Two-sided Matching, Who Marries Whom? And what Happens upon Divorce?

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Abstract

Conventional two-sided matching game is a one-period game. In this note, we contribute to the existing literature by examining a multi-period two-sided matching problem allowing for the possibility of a divorce. We assume that the matching game is played repeatedly and the payoff matrix changes over time. It is shown that the rule of divorce will affect the equilibrium of a marriage game. An empirical implication of our result is that a country with a well-developed financial market will have a better marital outcome as compared to a less-developed country.

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1 Introduction

The Two-sided matching theory has ample applications. In particular, the theory is widely applied to model the marriage market. However, most studies in the existing literature focus on a single-period matching game. In the conventional studies, it is usually assumed that each player chooses a partner to maximize their expected payoff in a one-shot manner (Becker, 1976, 1981; Roth and Sotomayor, 1990). In the real world, state of nature changes over time and a divorce may happen (Gale and Shapley, 1962; Becker et al., 1977; McAfee, 1992; Weiss, 1997). A one-shot two-sided matching game may not be able to characterize the divorce behavior. Therefore, a more complete matching game should also consider the possibility of a change of the state of nature.

This note extends the conventional one-shot matching game to a dynamic T-period ($1 < T < \infty$) game allowing for the possibility of a divorce. We consider two popular forms of divorce, namely, divorce that requires mutual consents and the unilateral (no-fault) divorce. We also compare the outcomes between the cases with and without a transfer.

Let $H = \{m_1, m_2, ..., m_n\}$ be a set of men, $F = \{f_1, f_2, ..., f_n\}$ be a set of women and E be the set of all conceivable outcomes. For instance, everybody remains single is an element in E. In an economy consisting of n men and n women, with heterosexual one-to-one marriage and remaining single is allowed, the number of elements in E is $|E| = \sum_{s=0}^{n} \binom{n}{s}^2 (n-s)!$[1]. The marriage market is said to be cleared if everybody gets married.Suppose preferences are substitutable[2] and suppose the state of nature is known in the beginning of each period. Without loss of generality, the return for remaining

[1] This comes from the facts that there are $\binom{n}{s}$ ways of picking s singles from n men or women, and $(n-s)!$ possible combinations of the remaining $(n-s)$ pairs of men and women.

[2] See Definition 6.2 of Roth and Sotomayor (1990).

single is assumed to be zero. Suppose the payoff of marriage is split evenly between husband and wife, Tables 1 and 2 show how much each individual will get in a marriage game with $T = n = 2$.

	f_1	f_2
m_1	2	3
m_2	9	11

Period 1

Table 1

	f_1	f_2
m_1	15	1
m_2	16	1

Period 2

Table 2

In period 1, all women prefer m_2 and all men prefer f_2. Thus, m_2 will choose f_2 and therefore m_1 will marry f_1 since getting married is assumed to be better than remaining single. In period 2, both men prefer f_1. Hence, f_1 marries m_2 and m_1 marries f_2 is the equilibrium. The equilibrium is optimal since it maximizes the total payoff in both periods. We will examine how the rule of divorce alters the equilibrium in a multi-period marriage game.

2 Mutual-Consent Divorce without a Transfer

Suppose the divorce rule is mutual consent without a transfer. For simplicity, we also assume that the discount rate is zero. In period 1, both women prefer m_2 and his decision determines the outcome of the game. If m_2 would like to maximize his return for period 1 only, he will choose f_2. As a result, m_1 will marry f_1. In period 2, however, m_2 would like to divorce f_2 and f_1 would like to divorce m_1. The problem is that m_1 does not want to be divorced. Consequently, f_1 cannot divorce m_1. Since m_2 cannot find someone to get married with if he divorces f_2, by the assumption that remaining single is worse than getting married, m_2 and f_2 will not divorce each other. As such, m_2 will only get a payoff of $12(11 + 1)$ in both periods, while he can get a payoff of $25(9+16)$ if he chooses f_1 in the first period. This implies that the outcome in each period in a multi-period marriage game may be different from the outcome of a single-period game.

3 Mutual-Consent Divorce with a Transfer

One may wonder if the example can yield an optimal outcome if a transfer is allowed. It can be shown that the result may remain unchanged even if an intra-temporal transfer and saving are allowed. To see this, assume that inter-couple and intra-couple transfers are allowed at zero transaction cost and that the payoff is not perishable. However, we assume that people cannot borrow against their future.

If m_2 marries f_2 in the first period, he will ask m_1 to divorce f_1 in the second period. The minimal compensation for m_1 to divorce f_1 is $14(15-1)$, so that he is indifferent between f_1 and f_2.

Despite the fact that f_1 would like to divorce m_1, she cannot afford 14. What she can afford is 1. This means that m_2 has to pay at least 13 to m_1, and m_2 can get at most $3(16-13)$ in period 2, so that the total for m_2 in both periods is $14(11+3)$.

However, if m_2 marries f_1 in period 1, he does not need to compensate anybody in period 2, so that he can make $25(9+16)$ totally. Thus, if nobody pays m_2 in period 1, he will not marry f_2. It should be mentioned that f_2 is willing to pay at most 8 to m_2 in period 1, but this is insufficient as $22(14+8)$ is less than 25.

Note that m_1 would also like to pay somebody to make himself not to marry f_2 in the first period in order to avoid getting a payoff of 1 in the second period. However, even if m_1 pays 2 to m_2, it is still not enough to bribe m_2 not to marry f_1 in period 1. Thus, m_1 would like make a contract with m_2 to charge him a lower compensation in period 2. Such a contract, however, is time inconsistent. Once m_2 has married f_2 in period 1, m_1 will not demand any compensation less than 14 in period 2.

Thus, the final outcome will be: m_2 marries f_1, m_1 marries f_2 in period 1 and nobody gets divorced in period 2. This is not a social optimal outcome. Therefore, in a society where people could not borrow against their future, the equilibrium marital status in a multi-period marriage game may not be optimal.

4 Unilateral Divorce

Under no-fault divorce, people get divorced as long as at least one party terminates the marriage contract.

If divorce is unilateral and if there is no transfer, m_2 does not need to compensate m_1 in period 2 as f_1 would like to divorce m_1. Thus, m_2 will marry f_2 in the first period to maximize his total return $(11 + 16)$.

People now become myopic. If a man knows that the woman he prefers in the next period is willing to marry him, the problem reduces to a single-period marriage game.

If transfer is feasible, then m_2 will choose f_2 in period 1, m_1 is willing to pay up to 14 to f_1 for not divorcing him in period 2. Now, m_2 needs to pay at least 13 to f_1 to attract her and he can get a payoff of 3 at most in the second period. The right of getting compensation goes to f_1 now. If m_2 marries f_1 in period 1, m_1 will seduce f_1 in period 2 by paying her up to $14(15 - 1)$, and f_1 will divorce m_2 if he does not react. In both cases, m_2 gets at most 3 in period 2. Hence, m_2 will marry f_2 in period 1 to maximize his first-period return.

Proposition 1: If

(i) Preferences are substitutable;

(ii) Intra-temporal transfer (a transfer within or across couples) is allowed;

(iii) Inter-temporal transfer (i.e., saving and borrowing against the future) is feasible;

then, the equilibrium marital status in a finite multi-period marriage-divorce game will be optimal irrespective of the form of divorce.

Proof.

Consider a T-period $(0 < T < \infty)$ game. Since the payoff is intratemporally transferable, the outcome in the last period will be in the core. Thus, in the last period, the payoff should be maximized.

Given that people know their marital status in period T and given the rule of divorce, they will make a choice in period $(T-1)$ to maximize the sum of the payoff in periods $(T-1)$ and T. By the same argument, the strategy adopted in period $(T-2)$ has to maximize the total payoff of the last three periods. By deduction, the first period decision maximizes the payoff of all the T periods. The payoff of all the T periods is maximized if and only if the payoff in every single period is maximized. Therefore, the outcome is optimal. ■

5 Conclusion

This note extends the conventional one-period two-side matching game to multi-period to allow for the possibility of a divorce. The model discussed here is simple and technically tractable. We show that in a society where people cannot borrow against their future, the game may not have an optimal outcome. It is found that if preferences are substitutable and if intra- and inter-temporal transfers are feasible, the equilibrium marital outcome maximizes the total payoff of a society. An important implication of our result is that a society with a well-developed financial market will have a better marital outcome as compared to a financially less-developed society.

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