradiction because the indifference curve that is tangent to this line must be at a point like D.

The Figure also makes clear that $IEP(R)$ must be to the left of $IEP(R'<R)$. Therefore when the policy maker increases R, IEP moves to the left and the production point $\hat{p}$ moves to the right. It follows that if there was equilibrium at the interest rate R' (point A in Figure 6) there is also equilibrium at the interest rate $R>R'$ (point B).

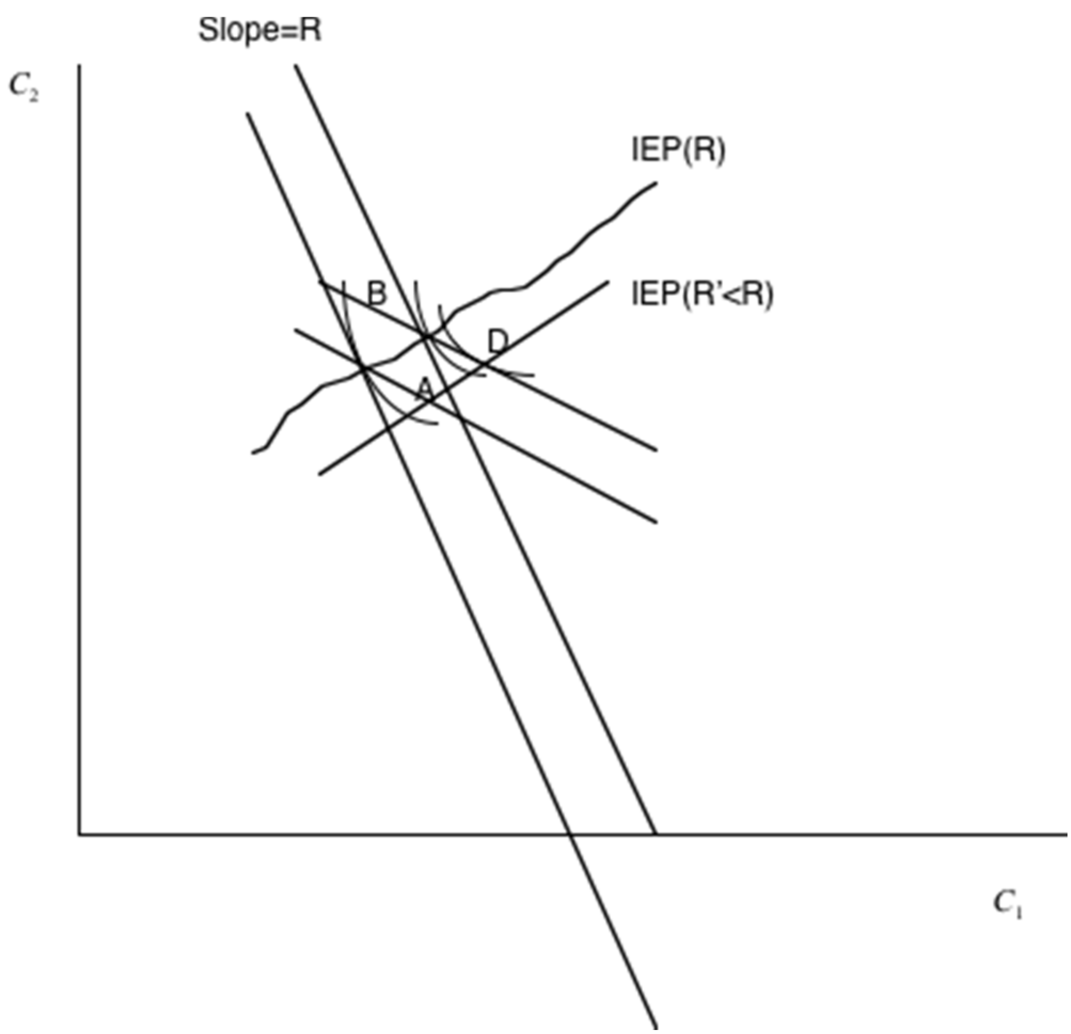

Figure 5: The income expansion path moves to the left when R goes up.

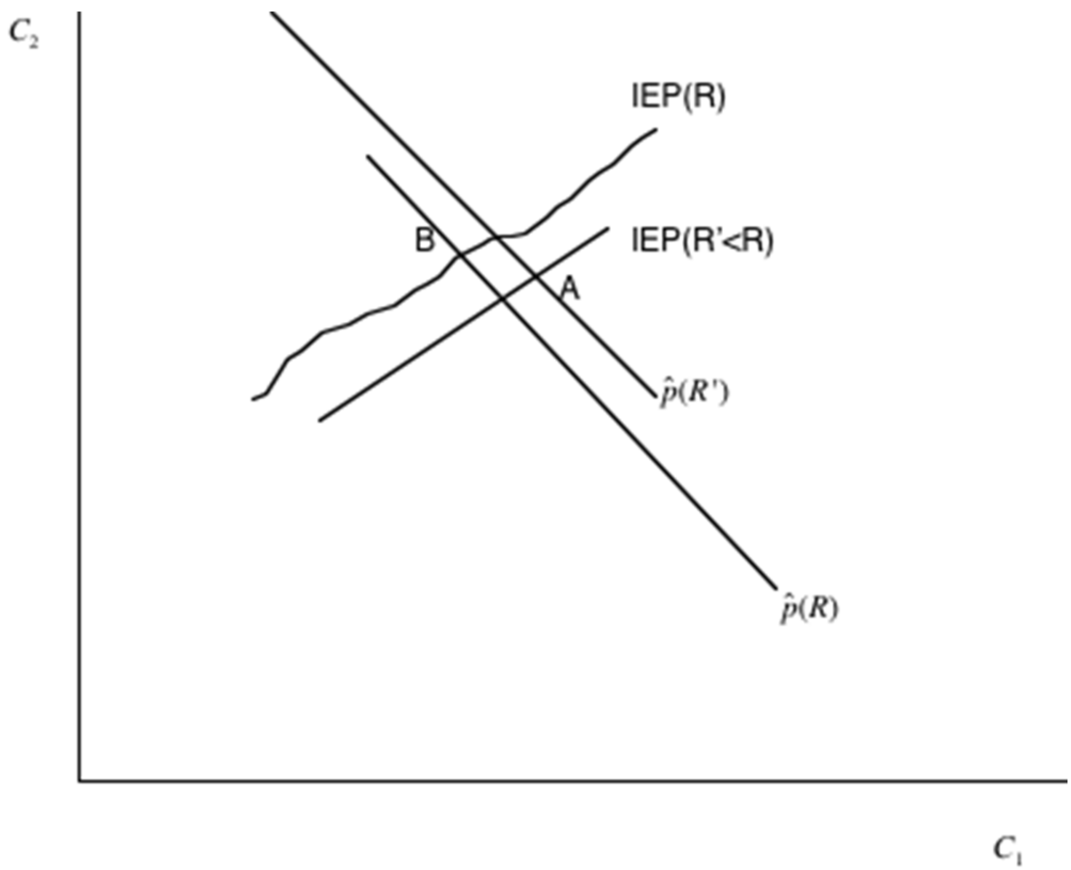

Figure 6: If there exists equilibrium when the policy maker announces R', then there exists an equilibrium when he announces $R > R'$.

We have shown that there exists equilibrium with $R = R^A$ that supports the autarkic outcome and that there exists equilibrium if $R \geq R^A$. To show the "only if" part of the Claim we need to show that there is no equilibrium if $R < R^A$. This can be shown with the help of Figure 7. I start with equilibrium with a gross interest R^A and the production point $\hat{p}(R^A)$. In this equilibrium we have the autarkic outcome with $m = \tau = 0$. I then reduce the gross interest to $R < R^A$. The production point moves to the left (to $\hat{p}(R)$ in the Figure) while the IEP moves to the right and therefore an intersection of $IEP(R < R^A)$ with a line that starts from $\hat{p}(R)$ is not possible. Note that this result implies the result cited above, namely that money may have value only if $R^A < 1$ and the underlying economy is inefficient.

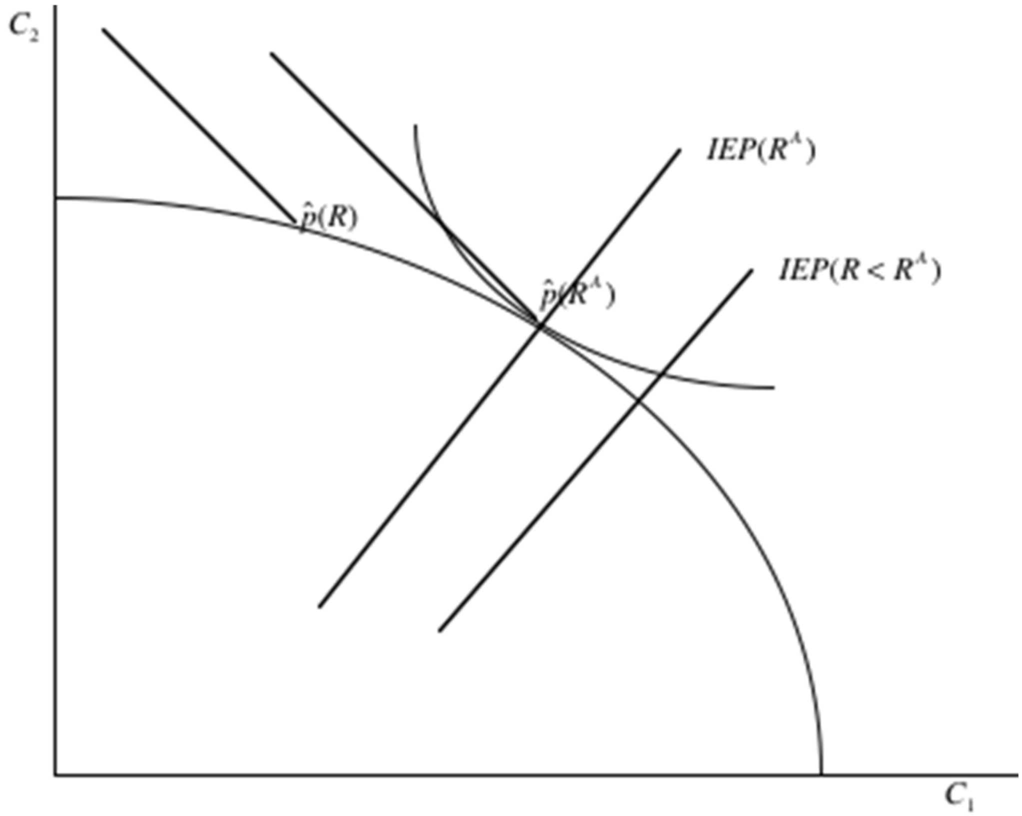

Figure 7: Equilibrium with $R < R^A$ does not exist.

3.1 PRIVATE BUBBLES

I now allow trade in Rock shares. As before the Rock does not yield any dividends and the dollar price of goods (the price level) is constant at the level of unity. The dollar price of shares increases at the rate of interest paid on money.

The young sows k units of his endowment and consumes C_1 units. He sells the rest of his endowment for m dollars and s dollars worth of Rock shares. His budget constraints are:

(3.6) $k+m+s+C_1 = x$; $C_2 = F(k)+Rm+Rs-\tau$ and $(k,m,s,C_1,C_2) \geq 0$.

where R is the gross interest and τ is the tax paid by old agents.

The above analysis does not change if the government accepts both money and shares as tax payments. But it is possible to rule out privately created bubbles when $R>1$ if the government commits to a no-bailout policy.

To show that this Claim holds in a steady-state equilibrium, let (m_t, s_t) denote the amount of dollars and the dollar value of stocks received by the agent born at time t and define equilibrium as follows.

A no-bailout steady-state equilibrium is a vector $(k, C_1, C_2, z, R, \tau)$ and a sequence $\{m_t, s_t\}$ that satisfy (3.2), (3.5), (3.6) and

(3.7) $$m_t + s_t = z \text{ for all } t$$

(3.8) $$rz = \tau$$

(3.9) $$s_{t+1} = Rs_t \geq 0$$

(3.10) $$m_{t+1} = Rm_t - \tau \geq 0$$

The requirement (3.10) is the no-bailout policy: It says that taxes must be paid in money only and the government will not bailout agents who cannot sell their Rock shares. As we have seen in the previous section (Claim 2) this requirement is necessary to rule out privately created bubbles. A similar argument can be applied here when $R>1$.

<u>Claim 4</u>: A no-bailout steady-state equilibrium with $s_0 > 0$ and $R>1$ does not exist.

To show this Claim, note that substituting (3.7) in (3.8) leads to: $rm_t = \tau - rs_t$. Substituting this in (3.10) leads to $m_{t+1} = m_t - rs_t < m_t$. Thus, when $s_t > 0$ and $r>0$, real balances decrease over time and eventually (3.10) will be violated.

Note however that we cannot rule out privately created bubbles when $r<0$.

Claim 5: When $R^A < 1$, there exists a no-bailout steady state equilibrium with $m_0 \geq \tau$, $s_0 > 0$ and $R^A < R \leq 1$.

To show this Claim note, that here z plays the role of m in section 3 and therefore Claim 3 implies that when $R > R^A$ there exists a steady state with $z = x - k - \hat{C}_1 > 0$ as in Figure 3. Since the interest rate is negative $m_{t+1} = m_t - rs_t < m_t$. Thus, real balances grow over time and (3.10) holds for all t. Note that the fraction of privately created bubble asset out of total financial assets goes to zero when $r < 0$.

Claims 4 and 5 implies that raising the real interest on money discourages private bubbles. This result is somewhat similar to the result in Farhi and Tirole (2010) who showed that bubbles are more likely to emerge when there is a greater need for liquidity. But in their setting liquidity is supplied inellastically by agents who live for one period and need to consume at that date. In our model, the government set the interest rate on money and when it set it low enough it may support private bubbles.

4. A NON-INDEXED OG EXCHANGE ECONOMY

Section 3 may be viewed as the analysis of a fully indexed OG economy in which the government could make promises about the real interest paid on money (indexed bonds) and the level of real taxes. I now consider an economy that is not indexed.

To insure the existence of bubbles, I assume an exchange economy with no real investment opportunities. The representative agent gets an endowment of x units of corn in the first period of his life but derives utility only from consumption in the second period. In equilibrium the young agent born at t sells his endowment for M dollars (the constant money supply) and S_t dollars worth of Rock shares. Thus, (4.1) $$M + S_t = P_t x \,,$$

where P_t is the dollar price of corn (the price level). The government pays nominal interest on money at the rate of i_m dollars per dollar and finances it by a lump sum tax of T dollars:

(4.2) $$i_m M = T$$

As in Blanchard (1979) the value of Rock shares may pop and the dollar value of the Rock evolves according to:

(4.3) $$S_t = \{\, S_{t-1}(1+i) \text{ with probability } \pi \text{ and zero otherwise}\}$$

And the expected rate of return on the two assets is the same:

(4.4) $$(1+i)\pi = 1 + i_m$$

A history is characterized by the date at which the bubble pops. Let $\{S_t^j, P_t^j\}_{t=0}^{\infty}$ denotes the value of the Rock and the dollar price of consumption when the bubble pops at time j (history j). The dollar value of the Rock in history j is thus: $(S_0^j = S_0, S_1^j = (1+i)S_0, ..., S_t^j = (1+i)^t S_0, ..., S_j^j = 0, S_{j+1}^j = 0, ...)$. The set of all possible histories are the sequences $\{\{S_t^j, P_t^j\}_{t=0}^{\infty}\}_{j=1}^{\infty}$.

Equilibrium is thus a vector of scalars $(i_m, i, M \ge 0, T, S_0 \ge 0)$ that satisfies (4.2) and (4.4) and sequences $\{\{S_t^j, P_t^j\}_{t=0}^{\infty}\}_{j=1}^{\infty}$ such that: (a) $M + S_t^j = P_t^j x$ for all (t, j), (b) $S_{t+1}^j = S_t^j(1+i)$ for all $t \ne j$, (c) $S_0^j = S_0$ and $S_j^j = 0$ for all j.

Claim 6: For any given $(i_m, M \geq 0, T = i_m M, S_0 \geq 0)$, there exists a unique equilibrium.

The proof is trivial. For each possible popping date j, the value of the bubble asset is: $S_t^j = S_0\left((1/\pi)(1+i_m)\right)^t$ for $t < j$ and $S_t^j = 0$ for $t \geq j$. We can then solve for $P_t^j = (1/x)(M + S_t^j)$.

Note that when $1 + i = {}^{1+i_m}/_{\pi} > 1$, the dollar value of the Rock grows over time and the total supply of liquid assets $M + S_t$ grows over time. In this case $P_t = (1/x)(M + S_t)$ increases until the private bubble pops. The price level declines when it pops and then remains constant. When $1 + i < 1$ the price level declines until the private bubble pops. In the special case $1 + i = 1$ prices are stable until they drop when the private bubble pops. Prices are stable after the pop so that the drop in the price level is a one-time event.

The example illustrates the difference between an indexed and a non-indexed economy. While in the indexed economy high real interest rate eliminates private bubbles in the steady state (assuming a no-bailout policy) here private bubbles can survive for any choice of the nominal interest rate (i_m) and a high nominal interest rate increases the importance of the private bubble in the pre-pop period.

I find the effect of i_m policy on private bubbles to be counter-intuitive. It may point to a difficulty in the standard modeling strategy. At the end of his seminal paper, Tirole (1985) made the following observation: “In a sense I have been considering the demand for bubbles. The supply is virtually unlimited. For example I am always willing to pretend that a drawing I made when I was young is worth $1000, say. However I doubt I will be successful in convincing others that they should invest in it. If I were famous, I might be able to do so.” (page 1093). In what follows I attempt to model the supply of bubble assets in a price-setting environment, using Tirole’s insight.

5. A NON-INDEXED *OG* PRODUCTIVE ECONOMY

In the above example (a) All policies are equivalent from the welfare point of view; (b) The popping of a bubble does not cause any real effect and (c) Once the bubble pops there is no other bubble that takes its place. I start by modifying the above example to address these issues.

Instead of an endowment economy, I assume here young agents who work and produce. The utility function of the representative agent born at time t is: $\beta c_{t+1} - v(L_t)$, where L_t is the amount of labor he supplies in the first period of his life, c_{t+1} is his consumption in the second period of his life and $\beta > 0$ can be interpreted as a parameter that determines time preference or the value of leisure.

I start with the case in which money (or government bonds) is the only asset. The government pays interest on money financed by a lump sum tax. The gross *nominal* interest on money is: $R = 1 + r$. Note that the notations here were used before for real magnitudes. I hope that this is not a problem because from now on everything is in nominal terms.

In the steady state the representative old agent holds M dollars before interest and tax payments. Unlike the previous (and subsequent) sections, here money may pop: In the case of panic that occurs with probability $1 - \pi$, no one wants to accept money, there is no trade and the output produced by the young is wasted.

The panic is self-fulfilling. After the panic the money that the young refused to accept is indeed worthless. Eventually, a new government is elected and issue new money. The new money is not better than the old one. It can also pop with probability π. But for some reason that will not be modeled here, the agents have faith in the new money and not in the old one.[3]

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[3] Alternatively, we may assume that the loss of confidence lasts for some time and then confidence in money is restored.

We may distinguish between two cases. In the first the government reacts immediately after the old money pops and “bailout” the old money: Each old dollar that pop is being bought by a new dollar and popping is neutral. The second case that will be analyzed here is when the government cannot react immediately and gives the new money in the next period to the next period’s old agents. The market for goods opens only when money works. The sequence of events is in Figure 8.

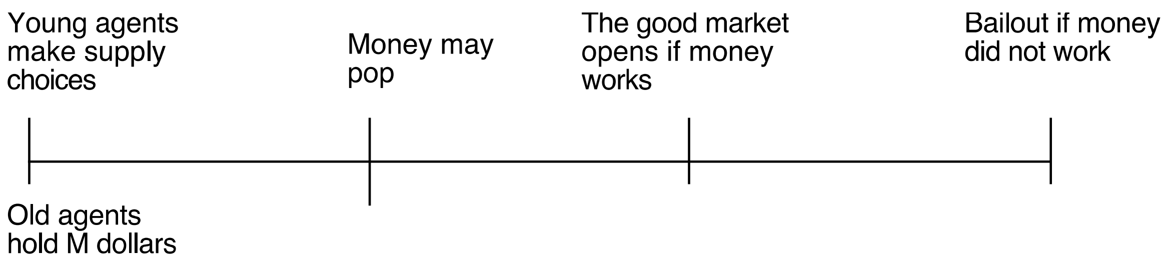

Figure 8: The sequence of events within the period

The dollar price of the good if the market opens is p (and in the steady state it does not change over time). I use $z = \pi R/p$ to denote the expected purchasing power of a dollar held at the beginning of the period.

At the end of the period (and before interest payments) the young agent will have pL dollars in the no panic state. In the panic state, he will get a transfer of M dollars. The young agent chooses the amount of labor by solving:

$$\max_L \beta\left(\pi pLz + (1-\pi)Mz\right) - v(L) \tag{5.1}$$

The first order condition for an interior solution to this problem is:

$$\beta\pi pz = \beta\pi^2 R = v'(L) \tag{5.2}$$

These equations say that the marginal cost should equal the (expected discounted) real wage $\beta\pi^2 R$. The intuition for the π^2 term in the real wage is as follows. The young agent invests effort and will reap the benefits if he sells (if money works in the current period) and if money works when he is old. The probability that this joint event will occur is π^2 and therefore the real wage is $\beta\pi^2 R$.

Market clearing requires:

(5.3) $$pL = M$$

Average capacity utilization (ACU) is the ratio between expected consumption and output. In equilibrium: $ACU = \pi$. A social planner that can choose $ACU = 1$ may achieve the first best by solving: $\max_L \beta L - v(L)$. The Fed cannot attain the first best because it must use imperfect money that may pop. I therefore consider the second best problem of a less powerful social planner that takes $ACU = \pi$ as given and chooses labor by solving:

(5.4) $$\max_L \pi\beta L - v(L)$$

The first order condition for the planner's problem is:

(5.5) $$v'(L) = \pi\beta$$

The equilibrium outcome (5.2) coincides with the planner's solution (5.5) if:

(5.6) $$R = 1/\pi$$

Note that when $\pi < 1$, the optimal interest rate in the survival state is different from the rate of population growth advocated by Samuelson (1958) and Diamond (1965) but the expected rate of return πR is equal to the rate of population growth.

Note also that when $R = 1$, there is a difference between the return to effort from the social and the individual's point of view. From the social point of view a unit produced will be consumed if money works in the current period and therefore the social benefits from producing a unit occurs with probability π. From the individual's point of view the benefits from a unit produced occurs only if money works in both periods, with probability π^2. When $R = 1$ and $\pi < 1$ there is thus a discrepancy between the social and the private point of views. When $R = 1/\pi$ the real wage is at the optimal level and the social and the individual points of view coincide.

Should the government bailout the old? The answer here is a trivial yes. And the sooner the government can restore money by bailout - the better. The issue is not trivial when the private sector can also create bubble assets.

5.1 PRIVATE BUBBLES

I attempt to model the supply of pure private bubbles that have no fundamentals. I follow Tirole (1985) in assuming that the creator of a bubble asset must be "famous". The underlying assumption is that a bubble asset must be easy to evaluate. Otherwise, when a bubble pops, there may be asymmetric information and a market for lemons problem as in Akerlof (1970). This is related to Stein (2010) who assumes that riskless assets yield utility because they are easy to evaluate. Unlike Stein, here assets do not enter the utility function and are "easy to evaluate" even when they pop.

Not everyone can be "famous". At each point in time the number of agents (firms or individuals) who can have the special attribute ("fame") is constant. After an asset pops, the agent who created it looses his "fame" but another agent takes his place and becomes “famous”. The agent with the newly acquired “fame” creates a new bubble asset.

This is modeled here by assuming a fixed number of slots that can be used to create bubble assets. Whenever a bubble asset pops the slot becomes vacant and after some time, another agent\asset occupies it. For example, typically there are only few international currencies. The main international currencies used to be gold and silver (with gold being more prominent). It is now the dollar and the euro (with the dollar being more prominent) and there are talks that this may change soon.[4]

I model the special attribute (“fame”, ”credibility”, “status”) as a commitment and transmission technology. A “famous” agent can commit to the probability distribution of the rate of return on the asset that he creates and can transmit information about it to all agents. He can also commit to low risk of asymmetric information: When the bubble pops everyone will know about it at the same time.

In the real world, agents invest resources in acquiring “fame”. A firm may do it by investing in buildings, advertisement and overcapacity. The attribute “fame” can also be acquired by securitization: A security backed by many assets maybe easier to evaluate than the underlying individual assets.[5] Here I simplify and assume that the commitment\transmission technology is allocated by a lottery and a firm that wins the lottery gets it at no cost. I assume that the government has the best technology and can commit to the highest survival probability. For simplicity and unlike the assumption in the previous section, I assume that the survival probability of the government’s bubble is 1. Private agents (firms) can commit only to lower survival probabilities.

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[4] There are other areas in which the number of “slots” is more or less constant and the identity of the occupier of the slot changes over time. In macroeconomics there are two main “slots”: Keynesian and Classical. New Keynesian replaced old Keynesian and real business cycle replaced monetarism. Also in religion there seems to be a relatively constant number of “slots”. Christianity replaced the pagan religion of the Greek and the Roman while Islam replaced the pagan religion of the Arabs.

[5] It is often argued that it was very difficult to evaluate mortgage-backed securities. But this is still much easier than the evaluation of individual mortgages. Here I assume that everyone was aware of the risk associated with buying the securities. This is of course a modeling device.

There are $\bar{N}$ status slots indexed by i. The government occupies the first slot. Private firms occupy slots $i > 1$. The survival of assets is determined by the number of "sun spots" that occur in the period. All the assets indexed $i > s$ pop and all the assets indexed $i \le s$ survive when the state (the number of "sun spots") is s. The probability that there will be s "sun spots" is Π_s. I use $q_i = \sum_{s \ge i} \Pi_s$ to denote the probability that asset i will survive and rank slots by the survival probabilities: $1 = q_1 > q_2, ..., q_{\bar{N}} > 0$.

The occupier of slot $i > 1$ announces the nominal growth in the survival state:

(5.7) $$g_i \le \bar{g}_i$$

I assume that the upper bounds $\bar{g}_i$ are constants that do not depend on R. I also assume: $q_i \bar{g}_i \ge q_{i+1} \bar{g}_{i+1}$ for $i > 2$. Thus more "famous" agents are able to promise a higher expected rate of return.

Note that the government has an advantage not only in the ability to promise a low popping probability but also in its ability to promise a high nominal return (R) in the survival state. This second advantage is related to its ability to collect taxes that cover the interest payments. The low popping probability advantage may also be related to the tax-collection advantage.

When $q_i \bar{g}_i < R$ there is no demand for the asset. Let $N(R) \le \bar{N}$ denote the number of slots that can potentially offer expected rate of return that is higher than R. Thus

(5.8) $$q_i \bar{g}_i < R \text{ if } i > N(R) \text{ and } q_i \bar{g}_i \ge R \text{ otherwise.}$$

Note that $N(R)$ is a decreasing function. This is different from Bernanke and Gertler (2001) who assume that bubbles are exogenous and exist regardless of the policy choice. Here existence is endogenous.

To simplify notation I use $n = N(R)$ to denote the number of viable assets (slots). The expected rate of return on asset $i \le n$ will be higher than the rate of return on money R, if $g_i > R/q_i$. To make the promise that money never pops credible, the government follows a policy that insures valued money and does not allow the crowding out of money by private assets. For example, it may impose reserve requirements on assets that threaten to crowd out money[6]. I assume that rather than being subject to government regulations the creator of the bubble operates under the constraint $q_i g_i \le R$. He chooses to announce the highest possible rate of growth under this constraint because he will not be able to sell his bubble otherwise. Thus, agent $i \le n$ announces:

(5.9) $$G_i = R/q_i$$

After a bubble asset $i > 1$ pops the value of the asset drops to zero and the value of the new firm that occupies the slot is I_i, where I_i is arbitrarily small (later it will be treated as zero). The price (dollar value) of asset $i > 1$ in state s at time t depends on its price at time $t-1$ in the following way:

(5.10) $$m_{it}^s(m_{it-1}) = \{G_i m_{it-1} = \frac{R}{q_i} m_{it-1} \text{ if } i \le s \text{ (with probability } q_i) \text{ and } I_i \text{ otherwise}\}.$$

I use $m_t^s(m_{t-1}) = (m_{2t}^s, ..., m_{nt}^s)$ to describe the prices of privately created assets at time t in state s, where $m_{t-1} = (m_{2t-1}, ..., m_{nt-1})$ is the beginning of period prices.

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6 Thus, whenever $q_i g_i > R$, a fraction θ_i of the value of the asset must be held at the central bank in the form of reserves that pay no interest, where the fraction θ_i is determined by: $(1-\theta_i) q_i g_i = R$. We may assume that the threat of reserve requirements or any other regulation induces firms to "stay under the radar" and satisfy the constraint in (7). Alternatively, we may interpret G_i as the nominal rate of growth net of the implicit tax imposed by the reserve requirement. The assumption that money has value because of government intervention is similar to the legal restrictions in Sargent and Wallace (1982). But here protecting money improves matters.

Government intervention: Stabilizing the price level requires intervention. Here I assume that the government (and the central bank) can react immediately after the new asset prices are realized (and before the beginning of trade in the goods market). It may print (high-powered) money, collect lump sum taxes (payable in money) and engage in open market operations (exchange high-powered money for other bubble assets). Formally, the government commits to a vector of policy reaction functions $\left(M_1(m_t^s),\sigma_2(m_t^s),...,\sigma_n(m_t^s)\right)$, where $M_1(m_t^s)$ is the post intervention amount of (high powered) money and $0 \le \sigma_i(m_t^s) \le 1$ is the fraction of asset $i > 1$ that the government will buy for money when the price of assets are m_t^s. Note that here the lump sum tax must be in terms of money but the government can change the amount of private bubbles by open market operations. Alternatively, the government could impose lump sum taxes directly in terms of the private bubbles.

The post intervention dollar value of asset $i > 1$ held by the representative old agent is:

(5.11) $$M_i^s(m_{t-1}) = \left(1-\sigma_i\left[m_t^s(m_{t-1})\right]\right) m_{it}^s(m_{t-1})$$

Figure 9 illustrates the sequence of events within the period. At the beginning of the period young agents make supply and price choices and old agents trade in assets. The old generation then gets interest payments on money, the state (s) is revealed, the government reacts and trade in the goods market follows.

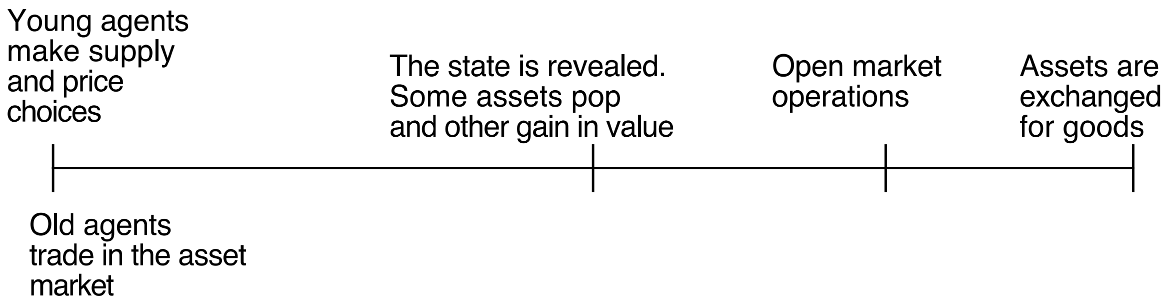

Figure 9: Sequence of events within the period

As in the Prescott (1975) model, the dollar prices of goods cannot be changed during the period and cheaper goods are sold first. There is no cash-in-advance constraint and assets are exchanged directly for goods. Thus a buyer that finds a unit at the price of P dollars can use P dollars worth of any asset to pay for it, where the dollar value of the assets in the goods market are determined by (5.10) after the state is observed.

The seller puts a price tag on each unit produced and these tags may be different across units. There are n cutoff prices $(P_{1t} < P_{2t} <,...,< P_{nt})$ where the cutoff price P_{it} clears a hypothetical market that will be described shortly. The seller expects that if he puts a price tag $P_{i-1t} < p \le P_{it}$ he will sell the good with probability q_i. Therefore P_{it} dominates any price $P_{i-1t} < p < P_{it}$ and we may limit the price choice of the seller to the n cutoff prices.

Let x_{it} denote the number of units with a price tag P_{it}. Total production (labor supply) is:

(5.12) $$L_t = \sum_{i=1}^{n} x_{it}$$

The expected consumption that the seller will get from a unit with a price tag P_{it} is $q_i P_{it} z_{t+1}^i$, where z_{t+1}^i is the expected purchasing power of a dollar at the beginning of next period, conditional on selling the unit ($i \le s$). The seller chooses x_{it} by solving:

(5.13) $$\max_{x_{it}\geq 0}\beta\left(\sum_{i=1}^{n} q_i x_{it} P_{it} z_{t+1}^{i}\right) - v\left(L_t = \sum_{i=1}^{n} x_{it}\right)$$

The first order conditions that an interior solution for this problem must satisfy are:

(5.14) $$\beta q_i P_{it} z_{t+1}^{i} = \beta P_{1t} z_{t+1}^{1} = v'(L_t)$$

Hypothetical Markets: I assume that the buyers form a line and arrive at the market sequentially according to their place in the line. Upon arrival, each buyer spends his entire portfolio of assets at the cheapest available price. From the sellers' point of view, the purchasing power that arrives, rather than the number of buyers, is relevant.

To simplify, I assume that the post intervention dollar value of the assets held by the representative old agent, $\sum_i M_i^s(m_{t-1})$ is increasing in s. The minimum amount that the old agents will spend is therefore $\Delta_1(m_{t-1}) = \min_s \sum_{i=1}^{s} M_i^s(m_{t-1}) = M_1^1(m_{t-1})$. The minimum additional amount that will be spent if $s>1$ is:
$\Delta_2(m_{t-1}) = \min_{s>1} \sum_{i=1}^{s} M_i^s - \Delta_{1t} = M_1^2 + M_2^2 - \Delta_1$. And in general,

(5.15) $$\Delta_i(m_{t-1}) = \min_{s>i-1} \sum_{j=1}^{s} M_j^s - \sum_{j=1}^{i-1}\Delta_j = \sum_{j=1}^{i} M_j^i - \sum_{j=1}^{i-1}\Delta_j .$$

Note that it is possible to compute $\Delta_i(m_{t-1})$ on the basis of information available at the beginning of the period because the government reaction functions are known.

The first Δ_1 dollars worth of assets that arrive buy in the first market (at the lowest price, P_{1t}). If $s>1$ then a second batch of Δ_2 dollars worth will open the second market and buy at the price P_{2t}. If $s > 2$ then a third batch of purchasing power will arrive and buy in the third market and so on.[7]

In equilibrium markets that open are cleared:

(5.16) $$P_{it}x_{it} = \Delta_i(m_{t-1})$$

The expected purchasing power of a dollar: I now calculate the expected purchasing power of a dollar held at the beginning of the period as a function of $(\Delta_1,...,\Delta_n)$. I use $\phi_i^s(m_{t-1}) = \Delta_i(m_{t-1})\left(\sum_{j=1}^{s}\Delta_j(m_{t-1})\right)^{-1}$ to denote the probability that a dollar worth of an asset will buy in market i when exactly s markets open. The expected purchasing power of a dollar held at the beginning of the period (before interest payments) is:

(5.17) $$z(m_{t-1},P_t) = R\sum_{s=1}^{n}\Pi_s\sum_{i=1}^{s}\frac{\phi_i^s(m_{t-1})}{P_{it}},$$

where $P_t = (P_{1t},...,P_{nt})$ is the vector of current period prices (of goods not assets). I use $P_{t+1}^s = (P_{1t+1}^s,...,P_{nt+1}^s)$ to denote expectations about next period prices if in the current period exactly s markets open. Using this notation, the expected purchasing power of a

[7] I assumed that the state (number of sunspots) is known before the beginning of trade but young sellers cannot change their prices in response to the information about the state. In this sense, prices are rigid. This assumption is not necessary for the main results. We can assume that sunspots appear sequentially and no one knows when the process will stop. Asset i survives if the number of sunspot is: $s \geq i$. Sellers accept asset i immediately after observing $s \geq i$. As a result money will buy in the first market, the closest substitutes will buy in the second and so on. In this version, prices are not rigid because a seller that accepts asset i does not know whether market $i+1$ will open or not. See Eden (1990, 1994) for a UST model that assumes flexible prices.

The assumption that sellers accept all assets directly for goods can also be relaxed. We can impose a cash-in-advance constraint in a sequential trade model with flexible prices. In the first market the young gets all the high-powered money. Then if they observe a second sunspot ($s \geq 2$) they go to the asset market and exchange the money they have for asset 2. The old who sell asset 2 go immediately to the goods market and use the money they have to buy goods. This process continues until the old have sold their entire holding of asset 2. Everyone then waits and sees whether a third sunspot will appear. If it does, the young go to the asset market and exchange the money they hold for asset 3 and so on.

dollar next period if exactly s markets open is: $z(m_t^s, P_{t+1}^s)$. The expected purchasing power of a dollar if market i opens ($i \leq s$) is:

(5.18) $$z_{t+1}^i = (1/q_i)\sum_{s=i}^{n}\Pi_s z(m_t^s, P_{t+1}^s)$$

where Π_s/q_i is the probability of state s conditional on $i \leq s$.

I now define equilibrium as follows.

Equilibrium is a gross interest R, a number of viable slots $n = N(R)$ and a vector of functions

$\left(m_i^s(m), \sigma_i\left[m^s(m)\right], M_i^s(m), P_i(m), x_i(m), \Delta_i(m), z(m), z^i(m); i, s = 1,...,n\right)$ from the beginning of the period asset prices m to R_+ such that:

(5.19) $m_i^s(m) = \{ G_i m_i$ if $i \leq s$ and I_i otherwise$\}$

(5.20) $M_1^s(m) > 0$

(5.21) $0 \leq \sigma_i\left[m^s(m)\right] \leq 1$ and $M_i^s(m) = \left(1 - \sigma_i\left[m^s(m)\right]\right) m_i^s(m)$, for $i > 1$

(5.22) $\beta q_i P_i(m) z^i(m) = \beta P_1(m) z^1(m) = v'\left(\sum_i x_i(m)\right)$

(5.23) $\Delta_1(m) = M_1^1(m)$ and $\Delta_i(m) = \sum_{j=1}^{i} M_j^i(m) - \sum_{j=1}^{i-1} \Delta_j(m)$

(5.24) $P_i(m) x_i(m) = \Delta_i(m)$

(5.25) $z(m) = R\sum_{s=1}^{n}\Pi_s \sum_{i=1}^{s}\frac{\Delta_i(m)}{P_i(m)}\left(\sum_{j=1}^{s}\Delta_j(m)\right)^{-1}$

(5.26) $z^i(m) = (1/q_i)\sum_{s=i}^{n}\Pi_s z\left(m^s(m)\right)$

Equilibrium condition (5.19) is the beginning of next period asset prices; condition (5.20) requires that money is not crowded out; condition (5.21) defines the dollar value of assets held by the public after the open market operations; (5.22) are the first order conditions that an interior solution to the young agent's problem must satisfy;

(5.23) defines the nominal demand for each of the hypothetical markets and (5.24) are market clearing conditions; (5.25) is the expected purchasing power of a dollar held at the beginning of the period and (5.26) is the expected purchasing power of a dollar if market i opens.

The equilibrium solution is relatively simple when the government adopts a policy that keeps the purchasing power of money constant. In this case, $z^i(m) = z^1(m) = z$ and (5.22) imply an expected nominal price that is the same across markets: $q_i P_i(m) = P_1(m)$. The solution of this case is in the Appendix to Eden (2011).

The optimal policy: Here money does not pop and the first best can be attained. The first best is a solution to (5.4) when $\pi = 1$: $\max_L \beta L - v(L)$. To achieve the first best the policy-maker must eliminate private bubbles. This can be done by buying the entire supply of private bubbles and using lump sum taxes to maintain a constant money supply. This will lead to a stable price level and if in addition the policy-maker chooses $R = 1$ he will get the first best outcome. This policy cannot be implemented if the policy-maker cannot identify private bubbles. It will also run into difficulties if private bubbles have some fundamentals.[8] I will elaborate on the policy issues in the next section.

Minimal intervention: To see the need for open market operations I consider the case in which the money supply is constant and the government (or the central bank) does not engage in open market operations. I thus assume that the government imposes a lump sum tax of rM dollars and chooses: $M_1^s(m) = M$ and $\sigma_i\left[m^s(m)\right] = 0$ for all m. In this case, $M_i^s(m) = m_i^s(m)$ for $i > 1$ and the nominal demand in the hypothetical markets are: $\Delta_1 = M$, $\Delta_2(m) = m_2^2 = G_2 m_2$, $\Delta_3(m) = m_3^3 = G_3 m_3$ and in general, $\Delta_i(m) = G_i m_i$. Since the vector of asset prices m, changes over time the nominal demand in markets $i > 1$

[8] For an extension of this model to the case in which private bubbles have some fundamentals, see Eden (2011).

changes over time. This will lead to fluctuations in prices and in the purchasing power of a dollar over time. Thus as in section 4, a policy of constant money supply is not consistent with price stability.

An increase in R may reduce $n = N(R)$ and a sufficiently high R will eliminate private bubbles. But it will distort the labor supply choice. When the first best cannot be attained by open market operations, the policy-maker may choose $R > 1$ to eliminate some private bubbles in a second best type analysis.

6. DISCUSSION

The literature on rational bubbles typically asks whether a Walrasian auctioneer can announce asset prices that are not strongly correlated with fundamentals but nevertheless clear markets. This approach usually does not assume a government role in supporting bubbles. But as the fiscal approach to the price level has taught us, the government can support bubbles even when the underlying economy is efficient. Here I distinguish between "good bubbles" and "bad bubbles" and focus on the question of whether the government can eliminate the "bad" ones, namely private bubbles.

In section 2-4 I explore the connection between the Friedman rule, the fiscal approach and bubbles. The connection is made in terms of a version of the IL model considered by Friedman (1969). At the Friedman rule the government collects lump sum taxes and burns the tax proceeds. Money is valued at the Friedman rule even when it does not yield any liquidity services (or non-pecuniary returns as defined by Friedman) because of a Ricardian equivalence type reasoning: the capital gains on money serve as hedge against the tax obligations. This is similar (but not identical) to the fiscal approach. When money is valued private bubbles may also have value unless there is a commitment to a no-bailout policy.

While the IL economy is useful for discussing the connections between related literatures, it is not useful for discussing interest rate policy. This requires an OG model in which the policy-maker can affect both the real and the nominal interest rates.

I distinguish between fully indexed ("real") OG economies and "nominal" OG economies. In the indexed economies, private bubbles may emerge if the underlying economy is not efficient and the interest on money (government bonds) is low.

In the non-indexed OG economy considered here, a high nominal interest rate does not eliminate private bubbles. It actually increases the importance of private bubbles in the pre-pop period. This non-intuitive result does not occur in section 5, where the suppliers of private bubbles face constraints on the type of promises they can make.

In section 5, privately created bubble assets are "bad" from the social point of view because when they pop some goods are not sold and capacity is not fully utilized.[9] There are of course other ways of modeling the cost of monetary or liquidity shocks but in all the models I know these shocks are "bad" and should be eliminated if possible.

The policy-maker can eliminate private bubbles by setting a high nominal interest rate but this may lead to (labor supply and real investment) distortions.

A second best type analysis should distinguish between discouraging the formation of bubbles and dealing with bubbles that are clearly identified. The policy-maker can discourage the formation of bubbles by a combination of moderately high interest rate and other measures such as capital gain tax (which has its own distortions). When this policy does not eradicate private bubbles some may develop. The optimal response to a bubble that is clearly identified, is to pop it immediately after it is identified by increasing the interest rate temporarily and then to inject money so as to minimize the impact on capacity utilization. The injection of money should not be to the owners of the bubble that pops so as not to encourage the formation of new bubbles.

[9] In a more general model privately created bubbles reduce welfare also in the pre-pop periods because they cause price dispersion.

The policy implications here are different from Bernanke and Gertler (2001) and Kocherlakota (2010). Bernanke and Gertler (2001) argue that: "Changes in asset prices should affect monetary policy only to the extent that they affect the central bank's forecast of inflation." Here the policy-maker should worry about the formation of bubbles in addition to forecasting inflation. Suppose for example, that private bubbles with a low popping probability is being created. This will have a negligible effect on the rate of inflation forecast but still when the bubble pops it can do substantial harm as it did in the recent great recession.

Kocherlakota (2010) argues that bailouts of financial institutions are inevitable and lead to excessive risk taking. He proposes to tax risk taking by financial institutions and to use a market-based method to estimate the excess risk caused by bailouts. "For a particular financial institution, the government should sell 'rescue bonds' paying a variable coupon linked to the size of the bailouts or other government assistance received by the institution or its owners." Here bubbles can be formed outside of the financial sector and the policy recommendation is not to bailout the owners of the bubble but to inject money by lump sum transfers.

The discussion here is related to the question of regulations designed to limit "money substitutes" and the desirability of a 100% reserve requirement. Hume (1752, p. 35) expressed "a doubt concerning the benefit of banks and paper-credit, which are so generally esteemed advantageous to every nation". He seems to favor regulations against paper (inside) money and argue (on page 36) for a government run bank. Simons (1948, p. 79-80) argued for "Financial reform (banking reform primarily) aiming at sharp differentiation between money and private obligations" and for "Increasing concentration on the hands of the central government of the power to create money and effective money substitutes".

The differentiation between money and private obligations requires the understanding of the nature of money. I think that if we could ask Henry Simons, he would stress the transactions role of money. But as was pointed out by Woodford (2003) this role has become less important in our technological advanced society. Here the distinct feature of money is that it is a bubble. Discouraging "money substitutes" therefore requires regulations that limit the ability of the private sector to create bubble assets.

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