E C O N O M I C S   B U L L E T I N

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We are grateful to an anonyms referee for his constructive comments.
Citation: Epstein, Gil S. and Shmuel Nitzan, (2005) "Size and distribution of prizes and efforts in contests." *Economics Bulletin*, Vol. 8, No. 10 pp. 1-10
Submitted: October 21, 2004. **Accepted:** September 27, 2005.
URL: http://www.economicsbulletin.com/2005/volume8/EB-04H00002A.pdf

Size and Distribution of Prizes and Efforts in Contests

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Abstract: The intensity of competition in contests is affected by the sum of the awarded prizes and by the prize distribution among the contestants. The current paper examines which of these two parameters has a larger effect on the players' extent of participation in the contest.

Keywords: Contests, Efforts, dstakes, distribution.

JEL Classification: D72.

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1. Introduction

The intensity of competition in contests is affected by the sum of the awarded prizes and by the prize distribution between the contestants. It seems that the larger the sum of the prizes and the more symmetric its distribution, the more intense is the competition; that is, the larger the efforts incurred by the contestants. In this paper we study the general class of such two-player variable contests and examine their effect on the contestants' efforts.

The efforts exerted in the contest deserve attention, first, because they can be interpreted as social costs and, second, because they can serve as a measure of an interest-group involvement in the contest. Of course, *ceteris paribus*, when a player is more involved in the contest, he has a higher probability of winning. In many cases it is in the public interest or in the interest of the ruling politicians to induce one of the participants in the contest to be more active, that is, be more involved in the contest. Typically, the provision of such an incentive is considered in the context of contests that arise when the government proposes some new public policy, e.g., some new form or degree of monopoly regulation, (Ellingsen, 1991), a tax reform or a trade policy, and the contest outcome determines whether the proposed policy is approved or rejected (Epstein and Nitzan, 2002, 2005).

Our results hinge on a fundamental equation that decomposes the total effect on individual effort into two sub-effects that correspond to the change in the two measures of intensity of competition. We show that the 'prize-distribution effect' is always larger than the 'size effect' (size of the sum of the prizes). The result states that when there is a change in both the size of the prizes and in their distribution, the direct incentives due to the change in the contestant's relative share is larger than the indirect incentives due to the relative change in the sum of the contest prizes. In particular, a contest on part of the GNP is going to affect waste (lobbying efforts) more through the contestants' direct distributional (inequality) incentives than through their indirect size (the size of the contested "cake") incentives.

2. The Variable Contest

In our contest there are two players that compete for different (or equal prizes). In general, one group may gain a higher benefit than the other from winning the contest.

The players engage in a contest that determines the probabilities of winning or losing the contest.[1]

The total amount of prizes in the contest is denoted by *V*. With probability $\Pr_i$ ($i = 1,2$) player *i* wins the contest and gains $\alpha_i V$, where $0 < \alpha_i < 1$ and $\alpha_1 + \alpha_2 = 1$. He loses the contest and gains no prize and gains no prize with probability $\Pr_j = 1 - \Pr_i$. Let x_i denote the effort of the risk-neutral player *i*. The expected net payoff of *i* is given by:

$$E(w_i) = \Pr_i \alpha_i V - x_i \qquad \forall\ i = 1,2 \tag{1}$$

Our primary concern is with question how do changes in the value of the total prize V and in its distribution affects the effort exerted by the players. To analyze this problem, we consider an exogenous variable *I* that affects both the value of the total prize *V* and the share each of the players may gain, α_i. Both α_i and V thus depend on the value of the parameter *I*: $\alpha_i(I)$ *and* $V(I)$.

Given the contestants' efforts, the probabilities of winning or losing the contest are obtained by the contest success function. As in Skaperdas (1992), it is assumed that $\frac{\partial \Pr_i(x_i, x_j)}{\partial x_i} > 0$, $\frac{\partial \Pr_i(x_i, x_j)}{\partial x_j} < 0$ and $\frac{\partial^2 \Pr_i(x_i, x_j)}{\partial x_i^2} < 0$ [2] (the latter inequality ensures that the second order conditions are satisfied). Since $\Pr_i(x_i, x_j) + \Pr_j(x_j, x_i) = 1$, $i \neq j$, it holds that

$$\frac{\partial^2 \Pr_i(x_i, x_j)}{\partial x_i \partial x_j} = -\frac{\partial^2 \Pr_j(x_j, x_i)}{\partial x_i \partial x_j}. \tag{2}$$

[1] Modeling the contestants as single agents presumes that they have already solved the collective action problem. The model thus applies to already formed interest groups.

[2] The function $\Pr_i(x_i, x_j)$ is usually referred to as a contest success function (CSF). The functional forms of the CSF's commonly assumed in the literature, see Nitzan (1994) and Skaperdas (1996), satisfy these assumptions.

By our assumptions, both players participate in the contest (x_1 and x_2 are positive). We therefore focus on interior Nash equilibria of the contest. Solving the first order conditions $\left(\frac{\partial E(w_i)}{\partial x_i} = 0 \;\forall\; i=1,2\right)$ we obtain:

$$\Delta_i = \frac{\partial \Pr_i(x_i, x_j)}{\partial x_i}\alpha_i(I)V(I) - 1 = 0 \quad \forall\; i \neq j \;\; and \;\; i, j = 1,2 \tag{3}$$

The first order conditions therefore require[3] that:

$$\frac{\partial \Pr_i}{\partial x_i} = \frac{1}{\alpha_i(I)V(I)} \;\; \forall\; i = 1,2 \tag{4}$$

By the expressions in (4) that determine the equilibrium efforts of the players and their probabilities of winning the contest and by the assumed properties of the CSF, we directly obtain that under a symmetric contest success function[4] ($\forall x_i, x_j, \Pr_i(x_i, x_j) = \Pr_j(x_j, x_i)$), the player with the higher stake makes a larger effort and has a higher probability of winning the contest. The probability of the socially more efficient outcome of the contest is therefore higher than the probability of the less efficient outcome. For a similar result see Baik (1994) and Nti (1999). This type of efficiency criterion has been used by Ellingsen (1991), Fabella (1995) and, more recently, by Hurley (1998).

3. Results

Let us now consider the effect of changes in I on the effort exerted by the players. These efforts deserve attention because they can be interpreted as social costs and because they represent each player's involvement in the contest that often becomes a direct target of the agent (usually the government) that controls I, the contest designer. In other words, when a player is more involved in the contest, his

[3] It can be easily verified that the second order conditions hold.

[4] Such symmetry implies that the two players share an equal ability to convert effort into probability of winning the contest.

probability of winning the contest becomes higher, which might coincide with the interest of the contest designer. The selection of the functions $\alpha_i(I)$ *and* $V(I)$, or the selection of I when these functions are given might therefore be of considerable significance to the contest designer. In many cases indeed it is in the public interest or in the interest of the ruling politicians to induce one of the contestants to be more involved in the public debate over issues such as monopoly regulation, some environmental policy, a tax reform or a new trade policy. Typically, the provision of such an incentive is considered in the context of contests on the approval or rejection of new policy proposals by the government.

A change in *I* affects both the total amount of prizes in the contest *V* and the share each of the players gains if he wins the contest. Notice that there are four different types of variability patterns corresponding to the four possible sign combinations of $\frac{\partial V}{\partial I}$ and $\frac{\partial \alpha_i}{\partial I}$: a change in *I* that reduces (increases) the prize to be divided between the players, that is, $\frac{\partial V}{\partial I} > 0$ $\left(\frac{\partial V}{\partial I} < 0\right)$ and redistributes benefits in favor of player *i* (player *j*) which implies an increase (a decrease) in the share of the prize that goes to player *i*, that is, $\frac{\partial \alpha_i}{\partial I} > 0$ $\left(\frac{\partial \alpha_i}{\partial I} < 0\right)$.

The effort of a contestant is determined not only by the effect of *I* on *V* and α_i, but also by the ability of contestant j to convert effort into probability of winning the contest. This ability can be represented by the marginal effect of a change in his effort on his winning probability. By assumption, this marginal effect is declining with his own effort. A change in his effort also affects, however, the marginal winning probability of his opponent *i*. The opponent *i* has an advantage in terms of ability if a change in *j*'s effort positively affects his marginal winning probability. In other words, a positive (negative) sign of the cross second-order partial derivative of $\Pr_i(x_i, x_j)$, $\frac{\partial^2 \Pr_i}{\partial x_j \partial x_i}$, implies that *i* has an advantage (disadvantage) when *j*'s effort changes. At some given combination of efforts (x_i, x_j), the ratio between the effect of a change in *j*'s effort on the marginal winning probability of *i* and the effect of a change in j's effort on his own ability, $\frac{\partial^2 \Pr_i}{\partial x_i \partial x_j} \Big/ \left(\frac{\partial^2 \Pr_j}{\partial x_j^{\,2}} \right)$, is therefore a local measure of the asymmetry between the abilities of i and j.

By differentiation of the first order conditions (see (3)), we get that the Nash equilibrium efforts satisfy the following conditions:

$$\frac{\partial x^*_i}{\partial I} = \frac{\frac{\partial \Delta_i}{\partial x_j}\frac{\partial \Delta_j}{\partial I} - \frac{\partial \Delta_j}{\partial x_j}\frac{\partial \Delta_i}{\partial I}}{\frac{\partial \Delta_i}{\partial x_i}\frac{\partial \Delta_j}{\partial x_j} - \frac{\partial \Delta_j}{\partial x_i}\frac{\partial \Delta_i}{\partial x_j}} \quad \forall i \neq j, \;\; i, j = 1,2 \tag{5}$$

Rewriting (5) together with (4), we obtain:

$$\frac{\partial x^*_i}{\partial I} = D\left\{ \frac{V'}{V}\left(\frac{\frac{\partial^2 \Pr_i}{\partial x_i \partial x_j}}{\frac{\partial^2 \Pr_j}{\partial x_j^{\,2}}} - \frac{\alpha_j}{\alpha_i} \right) + \frac{\alpha_j'}{\alpha_j}\left(\frac{\frac{\partial^2 \Pr_i}{\partial x_i \partial x_j}}{\frac{\partial^2 \Pr_j}{\partial x_j^{\,2}}} + \left(\frac{\alpha_j}{\alpha_i}\right)^2 \right) \right\} = \tag{6}$$

$$= D\left\{ \frac{V'}{V}\left(\frac{\frac{\partial^2 \Pr_i}{\partial x_i \partial x_j}}{\frac{\partial^2 \Pr_j}{\partial x_j^{\,2}}} - \frac{\alpha_j}{\alpha_i} \right) - \frac{\alpha_i'}{\alpha_i}\frac{\alpha_i}{(1-\alpha_i)}\left(\frac{\frac{\partial^2 \Pr_i}{\partial x_i \partial x_j}}{\frac{\partial^2 \Pr_j}{\partial x_j^{\,2}}} + \left(\frac{\alpha_j}{\alpha_i}\right)^2 \right) \right\} \quad \forall i \neq j, \;\; i, j = 1,2$$

where

$V' = \left(\frac{\partial V}{\partial I}\right)$, $\alpha_i' = \left(\frac{\partial \alpha_i}{\partial I}\right)$ and $D = \left(V\alpha_j \frac{\partial^2 \Pr_j}{\partial x_j^{\,2}} \left(\frac{\partial^2 \Pr_j}{\partial x_j^{\,2}}\frac{\partial^2 \Pr_i}{\partial x_i^{\,2}} - \frac{\partial^2 \Pr_i}{\partial x_i \partial x_j}\frac{\partial^2 \Pr_j}{\partial x_i \partial x_j} \right)^{-1} \right) < 0$.

In this equation one can clearly distinguish between the separate effects on i's effort of a change in the relative size of the prize and of a change in the relative share of i's stake. The change in the relative size of the prize is given by *V'/V* (see the first component in the RHS of the equation). There may be many measures for the change in the share of i's stake. We consider the measure that takes into account both the relative (percentage) change in i's share of the stake, $\frac{\alpha_i'}{\alpha_i}$, and the distribution of the stakes, $\frac{\alpha_i}{\alpha_j}=\frac{\alpha_i}{(1-\alpha_i)}$. The former element by itself is inadequate, because it doesn't provide all relevant information. For example, $\frac{\alpha_i'}{\alpha_i}$ may equal 10%, however, it is not clear 10% of what? We thus factor this element by $\frac{\alpha_i}{1-\alpha_i}$, which means that the measure we use is $\frac{\alpha_i'}{\alpha_i}\frac{\alpha_i}{(1-\alpha_i)}$ (see the second component in the RHS of the equation). It is clear that the weight of the effect of the change in the relative size of the prize is ***smaller*** than the weight of the effect of the change in the relative share of j's stake:

$$\frac{\frac{\partial^2 \Pr_i}{\partial x_i \partial x_j}}{\frac{\partial^2 \Pr_j}{\partial x_j^2}}-\frac{\alpha_j}{\alpha_i} < \frac{\frac{\partial^2 \Pr_i}{\partial x_i \partial x_j}}{\frac{\partial^2 \Pr_j}{\partial x_j^2}}+\left(\frac{\alpha_j}{\alpha_i}\right)^2 .$$ However, $$\left|\frac{\frac{\partial^2 \Pr_i}{\partial x_i \partial x_j}}{\frac{\partial^2 \Pr_j}{\partial x_j^2}}-\frac{\alpha_j}{\alpha_i}\right| > \left|\frac{\frac{\partial^2 \Pr_i}{\partial x_i \partial x_j}}{\frac{\partial^2 \Pr_j}{\partial x_j^2}}+\left(\frac{\alpha_j}{\alpha_i}\right)^2\right|$$

only if $\frac{\frac{\partial^2 \Pr_i}{\partial x_i \partial x_j}}{\frac{\partial^2 \Pr_j}{\partial x_j^2}}-\frac{\alpha_j}{\alpha_i}<0$. Hence,

Proposition: *The effect on i's effort of a change in the prize distribution is always larger than the effect on i's effort of a change in the sum of the prizes.*
Moreover, if the effect of the a change in the sum of the prize is <u>stronger</u> *than the effect of the change in the prize distribution, then the net effect of the change is negative.*

The proposition states that when there is a change in both the size of the prizes and in their distribution, the direct incentives corresponding to the change in the contestant's relative share are larger than the indirect incentives corresponding to the change in the contest prize. On the other hand, if the effect of the relative share is weaker than the indirect incentives corresponding to the change in the contest prize, then the effect of the change in the size is negative. In particular, if both weights are positive, then in case of a contest for part of the GNP, the waste (lobbying efforts) are affected more through the contestants' direct distributional (relative inequality) incentives than through the indirect size (the size of the contested "cake") incentives.

To illustrate the implications of the proposition, consider the case where player *i* is the weak player, both in terms of the share he gains in case of winning the contest and in terms of the equilibrium probability of winning, that is, $\alpha_j > \alpha_i$ and $\frac{\partial^2 \Pr_i}{\partial x_i \partial x_j} < 0$. Now suppose that the relative change in player *j's* share equals the relative change in the size of the prizes: $\frac{\alpha_j'}{\alpha_j} = \frac{V'}{V}$ or putting it differently $\frac{\alpha_i'}{\alpha_i}\frac{\alpha_i}{\alpha_j} == \frac{V'}{V}$ and that both *j*'s share and the sum of the prizes are reduced, that is, $\alpha_j' < 0 \ \left(\alpha_j' > 0\right)$ and $V' < 0$. In such a case player *j* reduces his effort. By applying (6), we obtain that $\frac{\partial x^*_i}{\partial I} > 0$ and $\frac{\partial x^*_j}{\partial I} < 0$. That is, although player *i* receives a higher share of the reduced aggregate prize in case of winning the contest, he increases his effort. The increased involvement of player *i* and the reduced involvement of player *j* increases *i*'s chances of winning the contest.

To conclude, we have shown that in a general two-player contest, a change in the relative share of the aggregate prize has a larger effect on the effort invested by a contestant in comparison to a change in the relative size of the aggregate prize. A contestant's behavior is always more sensitive to a change in intensity of competition as measured by $\frac{\alpha_i'}{\alpha_i}\frac{\alpha_i}{\left(1-\alpha_i\right)}$ relative to a change in the intensity of competition as measured by *V'/V* .

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