# Bundling by Competitors and the Sharing of Profits 

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#### Abstract

We discuss the effects of bundling two goods offered by two symmetric firms. This situation requires the use of some sharing rule for the profits from the sales of the bundle. We show that the choice of this rule may have substantial effects on prices and profits - even if the possible rules eventually yield equal shares. In particular, the use of the a priori equal sharing rule yields lower prices and profits, than a price weighted sharing rule. When competitors bundle, they can implicitly cooperate via the setting of the profit sharing rule and increase their profits at the expense of consumers. This issue calls for some further attention by regulators.


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## 1. Introduction

Bundling is a pervasive policy used by firms. Beginning with Stigler (1968), most of the literature considers bundling by a monopolist as a tool for price discrimination (Adams and Yellen, 1976, Matutes and Regibeau, 1992, McAfee, McMillan and Whinston, 1989, Schmalensee, 1984, Spence, 1980), especially when the valuations of the two bundled commodities are negatively correlated, but also when they have independent valuations (McAfee, McMillan and Whinston, 1989). The gains decrease with positive correlation and vanish when this correlation is perfect. More recently, Nalebuff $(1999,2004)$ showed that the main gains for the bundling producer come from entry deterrence. In particular, he proved that the pure price discrimination effect may increase the profit of a bundling monopolist by about nine percent, but if he can make entry deterrence effective, his profits can more than double.

In this paper, we look at pure welfare effects of competitive bundling by competitors. We consider bundles of goods, produced by two competing firms, and allow for a mixed policy where, in addition to offering the bundle, each participating firm continues to offer its own good.

Such practices are common in the cable television industry, where broadcast networks (such as ABC, CBS, or NBC) or cable television networks (MTV, CNN) bundle channels (HBO, SHO etc.) into services where, usually, each channel can also be bought à la carte. See Crawford (2000, 2004), and Coppejans and Crawford (1999) for an analysis of this case. In 2003, Comcast, the largest US cable operator, decided to offer discount packages to customers subscribing simultaneously to its Internet access and to cable television services. The Comcast bundle forced its Internet customers to also sign up for its cable services while allowing its cable customers not to sign up to its Internet services. ${ }^{1}$ Bundles composed of broadband and Internet access services are being offered as joint ventures between ISP's and cable companies. Telecommunication companies bundle local and long distance call, cellular or wireless services provided by different carriers. See Linhart et al. (1995) for an analysis.

The practice of bundling by competitors is also observed more and more frequently in the world of tourism, transportation, culture and entertainment. In particular, museum passes, which give visitors (tourists or residents) unlimited access to a list of participating museums during a limited period of one to several days (perhaps a whole year for residents) have become very common in many cities and countries, or even across nations. Since this happens mainly in the not-for-profit sector, no anti-competitive concerns have been brought up so far. See Ginsburgh and Zang (2003, 2004). New marketing models in scientific publishing, such as those employed by JSTOR, offer to libraries subscriptions to bundles of scientific journals offered by various publishers, and constitute another example of bundling by competitors. This will even get more powerful with CrossRef, evoked by Reed Elsevier (2005, pp. 54-55) as "a wholly independent association of over 1,400 scholarly and professional publishers that cooperate to provide reference links into and out of their electronic content [and which allows] researchers to navigate the online literature at the article level."

Bundling by competitors raises several interesting problems, the first of which is price setting. Throughout most of the literature, prices are set via joint profit maximization by the single firm, offering both the individual goods and the bundle. In our case, individual prices are set by each firm, while the price of the bundle is set jointly by the participating firms or their common agent. A plausible modeling solution is to have a jointly owned subsidiary (called here the bundle identity), which introduces the bundle and conducts its pricing, marketing and sales. Naturally, this subsidiary will maximize its own profit and will share it

[^1]among its owners. This leads to a two-stage pure strategy non-cooperative game in which the bundling entity sets the price of the bundle in the first stage, while each partner sets the price for his own product (also offered as a part of the bundle) in the second stage, taking into account his share in the profits of the bundle entity.

This brings us to the second problem, related to the sharing of the profits generated by the bundle. If the bundled goods are produced by competing firms or by different divisions of a firm managed as profit centers, then it becomes important to devise a sharing rule, if only for accounting purposes. Linhart et al. (1995) and Ginsburgh and Zang (2003) have independently shown that the Shapley value (Shapley, 1953) is a convenient allocation rule applicable for some of these cases, namely when each bundled good is usable only once, when the buyer can also choose not to consume all the goods in the bundle and when her actual consumption can be recorded (for accounting purposes). All these properties are satisfied by museum passes, as well as long distance calls and electronic journal bundles.

In general, our models are such that their analytical solutions are simply numbers. However, due to analytical complexities, solutions can only be obtained by numerical computations. We consider the case of two symmetric producers, each producing a single good or service at zero marginal production cost, and where the reservation prices of potential customers are uniformly distributed and independent. The two producers cooperate in introducing the bundle, but continue to offer their individual products. Hence a profit sharing rule is needed. Following a description of the setup in Section 2, we consider, in Section 3, the effects of the sharing rule when firms offer their individual goods and the bundle is managed by the bundle entity. We analyze two such sharing rules: a priori equal sharing, where profits are split "as is" between the owning partners, and price weighted sharing, where the prices of the individual goods are taken as weights in the sharing formula. In our symmetric two-firm settings both rules end up sharing profits equally. However, the resulting effects on prices and profits are dramatically different when the two firms take the mechanisms of the allocation rules into their optimizing behavior. Indeed, under a priori equal sharing, profits and prices are lower (sometimes substantially so) than under price weighted sharing. Thus, if it is up for producers to decide on the sharing rule, then, for society, the worst option will be chosen. In Section 4 we discuss the effects of positive or negative correlation between products' valuations.

The implications are obvious. It is well known that, as in the monopoly case, bundling by competitors might discourage entry. Here, in addition, bundling competitors can implicitly cooperate to affect product prices and increase their profits by selecting the profit sharing mechanism that is more profitable for them. This might call for intervention by the regulator.

## 2. The General Setup

Following Nalebuff, we assume that there are two firms producing two goods A and $B$ at zero marginal cost. Customers' willingness to pay is uniformly distributed on the $(0,1) \times(0,1)$ square (market of size 1 for each good) and there are no budget considerations, implying that the valuations for the two goods are independent. We assume that entry in not possible and hence each firm is a monopoly in its own product market. Each consumer buys one unit of either A , or B or one unit of the bundle at prices $C_{A}, C_{B}$ and $C$, respectively. Figure 1 shows how the $(0,1) \times(0,1)$ square will be partitioned.

Figure 1. Segmentation of the market with mixed bundling


Customers will be partitioned as follows: (i) Those whose reservation price for A is not smaller than $C_{A}$ and whose reservation price for B is not greater than $C-C_{A}$ will purchase A only. (ii) Those whose reservation price for B is not smaller than $C_{B}$ and whose reservation price for A is not greater than $C-C_{B}$ will purchase B only. (iii) Those whose reservation price for the bundle (the sum of the individual products reservation prices) is not smaller than $C$, and their reservation prices for A and B are not smaller than $C-C_{B}$ and $C-C_{A}$ respectively, will purchase the bundle.

Using the figure, the number of buyers of $A$ and $B$ is $\left(1-C_{A}\right)\left(C-C_{A}\right)$ and $\left(1-C_{B}\right)\left(C-C_{B}\right)$, respectively, while $\frac{1}{2} \times\left((2-C)^{2}-\left(1-C_{A}\right)^{2}-\left(1-C_{B}\right)^{2}\right)$ will buy the bundle. It follows that the profit generated by the bundle is given by

$$
\begin{equation*}
\Pi\left(C_{A}, C_{B}, C\right)=C \times \frac{1}{2} \times\left((2-C)^{2}-\left(1-C_{A}\right)^{2}-\left(1-C_{B}\right)^{2}\right) \tag{1}
\end{equation*}
$$

while the profits generated from the sales of the individual products are:

$$
\begin{align*}
& \prod_{A}\left(C_{A}, C_{B}, C\right)=C_{A}\left(1-C_{A}\right)\left(C-C_{A}\right),  \tag{2}\\
& \prod_{B}\left(C_{A}, C_{B}, C\right)=C_{B}\left(1-C_{B}\right)\left(C-C_{B}\right) . \tag{3}
\end{align*}
$$

Naturally, prices should satisfy the following intuitive relations: $C \leq C_{A}+C_{B}, C_{A} \leq C, C_{B} \leq C$.

## 3. Bundling by Competitors -- The Sharing Rules

We assume that the two firms cooperate in offering the bundle while maintaining competition in the individual products. We therefore need to specify how the bundling operation works and how its profits are distributed, that is, the sharing mechanism. When the two firms are symmetric, each reasonable sharing mechanism will end up with an equal split. However, since firms behave strategically, they implicitly take into account the marginal effects of the sharing mechanism. This might generate different equilibrium outcomes for
different sharing rules. Hence, if the choice of the sharing mechanism is not regulated, then firms will agree on the mechanism that generates larger profits, without paying attention to a possible decrease in consumer surplus.

We assume that the bundling operation is managed independently by an autonomous profit maximizing entity, called the bundle entity, owned by the two firms, and operating at zero marginal cost. The profit of the entity is distributed among its owners according to some pre-specified rule to be discussed later. Both firms set their prices to maximize their own profit, which includes the profit distributed by the bundle entity. This leads to the following two-stage non-cooperative game.
Stage 1: The bundle entity determines $C$, the bundle price (anticipating the reaction of the individual firms in stage 2).
Stage 2: Given the bundle price, firms set their individual product prices $C_{A}$ and $C_{B}$.
In the sequel, we consider symmetric sub-game perfect Nash equilibria in pure strategies for the above game.

The stage 1 profit function is (1), where $C_{A}$ and $C_{B}$ are determined in stage 2 .
Hence, the stage 1 problem, solved by the bundle entity, anticipating the stage 2 reaction, is

$$
\begin{equation*}
\underset{0 \leq C \leq C_{A}+C_{B}}{\operatorname{Max}} \Pi\left[C_{A}(C), C_{B}(C), C\right] . \tag{4}
\end{equation*}
$$

The stage 2 profit functions $\prod_{A}$ and $\prod_{B}$ vary according to the specific profit distribution rule that applies. The basic structure of these profit functions is:

$$
\begin{align*}
& \Pi_{A}\left(C_{A}, C_{B}, C\right)=C_{A}\left(1-C_{A}\right)\left(C-C_{A}\right)+\beta_{A} \Pi\left(C_{A}, C_{B}, C\right),  \tag{5}\\
& \prod_{B}\left(C_{A}, C_{B}, C\right)=C_{B}\left(1-C_{B}\right)\left(C-C_{B}\right)+\beta_{B} \Pi\left(C_{A}, C_{B}, C\right), \tag{6}
\end{align*}
$$

where $\beta_{A}$ and $\beta_{B}$ are the shares of A and B in the profits of the bundle entity. Note that both can be either constants or functions of $C_{A}, C_{B}$ and $C$, and that, in general, $\beta_{A}$ and $\beta_{B}$ may or may not sum up to 1 . In the cases discussed below we assume that the $\beta$ s do sum up to 1 .

We now turn to discuss two potential sharing rules. These are the a priori equal sharing rule, and the price weighted sharing rule. ${ }^{2}$ We assume that customers who purchase the bundle consume both goods. This is a natural assumption since these are also available individually and the pricing is such that the consumer does not purchase the bundle unless she intends to consume both goods.

## A priori Equal Sharing

In this case, under the assumption that each bundle buyer consumes both A and B, this rule will share the income from the bundle equally between the two firms regardless of their individual actions. Hence $\beta_{A}=\beta_{B}=1 / 2$, and it follows from (5) and (6) that the stage 2 profit functions are:

$$
\begin{align*}
& \prod_{A}\left(C_{A}, C_{B}, C\right)=C_{A}\left(1-C_{A}\right)\left(C-C_{A}\right)+\frac{1}{2} \Pi\left(C_{A}, C_{B}, C\right) \\
& \quad=C_{A}\left(1-C_{A}\right)\left(C-C_{A}\right)+\frac{1}{4} \times C \times\left((2-C)^{2}-\left(1-C_{A}\right)^{2}-\left(1-C_{B}\right)^{2}\right),  \tag{7}\\
& \prod_{B}\left(C_{A}, C_{B}, C\right)=C_{B}\left(1-C_{B}\right)\left(C-C_{B}\right)+\frac{1}{2} \Pi\left(C_{A}, C_{B}, C\right) \\
& \quad=C_{B}\left(1-C_{B}\right)\left(C-C_{B}\right)+\frac{1}{4} \times C \times\left((2-C)^{2}-\left(1-C_{A}\right)^{2}-\left(1-C_{B}\right)^{2}\right), \tag{8}
\end{align*}
$$

where $\Pi$ is given by (1). The stage 2 optimization problem for firm $A$ is:

[^2]\[

$$
\begin{array}{ll}
\operatorname{Max}_{C_{A}} & \prod_{A}\left(C_{A}, C_{B}, C\right) \\
\text { s. t. } & 0 \leq C_{A} \leq 1,  \tag{9}\\
& C_{A} \leq C .
\end{array}
$$
\]

Firm B's stage 2 problem is similar and there is also a combined constraint $C \leq C_{A}+C_{B}$, which, given that firms are identical, can be expressed as $C_{A} \geq C / 2$, and $C_{B} \geq C / 2$. The stage 1 optimization problem is (4). Assuming interior solutions, the numerical solution is reported in Table 1, column (2).

## Prices Weighted Sharing

This rule shares the joint income of the bundle entity proportionally to the individual prices of the two goods. This implies $\beta_{A}=C_{A} /\left(C_{A}+C_{B}\right)$ and $\beta_{B}=C_{B} /\left(C_{A}+C_{B}\right)$. By (5) and (6), the second stage profits are

$$
\begin{align*}
& \prod_{A}\left(C_{A}, C_{B}, C\right)=C_{A}\left(1-C_{A}\right)\left(C-C_{A}\right)+\frac{C_{A}}{C_{A}+C_{B}} \times \Pi\left(C_{A}, C_{B}, C\right) \\
& \quad=C_{A}\left(1-C_{A}\right)\left(C-C_{A}\right)+\frac{C_{A}}{C_{A}+C_{B}} \times \frac{C}{2}\left((2-C)^{2}-\left(1-C_{A}\right)^{2}-\left(1-C_{B}\right)^{2}\right),  \tag{10}\\
& \prod_{B}\left(C_{A}, C_{B}, C\right)=C_{B}\left(1-C_{B}\right)\left(C-C_{B}\right)+\frac{C_{B}}{C_{A}+C_{B}} \times \Pi\left(C_{A}, C_{B}, C\right) \\
& \quad=C_{B}\left(1-C_{B}\right)\left(C-C_{B}\right)+\frac{C_{B}}{C_{A}+C_{B}} \times \frac{C}{2}\left((2-C)^{2}-\left(1-C_{A}\right)^{2}-\left(1-C_{B}\right)^{2}\right), \tag{11}
\end{align*}
$$

The second stage optimization problem for firm A is (9), where $\prod_{A}$ is now given by (10). Firm B's stage 2 problem is similar and the combined equilibrium solution determines $C_{A}$ and $C_{B}$ respectively as functions of $C$. The stage 1 problem is (4). This leads eventually to the numerical solution reported in Table 1, column (3).

Table 1. Summary of main numerical results

| (1)Indicator | Operating mode |  | (4) |
| :---: | :---: | :---: | :---: |
|  | (2) | (3) |  |
|  | A priori equal sharing | Prices weighted sharing | Percentage change (3)/(2) |
| $C_{A}=C_{B}=$ | 0.403 | 0.725 | 80\% |
| C | 0.647 | 0.921 | 42\% |
| $\prod_{A}=\prod_{B}=$ | 0.24 | 0.272 | 13\% |
| $\Pi=$ | 0.361 | 0.467 | 29\% |
| Producers' profits | 0.479 | 0.545 | 14\% |

Note: Detailed computations are available from the authors.
Note that the a priori equal sharing rule yields the lowest prices. However, producers are more likely to use the weighted sharing rule since it yields higher industry profits.

We state these findings in the following Proposition.

## Proposition.

1. The a priori equal sharing rule yields lower prices and profits as compared to the prices weighted sharing.
2. Both allocation mechanisms end up splitting the profit of the bundle entity equally between the two firms. However, since the nature of the sharing rule affects the final outcome, its choice can be used by the firms to their advantage.

An intuitive explanation to these results follows from comparing the weight factors multiplying (sharing) the bundle entity profit $\Pi\left[C_{A}, C_{B}, C\right]$ in the two sharing cases considered. When a priori equal sharing applies, it is the constant factor of 0.5 in expressions (7) (and similarly in (8)). When price weighted sharing applies, it is the weight $C_{A} /\left(C_{A}+C_{B}\right)$ in expression (10) (and similarly in (11)). As the derivative of the prices weight $C_{A} /\left(C_{A}+C_{B}\right)$ w.r.t $C_{A}$ is the positive expression $C_{B} /\left(C_{A}+C_{B}\right)^{2}$, it follows that, in the equilibrium solution corresponding to a priori equal sharing, when the constant weight of 0.5 is replaced by the above varying prices weight, it becomes marginally beneficial for A (and similarly for B ) to increase her price $C_{A}$ and hence her weight factor $C_{A} /\left(C_{A}+C_{B}\right)$, and subsequently her profit. Thus, the equilibrium solution corresponding to a priori equal sharing, can not hold as equilibrium under price weighted sharing, and the initial deviation from this equilibrium is price increasing. Of course, this does not necessarily imply that the resulting price weighted sharing equilibrium will generate higher prices and profits, as our calculations indeed finally show. But at least this indicates a direction.

By choosing the weighted price sharing, producers can ensure themselves a profit of 0.545 . This can be shown to be only $0.7 \%$ smaller than the maximum possible industry profit of 0.549 attained by a monopoly that is offering the bundle as well as the two individual products. Producers might argue that both sharing rules eventually lead to an equal split. However, since the formulas utilized by the two rules are different, outcomes will also be.

## 4. Correlated valuations

We consider two extreme cases of customers' correlated product valuations. These are the cases of perfect positive and perfect negative correlations of valuations. The analysis of intermediate cases should yield intermediate results.

For the first case, we assume that customers are uniform and their willingness to pay for A and B are perfectly positively correlated. In the second case, we assume that customers’ willingness to pay for A and B are perfectly negatively correlated. This leads to the results displayed in Table 2, columns 2-7.

The conclusion is that perfect positive correlation of preferences leads to identical results on prices and profits for the two cases examined. Under perfect negative correlation, bundling yields higher prices and profits (as compared to the other two cases considered here). Thus, consumers are substantially worse off. We also observe that although, for perfectly negatively correlated preferences, both sharing mechanisms leave consumers with very high prices, a priori equal sharing leads to lower prices than prices weighted sharing.

This has particular importance for the case of scientific publishing, where several competing firms join in offering to bundle their journals. An interesting question is whether bundling in scientific publishing exhibits positive, negative or no correlation. Definitely, there is some positive correlation, in particular among the top class journals. A strong group in a certain specific area will want to have all the good and some of the less good journals.

On the other hand, across disciplines one might find some negative correlation, ${ }^{3}$ but potentially, from the demand point of view, simply no correlation.

Table 2. Summary of results: Perfectly positively and negatively correlated valuations

| Indicator | Perfectly positively correlated valuations |  |  | Perfectly negatively correlated valuations |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | (2) | (3) | (4) | (5) | (6) | (7) |
|  | A priori equal <br> sharing | Weighted prices sharing | Percentage <br> change: (3) over (2) | A priori equal sharing | Weighted prices sharing | Percentage change (6)/(5) |
| $C_{\text {A }}=C_{B}=$ | $\begin{aligned} & C_{A} \geq 0.5 \\ & C_{B} \geq 0.5 \end{aligned}$ | $\begin{aligned} & C_{A}=\infty \\ & C_{B}=\infty \end{aligned}$ | undefined | 0.75 | 0.854 | 14\% |
| C | 1.00 | 1.00 | 0\% | 1.00 | 1.000 | 0\% |
| $\prod_{A}=\prod_{B}=$ | 0.25 | 0.25 | 0\% | 0.4375 | 0.479 | 9\% |
| $\Pi=$ | 0.50 | 0.50 | 0\% | 0.5 | 0.707 | 41\% |
| Producers' profits | 0.50 | 0.50 | 0\% | 0.875 | 0.957 | 9\% |

Note: Detailed computations are available from the authors.

## 5. Conclusions

Our result is concerned with the case in which the bundle is created by competitors and is sold by a profit maximizing entity that is owned by the two firms which competitively continue selling their individual goods. Then, when the two goods are either not correlated or perfectly negatively correlated, we show that the choice of the method to share the joint profit could lead to drastically different outcomes though the profit of the bundle entity is, eventually, shared equally between the two firms. One mechanism, a priori equal sharing, leads to lower prices and profits as compared to prices weighted sharing. Hence, regulation, focusing on the sharing mechanism, might become warranted here, even in the case of an oligopoly.

It is likely that the gist of our results will generalize to situations where there are more than two symmetric firms, selling one good each, though computations could become cumbersome. More work is needed to deal with cases in which (some) firms sell and bundle more than one good, and cases in which there is asymmetry.

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[^1]:    ${ }^{1}$ See Randolph May, The storm over broadband bundling, http://news.com.com/2010-1071-997226.html. (last accessed in December 2003).

[^2]:    ${ }^{2}$ We note that in a broader Game Theory perspective, these rules correspond to sharing according to the Shapley value (Shapley 1953) and the weighted Shapley value, respectively.

[^3]:    ${ }^{3}$ Budget constraints, that libraries nowadays face more than ever, tend to induce outcomes that are similar to those attained when there is negative correlation.

