E C O N O M I C S   B U L L E T I N

Launhardt's early contributions to the spatial monopoly model

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Abstract

This paper shows that an early appearance of the formal spatial monopoly model is in Chapter 27 of Launhardt's 1985 book, Mathematical Principles of Economics (1993). The well-known spatial monopoly model developed by the pioneering works of Beckmann (1968, pp. 32-33, p. 51) and Greenhut and Ohta (1975, pp. 23-27) was anticipated by Launhardt. Launhardt should be given credit for it.

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1. Introduction

In past years there have appeared a large number of articles that attempted to incorporate distance and transport cost into the traditional monopoly model.[1] These studies were stimulated by the pioneering works of Beckmann (1968), and Greenhut and Ohta (1975) (henceforth GO). One of interesting and important issues in these writings is the impact of economic space on the spatial monopoly pricing. Assume that (i) a monopolistic firm is located at a point on an unbounded plain where buyers are distributed evenly; (ii) individual demand functions are linear and identical in all locations; (iii) the freight rate is proportional to distance; (iv) marginal production cost is constant. Beckmann (1968) and GO (1975) derived the aggregate spatial demand in a circular market and set up a spatial monopoly model to examine the economic decision of a monopoly. They showed that the profit-maximizing mill price of spatial monopoly is one-fourth maximal demand price plus three-fourths marginal cost. It is less than the profit maximizing mill price of non-spatial monopoly that is the average of maximal demand price plus marginal cost.

The purpose of this paper is to show that an early appearance of the spatial monopoly model is in Chapter 27 of Launhardt's 1885 book entitled *Market Area for the Sale of Goods*. In fact, Launhardt obtained the Beckmann and GO results in the course of investigating the effects of economic space on mill price, output and profit, and then after almost eighty years Beckmann (1968) and GO (1975) independently arrived at Launhardt's result.

2. Launhardt's spatial monopoly model

Following Launhardt, consider a monopolistic firm produces the goods in a certain location and "the buyers are evenly distributed over the market area", (Launhardt, 1993, p. 143). "The goods will be sold everywhere within the maximum feasible distance within *the dispatch borders*, so that the market area for a good which can be produced in a given location in unlimited quantity will form a circle, given that the respective economic conditions are equal in all directions, with a radius which equals the furthest *dispatch distance*" (Launhardt, 1993, p. 142). Based on the quadratic utility function, Launhardt derived the individual demand function that can be specified as:[2]

$$q = a - b(p + tr),\ a > 0,\ b > 0 \tag{1}$$

where q is quantity demanded at any buying point on the plain, p is the mill price, t is the freight rate, r is the distance from the buyer to the seller, p + tr is the full-price paid by

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[1]. For a detailed listing of works related to the spatial monopoly model see the excellent books from Greenhut, Norman and Hung (1987) and Ohta (1988).

[2]. Launhardt assumed that the utility function for the individual buyer is $U = \alpha x - \alpha_1 x^2$, $\alpha > 0$, $\alpha_1 > 0$, $x \geq 0$ and $\alpha > 2\alpha_1 x$. He pointed out that "[t]his approximate form fulfills everything that is known about the utility function in that for x = 0, and for $x = \alpha/\alpha_1$, U equals zero, and for $x = \alpha/2\alpha_1$, it reaches a maximum." (Launhardt, 1993, p. 30). Via the optimal choice condition, $MU_x/(p + fz) = w$, Launhardt obtained the individual demand function as: $x = (\alpha/2\alpha_1) - (w/2\alpha_1)(p + fz)$, where p is he mill price, f is the freight rate, z is the buyer's located distance from the seller site, and w is marginal utility of money (income). For details, see Launhardt (1993, p. 142, p. 161 and p. 172). Let $(\alpha/2\alpha_1) = a$, $(w/2\alpha_1) = b$, f = t and z = r. We at once obtain the individual demand function (1).

the buyer at distance r, a and b are given constant, and a/b is the maximal demand price.[3] It should be noted that the notation used by Launhardt is rather awkward for the modern reader and potentially misleading. To ease the comparison between Launhardt's model and Beckmann's and GO's models, we use Beckmann's notations throughout the paper.

If the buyers are evenly distributed over the market area and per surface unit has one buyer,[4] the aggregate spatial demand is

$$Q = 2\pi\int_{0}^{R^*} (a - bp - btr)r\,dr = 2\pi\{[(a - bp)R^{*2}/2] - (btR^{*3}/3)\} \qquad (2)$$

Launhardt pointed out that "the maximum dispatch distance, R*, at which goods become so expensive that demand ceases is determined", and "because the longest dispatch distance is $R^* = (a - bp)/bt$, …, it follows that" (Launhardt, 1993, p. 141)

$$Q = (\pi/3b^2t^2)(a - bp)^3 \qquad (3)$$

The first and second derivatives of Q with respect to p yields

$$dQ/dp = -(\pi/bt^2)(a - bp)^2 < 0 \qquad (4)$$
$$d^2Q/dp^2 = 2(\pi/t^2)(a - bp) > 0 \qquad (5)$$

where $a > bp$. Thus, the aggregate spatial demand is convex to the origin regardless of the linear individual demand curve. This result is consistent with GO's (Greenhut and Ohta, 1975, pp. 24-25).

Launhardt specified the cost of production as:

$$C = cQ \qquad (6)$$

where c = MC = AC is a constant. It should be noted that Beckmann (1968) and GO (1975) specified $C = cQ + F$, where F = fixed cost.

If the seller adopts the f.o.b. mill pricing policy, the profit–maximizing problem can be formulated as:

$$\text{Max } G = (p - c)Q$$
$$= (\pi/3b^2t^2)(p - c)(a - bp)^3 \qquad (7)$$

where p is the choice variable. In the case where t = 1, the expression of equation (7) is identical to Beckmann's "gross profit before fixed cost)", (Beckmann, 1968, p.51).[5]

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[3]. In Beckmann, he states that "a – bp, demand curve". (Beckmann, 1968, p. 51). In fact, this should be read "a – b(p + r), demand curve". Note that Beckmann assumes that t = 1.

[4]. Launhardt assumed that per surface unit has n buyers. For simplicity, we assume that n = 1.

[5]. In page 51 of Beckmann (1968), $G = (1/6b^2)(p - c)(a - bp)^3$ should be read $G = (\pi/3b^2)(p - c)(a - bp)^3$. It should also be noted that in the same page $G = (p - c)[(1/2b^2)(a - bp)^3 - (b/3b^3)(a - bp)^3]$ should be read $G = 2\pi (p - c)[(1/2b^2)(a - bp)^3 - (b/3b^3)(a - bp)^3]$.

Utilizing the standard profit maximization procedure, we obtain the first-order and the second-order sufficient conditions as:

$$dG/dp = (\pi/3b^2t^2)[(a - bp)^3 - 3b(p - c)(a - bp)^2] = 0 \quad (8)$$
$$dG^2/dp^2 = - 4b(\pi/3b^2t^2)\,(a - bp)^2 < 0 \quad (9)$$

Since the second-order sufficient condition is met, we can solve (8) for the optimal mill price. To this end, following Beckmann (1968, p. 51), we rewrite (8) as:[6]

$$(a - bp)^3 - 3b(p - c)(a - bp)^2 = 0 \quad (10)$$

Solving (10), Beckmann obtained the optimal mill price as:

$$p^* = (1/4)(a/b) + (3/4)c \quad (11)$$

In other words, the profit-maximizing mill price is one-fourth maximal demand price plus three-fourths marginal cost. This result is identical to Launhardt's (1993, p. 146) and GO's. (Greenhut and Ohta, 1975, p. 28).[7]

Launhardt didn't consider the mill price, p, as a choice variable. He assumed that the fixed cost is zero and utilized the profit margin (g) = the mill price (p) - average cost (c) as the choice variable. According to Launhardt,

> If a businessman can exploit the production of certain goods as a *monopoly* then he will determine the profit margin in such a way that his total profit will reach a maximum. This most favorable profit marginal arrived at for sales within the market area by differentiation of [the profit function] with respect to the [profit margin], (Launhardt , 1993, p. 145).

Substituting g = p – c into (6), we can rewrite the objective function as:

$$\text{Max} \quad G = g(\pi/3b^2t^2)(a - bg - bc)^3 \quad (12)$$

where g is the choice variable. The first-order and second-order conditions are

$$dG/dg = (\pi/3b^2t^2)\,(a - bg - bc)^2[(a - bg - bc) - 3bg] = 0 \quad (13)$$
$$d^2G/dg^2 = - 4(\pi/3bt^2)\,(a - bg - bc)^2 < 0 \quad (14)$$

Since the second-order condition is satisfied, we can solve (13) for the optimal profit margin as:

$$g^* = (1/4)(a/b) - (1/4)c \quad (15)$$

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[6]. In Beckmann (1968, p.51), $(a - bp)^3(p - c) - (p - c)(a - bp)^2 3 = 0$ should be read $(a - bp)^3(p - c) - (p - c)(a - bp)^2 3b = 0$.

[7]. GO specified the individual demand as $q = (1/d)(a - p - r)$. For details of GO's model, see appendix.

Combing (14) with p = g + c, we at once obtain

$$p^* = (1/4)(a/b) + (3/4)c \qquad (16)$$

Virtually, (16) is identical to (11). Launhardt concluded that "[I]f (buyers) were spread over a market area, they would have to pay …a price of c + g* = (1/4)(a/b) + (3/4)c", (Launhardt, 1993, p. 146).

The above analysis shows that Launhardt did clearly anticipate the Backmann – GO optimal mill pricing results.

3. Conclusions

In his assessment of Launhardt's contributions to economic theory, Blaug pointed out that "[to] anyone interested in the fascinating topic of multiple discoveries in science, and the associated question of why some figures are systematically neglected, Launhardt's case affords a rich example." (Blaug, 1986, p. 123). Along this line, this note has shown Launhardt as one of earliest contributors to the spatial monopoly theory. It is not surprising that Launhardt's work has not been widely appreciated because of his obscure style and the inaccessibility of his writings.[8] As such, our analysis of Launhardt's spatial monopoly model provides a good example of the fact that the same scientific discovery can be realized independently by geniuses of different time. Beckmann (1968) and GO (1975) independently developed the spatial monopoly model and used this model as a vehicle to investigate spatial price policies, the market area and the plant location problems. Apparently, they were unaware that their basic model and conclusions had already been anticipated and given by Launhardt.

Launhardt's place in the history of spatial monopoly theory is thus incontestable, and his name deserves to be mentioned as the most important early precursor of modern spatial economics.

4. Appendix

The following model is an excerpt from Greenhut and Ohta's *Theory of Spatial Pricing and Market Areas* (1975, p. 28). To ease the comparison between Launhardt's model that we present in the text and GO's model, we use Beckmann's notations.

Following assumptions (i) – (iv), GO further assumed that the freight rate is equal to one and the cost of production is C = cQ + F, where F is fixed cost. GO specified individual demand as q = (1/d)(a – p – r), where d is a constant. Thus, the maximal radius of the circle is R* = (a – p). With this in mind, they present their model as follows.

"In the case of a continuous distribution of buyers over a plain, the aggregate demand is given by

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8. Only two Launhardt's books exist in published English translations: *The Principles of Railway Location*, (1900, 1902) and *Mathematical Principles of Economics*, (1993). It may be noted that *The Principles of Railway Location* was out of print long time ago.

$$Q = \int_0^{R^*} 2\pi r(a - p - r)(1/d)dr \qquad (2\text{-}27)$$
$$= \pi[(a - p)R^{*2} - (2/3)R^{*3}]/d$$

Profits then are specified as

$$\Pi = (p - c)Q - F \qquad (2\text{-}28)$$
$$= \pi(p - c)[(a - p)R^{*2} - (2/3)R^{*3}]/d - F$$

The first-order condition for maximization is

$$d\,\Pi/dp = \pi[(a + c - 2p)R^{*2} - (2/3)R^{*3}]/d = 0 \qquad (2\text{-}29)$$
$$p = [(a + c)/2] - (R^*/3) \qquad (2\text{-}30)$$

Again, via $a = p + R^*$ this solution may be rewritten as

$$p = (c + R^*)/3 \qquad (2\text{-}30)'$$
$$p = (a + 3c)/4 \qquad (2\text{-}30)''$$

as derived in (2-22) above." (GO, 1975, p.28).

Obviously, GO's model is similar to Launhardt's model.

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