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This paper studies optimal taxation in dynamic economies with a simple form of human capital accumulation as considered in Bull (1993). We show that in a Ramsey equilibrium along any balanced growth path, the taxes on wage income and (physical) capital income must be zero. Under the assumption on preferences of Bull (1993), we extend his result by showing that along a balanced growth all optimal taxes are necessarily zero.

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1 Introduction

This paper studies optimal taxation in dynamic economies with a simple form of human capital accumulation as considered in Bull (1993). Human capital is modeled like physical capital, with the important exception that human capital is used in the production sector together with raw labor as one factor and can therefore, in contrast to physical capital, not be taxed separately. Bull (1993) only showed that under an additional assumption to the utility function setting all taxes to zero is a possible balanced growth path solution of the first order conditions but did not rule out the existence of other solutions. We show that in a Ramsey equilibrium along any balanced growth path, the taxes on wage income and (physical) capital income must be zero without the additional assumption. We point out that this result allows for a straightforward interpretation in the spirit of the well-known zero tax results of Chamley (1986) and Judd (1985) once one recognizes that the labor income and the consumption tax play a different role here compared to the setup without human capital. With the additional assumption on preferences as imposed in Bull (1993), we extend his result by showing that zero taxes are the only balanced growth path solution of the Ramsey problem[1].

2 The Model

Representative agent There is a infinitely lived representative agent in a single-good economy. The agent has preferences over consumption (public and private) and leisure time. The utility function is time-separable and given in each period by $U(c, g, n) = g^Z \Psi(\frac{c}{g}, n) + X$ where c is private consumption, g public expenditures, $0 \leq n \leq 1$ time spent on working, $Z > 0$, X is an arbitrary constant and Ψ is an arbitrary function. As shown by Bull (1993) this class of utility functions is consistent with balanced growth. Physical capital, k_t, and human capital, h_t, are created simply by saving a portion of output, x_t^k and x_t^h, until the subsequent period.

$$k_{t+1} = (1 - \delta^k)k_t + x_t^k,$$
$$h_{t+1} = (1 - \delta^h)h_t + x_t^h.$$

$\delta^k \geq 0$ and $\delta^h \geq 0$ are the rates at which physical and human capital depreciate. The agent´s optimization problem is (given an initial stock of physical and human capital and bonds in the first period, k_0, h_0 and b_0) to maximize utility with respect to consumption, working time, investment into physical capital, human capital and government bonds (b_t). The maximization problem is subject to the budget constraint per period:

$$\begin{aligned} &\max_{\{c_t, n_t, k_{t+1}, h_{t+1}, b_{t+1}\}_{t=0,1,2...}} \sum_{t=0}^{\infty} \beta^t [g_t^Z \Psi(\frac{c_t}{g_t}, n_t) + X] \\ &\text{s.t.} \quad \sigma_t c_t + k_{t+1} + h_{t+1} + b_{t+1} \leq (1 + (1 - \tau_t)W_t n_t - \delta^h)h_t \\ &\qquad\qquad + (1 + (1 - \theta_t)R_t - \delta^k)k_t + r_t b_t, \quad \forall t. \end{aligned} \tag{1}$$

where σ_t the gross consumption tax (so $\sigma_t = 1$ means consumption is untaxed), W_t the wage rate, τ_t the wage tax, R_t the capital rent, θ_t the capital tax and r_t the gross return on bonds. Human capital is used to produce together with raw time ¨efficiency units¨ of labor,

[1] Jones, Manuelli and Rossi (1997) consider a more complicated two sector growth model of human capital accumulation. Note that in their model, human and physical capital are not simple substitutes produced directly from the final consumption good which makes their analysis different from the one presented here.

$e_t := n_t h_t$. The representative agent takes prices, government spending and tax rates as given. The necessary conditions for an interior solution of the consumer´s problem are given by:

$$\sigma_t g_t \frac{-\Psi_2(t)}{\Psi_1(t)} = (1-\tau_t) W_t h_t, \tag{2}$$

$$\frac{\sigma_{t+1} g_t^{Z-1} \Psi_1(t)}{\sigma_t g_{t+1}^{Z-1} \Psi_1(t+1)} = \beta(1 - \delta^k + (1-\theta_{t+1}) R_{t+1}) = \beta r_{t+1}, \tag{3}$$

$$\frac{\sigma_{t+1} g_t^{Z-1} \Psi_1(t)}{\sigma_t g_{t+1}^{Z-1} \Psi_1(t+1)} = \beta(1 - \delta^h + (1-\tau_{t+1}) W_{t+1} n_{t+1}). \tag{4}$$

By using (2) the last equation can be reformulated as:

$$\sigma_t^{-1} g_t^{Z-1} \Psi_1(t) = \sigma_{t+1}^{-1} g_{t+1}^{Z-1} \Psi_1(t+1) \beta(1-\delta^h) - \beta g_{t+1}^Z \Psi_2(t+1) \frac{n_{t+1}}{h_{t+1}}. \tag{5}$$

Using equation (2) - (4) to replace the after-tax rates of return in the budget constraint of the representative agent yields:

$$\begin{aligned} & \sigma_t^{-1} g_t^{Z-1} \Psi_1(t)(\sigma_t c_t + k_{t+1} + h_{t+1} + b_{t+1} - (1-\delta^h) h_t) \\ \leq \quad & - g_t^Z \Psi_2(t) n_t + \sigma_{t-1}^{-1} g_{t-1}^{Z-1} \Psi_1(t-1) \beta^{-1} (k_t + b_t). \end{aligned} \tag{6}$$

Representative firm A representative firm produces the consumption good with a constant returns to scale production function $f(k_t, e_t)$. Profit maximization implies:

$$R_t = f_1(k_t, n_t h_t), \tag{7}$$

$$W_t = f_2(k_t, n_t h_t). \tag{8}$$

The Ramsey Problem An allocation $\{k_t, h_t, c_t, n_t, g_t\}_{t=0}^{\infty}$ is *feasible* if it fulfills in all periods the resource constraint of the economy:

$$c_t + g_t + h_{t+1} + k_{t+1} \leq f(k_t, n_t h_t) + (1-\delta^h) h_t + (1-\delta^k) k_t. \tag{9}$$

A *competitive equilibrium* consists of a feasible allocation $\{k_t, h_t, c_t, n_t, g_t\}_{t=0}^{\infty}$, a strictly positive and bounded price system $\{W_t, R_t, r_t\}_{t=0}^{\infty}$, and a government policy $\{g_t, \tau_t, \theta_t, \sigma_t, b_t\}_{t=0}^{\infty}$ such that: (i) Given the price system and the government policy: the allocation solves the firm's and the household's maximization problems in each period. (ii) Given the price system and the feasible allocation, the government policy satisfies the government budget constraint in each period.
Given k_0, b_0 and h_0 the *Ramsey problem* for the government is to choose a competitive equilibrium which maximizes the utility of the representative agent.
Hence the government maximizes the utility of the agent with respect to the agent´s first order conditions given by equations (2)-(4), the budget constraint of the agent, the first order conditions of the firm and the resource constraint of the economy. Applying the primal approach to the Ramsey problem [Lucas and Stokey (1983)] the agent's and the firm's first order conditions will be used to define the tax rates $\{\theta_t, \tau_t\}_{t=0}^{\infty}$ and the prices $\{W_t, R_t, r_t\}_{t=0}^{\infty}$. The government can now be thought of as directly choosing an allocation $\{k_{t+1}, h_{t+1}, c_t, n_t, g_t\}_{t=0}^{\infty}$, the stream of government bonds $\{b_{t+1}\}_{t=0}^{\infty}$ and the stream of inverse consumption tax rates $\{z_t\}_{t=0}^{\infty} := \{\sigma_t^{-1}\}_{t=0}^{\infty}$, and maximizing the utility with respect to the resource constraint which

is unchanged, the adjusted budget constraint of the agent (6) and the Euler equation for human capital accumulation (5) which cannot be used to eliminate the consumption tax σ_t:

$$\max_{\{k_{t+1},h_{t+1},c_t,n_t,g_t,b_{t+1},z_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t [g_t^Z \Psi(\frac{c_t}{g_t}, n_t) + X]$$

subject to:

$$\begin{aligned}
\lambda_t: &\quad z_t g_t^{Z-1} \Psi_1(t)(z_t^{-1} c_t + k_{t+1} + h_{t+1} + b_{t+1} - (1-\delta^h)h_t) \\
&\quad \le -g_t^Z \Psi_2(t) n_t + z_{t-1} g_{t-1}^{Z-1} \Psi_1(t-1)\beta^{-1}(k_t + b_t), \\
\mu_t: &\quad c_t + g_t + h_{t+1} + k_{t+1} \le f(k_t, n_t h_t) + (1-\delta^h)h_t + (1-\delta^k)k_t, \\
\nu_t: &\quad z_t g_t^{Z-1} \Psi_1(t) = z_{t+1} g_{t+1}^{Z-1} \Psi_1(t+1)\beta(1-\delta^h) - \beta g_{t+1}^Z \Psi_2(t+1)\frac{n_{t+1}}{h_{t+1}},
\end{aligned}$$

where $\beta^t\lambda_t, \beta^t\mu_t, \beta^t\nu_t$ are Lagrange multipliers associated with the constraints. We omit further conditions that are relevant for the initial period and focus only on the periods $t > 0$.

3 Results

Balanced Growth Path Suppose that there is a balanced growth path with an endogenous given non-negative growth rate ξ. From the resource constraint of the economy, the first order conditions of the firm and the budget constraint of the agent follows that a balanced growth path with growth rate $\xi \ge 0$ is characterized by: $\xi = \frac{f(t+1)}{f(t)} - 1 = \frac{k_{t+1}}{k_t} - 1 = \frac{b_{t+1}}{b_t} - 1 = \frac{c_{t+1}}{c_t} - 1 = \frac{h_{t+1}}{h_t} - 1 = \frac{g_{t+1}}{g_t} - 1$ and $n_t = n$, $W_t = W$, $R_t = R$, $r_t = r$, $\tau_t = \tau$, $\sigma_t = \sigma$ and $\theta_t = \theta$.

Proposition 1 *It is necessary for optimality that taxes on wage and capital income are equal to zero along any balanced growth path: $\tau = \theta = 0$.*
The optimal consumption tax is given by: $\sigma = -W\frac{h_t}{g_t}\frac{\Psi_1}{\Psi_2}$ which does not in general imply that the consumption tax is equal to zero ($\sigma = 1$).

We will introduce an additional condition[2] on the utility function of the agent:

Condition 1 $\frac{c}{g}\frac{\Psi_{12}(\frac{c}{g},n)}{\Psi_2(\frac{c}{g},n)} = 1 + \frac{c}{g}\frac{\Psi_{11}(\frac{c}{g},n)}{\Psi_1(\frac{c}{g},n)}$.

Proposition 2 *The consumption tax along a balanced growth path is equal to zero if and only if condition 1 holds.*

This result implies that all taxes have to be zero along a balanced growth path and government expenditures will be financed by the return on (negative) bonds.

4 Intuition and Conclusion

Bull (1993) only proved that under condition 1 zero taxes are a possible solution of the Ramsey problem along a balanced growth path. We prove the more general result that under condition

[2]This condition is consistent with standard utility functions of the form $U(c,n,g) = g^Z \frac{(\frac{c}{g})^{1-\theta}}{1-\theta} v(n)$, for $\theta > 0$, and $U(c,n,g) = \ln\frac{c}{g} + v(n) + Z\ln g$, for $\theta = 1$, where $v(n)$ is any differentiable function in n.

1 all taxes are along a balanced growth path necessarily zero, and that even without condition 1 taxes on capital and labor income are in any case zero along a balanced growth path.

This last fact can be easily explained by noting that for the zero tax on labor income, the same mechanism as for the zero capital income tax is at work. To see this, note that physical and human capital are completely symmetric in this model from a technological perspective. Hence, if it is not optimal to tax physical capital, it should also not be optimal to tax human capital. However, the tax rate on the marginal product of human capital (which is the marginal product of efficiency units of labor), is also the tax on the marginal product of raw labor, and thus enters also in the static consumption leisure choice (see (2)). At first sight, this seems to make it difficult to pin down the intertemporal margin for human capital accumulation without affecting the static consumption leisure choice. However, the consumption tax enters also in (2), and thus *any* consumption leisure wedge can be implemented for *any* given wage tax. Since the consumption tax on a balanced growth path is time invariant, it does not distort the intertemporal margins in (3) and (4). Thus, it is possible to pin down the intratemporal and intertemporal margins separately and the same logic as in Chamley (1986) or Judd (1985) can be applied to see that $\tau = 0$.

Note that here in contrast to Chamley (1986) or Judd (1985), having both labor income and consumption taxes does not lead to an indeterminacy, but the labor income tax acts as a tax on the return of human capital and the consumption tax is used to distort the consumption leisure decision efficiently.

To see the impact of human capital, compare this model with the corresponding exogenous growth model without human capital of Chamley (1986). The wage tax, which is equivalent to the consumption tax in such a simple setup, is also equal to zero if and only if the following condition 2 holds: $\frac{c_t}{g_t}\frac{\Psi_{11}}{\Psi_1} + \frac{\Psi_{12}}{\Psi_1}n = \frac{c_t}{g_t}\frac{\Psi_{12}}{\Psi_2} + \frac{\Psi_{22}n}{\Psi_2}$ which is a much stronger condition than condition 1. If we assume for instance like in Barro and Sala-i-Martin (2004) that the disutility of work takes a constant-elasticity form $\omega(n) = -\xi n^{1+\sigma}$ condition 2 is fulfilled for utility functions of the form $u(c,n) = \frac{c^{1-\theta}\exp[(1-\theta)\cdot\omega(n)]-1}{1-\theta}$ if and only if $\sigma = -1$ and hence $\omega(n) = -\xi$ which cannot model the disutility of work. Whereas condition 1 is fulfilled for any differentiable function $\omega(n)$ in n.

5 Proofs

Proposition 1:
The first-order conditions for the government with respect to b_{t+1}, k_{t+1} and z_t along the balance growth path can be shown to reduce to $\lambda_t = \lambda_{t+1} =: \lambda$ and

$$k_{t+1}: \quad \frac{\mu_t}{\mu_{t+1}} = \beta(f_1 + 1 - \delta^k), \tag{10}$$

$$z_t: \quad (\nu_t - \nu_{t-1}(1-\delta^h)) = -\lambda(h_{t+1} - h_t(1-\delta^h)). \tag{11}$$

It follows[3] that ν_t has to grow with the same growth rate as h_t: $\frac{\nu_t}{\nu_{t-1}} - 1 = \frac{h_t}{h_{t+1}} - 1 = \xi$. Reformulation of (11) yields $\frac{\nu_t}{h_{t+1}}(1 - \frac{\nu_{t-1}}{\nu_t}(1-\delta^h)) = -\lambda(1 - \frac{h_t}{h_{t+1}}(1-\delta^h))$. Hence it follows for a positive growth rate $(\xi = \frac{h_{t+1}}{h_t} - 1 \geq 0)$:

$$\frac{\nu_t}{h_{t+1}} = -\lambda. \tag{12}$$

[3]This follows from (11) by solving this difference equation for v_t. Given that $h_t = c\xi^t$ for some constant $c > 0$, one can solve the corresponding first order linear difference equation which yields the desired conclusion.

The first order conditions with respect to human capital are:

$$h_{t+1}: \quad \mu_t - \mu_{t+1}(1-\delta^h + f_2 n)\beta = \lambda\sigma^{-1} g_t^{Z-1}\Psi_1 - \lambda\beta\sigma^{-1} g_{t+1}^{Z-1}\Psi_1(1-\delta^h) - \nu_t\beta g_{t+1}^Z \Psi_2 \frac{n}{h_{t+1}^2}.$$

By using equation (12) this can be rewritten as: $\mu_t - \mu_{t+1}(1-\delta^h + f_2 n)\beta = \lambda[\sigma^{-1} g_t^{Z-1}\Psi_1 - \beta\sigma^{-1} g_{t+1}^{Z-1}\Psi_1(1-\delta^h) + \beta g_{t+1}^Z \Psi_2 \frac{n}{h_{t+1}}]$. The term on the right side is equal to zero because of first order condition (5) of the agent along the balanced growth path. Thus the equation can be abbreviated to:

$$\frac{\mu_t}{\mu_{t+1}} = \beta(1-\delta^h + f_2 n). \tag{13}$$

The first order conditions with respect to consumption take the following form:

$$\begin{aligned} c_t: \quad & g_t^{Z-1}\Psi_1 - \lambda g_t^{Z-1}\Psi_1 - \mu_t + \lambda\sigma^{-1} g_t^{Z-2}\Psi_{11}(k_{t+1} + b_{t+1}) \\ & + \lambda(-g_t^{Z-1}\Psi_{21} n - \sigma^{-1} g_t^{Z-2}\Psi_{11}(t)(\sigma c_t + k_{t+1} + h_{t+1} + b_{t+1} - (1-\delta^h)h_t)) \\ & - \nu_t\sigma^{-1} g_t^{Z-2}\Psi_{11} + \nu_{t-1}(\sigma^{-1} g_t^{Z-2}\Psi_{11}(1-\delta^h) - g_t^{Z-1}\Psi_{21}\frac{n}{h_t}) = 0. \end{aligned}$$

It can be directly seen that some terms will drop out. Using equation (12) yields:

$$\frac{\mu_t}{g_t^{Z-1}} = (1-\lambda)\Psi_1 - \lambda\frac{c_t}{g_t}\Psi_{11}. \tag{14}$$

It follows that the term $\frac{\mu_t}{g_t^{Z-1}}$ has to be a constant because the term on the right side of the last equation is constant. Thus μ_t has the same growth rate as g_t^{Z-1}: $\frac{\mu_{t+1}}{\mu_t} = (\frac{g_{t+1}}{g_t})^{Z-1}$. Hence comparing equation (13) with the first order condition (4) of the agent along the balanced growth path and equation (10) with the agent's first order condition (3) respectively yields: $\tau = 0$ and $\theta = 0$.

Proposition 2:

The first order conditions with respect to labor time take the form:

$$\begin{aligned} n_t: \quad & g_t^Z\Psi_2 + \lambda[-g_t^Z\Psi_2 - g_t^Z\Psi_{22} n - \sigma_t^{-1} g_t^{Z-1}\Psi_{12}(\sigma_t c_t + k_{t+1} + h_{t+1} + b_{t+1} - (1-\delta^h)h_t) \\ & + \sigma_t^{-1} g_t^{Z-1}\Psi_{12}(k_{t+1} + b_{t+1})] + \mu_t f_2 h_t - \nu_t\sigma_t^{-1} g_t^{Z-1}\Psi_{12} \\ & + \nu_{t-1}[\sigma_t^{-1} g_t^{Z-1}\Psi_{12}(1-\delta^h) - g_t^Z\Psi_{22}\frac{n}{h_t} - g_t^Z\Psi_2 h_t^{-1}] = 0. \end{aligned}$$

Substituting $-\lambda$ for $\frac{\nu_t}{h_{t+1}}$ and dropping some terms yields: $f_2\frac{h_t}{g_t}\frac{\Psi_1}{\Psi_2} = (\frac{\mu_t}{g_t^{Z-1}})^{-1}\,[\lambda\frac{c_t}{g_t}\frac{\Psi_{12}\Psi_1}{\Psi_2} - \Psi_1]$. Using equation (14) this can be rearranged to:

$$f_2\frac{h_t}{g_t}\frac{\Psi_1}{\Psi_2} = \frac{-\Psi_1 + \lambda\Psi_1\frac{\Psi_{12}}{\Psi_2}\frac{c_t}{g_t}}{\Psi_1 - \lambda[\Psi_1 + \frac{c_t}{g_t}\Psi_{11}]}. \tag{15}$$

From the agent's first order condition (2) along a balanced growth path we obtain the gross consumption tax in the following form: $\sigma = -f_2\frac{h_t}{g_t}\frac{\Psi_1}{\Psi_2}$. Using equation (15) it follows:

$$\sigma = \frac{\Psi_1 - \lambda\Psi_1\frac{\Psi_{12}}{\Psi_2}\frac{c_t}{g_t}}{\Psi_1 - \lambda[\Psi_1 + \frac{c_t}{g_t}\Psi_{11}]}.$$

Hence consumption will not be taxes (i.a. $\sigma = 1$) if and only if $\Psi_1 + \frac{c_t}{g_t}\Psi_{11} = \frac{\Psi_{12}}{\Psi_2}\Psi_1\frac{c_t}{g_t}$ which is exactly condition 1.

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