E C O N O M I C S   B U L L E T I N

On two basic properties of equilibria of voting with exit

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Abstract

We consider the problem of a society whose members must choose from a finite set of alternatives. After knowing the chosen alternative, members may reconsider their membership. Thus, they must take into account, when voting, the effect of their votes not only on the chosen alternative but also on the final composition of the society. We show that, under plausible restrictions on preferences, equilibria of this two-stage game satisfy stability and voter's sovereignty.

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1 Introduction

Most of voting theory studies the static problem of how societies select an alternative from a given set of potential choices. However, the set of members belonging to a society often evolve over time. Moreover, this evolution partly depends on the selected alternative. If membership is voluntary, members might leave the society if the chosen alternative makes it undesirable. This, in turn, might cause that other members (who also care about who belongs to the society) might now find undesirable to belong to the society and leave as well.[1] We model this strategic problem as a two-stage game in which members first choose (by a voting procedure) an alternative and then, after knowing the chosen alternative, they decide whether to stay or to exit the society. We show that, under plausible restrictions on preferences, the equilibria of this game satisfy two basic properties. We first show that, whenever preference profiles are monotonic, any equilibrium is stable in the sense that (after knowing the chosen alternative and the final composition of the society) members who have decided to remain in the society do not want to exit (internal stability) and members who have decided to leave the society do not want rejoin it (external stability). Second, we show that for the case of a society using voting by committees to select its new members (as in Barberà, Sonnenschein, and Zhou, 1991), and provided that preference profiles are also candidate separable, any undominated equilibrium strategy satisfies voter's sovereignty in the sense that unanimously good candidates are elected and unanimously bad candidates are not.

The paper is organized as follows. Section 2 contains the preliminaries and Section 3 presents the results.

2 Preliminaries

Let $N = \{1, ..., n\}$ be the initial set of *members* of a society who must first choose an *alternative* from a non-empty set X and then, knowing the chosen alternative $x \in X$, decide to stay or to leave the society. A *final society* $[x, S]$ consists of the chosen alternative $x \in X$ and the subset of members $S \in 2^N$ that have chosen to remain in the society. Members have *preferences* over $X \times 2^N$, the set of all possible final societies. Each member $i \in N$ has a *preference relation* R_i over $X \times 2^N$, where R_i is a complete, reflexive and transitive binary relation (P_i and I_i are the strict and indifference preference relations induced by R_i) satisfying the following four conditions:

(C1) Strictness: For all $x, y \in X$ and $S, T \in 2^N$ such that $i \in S \cap T$ and $[S, x] \neq [T, y]$,

[1]See Barberà, Maschler, and Shalev (2001), Barberà and Perea (2002), Granot, Maschler, and Shalev (2002), Berga, Bergantiños, Massó, and Neme (2004, 2006, and 2007), and Massó and Nicolò (2008) for dynamic analisys of voting.

either $[x,S]\, P_i\, [y,T]$ or $[y,T]\, P_i\, [x,S]$.

(C2) Indifference: For all $x \in X$ and all $S \in 2^N$, $i \notin S$ if and only if $[x,S]\, I_i\, [x,\emptyset]$. Moreover, for all $x, y \in X$, $[x,\emptyset]\, I_i\, [y,\emptyset]$.

(C3) Non-initial Exit: If $\emptyset \in X$, then $[\emptyset, N]\, P_i\, [\emptyset, N\backslash \{i\}]$.

(C4) Monotonicity: For all $x \in X$ and all $T \subsetneq T' \subset N$ such that $i \in T$, $[x, T']\, P_i\, [x,T]$.

Monotonicity means that members consider the exit of other members undesirable, independently of the chosen alternative. Notice that monotonicity does not impose any condition when comparing two final societies with different chosen alternatives. In particular, monotonicity admits the possibility that member i prefers to belong to a smaller society.

Let $\mathcal{R}_i$ be the set of all such preference relations for member i and let $\mathcal{R} = \mathcal{R}_1 \times ... \times \mathcal{R}_n$. We call $R_i \in \mathcal{R}_i$ a *monotonic preference relation* and $R = (R_1, ..., R_n) \in \mathcal{R}$ a *monotonic preference profile.*

First, to choose an alternative from the set X each member i has to select a particular message (vote) m_i from a given set M_i. A *voting procedure* is a mapping $v : M_1 \times ... \times M_n \to X$. Observe that if $M_i = \mathcal{R}_i$ for all $i \in N$, v is a social choice function.

Second, assume that $x \in X$ has already been chosen by a voting procedure v. To avoid to go into the specific details of the exit decisions (the order in which members have to make their exit decision, as well as their information about the other's decisions) we define recursively the set of members leaving the society after x is chosen.

Define the set $EA^1(x) = \{i \in N \mid [x, N\backslash \{i\}]\, P_i\, [x,N]\}$, or equivalently, $\{i \in N \mid [x,\emptyset]\, P_i\, [x,N]\}$. Namely, $EA^1(x)$ is the set of members who want to exit when x is chosen even when the other members stay. Notice that by (C4), they want to exit independently of the exit decision of the other members. Let $t \geq 1$ and assume $EA^{t'}(x)$ has been defined for all t' such that $1 \leq t' \leq t$. Then,

$$EA^{t+1}(x) = \left\{ i \in N\backslash \left(\bigcup_{t'=1}^{t} EA^{t'}(x) \right) \mid [x,\emptyset]\, P_i \left[x, N\backslash \left(\bigcup_{t'=1}^{t} EA^{t'}(x) \right) \right] \right\}.$$

At each step, all members who would like to exit do so, given that x has been chosen, and the current society is formed by all members who in all previous steps wanted to stay (*i.e.*, the most optimistic circumstance). Let t_x be either equal to 1 if $EA^1(x) = \emptyset$ or else be the smallest positive integer satisfying the property that $EA^{t_x}(x) \neq \emptyset$ but $EA^{t_x+1}(x) = \emptyset$. Then, define the *exit set after* x as $EA(x) = \bigcup_{t=1}^{t_x} EA^t(x)$.

Observe that this set only depends on the preference profile R. Motivation and some of its properties can be found in Berga, Bergantiños, Massó, and Neme (2006). In particular, $EA(x)$ is the set of members leaving the society if exit is sequential (and they play according to the unique subgame perfect Nash equilibrium of the subgame starting at x) and it is independent of the ordering in which members decide (sequentially) whether to stay or to exit. The set $EA(x)$ also coincides with the set of members leaving the society if exit is simultaneous and players eliminate iteratively dominated strategies.

Now, given any voting procedure $v : M \rightarrow X$, we model our voting problem with exit as the normal form game $\Gamma^v = (N, M, R, o^v)$ where o^v is the outcome function such that for each $m \in M$, $o^v(m) = [v(m), N \backslash EA(v(m))]$ is the final society. Observe that a Nash Equilibrium (NE) m^* of Γ^v imposes to members, through $(EA(x))_{x \in X}$, a minimal rational behavior in all subgames starting at any x (subgame perfection, for instance, if exit is sequential).

Later on we will focus on a particular instance of our general problem by introducing the possibility of exit in the framework studied by Barberà, Sonnenschein, and Zhou's (1991), which corresponds to consider $X = 2^K$, where K is a finite set of candidates to become new members of the society, and to consider the normal form game $\Gamma^{vc} = (N, M, R, o^{vc})$, where $M_i = 2^K$ for all $i \in N$ (each member votes for a subset of candidates) and letting the voting procedure $vc : (2^K)^N \rightarrow 2^K$ be voting by committees. Following Barberà, Sonnenschein, and Zhou (1991) voting by committees are defined by a collection of families of winning coalitions (committees), one for each candidate, $\mathcal{W} = (\mathcal{W}_k)_{k \in K}$. Members vote for a subset of candidates. To be elected, a candidate must get the vote of all members of some coalition among those that are winning for that candidate. Formally, a *committee for* k, denoted by $\mathcal{W}_k$, is a non-empty family of non-empty coalitions of N satisfying coalition monotonicity ($S \in \mathcal{W}_k$ and $S \subset T$ imply $T \in \mathcal{W}_k$). Given a committee $\mathcal{W}_k$ its set of minimal winning coalitions is $\mathcal{W}_k^m \equiv \{S \in \mathcal{W}_k \mid T \notin \mathcal{W}_k \text{ for all } T \subsetneq S\}$. A voting procedure $vc : \left(2^K\right)^N \rightarrow 2^K$ is *voting by committees* if there exists $(\mathcal{W}_k)_{k \in K}$ such that for all $(S_1, ..., S_n) \in \left(2^K\right)^N$ and all $k \in K$,

$$k \in vc(S_1, ..., S_n) \Longleftrightarrow \{i \in N \mid k \in S_i\} \in \mathcal{W}_k.$$

We say that vc has no *dummies* if the corresponding committee $\mathcal{W}$ has the property that for all $k \in K$ and all $i \in N$ there exists $S \in \mathcal{W}_k^m$ such that $i \in S$.

Barberà, Sonnenschein, and Zhou (1991) show that for the problem of choosing new members of the society (without exit), voting by committees is the class of strategy-proof and onto social choice functions on the domain of separable preferences. We now translate to our setting with exit the concept of separable preferences. Given $R_i \in \mathcal{R}_i$ and $y \in K$, we say that candidate y is *good* for i according to R_i whenever $[\{y\}, N] \, P_i \, [\emptyset, N]$; otherwise, we say that candidate y is *bad* for i according to R_i. Denote by $G(R_i)$ and $B(R_i)$ the set of good and bad candidates for i according to R_i, respectively. Given $R \in \mathcal{R}$, let $G(R) = \bigcap_{i \in N} G(R_i)$ the set of unanimously good candidates and $B(R) = \bigcap_{i \in N} B(R_i)$ the set of unanimously bad candidates.

Candidate Separability: A preference R_i is *candidate separable* if for all $S \subset K$ and $y \in K \backslash S$, and for all $T \subset N$ such that $i \in T$, $[S \cup \{y\}, T] \, P_i \, [S, T]$ if and only if $y \in G(R_i)$.

Let $\mathcal{S}_i \subset \mathcal{R}_i$ be the set of *monotonic and candidate separable preference relations* of i and let $\mathcal{S} = \mathcal{S}_1 \times ... \times \mathcal{S}_n$.

3 Results

We first show that for any voting procedure v, all Nash equilibria (NE) of Γ^v satisfy two stability properties. The first one is internal stability which says that members who remain in the society do not want to exit. The second one is external stability which says that members who leave the society do not want to rejoin it (see Berga, Bergantiños, Massó, and Neme (2004) for a motivation, definition and analysis of these properties in a more general framework). Formally,

Internal Stability: A strategy profile $m \in M$ satisfies *internal stability* if $i \in N \backslash EA\left(v\left(m\right)\right)$ implies $\left[v\left(m\right), N \backslash EA\left(v\left(m\right)\right)\right] P_i \left[v\left(m\right), \emptyset\right]$.

External Stability: A strategy profile $m \in M$ satisfies *external stability* if $i \notin N \backslash EA\left(v\left(m\right)\right)$ implies $\left[v\left(m\right), \emptyset\right] P_i \left[v\left(m\right), N \backslash EA\left(v\left(m\right)\right) \cup \{i\}\right]$.

Proposition 1 states that, for any voting procedure $v : M \to X$, all NE of Γ^v satisfy internal and external stability.

Proposition 1 *Let $m \in M$ be a NE of $\Gamma^v = (N, M, R, o^v)$, where $R \in \mathcal{R}$. Then, m satisfies internal and external stability.*

Proof Let m be a NE of Γ^v and assume first that $i \in N \backslash EA\left(v\left(m\right)\right)$. Hence, $i \notin EA^{t_{v(m)}+1}\left(v\left(m\right)\right)$. By (C2), $\left[v\left(m\right), N \backslash EA\left(v\left(m\right)\right)\right] P_i \left[v\left(m\right), \emptyset\right]$. Thus, m satisfies internal stability.

Assume now that $i \notin N \backslash EA\left(v\left(m\right)\right)$. Therefore, there exists t such that $i \in EA^t\left(v\left(m\right)\right)$. Hence, $\left[v\left(m\right), \emptyset\right] P_i \left[v\left(m\right), N \backslash \left(\bigcup_{t'=1}^{t-1} EA^{t'}\left(v\left(m\right)\right)\right)\right]$. Since $N \backslash EA\left(v\left(m\right)\right) \subset N \backslash \left(\bigcup_{t'=1}^{t-1} EA^{t'}\left(v\left(m\right)\right)\right)$ and R_i is monotonic,

$$\left[v\left(m\right), N \backslash \left(\bigcup_{t'=1}^{t-1} EA^{t'}\left(v\left(m\right)\right)\right)\right] P_i \left[v\left(m\right), \left(N \backslash EA\left(v\left(m\right)\right)\right) \cup \{i\}\right].$$

By transitivity of P_i, $\left[v\left(m\right), \emptyset\right] P_i \left[v\left(m\right), \left(N \backslash EA\left(v\left(m\right)\right)\right) \cup \{i\}\right]$. Thus, m satisfies external stability. ■

Internal stability follows immediately from the definition of $EA(x)$, independently of the monotonicity of the preference profile. However, the example below illustrates the fact that if the preference profile is non-monotonic, a NE of Γ^v may not satisfy external stability.

Example Let $N = \{1, 2, 3\}$ be a society whose members have to decide whether or not to admit candidate y as a new member of the society (*i.e.*, $X = \{\emptyset, y\}$). Let the voting procedure vc^1 be voting by quota 1; that is, y is chosen if and only if at least a member votes for it. Consider first the non-monotonic preference profile R, additively representable by the following table

	u_1	u_2	u_3
1	1	-8	1
2	2	5	-10
3	4	12	15
y	100	-7	-8

,

where the number in each cell represents the utility each member $i \in N$ assigns to members in N, as well as to candidate y (we normalize by setting $u_i(\emptyset) = 0$ for all $i \in N$ and by saying that if $i \notin T$ then, the utility of $[x, T]$ is 0). That is, for all $i \in N$, all $x, x' \in \{\emptyset, y\}$, and all $T, T' \in 2^N$, $[x, T]\, P_i\, [x', T']$ if and only if

$$\begin{cases} \sum\limits_{j \in T} u_i(j) + u_i(x) > \sum\limits_{j \in T'} u_i(j) + u_i(x') & \text{if } i \in T \cap T' \\ \sum\limits_{j \in T} u_i(j) + u_i(x) > 0 & \text{if } i \in T \text{ but } i \notin T'. \end{cases}$$

Notice that, by the indifference condition (C2), if $i \notin T$ and $i \notin T'$ then, $[x, T]\, I_i\, [x', T']$. Notice that R_2 and R_3 are not monotonic ($[\emptyset, \{2, 3\}]\, P_2\, [\emptyset, N]$ and $[\emptyset, \{1, 3\}]\, P_3\, [\emptyset, N]$). Clearly $EA(\emptyset) = \emptyset$. Moreover, $EA^1(y) = \{3\}$, $EA^2(y) = \{2\}$, and $EA^3(y) = \emptyset$. Hence, $EA(y) = \{2, 3\}$. Let m be such that $vc^1(m) = \emptyset$. Then, $m_i = \emptyset$ for all $i \in N$. If member 1 votes for y instead of voting for $\emptyset$, $vc^1(y, m_{-1}) = y$ and hence,

$$\left[vc^1(y, m_{-1}), N \backslash EA\left(vc^1(y, m_{-1})\right)\right] = [y, \{1\}]\, P_1\, [\emptyset, N] = \left[vc^1(m), N \backslash EA\left(vc^1(m)\right)\right],$$

which means that m is not a NE of Γ^{vc^1}.

It is easy to see that $[y, \{1\}]$ is the final society generated by the NE strategy $m^* = (y, \emptyset, \emptyset)$. Moreover, it is the unique final society that can be generated by a NE of Γ^{vc^1}. But m^* does not satisfy external stability because $[y, \{1, 3\}]\, P_3\, [y, \emptyset]$. □

We now ask whether in the context of selecting new members of the society, any NE of the game $\Gamma^{vc} = (N, \left(2^K\right)^N, R, o^{vc})$ satisfies the property that unanimously good candidates are chosen while unanimously bad ones are not. Formally,

Voter's Sovereignty: A strategy profile $m \in M$ of $\Gamma^{vc} = (N, \left(2^K\right)^N, R, o^{vc})$ satisfies *voter's sovereignty* if $G(R) \subset vc(m) \subset K \backslash B(R)$.

Proposition 2 *Let $vc : \left(2^K\right)^N \to 2^K$ be a voting by committees without dummies and let $R \in \mathcal{S}$. Then, the strategy m_i of voting for a common bad ($m_i \cap B(R) \neq \emptyset$) and the strategy $\tilde{m}_i$ of not voting for a common good ($G(R) \cap (K \backslash \tilde{m}_i) \neq \emptyset$) are dominated strategies in Γ^{vc}.*

Proof We will only show that to vote for a common bad is a dominated strategy. The proof that to not vote for a common good is also a dominated strategy is similar and left to the reader. Let $i \in N$ and $m_i \in 2^K$ be such that $y \in m_i \cap B(R)$. We will show that the strategy $m_i' = m_i \backslash \{y\}$ dominates m_i. Fix $m_{-i} \in M_{-i}$ and consider the two subsets of candidates $vc(m)$ and $vc(m) \setminus \{y\}$. We first prove the following claim:

Claim: $EA(vc(m) \setminus \{y\}) \subset EA(vc(m))$.

Proof of the Claim: By definition, $EA(vc(m) \setminus \{y\}) = \bigcup_{t=1}^{T'} EA^t(vc(m) \setminus \{y\})$ and $EA(vc(m)) = \bigcup_{t=1}^{T} EA^t(vc(m))$, where $T' = t_{vc(m) \setminus \{y\}}$ and $T = t_{vc(m)}$. We first establish that $EA^1(vc(m) \setminus \{y\}) \subset EA(vc(m))$. Assume $j \in EA^1(vc(m) \setminus \{y\})$. Then,

$$[vc(m) \setminus \{y\}, \emptyset] \, P_j \, [vc(m) \setminus \{y\}, N]. \tag{1}$$

Since $y \in B(R_j)$ and R_j is candidate separable, $[vc(m) \setminus \{y\}, N] \, P_j \, [vc(m), N]$. Therefore, by (C2), (1), and transitivity of R_j we conclude that

$$[vc(m), \emptyset] \, P_j \, [vc(m), N].$$

Thus, $j \in EA^1(vc(m)) \subset EA(vc(m))$. Assume now that $EA^t(vc(m) \setminus \{y\}) \subset EA(vc(m))$ for all $t = 1, ..., t_0 - 1$, where $2 \leq t_0 \leq T'$. We now prove that $EA^{t_0}(vc(m) \setminus \{y\}) \subset EA(vc(m))$. Suppose not. Then, there exists $j \in EA^{t_0}(vc(m) \setminus \{y\})$ such that $j \notin EA(vc(m))$. Since $j \in EA^{t_0}(vc(m) \setminus \{y\})$,

$$[vc(m) \setminus \{y\}, \emptyset] \, P_j \left[vc(m) \setminus \{y\}, N \backslash \left(\bigcup_{t=1}^{t_0-1} EA^t(vc(m) \setminus \{y\})\right)\right].$$

Then,

$$\left[vc(m) \setminus \{y\}, N \backslash \left(\bigcup_{t=1}^{t_0-1} EA^t(vc(m) \setminus \{y\})\right)\right] P_j \, [vc(m) \setminus \{y\}, N \backslash EA(vc(m))]$$

because preferences are monotonic and $\bigcup_{t=1}^{t_0-1} EA^t(vc(m) \setminus \{y\}) \subset EA(vc(m))$ by assumption. Since $y \in B(R_j)$ and R_j is candidate separable,

$$[vc(m) \setminus \{y\}, N \backslash EA(vc(m))] \, P_j \, [vc(m), N \backslash EA(vc(m))].$$

Moreover,

$$[vc(m), N \backslash EA(vc(m))] \, P_j \, [vc(m), \emptyset]$$

because $j \notin EA(vc(m))$. Hence, by transitivity of R_j, $[vc(m) \setminus \{y\}, \emptyset] \, P_j \, [vc(m), \emptyset]$, which contradicts (C2). Therefore, the Claim is proved.

We now compare the outcomes $o^{vc}(m_i', m_{-i})$ and $o^{vc}(m_i, m_{-i})$ in the three following mutually exclusive cases:

Case 1: $i \in EA(vc(m) \setminus \{y\})$. By the above Claim, $i \in EA(vc(m))$. Therefore, by (C2), $o^{vc}(m_i', m_{-i}) I_i o^{vc}(m_i, m_{-i})$.

Case 2: $i \notin EA(vc(m) \setminus \{y\})$ and $i \in EA(vc(m))$. Hence,

$$[vc(m) \setminus \{y\}, N \backslash EA(vc(m) \setminus \{y\})] P_i [vc(m) \setminus \{y\}, \emptyset] I_i [vc(m), \emptyset].$$

Since $vc(m_i', m_{-i})$ is equal to either $vc(m)$ or $vc(m) \setminus \{y\}$,

$$\begin{aligned} o^{vc}(m_i', m_{-i}) &= [vc(m_i', m_{-i}), N \backslash EA(vc(m_i', m_{-i}))] \\ &R_i\ [vc(m_i, m_{-i}), N \backslash EA(vc(m_i, m_{-i}))] \\ &= o^{vc}(m_i, m_{-i}). \end{aligned}$$

Case 3: $i \notin EA(vc(m) \setminus \{y\})$ and $i \notin EA(vc(m))$. Hence,

$$\begin{aligned} [vc(m) \setminus \{y\}, N \backslash EA(vc(m) \setminus \{y\})] \quad &P_i \quad [vc(m) \setminus \{y\}, N \backslash EA(vc(m))] \\ &P_i \quad [vc(m), N \backslash EA(vc(m))], \end{aligned}$$

where the two strict preferences follow from monotonicity (and the above Claim) and candidate separability of R_i, respectively.

Since vc is without dummies we can find $I \in \mathcal{W}_y^m$ such that $i \in I$. Take $m_j^* = \{y\}$ for all $j \in I \setminus \{i\}$, $m_j^* = \emptyset$ for all $j \in N \setminus I$, and $m_i' = \emptyset$. Remember that $y \in m_i$. Then, $vc(m_i, m_{-i}^*) = \{y\}$ and $vc(m_i', m_{-i}^*) = \emptyset$, and hence, by (C3), $i \notin EA(vc(m_i, m_{-i}^*) \setminus \{y\}) = EA(vc(m_i', m_{-i}^*)) = EA(\emptyset) = \emptyset$. By (C2) and (C3), if $i \in EA(y)$ then

$$o^{vc}(m_i', m_{-i}^*) = [N, \emptyset] P_i [\emptyset, \emptyset] I_i [\{y\}, \emptyset] I_i [\{y\}, N \setminus EA(y)] = o^{vc}(m_i, m_{-i}^*).$$

Since $y \in B_K(R_i)$ and $R_i \in \mathcal{S}_i$, if $i \notin EA(y)$ then

$$o^{vc}(m_i', m_{-i}^*) = [\emptyset, N] P_i [\{y\}, N \setminus EA(y)] = o^{vc}(m_i, m_{-i}^*).$$

In both cases, $o^{vc}(m_i', m_{-i}^*) P_i o^{vc}(m_i, m_{-i}^*)$. Therefore, $o^{vc}(m_i', m_{-i}) R_i o^{vc}(m_i, m_{-i})$ for all m_{-i} and there exists at least one $m_{-i}^* \in M_{-i}$ for which $o^{vc}(m_i', m_{-i}^*) P_i o^{vc}(m_i, m_{-i}^*)$. Thus, strategy m_i is dominated by strategy m_i'. ■

Remark In Proposition 2 we assumed that the voting by committees vc had no dummies. Notice that if member i is a dummy for y, then to vote m_i and to vote $m_i \setminus \{y\}$ are equivalent strategies for member i because, independently of what the rest of members are voting, a vote of m_i or $m_i \setminus \{y\}$ leads to the same final outcome.

The next corollary is an immediate consequence of Proposition 2.

Corollary *Let $m \in M$ be an undominated NE of $\Gamma^{vc} = \left(N, \left(2^K\right)^N, R, o^{vc}\right)$ where $R \in \mathcal{S}$ and vc is voting by committees without dummies. Then, m satisfies voter's sovereignty.*

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