E C O N O M I C S   B U L L E T I N

Can a Time−to−Plan Model explain the Equity Premium Puzzle

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Abstract

This paper proposes a quantitative evaluation of the time−to−plan technology in order to investigate up to which point this mechanism could constitute a satisfactory alternative to the well−known capital adjustment cost technology. We show that the time−to−plan mechanism reproduces a realistic risk−free rate, whilst being capable of generating a substantial equity premium. About the model's explanation of the business cycle, it turns out that the model predicts a perfectly positive and significant correlation between employment and output.

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Introduction

In dynamic general equilibrium models, the investment technology plays an important role because it allows to generate capital gains which are a necessary ingredient in reproducing the volatility of risky assets returns.

Boldrin, Christiano and Fisher (1995, 2001), following Christiano and Todd (1996), propose a mechanism that consists in abandoning the hypothesis according to which one period is enough for the construction of a new unit of capital. The idea is that an investment project is spread out over several periods. The introduction of delays in the capital accumulation technology does not modify the firms' behavior. Firms are still supposed to decide how much to invest at each period . However, in the presence of time-to-plan delays, the current gross investment is just a subset of a larger set of investment projects initiated at previous dates. That is, the date t investment concerns the production to be realized for the four future quarters (when one considers a delay of four periods). It follows that the capital supply at the current date, is absolutely inelastic to the price of a unit of capital at the same date: this allows for variations in the price of the installed capital. It is by this channel that capital gains are introduced into the model. As stated in Christiano and Todd (1996), the "time-to-plan" mechanism differs from the "time-to-build" mechanism proposed by Kydland and Prescott (1982). The former implies that an investment project needs fewer resources over the first periods of its implementation. On the contrary, the "time-to-build" hypothesis assumes that the quantity of resources devoted to the investment realization is uniform troughout time.

This paper proposes a quantitative evaluation of the time-to-plan technology. We particularly want to investigate up to which point this mechanism could constitute a satisfactory alternative to the well-known capital adjustment cost technology. As pointed out by Boldrin, Christiano and Fisher (2001), when endogenous labour choice is considered, capital adjustment costs involve counterfactual results both for labor (which is countercyclical) and output (which is not persistent). We first evaluate the model's ability to reproduce the main asset returns facts. We show that the time-to-plan mechanism reproduces a realistic risk-free rate, whilst being capable of generating a substantial equity premium. Moreover, the equity return volatility is relatively close to its empirical value. We are also interested in the model's explanation of the business cycle. Despite the very average results obtained in reproducing the relative volatilities, the model predicts a perfectly positive and significant correlation between employment and output.

The remainder is organized as follows. Section 1 presents the model and the time-to-plan principle of modelling. Section 2 briefly sketches the quantitative methodology. Section 3 exposes the results. The last section concludes.

1 The Model

The following model is similar to a one-sector neoclassical model in discrete time. The representative household is assumed to value leisure and make an work-leisure choice. Its labour supply is thus variable.

1.1 Households

The representative household solves the following program,

$$\max_{\{C_t,\ell_t\}} \quad \mathrm{E}_t \sum_{k=0}^{\infty} \beta^k \left[\log\left(C_{t+k} - \eta C_{t-1+k}\right) + \psi \ell_{t+k}\right] \tag{1}$$

$$\text{sc.} \quad W_t N_t + a_t \left(V_t + D_t\right) = C_t + a_{t+1} V_t \tag{2}$$

Households derive utility from consumption C_t and leisure $\ell_t = 1 - N_t$, where N_t represents labor. When for $\eta > 0$, preferences are characterized by a simple habit formation. E_t is the conditional expectation operator, β is the discount factor, W_t is the real wage rate, a_t is a vector of financial assets held at time t, and V_t is a vector of asset prices, and D_t is dividends.

1.2 Firms

When time-to-plan delays are considered, the optimal policy function on investment and physical capital is modified. Indeed, the time necessary for the construction of a new capital unit is spread out over several periods. Consequently, at each period gross investment, I_t, is a weighted sum of investment projects initiated at n previous periods. Formally the investment technology writes,

$$I_t = \phi_1 X_t + \phi_2 X_{t-1} + \phi_3 X_{t-2} + \phi_4 X_{t-3}, \quad \text{and } \phi_i \geq 0, \quad i = 1,2,3,4 \tag{3}$$

where ϕ_i are the weighted coefficients of the projects according to their degree of maturity. As states in (3), the current investment, certainly depends on a level of resources decided t, that is $\phi_1 X_t$, but especially depends on past decisions concerning X_{t-i}. When $\phi_1 = 1$ and $\phi_2 = \phi_3 = \phi_4 = 0$, one obtains the linear technology commonly used in standard business cycle models. The time-to-plan technology implies the following parametrization,

$$\phi_1 = 0.01; \phi_2 = 0.33; \phi_3 = 0.33; \phi_4 = 0.33 \tag{4}$$

As suggested by (4), resources initially devoted to the project's inception, are weaker than the level of resources necessary at the end of the project. One usually assumes that,

$$\phi_1 + \phi_2 + \phi_3 + \phi_4 \equiv 1.$$

Considering the delay necessary for the construction of the new capital, the investment technology implies that net investment of period $t+3$, that is X_t writes,

$$K_{t+4} - (1-\delta) K_{t+3} = X_t, \tag{5}$$

where δ is the depreciation rate of capital, K_t is the stock of capital and X_t is net investment. So, the level of investment, $\phi_1 X_t$, must be applied in period t, $\phi_2 X_t$ must be applied in period

$t+1$, and $\phi_3 X_t$ must be applied in period $t+3$. Consequently, once initiated, the scale of an investment project can be neither expanded nor contracted. As a result, at each period, the capital stock is completely inelastic to the price of one unit of installed capital.

Given the previous elements, the representative firm that experiments time-to-plan delays has to choose how much labor (N_t) to hire, how much to invest (X_t), and the level of the next period's capital stock (K_{t+4}), in order to maximize the value of the firms to the owners, that is the present discounted value of current and future expected cash flows,

$$\max_{\{N_t, X_t, K_{t+4}\}} \quad \mathrm{E}_t \sum_{s=0}^{\infty} \beta^s \frac{\Lambda_{c,t+s}}{\Lambda_{c,t}} \left(Z_{t+s} K_{t+s}^{\alpha} \left(g^{t+s} N_{t+s} \right)^{1-\alpha} - W_{t+s} N_{t+s} - I_{t+s} \right) \tag{6}$$

$$\text{sc.} \, (3) \ \text{ and } \ (5) \, .$$

where $\beta^s \left(\Lambda_{c,t+s} / \Lambda_{c,t} \right)$ is the marginal rate of substitution of the firms owners, g is the deterministic technical progress trend, and the law of motion of technology Z_t is,

$$\log \left(Z_t \right) = (1-\rho) \log (Z) + \rho \log \left(Z_{t-1} \right) + \varepsilon_t, \quad \varepsilon_t \sim iid \left(0, \sigma_{\varepsilon}^2 \right) . \tag{7}$$

1.3 Asset returns

Prices and rates of return derive from the solution to each agent's optimization problem. To study asset prices, we use the following standard definitions. Dividends are,

$$D_t = Y_t - W_t N_t - I_t \tag{8}$$

the risk-free rate is,

$$r_t^f = \frac{\Lambda_{c,t}}{\beta \mathrm{E}_t \Lambda_{c,t+1}} - 1, \tag{9}$$

where $\Lambda_{c,t}$ is the Lagrange multiplier associated with the household's resource constraint, which also operates in the intertemporal marginal rate of substitution of the owners of the firm. This multiplier is the derivative of expected present discounted utility with respect to C_t. The rate of return on equity is,

$$r_{t+1}^e = \frac{V_{t+1} + D_{t+1}}{V_t} - 1. \tag{10}$$

2 Quantitative methodology

2.1 Solution method

The model is solved using the undetermined coefficients method of Christiano (2002), which is a synthesis of the approaches proposed Blanchard and Kahn (1980), King, Plosser and Rebelo (1988) and others. This method is particularly suitable for our purpose because it can easily accommodate a model which integrates "jump" variables depending on different information sets. Moreover, this method is particularly suited resolving models with lagged endogenous state variables, as with the time-to-plan investment delays.

Christiano's solution method implies the loglinearization of the first order conditions. Such a method is known for imposing equality between the rates of return of different assets. This disqualifies such a method for studying equity premium. We choose to follow Jermann's (1998) method which combines the loglinear solution with non-linear asset pricing formulae to study asset returns in dynamic general equilibrium models involving several endogenous state variables.

2.2 Calibration

Calibration is organized in two steps. We start by imposing the conventional long-run restrictions: $g = 1.004$, $\delta = 0.021$, $(1-\alpha) = 0.64$. Given the utility function, ψ is fixed in order to get $N = 0.30$. The persistence parameter, ρ, equals 0.95. In a second step, we choose to estimate the value of the following set of parameters $J = \{\eta, \beta, \sigma_\varepsilon\}$. To this end, we use the following minimum-distance criteria:

$$\mathcal{M}(J) = [\widehat{v}_T - \mathrm{g}(J)]' \mathrm{V}_T [\widehat{v}_T - \mathrm{g}(J)] \tag{11}$$

where $\widehat{v}_T$ is (3×1) vector composed of the sample average of quarterly observations on the risk-free rate, the equity premium and the standard deviation of the cyclical component of the US output. V_T is a (3×3) weighting diagonal matrix which is composed of the inverse of the variance of the statistics in $\widehat{v}_T$. Finally, $g(J)$ is the model's implied average quarterly mean risk-free rate, equity premium and output standard deviation conditional on $J = \{\eta, \beta, \sigma_\varepsilon\}$ and the value of the other parameters. The components of g are obtained by taking the average over 1000 simulations, each 300 quarters long. In practice, we compute $\mathcal{M}$ for a grid of values for $\eta = [0, 0.9]$, $\beta = [0.99, 0.99999]$, $\sigma_\varepsilon = [0, 0.03]$, then take the values $\left\{\hat{\eta}, \hat{\beta}, \hat{\sigma}_\varepsilon\right\}$ that minimize $\mathcal{M}$.

As stated by Table 1, which summarizes the calibration procedure, the values are,

$$\{0.82,\ 0.99952,\ 0.0102\} \tag{12}$$

These estimates are quite standard: the habit parameter is close the value reported by Jermann (1998) and Boldrin, Christiano and Fisher (2001). The standard deviation of the shock is near to one percent which is quite acceptable.

3 Results

Given the calibration procedure we will organize our comments in two steps. Let us start by analyzing the implications on asset returns stylized facts. A general result is that the time-to-plan model provides a good explanation for asset returns facts (Table 2). The theoretical mean risk-free rate equals 1.79%. The empirical equity premium is well accounted for (6.04%

against 6.01% in the data). The capital gain effect is quite significative since the equity return volatility is 14.40% against 15.8% in the observed data.

What are the implications for the business cycle ? We analyze the model's behavior in reproducing the second order moments of the cyclical components of consumption, investment employment and output. The cyclical component is obtained by applying the HP filter on the logged series. The relative volatility of investment is pretty good, whereas consumption's volatility is understated. This is due to the habit formation which reduces the consumption volatility. The habit persistence, as a necessary condition to reproduce the empirical equity premium implies a volatility of consumption which is too low . For employment, the relative volatility is overstated. We conclude that the model performance in explaining the variable volatilities remains very average.

The model works better at reproducing the comovement of output with its components. The strength and the sign of the instantaneous correlation with output are close to their empirical counterpart. A salient result concerns the correlation between output and employment. Empirically this correlation is 0.79. The model predicts 0.69 which means labor is perfectly procyclical. On this dimension, the time-to-plan model avoid one of the main criticism formulated against the capital adjustment cost mechanism, which predicts that employment is countercyclical.

Some counterfactual results must be indicated. The risk-free rate is too volatile compared to the empirical data. Finally, while the model heavily overestimates the persistence of the consumption growth rate (due to the habit formation in consumption) it turns out to be completely unable to generate the degree of persistence observed in the output growth rate.

Conclusion

The time-to-plan mechanism associated with habit formation constitutes a decisive step forward in the integrated analysis of business cycle and asset returns. This model provides a satisfactory explanation of asset returns when labour supply is endogenous. The main correlations with the output component are satisfactingly reproduced, particularly the instantaneous correlation between output and employment.

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TABLE 1. PARAMETERS VALUE

notation	value	interpretation
δ	0.021	rate of depreciation
α	0.36	capital share in output
ρ	0.95	shock persistence parameter
g	1.004	technical progress growth rate
σ_ε	0.0102	shock standard-deviation
β	0.99952	subjective discount factor
η	0.82	habit formation persistence

TABLE 2. RESULTS

Asset returns	Data	Model
Mean value of		
risk free rate	1.19	1.79
equity premium	6.01	6.04
Standard deviation of		
risk free rate	1.41	4.30
return on equity	15.87	14.40
Sharpe Ratio	0.37	0.41
Business cycle	Data	Model
Relative standard deviation with output of		
consumption	0.37	0.18
investment	2.15	2.62
employment	0.72	1.08
Correlation with output of		
consumption	0.77	0.69
investment	0.91	0.99
employment	0.79	0.69
Persistence	Data	Model
Autocorrelation of the growth rate of		
output	0.32	0.073
consumption	0.31	0.62