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Salvaging The Linkage Principle In Private−Value Auction For A Single Object

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Abstract

There are a number of examples in the auction literature (Perry and Reny, 1999, and Krishna, 2002) where releasing the seller's private information can lead to a lowering of expected revenue. On the other hand, releasing information always increases welfare. Levin and Smith (1994) point out that when entry is endogenous (and the entry decision is made before one observes one's signal), revenue and welfare objectives coincide. Therefore, revealing information will always increase revenue when we can use entry fees to capture all bidders' rents.

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1. Introduction

One of the biggest discoveries of auction theory after the revenue equivalence theorem is the linkage principle. The linkage principle suggests that the more the information provided to the bidders in an auction the greater the revenue (Milgrom and Weber, 1982). Recently, however, the linkage principle has been shown to be violated in the multi-unit framework. More precisely, Perry and Reny (1999) show that in multi-unit auctions revealing the seller's information may lead to a change in the ordering of the bids, thereby, lowering the expected revenue. Krishna (2002) shows that the switch in the ranking of bids that drives the example of Perry and Reny does not need multiple units to take place. It can happen in single object auctions, as well, thereby violating the linkage principle. In Krishna's example with two bidders only one bidder has a private value (in Perry and Reny both bidders have a private value, but only for the second unit). However, as Board (2004) shows, even that is not necessary. The change in the ranking of bids as a result of seller's private information is possible when both bidders have private values which in turn may cause a violation of the linkage principle. Moreover, the effect of the change in the ranking of bids is the strongest when there are two bidders in the auction (in the sense that it cannot be balanced by any other change).

In this paper, we consider auctions where entry of bidders is endogenously determined due to the presence of entry costs. We show that the linkage principle continues to hold when entry is endogenous even if revelation of the seller's private information gives rise to a switch in the ranking of the bids in the auction. Furthermore, we show that if the seller can charge the optimal entry fee revelation of the seller's information always increases the expected revenue, thereby restoring the linkage principle. The intuition behind this is simple. When entry is endogenous, the objectives of the seller and the social planner become perfectly aligned even when bidders are asymmetric. Therefore, the seller prefers the efficient outcome that is brought about by the release of the private information. This intuition extends to the case of more than two bidders, as well.

2. The Linkage Principle With Entry

The formal model of entry is the asymmetric version of the entry model of Levin and Smith (1994). A seller has an indivisible item and a privately observed random signal s. There are 2 potential bidders. The bidders simultaneously choose entry probabilities p_1 and p_2. When bidder i enters the auction he incurs a cost of entry c_i and observes a private signal X_i that is a random variable. The participating bidders then bid in an English auction with a zero reserve. The value of the object to bidder i is $u_i(X_i, s)$. It is possible, for instance, that $E_s u_1(X_1, s) > E_s u_2(X_2, s)$ but that $u_1(X_1, s) < u_2(X_2, s)$ where E_Z is the expectation operator over Z. Thus, a release of information may lead to a switch of private values (which is then bound to cause a violation of the linkage principle in the absence of entry costs). In short, we do not make any assumption to preserve the linkage principle in the case where two bidders participate with probability one in the auction.

We say that entry is *endogenous* in the auction when the participation probability of each bidder depends on the participation probability of the other bidder. In other words, entry is endogenous if in equilibrium bidder i enters with probability p_i and a change in p_i makes bidder j's choice of participation probability p_j suboptimal. In such a situation a bidder typically earn's a zero ex ante payoff. Therefore, in general, we will say that participation in the auction is endogenous if in equilibrium bidder i ($i = 1, 2$) is indifferent between participating and not participating. Since in the auction itself bidding the expected private value of the object is a weakly dominant strategy we formally define participation to be endogenous in the case where the seller's information is not released, if for $i \neq j$ we have

$$\begin{aligned} (1 - p_j) E_{X_i, s} u_i(X_i, s) + p_j P[E_s u_i(X_i, s) > E_s u_j(X_j, s)] \\ \times E_{X_i, X_j}[E_s u_i(X_i, s) - E_s u_j(X_j, s) | E_s u_i(X_i, s) > E_s u_j(X_j, s)] &= c_i \end{aligned} \tag{1}$$

Note that in this case entry need not always be endogenous. Our first result will, however, be true only when entry is endogenous, *i.e.*, the parameters of the model allow the above condition to be satisfied for some $p_i \in [0, 1]$, $i = 1, 2$.

If the seller can also charge a bidder specific entry fee e_i that is payable at the time of entry and the entry fees are set at the optimal level to maximize revenue then the condition

$$\begin{aligned}
&(1-p_j)E_{X_i,s}u_i(X_i,s) + p_jP[E_su_i(X_i,s) > E_su_j(X_j,s)] \\
&\times E_{X_i,X_j}[E_su_i(X_i,s) - E_su_j(X_j,s)|E_su_i(X_i,s) > E_su_j(X_j,s)] \;=\; c_i + e_i
\end{aligned} \tag{2}$$

must be satisfied.[1] Observe that when the seller can charge the optimal entry fee the above equation will always be satisfied, *i.e.*, entry will always be endogenous, whenever the bidders find it worthwhile to participate in the auction in the absence of the entry cost and entry fees (so that the linkage principle is a relevant issue in the auction).[2]

Rather than thinking of the seller as setting the optimal entry fee, let us think of her as setting the participation rates and collecting the corresponding entry fees. Setting of optimal entry fees in that case is synonymous with setting the optimal participation rates. Now equation (2) can be rewritten as

$$\begin{aligned}
&(1-p_j)E_{X_i,s}u_i(X_i,s) + p_jP[E_su_i(X_i,s) > E_su_j(X_j,s)] \\
&\qquad\qquad \times E_{X_i,X_j}[E_su_i(X_i,s)|E_su_i(X_i,s) > E_su_j(X_j,s)] - c_i \\
=\;& p_jP[E_su_i(X_i,s) > E_su_j(X_j,s)]E_{X_i,X_j}[E_su_j(X_j,s)|E_su_i(X_i,s) > E_su_j(X_j,s)] + e_i
\end{aligned}$$

which upon being multiplied by p_i and summed for $i = 1, 2$ becomes

$$\begin{aligned}
&p_1(1-p_2)E_{X_1,s}u_1(X_1,s) + p_2(1-p_1)E_{X_2,s}u_2(X_2,s) \\
&\qquad + p_1p_2E_{X_1,X_2}[\max\{E_su_1(X_1,s), E_su_2(X_2,s)\}] - p_1c_1 - p_2c_2 \\
=\;& p_1p_2E_{X_1,X_2}[\min\{E_su_1(X_1,s), E_su_2(X_2,s)\}] + p_1e_1 + p_2e_2
\end{aligned}$$

[1] Note that the equilibrium entry probabilities are determined by the equilibrium condition (1) when the seller cannot charge an entry fee and condition (2) when the seller can charge an entry fee satisfy the inequalities $0 \le p_i \le 1$, $i = 1, 2$. Thus, whether a pure strategy or a mixed strategy equilibrium exists depends on the value of p_1 and p_2 determined by the equilibrium condition. For instance, if the equilibrium conditions give $p_1 = p_2 = 1$ it is a pure strategy equilibrium, and it is a mixed strategy equilibrium when $0 < p_i < 1$, $i = 1, 2$. In particular, the existence of a given type of equilibrium depends on the existence of the relevant solution of the equilibrium condition.

[2] In other words, unlike Proposition 1, Proposition 2 in the next section will hold very generally.

where the right hand side is the expected revenue to the seller and the left hand side is the expected social surplus. Similarly, when the seller's information is released the corresponding equation becomes

$$\begin{aligned} & \hat{p}_1(1-\hat{p}_2)E_{X_1,s}u_1(X_1,s) + \hat{p}_2(1-\hat{p}_1)E_{X_2,s}u_2(X_2,s) \\ & \qquad + \hat{p}_1\hat{p}_2E_{X_1,X_2,s}[\max\{u_1(X_1,s),u_2(X_2,s)\}] - \hat{p}_1c_1 - \hat{p}_2c_2 \\ = \;& \hat{p}_1\hat{p}_2E_{X_1,X_2,s}[\min\{u_1(X_1,s),u_2(X_2,s)\}] + \hat{p}_1\hat{e}_1 + \hat{p}_2\hat{e}_2 \end{aligned}$$

Clearly, the participation rates that maximize the right hand side of the equations also maximize the left hand side of the equations. Now suppose that p_i-s and $\hat{p}_i$-s maximize the left hand sides of the above two equations. We have, since "max" is a convex function, that

$$\begin{aligned} & \hat{p}_1(1-\hat{p}_2)E_{X_1,s}u_1(X_1,s) + \hat{p}_2(1-\hat{p}_1)E_{X_2,s}u_2(X_2,s) \\ & \qquad + \hat{p}_1\hat{p}_2E_{X_1,X_2,s}[\max\{u_1(X_1,s),u_2(X_2,s)\}] - \hat{p}_1c_1 - \hat{p}_2c_2 \\ \geq \;& p_1(1-p_2)E_{X_1,s}u_1(X_1,s) + p_2(1-p_1)E_{X_2,s}u_2(X_2,s) \\ & \qquad + p_1p_2E_{X_1,X_2,s}[\max\{u_1(X_1,s),u_2(X_2,s)\}] - p_1c_1 - p_2c_2 \\ \geq \;& p_1(1-p_2)E_{X_1,s}u_1(X_1,s) + p_2(1-p_1)E_{X_2,s}u_2(X_2,s) \\ & \qquad + p_1p_2E_{X_1,X_2}[\max\{E_su_1(X_1,s),E_su_2(X_2,s)\}] - p_1c_1 - p_2c_2. \end{aligned}$$

It follows that

$$\begin{aligned} & \hat{p}_1\hat{p}_2E_{X_1,X_2,s}[\min\{u_1(X_1,s),u_2(X_2,s)\}] + \hat{p}_1\hat{e}_1 + \hat{p}_2\hat{e}_2 \\ \geq \;& p_1p_2E_{X_1,X_2}[\min\{E_su_1(X_1,s),E_su_2(X_2,s)\}] + p_1e_1 + p_2e_2. \end{aligned} \tag{3}$$

When entry is endogenous, so that without the entry fee equation (1) is satisfied in equilibrium, then it can be checked that the optimal entry fees are equal to zero. Therefore, in that case, the equilibrium participation rates are the same regardless of whether the seller can or cannot charge entry fees. In other words, when entry is endogenous we have

$$\hat{p}_1\hat{p}_2E_{X_1,X_2,s}[\min\{u_1(X_1,s),u_2(X_2,s)\}] \geq p_1p_2E_{X_1,X_2}[\min\{E_su_1(X_1,s),E_su_2(X_2,s)\}].$$

Formally, we have the following result.

Proposition 1. If entry in the auctions is endogenous, with and without the seller releasing her private signal, then the expected revenue to the seller increases when her private information is released.

In fact when the seller can charge the optimal entry fees equation (2) holds regardless of whether or not entry is endogenous in the absence of the entry fees. Thus, we have the following result.

Proposition 2. If the seller can charge the bidder specific optimal entry fees in the auction then a public release of the seller's information increases the expected revenue.

3. Discussion

The results can be viewed from the perspective of the entry costs. When the entry costs are small entry is not endogenous in the sense that a bidder need not care about the participation rate of the other bidder. In that case, the linkage principle may fail. However, if entry is endogenous then the linkage principle continues to hold. Moreover, if the seller can charge the optimal entry fees in the auction then the release of seller's information increases the expected revenue regardless of the entry costs.

While endogenizing entry in the asymmetric single-object private-value auctions returns the linkage principle, it is not difficult to construct examples to show that this need not be enough to salvage the linkage principle in the multi-unit case Perry and Reny (1999). What or what else is necessary to restore the linkage principle in that case continues to remain an open question.

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