ECONOMICS BULLETIN

Endogenous Timing in a Mixed Duopoly: The Managerial Delegation Case

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Abstract

We introduce managerial delegation into Pal's (1998) model and examine the impact of the introduction of managerial delegation on endogenous timing in a mixed duopolistic model for differentiated goods. We show that a public firm and a private firm choose quantities sequentially in the equilibrium of our model. Thus, we find that the Pal's (1998) results are robust against managerial delegation.

We are grateful to the associate editor (Patrick Legros) and an anonymous referee for their helpful comments and suggestions. We appreciate the financial support from the Japanese Ministry of Education, Culture, Sports, Science and Technology under Waseda University 21st-COE GLOPE project. Needless to say, we are responsible for any remaining errors.
Citation: Nakamura, Yasuhiko and Tomohiro Inoue, (2007) "Endogenous Timing in a Mixed Duopoly: The Managerial Delegation Case." *Economics Bulletin*, Vol. 12, No. 27 pp. 1-7
Submitted: September 29, 2007. **Accepted:** October 19, 2007.
URL: http://economicsbulletin.vanderbilt.edu/2007/volume12/EB-07L20010A.pdf

1 Introduction

This paper presents a theoretical analysis of mixed duopoly where a public firm and a private firm choose whether to set their own quantities sequentially or simultaneously. The literature on mixed oligopoly mainly assumes that the order of firms' moves is exogenous. Since an alternative order of moves gives rise to different results in mixed oligopoly, it is significantly important to endogenously examine the timing at which they choose their quantities/prices. The issue of the endogenous order in mixed oligopoly has been analyzed by Pal (1998), Matsumura (2003), Lu (2006), and Barcena-Ruiz (2007). In his pioneering work, Pal (1998) considered the situation in which firms decide quantities. Matsumura (2003) and Lu (2006) extended the analysis to investigate the competition between a domestic public firm and foreign private firms, and Barcena-Ruiz (2007) followed them by considering the model of price competition. All of the above works adopt the observable delay game of Hamilton and Slutsky (1990) in their analysis. This paper also focuses on the role of the endogenous timing of firms' moves using the observable delay game along the lines of these works.

In the four above mentioned papers of Pal (1998), Matsumura (2003), Lu (2006), and Barcena-Ruiz (2007), the public firm and private firms are all assumed to be an entrepreneurial ones, that is, each managerial decision-making process is enforced by the owner of each firm. The literature on mixed oligopoly has paid scant attention to investigating the management of firms, whereas there have been numerous contributions on the managerial incentive contract in private oligopoly since the pioneering works of Fershtman and Judd (1987) and Sklivas (1987). The papers by Barros (1995), White (2001), and Nishimori and Ogawa (2005) are notable exceptions. Separating the ownership and management within both of public and private firms, in each paper, Barros and White modeled the incentive contracts that are linear in terms of the firm's profit and sales revenue à la Fershtman and Judd (1987) and Sklivas (1987), so-called FJS contract. Nishimori and Ogawa considered the interaction between the length of incentive contracts and market behavior using a two-period mixed oligopolistic framework. However, the literature has not considered the issue of what type of competition occurs in the equilibrium in the case of introducing managerial delegation within both a public and private firm, when they can endogenously choose their moves.

The purpose of this paper is primarily to examine whether Pal's (1998) results are robust when managerial delegation is introduced in a mixed duopolistic model for differentiated goods. In the literature on private duopoly, Lambertini (2000) adopted the same approach as ours. In this paper, we extend the scope of the model through the application of Hamilton and Slutsky (1990) in the context of mixed oligopoly, by introducing the FJS contract as in Lambertini (2000). In contrast to the case of entrepreneurial firms, each firm's owner delegates the output decision to a manager. Each manager sets the output to maximize his/her payoff defined by an incentive contract provided by the firm's owner. Our interest lies in which order of moves will lead to an equilibrium in the case of introducing managerial delegation. The results obtained in this paper are similar to those in Pal (1998). We find that a public firm and a private firm choose quantities sequentially in our delegation model. Thus, in the context of a quantity setting mixed duopoly where a public firm and a private firm are domestic, we find that the results in the observable delay game are robust against the FJS contract.

The rest of the paper is organized as follows. In Section 2, we formulate the basic setting of the model. In Section 3, we analyze three types of competition of fixed timing and present the equilibrium in our model. Section 4 presents some concluding remarks.

2 Model

We consider a mixed duopolistic model using a standard product differentiation model as in Singh and Vives (1984). The model consists of a public firm and a private firm, both of which produce a differentiated good. In the rest of this paper, we often refer to the public (private) firm as Firm 0 (Firm 1) and the owner of the public (private) firm as Owner 0 (Owner 1). A representative consumer's utility is denoted as[1]

$$U(q_0, q_1) = a(q_0 + q_1) - \frac{1}{2}(q_0^2 + 2bq_0q_1 + q_1^2), \qquad b \in [0, 1),$$

where q_i is the quantity of good i $(= 0, 1)$, and b represents the degree of product differentiation. Then, the inverse demand functions are given by

$$p_i = a - q_i - bq_i, \qquad b \in [0, 1), \quad i, j = 0, 1, \quad i \neq j,$$

where p_i is the price of good i $(= 0, 1)$. We assume that both the two firms have constant marginal costs of production. However, it is assumed that Firm 0 is less efficient than Firm 1, and thus, the Firm 0's marginal cost is $c > 0$, whereas Firm 1's marginal cost is normalized to 0. The profits of the Firms 0 and 1 are given by

$$\Pi_0 = (a - q_0 - bq_1 - c)q_0 \quad \text{and} \quad \Pi_1 = (a - q_1 - bq_0)q_1, \quad \text{respectively.}$$

As usual, social welfare, denoted by W, is measured as the sum of consumer surplus (CS) and producer surplus (PS):

$$W = CS + PS.$$

where $PS = \Pi_0 + \Pi_1$, and consumer surplus is given by

$$CS = \frac{1}{2}(q_0^2 + 2bq_0q_1 + q_1^2).$$

We assume that Owner 0 is a welfare maximizer, whereas Owner 1 maximizes his/her own firm's profit.

To formalize managerial delegation, we mainly follow Lambertini (2000). The owners can assess the performance of their managers according to two observable indicators, the output and profit of the firm. In this case, Firm i considers the following indicator:

$$V_i = \Pi_i + \theta_i q_i, \qquad \theta_i \in \mathbb{R}, \quad i = 0, 1,$$

where parameter θ_i identifies the weight attached to the value of sales (delegation parameter). The manager of Firm i can maximize his/her payoff by choosing the output q_i that maximizes V_i. This can be supported by the assumption that the payoff to the manager of Firm i is represented

[1] We assume that $b < 1$ to assure that the function $U(q_0, q_1)$ is strictly concave, however, the following results hold even though each of the two firms produces a single homogeneous good, that is $b = 1$.

as $\lambda_i + \mu_i V_i$ for some real number λ_i and some positive number μ_i. Similar to Lambertini (2000) and White (2001), we assume that the payoffs to the managers are negligible as compared to profits, because we emphasize the impact of managerial delegation on the equilibrium outcomes.

We propose the following three-stage game as an application of the observable delay game of Hamilton and Slutsky (1990). Note that the third stage consists of two periods; let the first (second) period denote period 1 (period 2). In the first stage, Owner i ($i = 0, 1$) independently chooses $t_i \in \{first,\ second\}$, where t_i indicates the time of which should be set output q_i. $t_i = first$ implies that Firm i's manager sets his/her own output in period 1 of the third stage, and $t_i = second$ implies that he/she sets his/her own output in period 2 of the third stage. In the second stage, each of Owners 0 and 1 simultaneously sets their respective firm's level of θ_i. In the third stage, each of the firm's managers selects the output in the period that the corresponding Owners 0 and 1 choose in the first stage. If both the owners choose the same period, the managers take decisions simultaneously. If Owner i chooses $t_i = first$ and the other j ($\neq i$) chooses $t_j = second$, the quantity competition is sequential. We adopt a subgame perfect Nash equilibrium, and thus the game is solved backward.

3 Result

In the third stage, the manager of each firm chooses its output q_i to maximize V_i ($i = 0, 1$). There are three subgames in this stage: the simultaneous game (Case S), the sequential game with Firm 0 as the leader (Case L), and the sequential game with Firm 0 as the follower (Case F). Thus, we analyze the equilibrium for each case. The equilibrium points are showed in Figure 1. In this figure, point $k \in \{S, L, F\}$ is the equilibrium point in Case k and $R_i(q_j)$ represents the reaction function of Firm i ($i, j = 0, 1;\ j \neq i$),

$$R_0(q_1) = \frac{a - c - bq_1 + \theta_0}{2}, \quad R_1(q_0) = \frac{a - bq_0 + \theta_1}{2}. \tag{1}$$

Moreover, $\overline{V}_i$ expresses an iso-payoff curve of the manager of Firm i.

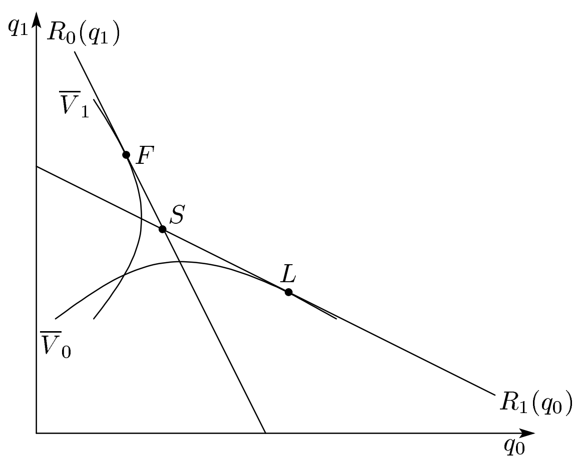

Figure 1: Equilibrium points in the third stage

In the second stage, the owner of each firm chooses θ_i to maximize their respective objective functions. Owner 0 maximizes social welfare, while Owner 1 maximizes the profit of Firm 1. Consequently, we obtain the equilibrium values of the delegation parameter θ_i^k $(i = 0, 1;\ k = S, L, F)$ as follows.

$$\text{Owner 0}: \begin{cases} \theta_0^S = \frac{a(8-8b+2b^3-b^4)-c(8-b^4)}{8-8b^2+b^4} & \text{in Case } S, \\ \theta_0^L = \frac{a(32-32b-24b^2+28b^3+2b^4-5b^5)-2c(16-12b^2+b^4)}{2(16-20b^2+5b^4)} & \text{in Case } L, \\ \theta_0^F = \frac{a(16-16b-4b^2+6b^3-b^4)-c(16-4b^2-b^4)}{16-20b^2+5b^4} & \text{in Case } F, \end{cases}$$

$$\text{Owner 1}: \begin{cases} \theta_1^S = \frac{2ab^2(1-b)+2b^3c}{8-8b^2+b^4} & \text{in Case } S, \\ \theta_1^L = \frac{4ab^2(1-b)+4b^3c}{16-20b^2+5b^4} & \text{in Case } L, \\ \theta_1^F = 0 & \text{in Case } F. \end{cases}$$

In the above three cases, the delegation parameter of Owner 0 can take a positive or negative value, whereas the parameter of Owner 1 does not take a negative value. This is because the characteristics of the delegation is similar to the ability to move first for Owner 1, as mentioned by Lambertini (2000). Thus, the owner chooses $\theta_1^j \geq 0$ to expand the output of Firm 1. In addition, when Firm 1 is the Stackelberg leader in the third stage (Case F), the owner chooses $\theta_1^F = 0$ because there is no need for expanding the output. However, for Owner 0, the delegation has a different meaning. The owner wishes to convert the objective of the manager into social welfare maximization, and thus, chooses the delegation parameter to move the equilibrium point in the third stage to the first best social welfare. For example, in the simultaneous game, since $R_0(q_1)$ shifts outward with the increase in θ_0 as described in (1), Owner 0 sets a positive parameter if the iso-welfare curve touches $R_1(q_0)$ in the right of the equilibrium point S; otherwise, it sets a negative parameter. This mechanism is analogous to partial privatization (*c.f.*, Matsumura, 1998). Figure 2 illustrates the mechanism. $R_W(q_0)$ denotes the reaction function of Firm 0 when its manager maximizes social welfare, $\overline{W}$ represents an iso-welfare curve, and E^S is the point at which the iso-welfare curve touches $R_1(q_0)$.

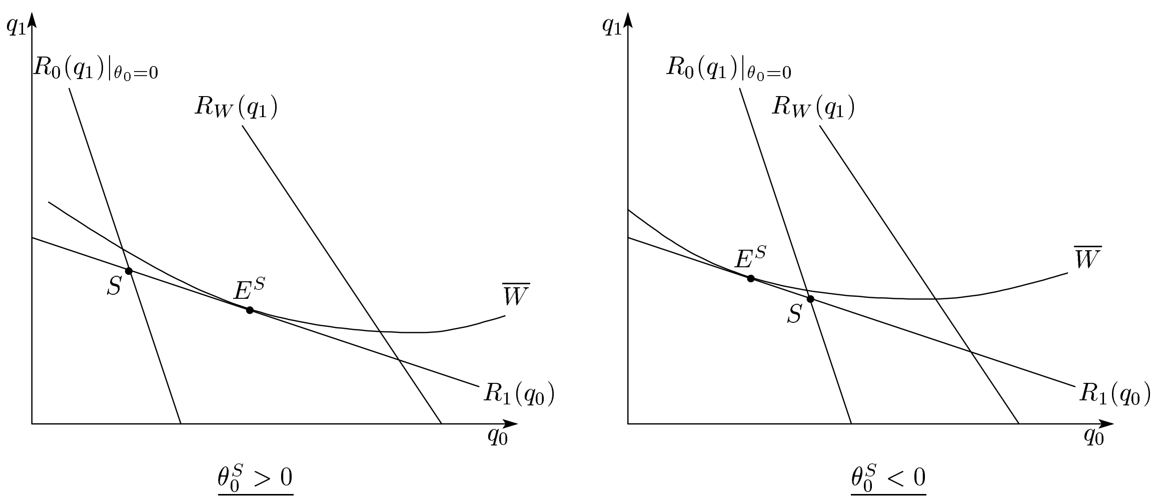

Figure 2: The choice of θ_0 in the simultaneous game

In the other two sequential games, the mechanisms are analogous to the simultaneous game. Thus, the delegation parameter of Owner 0 can take a positive or negative value.

The equilibrium payoffs of both the owners in the second stage are as follows.

Owner 0:

$$\begin{cases} W^S = \frac{\left[\begin{array}{c} a^2(112-96b-132b^2+104b^3+40b^4-24b^5-5b^6+2b^7) \\ -2ac(64-48b-80b^2+52b^3+28b^4-12b^5-4b^6+b^7)+4c^2(16-20b^2+7b^4-b^6) \end{array}\right]}{2(8-8b^2+b^4)^2} & \text{in Case } S, \\ W^L = \frac{\left[\begin{array}{c} a^2(448-384b-752b^2+608b^3+404b^4-296b^5-77b^6+50b^7) \\ -2ac(256-192b-448b^2+304b^3+256b^4-148b^5-52b^6+25b^7)+4c^2(64-112b^2+64b^4-13b^6) \end{array}\right]}{2(16-20b^2+5b^4)^2} & \text{in Case } L, \\ W^F = \frac{a^2(28-24b-12b^2+8b^3+b^4)-2ac(16-12b-8b^2+4b^3+b^4)+c^2(16-8b^2+b^4)}{2(16-20b^2+5b^4)} & \text{in Case } F, \end{cases}$$

Owner 1:

$$\begin{cases} \Pi_1^S = \frac{8(2-b^2)(a-ab+bc)^2}{(8-8b^2+b^4)^2} & \text{in Case } S, \\ \Pi_1^L = \frac{4(2-b)(2+b)(4-3b^2)(a-ab+bc)^2}{(16-20b^2+5b^4)^2} & \text{in Case } L, \\ \Pi_1^F = \frac{2(2-b)^2(2+b)^2(2-b^2)(a-ab+bc)^2}{(16-20b^2+5b^4)^2} & \text{in Case } F. \end{cases}$$

These payoffs can be ordered according to the following ranking:

$$\text{Owner 0:} \quad W^L > W^F > W^S, \tag{2}$$

$$\text{Owner 1:} \quad \Pi_1^F > \Pi_1^L > \Pi_1^S. \tag{3}$$

Meanwhile, if the owners directly manage their respective firms (in other words, V_0 and V_1 are replaced by W and Π_1, respectively), the payoff ranking of the three cases is as follows (Pal, 1998).

$$\text{Owner 0:} \quad \widehat{W}^F > \widehat{W}^L > \widehat{W}^S,$$

$$\text{Owner 1:} \quad \begin{cases} \widehat{\Pi}_1^F > \widehat{\Pi}_1^L > \widehat{\Pi}_1^S & \text{if} \quad b < \frac{2\sqrt{2}}{3}, \\ \widehat{\Pi}_1^L > \widehat{\Pi}_1^F > \widehat{\Pi}_1^S & \text{otherwise}, \end{cases}$$

where the hat ($\widehat{\ }$) represents the case in which there is no delegation and the firms are managed by their owners. The ranking of Owner 0 changes with the introduction of the delegation. In this no-delegation case, Owner 0 desires to move late in order to reduce his/her own firm's output without greatly reducing total output and to cut down the total cost. However, in the above delegation case, Owner 0 wishes to move first since he/she cannot control the firm's output completely. On the other hand, when the substitutability of the goods is sufficiently large, the ranking of Owner 1 also changes.

In the first stage, both the owners choose the timing of setting their own outputs in the third stage. This game can be described in the following matrix. Thus, by the payoff rankings (2) and (3), the equilibria in this stage are $(t_0, t_1) = (first,\ second)$ *and* $(second,\ first)$. Therefore, we obtain the following proposition.

		Owner 1	
		first	second
Owner 0	first	$(W^S,\ \Pi_1^S)$	$(W^L,\ \Pi_1^L)$
	second	$(W^F,\ \Pi_1^F)$	$(W^S,\ \Pi_1^S)$

Matrix 1: The choice of the timing

Proposition 1. *There are two subgame perfect Nash equilibria in the three-stage game with delegation. In one equilibrium, the timing of the two firms is* $(t_0, t_1) = (first,\ second)$ *and the delegation parameters, the quantities of both the firms* ($q_i^k,\ i = 0, 1;\ k = L, F$)*, and the payoffs of both the owners are as follows:*

$$\theta_0^L = \frac{a(32 - 32b - 24b^2 + 28b^3 + 2b^4 - 5b^5) - 2c(16 - 12b^2 + b^4)}{2(16 - 20b^2 + 5b^4)}, \qquad \theta_1^L = \frac{4ab^2(1-b) + 4b^3 c}{16 - 20b^2 + 5b^4},$$

$$q_0^L = \frac{a(16 - 12b - 8b^2 + 5b^3) - 8c(2 - b^2)}{16 - 20b^2 + 5b^4}, \qquad q_1^L = \frac{2(2-b)(2+b)(a - ab + bc)}{16 - 20b^2 + 5b^4},$$

$$W^L = \frac{\begin{bmatrix} a^2(448-384b-752b^2+608b^3+404b^4-296b^5-77b^6+50b^7) \\ -2ac(256-192b-448b^2+304b^3+256b^4-148b^5-52b^6+25b^7)+4c^2(64-112b^2+64b^4-13b^6) \end{bmatrix}}{2(16 - 20b^2 + 5b^4)^2},$$

$$\Pi_1^L = \frac{4(2-b)(2+b)(4-3b^2)(a - ab + bc)^2}{(16 - 20b^2 + 5b^4)^2}.$$

In the other equilibrium, the timing is $(t_0, t_1) = (second,\ first)$ *and the respective values are*

$$\theta_0^F = \frac{a(16 - 16b - 4b^2 + 6b^3 - b^4) - c(16 - 4b^2 - b^4)}{16 - 20b^2 + 5b^4}, \qquad \theta_1^F = 0,$$

$$q_0^F = \frac{a(16 - 12b - 8b^2 + 4b^3 + b^4) - c(4 - b^2)^2}{16 - 20b^2 + 5b^4}, \qquad q_1^F = \frac{2(2-b)(2+b)(a - ab + bc)}{16 - 20b^2 + 5b^4},$$

$$W^F = \frac{a^2(28 - 24b - 12b^2 + 8b^3 + b^4) - 2ac(16 - 12b - 8b^2 + 4b^3 + b^4) + c^2(16 - 8b^2 + b^4)}{2(16 - 20b^2 + 5b^4)},$$

$$\Pi_1^F = \frac{2(2-b)^2(2+b)^2(2-b^2)(a - ab + bc)^2}{(16 - 20b^2 + 5b^4)^2}.$$

These two subgame perfect Nash equilibria $(first,\ second)$ and $(second,\ first)$ are the same in the no-delegation case (Pal, 1998). Therefore, Pal's (1998) result is robust against the case of delegation to managers when the managers are being delegated the output decision.

4 Conclusion

This paper examined a mixed duopolistic model using the observable delay game of Hamilton and Slutsky (1990). We introduced managerial delegation à la Fershtman and Judd (1987) and Sklivas (1987) into the model of Pal (1998) and confirmed the robustness of the results obtained in Pal (1998) against such a extension. We showed that two managerial firms sequentially set outputs in the equilibrium. Therefore, the use of simultaneous quantity choice models with managerial delegation as in Barros (1995) and White (2001) cannot be justified through the endogenous timing game considered in this paper.

In this paper, as well as most existing works in this field, we assume that a private firm is domestic. The interesting extension of our model is to examine the case that a public firm competes against a foreign private firm. In this case, the owner of the public firm takes consumer surplus more seriously than the private firm's profit. Thus, the existence of the foreign private firm may change the decision making of the owner of the public firm and, consequently, the equilibrium outcomes as well. This is left for future research.

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