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An Extended Solow Growth Model with Emigration: Transitional Dynamics and Skills Complementarity

Patrice Pieretti
CREA, University of Luxembourg

Benteng ZOU
CREA, University of Luxembourg

Abstract

In this paper we develop an extended Solow growth model with skilled labor emigration which aggregates different labor types from strict complementarity to perfect substitution. Except in two particular cases, balanced growth paths can only be attained asymptotically. We therefore derive an analytical characterization of the transitional dynamics of the model. We are thus able to study the impact of labor elasticity of substitution on the time pattern of per capita income in the country that experiences brain drain. Simulations show that the shape of per capita trajectory depends crucially on the degree of complementarity (substitutability) between labor skills. Given that no income trajectory dominates the others, there is room for policy issues by influencing the elasticity of substitution (Klump and Preissler, 2000).

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1 Introduction

The relevance of complementarities between skilled and unskilled labor in the understanding of the effect of outward flows of skilled people on home country output has been raised by some authors (Piketty, 1997; Beine, Docquier and Rapoport, 2001). In the same vein, Saint-Paul (2004) tries to calculate the impact of European expatriates flows in the USA on the per capita income of the source country, by assuming imperfect substitution between migrants and non migrants. This analysis is static in nature and is restricted to the case of unit substitution elasticity between the different labor types.

To our knowledge however, there is no literature that formally explores the role of elasticity of substitution between skilled and unskilled labor in the discussion about the impact of brain drain on the growth performance of the country of origin.

In this paper we try to fill that gap by developing an extended Solow growth model that aggregates different types of labor skills in order to grasp the full range of substitution degrees from strict complementarity to perfect substitution. The derivation of balanced growth paths shows that except particular cases these paths can only be attained asymptotically. This justifies the need for using transitional dynamics. We therefore develop a full analytic characterization of the transitional dynamics of output and income in the sending country for all possible values taken by the elasticity of substitution between skilled and unskilled workers. We are thus able to study the impact of labor elasticity of substitution on the time pattern of per capita income in a country that experiences brain drain. Simulations show that the shape of per capita trajectory depends crucially on the degree of complementarity (substitutability) between labor skills. Low skill substitution induces high per capita income levels in the short and middle term and very low levels in the long term. High skill substitution induces a smoother (per capita) income pattern with no income collapse in the long run. Given that no income trajectory dominates the others, there is room for policy issues by influencing the elasticity of substitution (Klump and Preissler, 2000).

2 The model

2.1 Labor force

Assume that in a small open economy working population consists of unskilled workers (N, with exogenous growth rate $n \geq 0$) who are supposed internationally immobile and skilled workers (R) representing potential emigrants.

Denote the emigration flow by E and assume that the proportion of emigration to total skilled workers is represented by ξ ($\xi = E/R$)[1]. The "natural" growth rate of skilled labor is given by g, which is exogenously given. The law of motion of skilled labor is expressed as follows: $\dot{R} = gR - E$. It follows that the growth rate of skilled workers that do not emigrate is:

$$r = \frac{\dot{R}}{R} = g - \xi. \tag{1}$$

Assume that the labor force is a synthetic measure of skilled and unskilled labor given by a CES function:

$$L = [bR^{-\beta} + (1-b)N^{-\beta}]^{-\frac{1}{\beta}}, \quad -1 < \beta < \infty, \tag{2}$$

where parameter $b \in (0, 1)$ changes as the substitution parameter β varies between -1 and ∞.

Denote the elasticity of substitution between skilled labor R and unskilled labor N by $\sigma = \frac{1}{1+\beta}$. The labor force index L that combines R and N is able to grasp the full range of substitution degrees starting from strict complementarity ($\sigma = 0$ or $\beta \to \infty$) to perfect substitution ($\sigma \to \infty$ or $\beta = -1$). Note that in the second case both labor-types are additive since they are perfectly interchangeable.

We close this section by noting that the total stock of emigrated people M is proportional to the total home country skilled worker R, with $M = \phi R$ ($\phi = \frac{\xi}{g-\xi}$). This relation derives from the fact that workers are supposed to live infintely[2].

2.2 Output, wages, and capital accumulation

National output derives from a classical constant returns production function :

$$Y = F(K, AL) = K^{\alpha}(AL)^{1-\alpha}, \tag{3}$$

where $0 < \alpha < 1$ is capital share and A is a technology parameter. Technological change is described by $A(t) = A(0)e^{g_A t}$, where $g_A \geq 0$ is an exogenous growth rate.

Profit maximization by competitive firms leads to following equilibrium wages for unskilled (w_N) and skilled labor (w_R) :

$$w_N = (1-\alpha)(1-b)F(K, AL)\left(\frac{N}{L}\right)^{-\beta} N^{-1}, \; w_R = (1-\alpha)bF(K, AL)\left(\frac{R}{L}\right)^{-\beta} R^{-1}. \tag{4}$$

It is easy to see that the wage ratio becomes:

$$\frac{w_R}{w_N} = \frac{b}{1-b}\left(\frac{N}{R}\right)^{1+\beta} = \frac{b}{1-b}\left(\frac{N(0)}{R(0)}\right)^{1+\beta} e^{(n-r)(1+\beta)t}, \tag{5}$$

[1]Note that if $\xi < 0$, there is immigration.

[2]$M(t) = \int_{-\infty}^{t} E(s)ds = \xi \int_{-\infty}^{t} R(s)ds = \frac{\xi}{g-\xi} R(t)$.

where the last equality comes from the assumption of exogenous growth of unskilled and skilled labor.

Wages of actual emigrants are supposed to exceed those of potential emigrants in a fixed proportion $(1+h)$. Assume that the emigrants repatriate a percentage θ of their wages to their home country. It follows that emigrant remittances are expressed by :

$$\theta w_M \cdot M = \theta w_M \cdot \phi R = \theta(1+h) w_R \cdot \phi R = \theta(1+h)\phi(1-\alpha) b F(K, AL) \left(\frac{R}{L}\right)^{-\beta}. \quad (6)$$

Following Solow's vein we assume that the saving rate s is positive and constant. Note that national income equals GDP $(F(K, AL))$ plus emigrant remittances :

$$NI(t) = F(K, AL) + \theta w_M \cdot M = F(K, AL)\left[1 + \theta\phi b(1+h)(1-\alpha)\left(\frac{R}{L}\right)^{-\beta}\right]. \quad (7)$$

Capital accumulation is thus given by :

$$\begin{aligned} \dot{K} &= s\left(F(K, AL) + \theta w_M \cdot M\right) - \delta K \\ &= sF(K, AL)\left[1 + \theta\phi b(1+h)(1-\alpha)\left(\frac{R}{L}\right)^{-\beta}\right] - \delta K, \end{aligned} \quad (8)$$

with initial condition $K(0)$ given, and $\delta(>0)$ is the constant depreciation rate of capital.

2.3 Growth dynamics and balanced growth path

In this subsection, we are going to study the evolution of capital accumulation (8), labor force (2), wages ((4), and the wage ratio (5) along transition and balanced growth paths. Note that balanced growth is reached when all growth rates of endogenous variables are constant. Let's denote the growth rate of variable X by g_X, and g_X^* its balanced growth rate.

The growth rate of capital g_K may be written as follows:

$$g_K = s\frac{Y}{K}\left[1 + \theta\phi b(1+h)(1-\alpha)\left(\frac{R}{L}\right)^{-\beta}\right] - \delta. \quad (9)$$

This relation shows that the existence of a balanced growth path is not automatically verified even if output and capital grow at the same rate, because $\left(\frac{R}{L}\right)^{-\beta}$ is not necessarily constant. In order to get the conditions on existence of balanced growth paths, we set $a(t) = b\left(\frac{R}{L}\right)^{-\beta}$ and study the dynamics of $a(t)$ by writing following differential equation :

$$\dot{a}(t) = -\beta a(t)(r - g_L), \quad (10)$$

An explicit solution[3] is given by:

$$a(t) = \frac{1}{\frac{1-a(0)}{a(0)} e^{\beta(r-n)t} + 1}. \tag{11}$$

On the other hand, we also have

$$\left(\frac{R(t)}{L(t)}\right)^{-\beta} = \left(\frac{R(0)}{L(0)}\right)^{-\beta} e^{-\beta \int_0^t (r - g_L(s))ds}. \tag{12}$$

Combining the above two equations, we obtain the transitional growth rate of aggregated labor force

$$g_L(t) = \frac{(1-a(0))(n-r)e^{\beta(r-n)t}}{[1-a(0)]e^{\beta(r-n)t} + a(0)} + r. \tag{13}$$

Due to the above explicit growth path of labor force, it is possible to deduce the necessary and sufficient conditions for the existence of balanced growth paths.

Proposition 1 *The necessary and sufficient conditions for the existence of balanced growth paths are* $g_K^* = g_Y^* = g_A + g_L^*$.

If $\beta = 0$, *or* $\beta \neq 0$ *but* $r = n$, *the economy reaches a balanced growth path within finite time, where*

$$g_L^* = \begin{cases} br + (1-b)n, & \beta = 0, \\ r = n, & \beta \neq 0. \end{cases}$$

If $\beta \neq 0$ *and* $r \neq n$, *balanced growth is reached asymptotically and following growth rate is obtained after an infinite time, where*

$$g_L^* = \begin{cases} r, & \beta \neq 0 \text{ and } (r-n)\beta < 0, \\ n, & \beta \neq 0 \text{ and } (r-n)\beta > 0. \end{cases}$$

Proof: see Appendix

Proposition 1 shows that balanced growth can be realized at a finite point in time in merely two particular cases. In all other cases ($\beta \neq 0$ and $r \neq n$), balanced growth can only be attained asymptotically. Since brain drain alters the growth rate between skilled and unskilled labor as we shall argue, the study of transitional dynamics becomes particularly relevant. In this context a first step has already been achieved by deducing equation (13), which gives an analytical description of the transitional growth rate of aggregated labor.

[3]The details are given in Appendix.

In the Appendix we deduce the analytical description of the transitional growth rate of capital accumulation that is expressed as follows:

$$g_K(t) = \left[\frac{e^{-\int_0^t G(s)ds}}{[g_K(0)+\delta]} + (1-\alpha)e^{-\int_0^t G(s)ds} \int_0^t e^{\int_0^\tau G(s)ds} d\tau \right]^{-1} - \delta, \tag{14}$$

where

$$G(t) = (1-\alpha)\left(g_A + g_L(t) + \delta\right) + \frac{\theta\phi(1+h)(1-\alpha)\dot{a}(t)}{1+\theta\phi(1+h)(1-\alpha)a(t)},$$
$$g_K(0) = s\frac{Y(0)}{K(0)}\left[1+\theta\phi(1+h)(1-\alpha)a(0)\right] - \delta.$$

Output growth is described by $g_Y = \alpha g_K + (1-\alpha)(g_A + g_L)$ along with equations (13) and (14). If we furthermore remember equation (7), we obtain a description of per capita income in the sending country given as follows:

$$\frac{NI(t)}{N(t)+R(t)} = \frac{Y(0)e^{\int_0^t g_Y(s)ds}}{N+R}\left[1+\theta\phi(1+h)(1-\alpha)a(t)\right]. \tag{15}$$

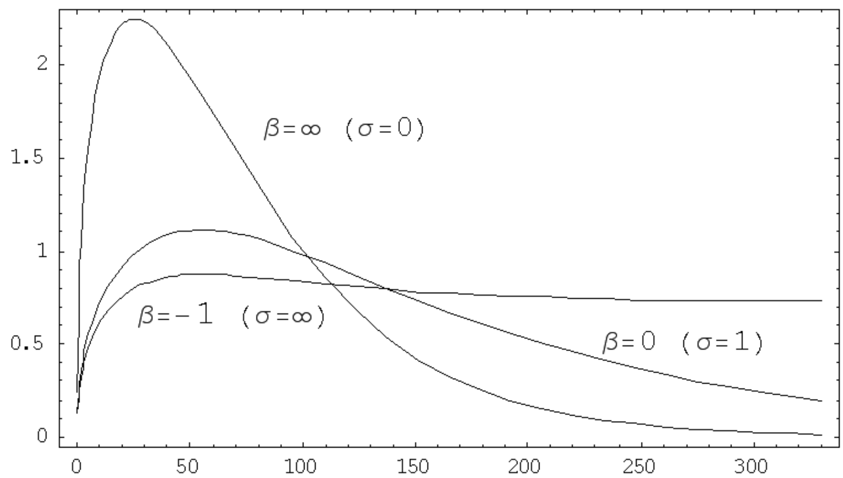

Figure 1: Elasticity effects on per capita income in the sending country

The simulations[4] in Figure 1 show the impact of alternative values for β on the evolution

[4]A similar idea on long-run ecomomic growth rate is also noticed by Groth et al (2006).

of per capita output when emigration lowers the growth rate of skilled labor[5]. It appears that a lower elasticity of labor substitution (higher degree of complementarity) raises the path of per capita income during a first stage but it lowers it after that period. One consequence is that there is no income time-pattern that strictly dominates the others. This leads us to the question about which elasticity scenario would be the most desirable for an economy that experiences brain-drain. In that context, the choice of an appropriate elasticity of substitution between different labor types could be a policy issue. As a matter of fact, Klump and Preissler (2000) regard the elasticity of factor substitution[6] as a variable that is not only technically given but may be strongly influenced by the institutional framework and more specifically by policy making. This could be achieved for instance by adjusting laws and regulations. For example, in Zambia, laws prohibiting nurses from prescribing medication and carrying out invasive procedures were amended and similarly, in Botswana, nurses have the authority to prescribe medication when a doctor is not present (Padarath A. et al. , 2003).

3 Conclusion

The aim of this note was basically to investigate the role of brain drain within an extended Solow growth model that explicitly takes into account the degree of substitutability between skilled (the potential emigrants) and unskilled workers.

We successfully derived balanced growth paths which, except particular cases, are only attainable asymptotically. We therefore developed a full analytic characterization of the transitional dynamics of output and income in sending country.

The derived analytic also served to provide a practical interest for studying the impact of emigration on output in the sending country. Assuming different labor elasticity scenarios the model generates various trajectories of per capita output where none strictly dominates the others. In that context, the choice of an appropriate elasticity of substitution between different labor types may be a policy issue.

In this short note we did not study the possible influence of brain drain on the incentive of low skilled people to emigrate too. This would be a question of interest if emigration widens the gap $(n - r)$ between the growth rate of unskilled and skilled workers who don't emigrate and if there is complementarity $(\beta > 0)$ between both labor types. In that scenario, it may be shown that the wage of unskilled people decreases heavily in the long run. This wage collapse may thus induce massive outflows of people trying to escape

[5]Prior to emigration, we assume that $n = r = 0,03$ since $\xi = 0$. The simulations are then run with an emigration rate of $\xi = 0,02$ what implies that $n > r = 0,01$.

[6]More precisely, the authors focus on substitution between capital and labor rather than on skill substitution.

absolute poverty[7].

Finally, we considered that the emigration flow was exogenously given. A way to relax this assumption may consist in supposing that agents optimally decide to acquire skills that enable that to emigrate in developed countries. Such an extension should however be considered in a future paper.

4 Appendix

4.1 Proof of Proposition 1

The proof is arranged as follows. First, we check the necessary and sufficient conditions of the existence for a balanced growth path. Then we give further specifications about the growth rate of aggregate labor by taking into account different cases concerning the parameters β, r and n.

Step 1.

Sufficiency is obvious. With $g_K^* = g_Y^*$, and $\beta(r - g_L^*) = 0$ (or $e^{-\beta(r-g_L^*)t} \to 0$) , all the endogenous variables grow at constant rates, which is balanced growth by definition.

Now, we prove the necessity. Suppose there is a balanced growth path, where capital, output and aggregated labor grow at constant rates g_K^*, g_Y^*, and g_L^* respectively. Then we have:

$$Y(t) = \bar{Y}e^{g_Y^* t},\ K(t) = \bar{K}e^{g_K^* t},\ L(t) = \bar{L}e^{g_L^* t},$$

where $\bar{X}$ denotes the level of variable X along a balanced growth path.

Therefore, due to (9), we have the relation

$$g_k^* + \delta = s\frac{\bar{Y}}{\bar{K}}e^{(g_Y^* - g_K^*)t}\left[1 + \theta\phi b(1+h)(1-\alpha)\frac{R(0)}{\bar{L}}e^{-\beta(r-g_L^*)t}\right],$$

where, the left hand side is always constant. Hence the equality holds if and only if $g_Y^* = g_K^*$ and $\beta(r - g_L^*) = 0,\ for\ t < \infty$, or $e^{-\beta(r-g_L^*)t} \to 0,\ for\ t \to \infty$.

Straightforward from the production function, and the fact that along balanced growth we have $g_Y^* = g_K^*$, it follows $g_Y^* = g_K^* = g_A + g_L^*$.

The system reaches its balanced growth path in finite time, if and only if $\beta(r - g_L^*) = 0$. If $\beta = 0$, intra-labor substitutability is unitary (and we have $L = R^b N^{1-b}$). Then the growth rate of aggregate labor force is $g_L(t) = br + (1-b)n = g_L^*$. If $\beta \neq 0$, we must have $r = g_L^*$, combining with (13), il follows $r = n = g_L^*$, which is showed in the next appendix.

[7] We thank a referee for having suggested this observation.

While $r \neq n$, there are several cases to study.

Step 2.

Case 1. $r < n$, $\beta > 0$. The infinite time limit of aggregate labor growth yields:

$$\lim_{t\to\infty} g_L(t) = \lim_{t\to\infty} \frac{(1-a(0))(n-r)e^{\beta(r-n)t}}{(1-a(0))e^{\beta(r-n)t}+a(0)} + r = r.$$

Moreover, $g_L(0) = (1-a(0))(n-r)+r > r$ and $g_L'(t) = \frac{-a(0)(1-a(0))\beta(n-r)^2 e^{\beta(r-n)t}}{\left[(1-a(0))e^{\beta(r-n)t}+a(0)\right]^2} < 0$ demonstrate that the growth rate of aggregate labor force is decreasing over time and will reach the lower bound as time goes to infinity ($t \to \infty$).

Case 2. The same argument for $r < n$, $-1 < \beta < 0$, we obtain $\lim_{t\to\infty} g_L(t) = n > r$, with $g_L(0) = (1-a(0))(n-r)+r > r$ and $g_L'(t) > 0$.

Case 3. As to $r > n$, $\beta > 0$, it follows the same as Case 2; and $r > n$, $-1 < \beta < 0$ is the same as Case 1. $\diamondsuit$

4.2 Proof of (11) and (13)

Suppose $\beta \neq 0$. The difference between skilled and aggregated labor force is

$$\begin{aligned} r - g_L &= \tfrac{\dot R}{R} - \tfrac{\dot L}{L} = \tfrac{\dot R}{R} - \left[br\left(\tfrac{R}{L}\right)^{-\beta} + (1-b)n\left(\tfrac{N}{L}\right)^{-\beta}\right] \\ &= (r-n)(1-b\left(\tfrac{R}{L}\right)^{-\beta}) = (r-n)(1-a(t)). \end{aligned}$$

Therefore the dynamics of $a(t)$ can be expressed as

$$\dot a(t) = -\beta(r-n)a(t)(1-a(t)),$$

with given $a(0) = b\left(\frac{R(0)}{L(0)}\right)^{-\beta}$. With standard ordinary differential equation technique, (11) can be obtained.

Combining (11) and (12), it follows

$$a(0)e^{-\beta\left(\int_0^t (r-g_L(s))ds\right)} = \frac{1}{\left(\frac{1-a(0)}{a(0)}e^{\beta(r-n)t}+1\right)}.$$

Taking logarithm

$$-\beta\left(\int_0^t (r-g_L(s))ds\right) = -\ln(a(0)) - \ln\left(\left(\frac{1-a(0)}{a(0)}e^{\beta(r-n)t}+1\right)\right),$$

Derivative with respect to t on both sides and rearranging the terms yields (13). $\diamondsuit$

4.3 Proof of (14)

From the production function, we derive that $g_Y = \alpha g_K + (1-\alpha)(g_A + g_L)$, and hence $g_Y - g_K = (\alpha - 1)g_K + (1-\alpha)(g_A + g_L)$.

Combining (9) and

$$Y(t) = Y(0)e^{\int_0^t g_Y(s)ds}, \ \ K(t) = K(0)e^{\int_0^t g_K(s)ds},$$

we obtain for any $t \geq 0$,

$$\begin{aligned} g_K(t) &= s\frac{Y(0)}{K(0)}e^{\int_0^t [g_Y(s)-g_K(s)]ds}\left[1+\theta\phi(1+h)(1-\alpha)a(t)\right] - \delta \\ &= s\frac{Y(0)}{K(0)}e^{\int_0^t (\alpha-1)g_K(s)ds}e^{\int_0^t (1-\alpha)(g_A+g_L(s))ds}\left[1+\theta\phi(1+h)(1-\alpha)a(t)\right] - \delta, \end{aligned}$$

especially

$$g_K(0) = s\frac{Y(0)}{K(0)}\left[1+\theta\phi(1+h)(1-\alpha)a(0)\right] - \delta.$$

Taking logarithm on both sides, it follows

$$\begin{aligned} \ln(g_K+\delta) &= \ln\left(s\frac{Y(0)}{K(0)}\right) + (\alpha-1)\int_0^t g_K(s)ds + (1-\alpha)\int_0^t \left(g_A + g_L(s)\right)ds \\ &\quad + \ln\left(1+\theta\phi(1+h)(1-\alpha)a(t)\right). \end{aligned}$$

Derivative with respect to time t and rearrange the terms:

$$\frac{\dot{g_K}}{g_K+\delta} = (\alpha-1)g_K(t) + (1-\alpha)\left(g_A + g_L(t)\right) + \frac{\theta\phi(1+h)(1-\alpha)\dot{a}(t)}{1+\theta\phi(1+h)(1-\alpha)a(t)}.$$

Let

$$V(t) = g_K(t) + \delta, \ \ V(0) = g_K(0) + \delta,$$

and

$$G(t) = -\delta(\alpha-1) + (1-\alpha)\left(g_A + g_L(t)\right) + \frac{\theta\phi(1+h)(1-\alpha)\dot{a}(t)}{1+\theta\phi(1+h)(1-\alpha)a(t)},$$

we have

$$\frac{\dot{V}}{V} = (\alpha-1)V + G(t),$$

which is a Bernoulli differential equation, and its solution is given by

$$V(t) = \left[\frac{e^{-\int_0^t G(s)ds}}{V(0)} + (1-\alpha)e^{-\int_0^t G(s)ds}\int_0^t e^{\int_0^s G(\tau)d\tau}ds\right]^{-1}.$$

We thus finish the proof of (14). $\diamondsuit$

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