Sectorial sift, inverted U-shaped fertility dynamics, and growth

Ken Tabata
Kobe City University of Foreign Studies

Kazuhiro Yamamoto
Graduate School of Economics, Osaka University

Abstract

This paper constructs a small open two-sector overlapping-generations model with the subsistence level of consumption of agricultural goods and explains the following key stylized facts in the process of economic development: increases followed by declines in fertility rate, increases in human capital investment for children, and a sectorial shift in labor from agriculture to manufacture.

Citation: Tabata, Ken and Kazuhiro Yamamoto, (2006) "Sectorial sift, inverted U-shaped fertility dynamics, and growth." *Economics Bulletin,* Vol. 10, No. 5 pp. 1-7
Submitted: June 16, 2006. **Accepted:** July 14, 2006.
URL: http://economicsbulletin.vanderbilt.edu/2006/volume10/EB-06J10003A.pdf

Sectorial Shift, Inverted U-shaped Fertility Dynamics, and Growth

Ken Tabata* Kazuhiro Yamamoto †

July 13, 2006

Abstract

This paper constructs a small open two-sector overlapping-generations model with the subsistence level of consumption of agricultural goods and explains the following key stylized facts in the process of economic development: increases followed by declines in fertility rate, increases in human capital investment for children, and a sectorial shift in labor from agriculture to manufacture.

Keywords: Demographic transition; Sectorial sift; Economic development
JEL classification: I28: J13 : O11

*Address: Kobe City University of Foreign Studies, 9-1, Gakuenhigashimachi, Nishi-ku, Kobe 651-2187, Japan; E-mail: tabataken@inst.kobe-cufs.ac.jp

†Corresponding author. Address: Graduate School of Economics, Osaka University, 1-7, Machikaneyama, Toyonaka, Osaka 560-0043, Japan; E-mail: yamamoto@econ.osaka-u.ac.jp

1 Introduction

The comprehensive historical data used in Dyson and Murphy (1985) suggests that the fertility rate increases at low income levels but decreases at high income levels. In addition, Mitchell (1981) has pointed out that a sectorial shift in labor from agriculture to manufacture occurs in the process of economic development. Kogel and Prskawetz (2001) and Tamura (2002) have focused on the sectorial shift in an attempt to explain the inverted U-shaped fertility dynamics.

This paper expands on the views presented by Tabata (2003), and provides a simple alternative model which generates the inverted U-shaped fertility dynamics and sectorial shift in labor from agriculture to manufacture in the process of economic development.

2 The model

Consider a two sector overlapping generations economy. There are two types of homogenous products, i.e., agricultural goods (good A) and manufactured goods (good M). Both goods are consumed, but only manufactured goods can be invested to increase the physical capital. The production of agricultural goods requires only labor input. In the agricultural sector, the production function is described by $Y_{At} = AL_{At}$, where Y_{At} is the output of the agricultural products in period t, A is the level of productivity, and L_{At} is the labor input in terms of efficiency units. On the other hand, the production of manufactured goods requires two inputs: capital and labor. In the manufacturing sector, the production function is described by $Y_{Mt} = ML_{Mt}^{\alpha}K_t^{1-\alpha}$ $(0 < \alpha < 1)$, where Y_{Mt} is the output of the manufactured products in period t, M is the level of productivity, L_{Mt} is the labor input in terms of efficiency units, and K_t is the amount of capital inputs.

This paper assumes that the market for agricultural goods and that for labor are closed at the domestic level and their prices are determined within the economy while manufacturing goods and physical capital are freely mobile at the international level. [1] Since capital is perfectly mobile, the interest rate r_t is set to be constant at the world interest rate r. Hence, due to profit maximization under perfect competition, the wage rate in the manufacturing goods sector w_{Mt} is also set to be constant at $w \equiv \alpha(\frac{1-\alpha}{r})^{\frac{1-\alpha}{\alpha}} M^{\frac{1}{\alpha}}$, where manufacturing goods are taken as the numeraire. On the other hand, the wage rate in the agricultural sector w_{At} is given by $w_{At} = p_t A$, where p_t is

[1] This paper focuses upon developing economies that do not have sufficient agricultural productivity to rely on the exports of goods for income growth.

the price of agricultural goods. Since workers move to the sector offering a higher wage rate, the perfect mobility of labor equalizes w_{At} and w_{Mt}. In other words,

$$w = p_t A. \tag{1}$$

Therefore, the price of agricultural goods p_t is also set to be constant at $p = \frac{w}{A}$.

The economy is populated by a large number of agents who live for three periods, namely, childhood, young age, and old age. The cohort whose young age is in period t is called generation t, and the size of generation t is denoted by N_t. After being raised and educated by their parents in the first period of life, agents born in period $t-1$ enter their second period of life with a given amount of human capital h_t. In the second period of life, agents are endowed with one unit of time, which is spent on rearing n_t identical children, on educating each child, u_t, and on working in the labor market, l_t, and obtaining wage income, wh_tl_t. Then, the agents retire in the third period of life.

Agents have preferences in relation to the consumption of agricultural goods in young age, c_{At}, manufactured goods in old age, c_{Mt+1}, the number of children they have, n_t, and the human capital level of their children, h_{t+1}. Following Kogel and Prskawetz (2001), we assume that agents have hierarchical preferences for agricultural products. Therefore, agents spend their incomes exclusively on agricultural goods up to the subsistence level, $x > 0$, and the demand for agricultural goods, which serve merely for survival, saturates at x. For analytical convenience, we only consider the case that the individual disposable income is sufficiently high to finance the subsistence level of consumption of agricultural products, x (i.e., $p_t x < wh_tl_t$), and each individual inelastically demands the same quantity, x (i.e., $c_{At} = x$). Then, the remaining income is saved for c_{Mt+1}, the consumption of manufactured goods in old age.

Therefore, the lifetime budget and time constraints of each agent are:

$$c_{Mt+1} = R(wh_tl_t - px), \tag{2}$$

$$l_t + (e + u_t)n_t = 1, \tag{3}$$

$$u_t \geq 0, n_t \geq 0, \tag{4}$$

where $R \equiv 1+r$. Following Becker (1965) and others, we assume that it takes fixed e and variable u_t amounts of time to raise and educate each child. In addition, we assume the following educational technology:

$$h_{t+1} = \eta(a + bh_t u_t)^\sigma, \tag{5}$$

where η, a, b, and σ express the parameters for educational technology. The above equation implies that the human capital level of parents h_t plays a crucial role to determine the effectiveness of the human capital investment in their children. [2]

Given these budget and time constraints from (2), (3), (4), and (5), agents in generation t decide their allocation of time in their young age so as to maximize the following utility function u^t:

$$u^t = \gamma \ln c_{Mt+1} + (1-\gamma)\ln n_t h_{t+1}, \tag{6}$$

where γ represents the agents' concern for their own consumption of manufactured goods.

Here, for the sake of clarity in the following discussion, we restrict our analysis to a set of parameters satisfying the following conditions:

$$\eta a^\sigma > \frac{x}{A}, \tag{7}$$

$$h_0 > \frac{x}{A}, \tag{8}$$

$$\frac{ebx}{A} \leq a. \tag{9}$$

Under these parameter conditions, [3] we obtain the following results.

$$u_t = \begin{cases} 0 & \text{if } h_t \leq h_u, \\ \frac{e\sigma b h_t - a}{(1-\sigma)bh_t} & \text{if } h_t > h_u, \end{cases} \tag{10}$$

$$n_t = \begin{cases} \frac{1-\gamma}{e}\left(1 - \frac{px}{wh_t}\right) & \text{if } h_t \leq h_u, \\ (1-\gamma)(1-\sigma)b\frac{h_t - \frac{p}{w}x}{ebh_t - a} & \text{if } h_t > h_u, \end{cases} \tag{11}$$

[2] When $u_t = 0$, the human capital level of the children becomes ηa^σ. Thus, the value of ηa^σ is interpreted as the innate ability of human beings.

[3] The explanation of these assumptions are presented in Appendix A which is available from the authors on request.

$$c_{Mt+1} = \gamma R(wh_t - px), \tag{12}$$

where $h_u \equiv \frac{a}{e\sigma b}$ and $p = \frac{w}{A}$ from equation (1).

In our model, the markets for agricultural products and labor are closed at the domestic level. Therefore, the market clearing conditions for these markets are:

$$AL_{At} = xN_t, \tag{13}$$

$$L_{At} + L_{Mt} = L_t, \tag{14}$$

where L_{At} (L_{Mt}) represents the labor demand in the agricultural (manufacturing) sector in terms of efficiency units and L_t satisfies the condition $L_t = l_t h_t N_t$. Therefore, by substituting (1), (3),(10),(11),(13), and $L_t = l_t h_t N_t$ into (14), we obtain the relative employment share of the manufacturing sector as follows:

$$\frac{L_{Mt}}{L_t} = \frac{\gamma(Ah_t - x)}{\gamma(Ah_t - x) + x}. \tag{15}$$

The above equation shows that $\frac{L_{Mt}}{L_t}$ is an increasing function of h_t. [4] Therefore, the employment share of the manufacturing sector increases monotonously with the accumulation of the per capita human capital, and the economy thus becomes steadily industrialized.

Using (5) and (10), the accumulation of the per capita human capital is represented as:

$$h_{t+1} = \begin{cases} \eta a^{\sigma} \equiv \psi_1(h_t) & \text{if } h_t \leq h_u, \\ \eta(\frac{\sigma}{1-\sigma})^{\sigma}(ebh_t - a)^{\sigma} \equiv \psi_2(h_t) & \text{if } h_t > h_u, \end{cases} \tag{16}$$

where $\psi_2'(h) > 0$, $\psi_2''(h) < 0$, and $\psi_2(h_u) = \eta a^{\sigma}$. The steady-state equilibrium is expressed as the stationary level of the per capita human capital h such that $h = \psi(h)$. Depending on the value of the parameters, there are several possible patterns of dynamics. However, in order to avoid some unnecessary lexicographic explanations, we only explain the interesting two cases described in Figs. 1 and 2. Fig. 1 shows the case in which the following condition holds:

$$(\eta\sigma eb)^{\frac{1}{1-\sigma}} < a < \sigma^{\frac{2\sigma}{1-\sigma}}(1-\sigma)^{\frac{1-2\sigma}{1-\sigma}}\eta^{\frac{1}{1-\sigma}}(eb)^{\frac{1}{1-\sigma}}. \tag{17}$$

[4] The induction of (15) is explained in Appendix B which is available from the authors on request.

In this case, there are three steady-state equilibria. Since $\eta a^\sigma \leq \frac{a}{\sigma eb}$, the graph of $\psi_1(h_t)$ intersects with the 45° line once. We denote this intersection as E_1 and define the level of h at E_1 as h^*. In addition, we can confirm that the graph of $\psi_2(h_t)$ has two intersections with 45° line under the condition of (17). Thus, we denote this intersection as E_2 (E_3) and define the level of h at E_2 (E_3) as h^{**} (h^{***}). From (10), (11) and (15), it is easily confirmed that E_1 (E_3) is the stable steady-state equilibrium characterized by "low income, low education, and low industrialization" ("high income, high education, and high industrialization"), while E_2 is the unstable steady-state equilibrium and corresponds to the threshold. When the initial per capita human capital h_0 is lower (higher) than h^{**}, the economy converges to E_1 (E_3). Therefore, the level of h_0 is crucial in this dynamics.

On the other hand, Fig. 2 shows the case in which the following condition holds:

$$a < (\eta\sigma eb)^{\frac{1}{1-\sigma}}. \tag{18}$$

In this case, since $\eta a^\sigma > \frac{a}{\sigma eb}$, only the graph of $\psi_2(h_t)$ intersects with the 45° line once. Thus, the dynamical system in Fig. 2 has a unique stable steady-state equilibrium E_3 characterized by "high income, high education, and high industrialization." [5]

Using (10), (11), and (15), Fig. 3 describes the evolution of the fertility rate (the upper), the human capital investment for children (the center), and the relative employment share of the manufacturing sector (the lower) when the economy rides on the dynamical path described in Fig. 2. Here, we focus on the case in which h_0 is lower than h_u. When the economy lies in the region $h_t < h_u$, agents do not allocate their time toward the human capital formation of their children, and the demand for children (the fertility rate) increases as the per capita income rises. On the other hand, when the economy reaches region $h_u < h_t < h^{***}$, the fertility rate decreases steadily and agents start to devote a fraction of their time to educating their children. Throughout this development process, the relative employment share of the manufacturing sector increases monotonously, and the economy steadily becomes industrialized.

The inverted U-shaped relationship between the fertility rate and economic development presented in Fig. 3 is consistent with the historically observed facts. For example, Dyson and Murphy (1985) has shown that the fertility rate continued to rise in most of Western Europe until the second half of the nineteenth century and then steadily declined. Moreover, the

[5]The rigorous proof of conditions defined in (17) and (18) is explained in Appendix C which is available from the authors on request.

rise in the relative labor share of the manufacturing sector is also one of the stylized facts in the process of economic development. For example, Mitchell (1981) has shown that the share of agricultural production in the total output steadily decreased in England, France, Germany, Sweden, and Italy during the nineteenth and twentieth centuries.

The mechanisms of inverted U-shaped fertility dynamics and sectorial shifts in labor from agriculture to manufacture are explained as follows. When the economy lies in the low-income levels, the first priority of agents is to secure the minimum level of consumption of agricultural goods for their survival. In this range, since human capital endowments are low, the productivity level of the agricultural sector is also low. Therefore, in order to supply enough agricultural products to satisfy the subsistence level of consumption in the economy, a large amount of labor must be allocated to the agricultural sector. Moreover, in this range, the human capital investment is not yet beneficial. Therefore, only the fertility rate increases as the income rises. When the economy reaches the region of high income levels, the human capital investment in children becomes beneficial. Then, agents start to allocate their time towards the human capital formation of their children. Since the returns from improving "child quality" become higher as the economy develops, the human capital investment in children steadily increases, and the demand for children (the fertility rate) decreases. Due to this human capital accumulation, the productivity level of the agricultural sector improves, and more labor can thus be released from the agricultural sector to the manufacturing sector. Moreover, the demands for manufactured products increase as the income rises. Therefore, sectorial shifts in labor from agriculture to manufacture occur in the process of economic development.

References

[1] Becker, G. S., 1965. A theory of the allocation of time. Economic Journal 75, 493-517.

[2] Dyson, T., Murphy, M., 1985. The onset of fertility transition. Population and Development Review 11, 399-440.

[3] Kogel, T., Prskawetz, A., 2001. Agricultural productivity growth and escape from the Malthusian trap. Journal of Economic Growth 6, 337-357.

[4] Mitchell, B., 1981. European Historical Statistics, 2nd ed., (New York).

[5] Tabata, K., 2003. Inverted U-shaped fertility dynamics, poverty trap, and growth. Economics Letters 81, 241-248.

[6] Tamura, R., 2002. Human capital and the switch from agriculture to industry. Journal of Economic Dynamics & Control 27, 207-242.

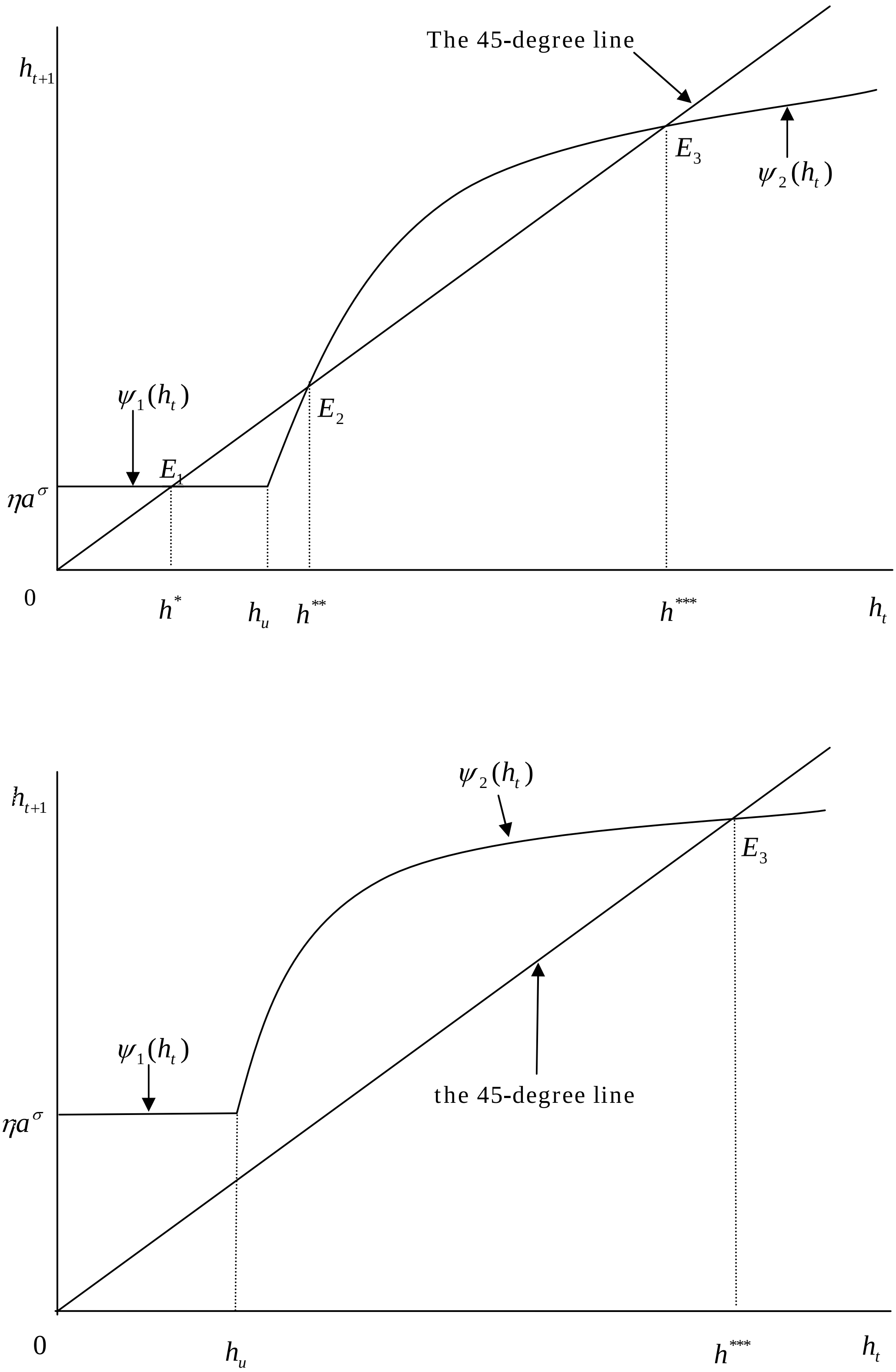

Fig. 2. Evolution of the per capita human capital

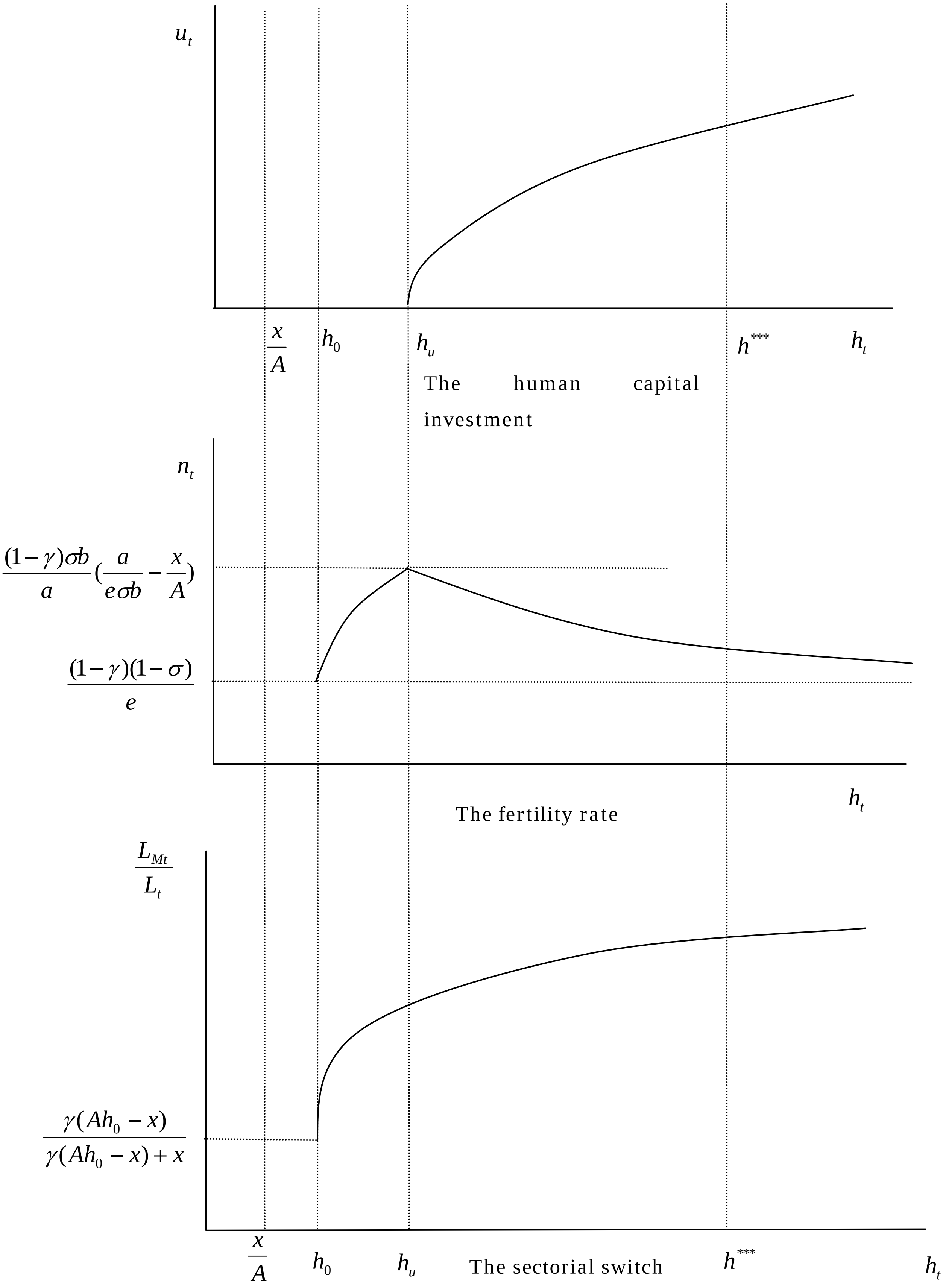

Fig. 3. changes in the evolution of the economy