E C O N O M I C S   B U L L E T I N

Habit Persistence and Money in the Utility Function

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Abstract

The paper introduces habit persistence in consumption decisions in an infinitely−lived agents monetary model where money enters in the utility of the agent. In this case, we show that the equilibrium is saddle path whereas Auray, Collard and Fève [2004] showed that the interplay between habit persistence and cash−in−advance generates real indeterminacy. These two distinct but commonly used framework do not lead to the same dynamic properties, therefore.

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Introduction

Monetary models, use either a cash–in–advance (CIA hereafter) constraint (see e.g. Clower [1967], Lucas and Stokey [1983] or Cooley and Hansen [1989] (for an application)) or the money in the utility function (MIUF hereafter) framework (See e.g. Sidrauski [1967], Matsuyama [1990] and Christiano, Eichenbaum and Evans [2004]) to introduce money. In this paper, we introduce intertemporal complementarities in consumption decisions in a monetary model and study its local dynamics properties. Money is held in the economy because the household face either a CIA constraint or because it directly enters in his utility function. More important is the fact that households' preferences are characterized by habit persistence, introducing time non–separability in the model. An important feature of our modelling of habit persistence, is that it is internalized by individuals whenever they decide upon their consumption plans.

Auray, Collard and Fève [2004] show that habit persistence generates real indeterminacy in a CIA economy. It results from the interplay between habit persistence and the CIA constraint, given a specific environment on the labor and asset markets. Indeed, when individuals face the same positive belief on future inflation, higher expected inflation leads them to substitute current for future consumption, thus increasing their habits. This translates into higher money demand for tomorrow when habit persistence is strong enough, putting upward pressure on prices. Then, inflation expectations become self–fulfilling. An immediate but misleading intuition is that when money and consumption are gross complement, the previous results should still hold as a CIA constraint actually reveals a strong complementarity between money and consumption. Related to this issue is the paper of Christiano et al. [2004]. They calibrate a monetary model with MIUF, exogenous money growth rule and habit formation and do not find real indeterminacy. Indeed, we show that when money is introduced in the utility function, habit persistence does not imply real indeterminacy. These two non–embodied monetary economies do not possess similar local dynamic properties, therefore. This may seem tautological. It seems important to know that these two commonly used framework lead to distinct conclusions, however.

The paper is organized as follows. A first section presents the MIUF economy and discusses the dynamic properties of this economy. A last section offers some concluding remarks.

1 MIUF and Habit Formation

We question the viability of habit persistence as a source of real indeterminacy when money is introduced *via* MIUF framework. Indeed, Christiano et al. [2004] calibrate a monetary model with MIUF, exogenous money growth rule and habit formation and do not find real indeterminacy.

1.1 The household behavior

The economy is comprised of a unit mass continuum of identical infinitely lived agents, so that we will assume that there exists a representative household in the economy. We consider an economy where money is held because the households derives utility from money holdings. Household preferences are then represented by the lifetime utility function[1]

$$E_t \sum_{\tau=0}^{\infty} \beta^\tau \left\{ \frac{1}{1-\sigma} \left[\left(s_{t+\tau}^\eta + a \left(\frac{M_{t+\tau+1}}{P_{t+\tau}} \right)^\eta \right)^{\frac{1-\sigma}{\eta}} - 1 \right] - h_{t+\tau} \right\}$$

with $a > 0$, $\eta < 1$ and $\sigma \in \mathbb{R}^+ \backslash \{1\}$.

We only depart from the standard MIUF model, in that we allow for habit persistence in the consumption behavior, and therefore introduce time non–separability in the utility function. In this paper, we follow Constantidines and Ferson [1991], Braun, Constantidines and Ferson [1993] and consider internal habit persistence specified in difference with one lag in consumption. More specifically, we assume that s_t takes the form

$$s_t = c_t - \theta c_{t-1} \quad \text{with} \quad \theta \in (0,1) \tag{1}$$

The standard MIUF model can then be simply retrieved by setting θ to zero, such that the model embeds the standard case.

The household enters period t with real balances[2] M_t/P_t brought into period t from the previous period, as a mean to transfer wealth from one period to another. The household supplies her hours on the labor market at the real wage w_t. During the period, she also receives a lump–sum transfer from the monetary authorities in the form of cash equal to N_t/P_t. These revenues are then used to purchase consumption goods c_t and to acquire money balances for the next period. Therefore, in each and every period, the representative household faces a budget constraint of the following form

$$c_t + \frac{M_{t+1}}{P_t} \leqslant w_t h_t + \frac{M_t}{P_t} + \frac{N_t}{P_t} \tag{2}$$

Denoting $m_{t+1} = M_{t+1}/P_t$, $z_t = s_t^{\eta-1} \psi_t^{1-\sigma-\eta}$, $\psi_t = (s_t^\eta + a m_{t+1}^\eta)^{1/\eta}$ and $s_t = c_t - \theta c_{t-1}$, the first order conditions for the problem write

$$z_t - \beta\theta E_t z_{t+1} = \lambda_t \tag{3}$$

$$\lambda_t w_t = 1 \tag{4}$$

$$a z_t \left(\frac{m_{t+1}}{s_t} \right)^{\eta-1} = \lambda_t - \beta E_t \frac{\lambda_{t+1}}{\pi_{t+1}} \tag{5}$$

1.2 The firm and the monetary authorities

Alike the household, we assume that there is a large number of identical firms, such that we make the assumption of a representative firm that produces the homogenous consumption good by means of labor according to the following constant returns–to–scale technology $y_t = h_t$. Profit maximization implies that, in equilibrium, the real wage will be constant and equal to 1.

[1]We consider alternative specifications of the utility function. In each case, we never find real indeterminacy stemming from habit persistence.

[2]Hereafter, uppercases will be used to refer to nominal variables, whereas lowercases will stand for real variables.

The monetary authorities supply money, M^s, exogenously and follow the simple money growth rule

$$M_{t+1}^s = \gamma_t M_t^s \tag{6}$$

where $\gamma_t \geqslant 1$ is the exogenous gross rate of money supply growth. Hence, the lump–sum transfer received by the household, N, is given by $N_t = M_{t+1}^s - M_t^s = (\gamma_t - 1)M_t^s$.

1.3 The Equilibrium

An equilibrium of this economy is a sequence of prices $\{\mathcal{P}_t\}_{t=0}^{\infty} = \{w_t, P_t\}_{t=0}^{\infty}$ and a sequence of quantities $\{\mathcal{Q}_t\}_{t=0}^{\infty} = \{c_t, y_t, h_t, M_t\}_{t=0}^{\infty}$ such that:

(i) given a sequence of prices $\{\mathcal{P}_t\}_{t=0}^{\infty}$ and a sequence of money growth $\{\gamma_t\}_{t=0}^{\infty}$, the household maximize her utility and the firm maximizes its profits;

(ii) given a sequence of quantities $\{\mathcal{Q}_t\}_{t=0}^{\infty}$ and a sequence of money growth $\{\gamma_t\}_{t=0}^{\infty}$, the markets clear in the sense

$$y_t = c_t = h_t \tag{7}$$
$$M_{t+1} = M_{t+1}^s \tag{8}$$

1.4 Dynamic properties of the economy

Since in this economy, the equilibrium real wage, w_t, is equal to one, the system (3)– (5) actually simplifies to

$$z_t - \beta\theta E_t z_{t+1} = 1$$
$$a z_t \left(\frac{m_{t+1}}{s_t}\right)^{\eta-1} = 1 - \beta E_t \frac{1}{\pi_{t+1}}$$

Straightforward but tedious algebra allow to approximate the local dynamics by the three log–linear equations[3]

$$\widehat{y}_t = \theta\widehat{y}_{t-1} + \frac{(1-\theta)(1-\omega)(1-\sigma-\eta)}{\omega\sigma + (1-\omega)(1-\eta)}\widehat{m}_t \tag{9}$$
$$\widehat{m}_{t+1} = \widehat{m}_t - \widehat{\pi}_t \tag{10}$$
$$E_t\widehat{\pi}_{t+1} = \frac{\sigma(\eta-1)(\pi-\beta)}{\beta(\omega\sigma + (1-\omega)(1-\eta))}\widehat{m}_{t+1} \tag{11}$$

where $\omega = \frac{\gamma(1-\theta)}{(\gamma(1-\theta)+(\gamma-\beta)(1-\beta\theta)\zeta)}$. Note that $\omega \in (0,1)$ as long as the average growth rate of money supply is greater or equal to zero ($\gamma \geq 1$) and for all $\theta \in (0,1)$. It should be clear that the two last equations (10)–(11) form an autonomous system with regard to the first equation (9). Further, any real indeterminacy can only result from a shift in the root of this later autonomous system. Hence, in order to shed light on the case for real

[3]Details are available in appendix.

indeterminacy, it suffices to study the system (10)–(11) whose characteristic polynomial is given by

$$P(x) = (1-x)(-\nu - x) + \nu = x(x-(1-\nu)) \quad \text{with} \quad \nu = \frac{\sigma(\eta-1)(\pi-\beta)}{\beta(\omega\sigma + (1-\omega)(1-\eta))}$$

Note that since in steady state $\pi = \gamma \geq 1$, $\beta \in (0,1)$, $\eta < 1$, $\sigma \in \mathbb{R}^+ \backslash \{1\}$ and $\omega \in (0,1)$ for all $\theta \in (0,1)$, we know that $\nu < 0$. It is also clear that this polynomial has one root equal to 0 and the other one is given by

$$x = 1 - \nu > 1$$

Hence, whatever the value for the habit persistence parameter, the system will display saddle path. This result points out two main findings. First, real indeterminacy essentially comes from the interplay between habit persistence and the CIA constraint. When money is introduced *via* MIUF, this dynamic property will disappear.[4] It follows that dynamics properties of monetary models will crucially depend on the way the money is introduced. When the habit persistence parameter is set to zero, we retrieve the cash constraint when real balance and consumption are complementary. Conversely, when $\theta > 0$, complementary implies that real balance are equal to the consumption index $s_t \equiv c_t - \theta c_{t-1}$. This differs from a framework where money is introduced *via* a CIA constraint, as now only a fraction of consumption is submitted to the cash constraint.

2 Conclusion

Christiano et al. [2004] calibrate a monetary model with MIUF, exogenous money growth rule and habit formation and do not find real indeterminacy. In this paper, we show that when money is introduced *via* MIUF in a model with habit persistence, the equilibrium is saddle path. Auray et al. [2004] show that habit persistence generates real indeterminacy in a CIA economy. It results from the interplay between habit persistence and the CIA constraint. These two non–embodied monetary economies do not possess similar local dynamic properties, therefore. This may seem tautological but it seems important to know that these two commonly used framework lead to distinct conclusions.

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[4]We also investigate the case where we introduced the CIA timing in the model as in Carlstrom and Fuerst [2001]. For our utility function, the model always generates saddle path, no matter the habit persistence parameter (see section B in appendix).

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Appendix

A Standard MIUF timing

Household preferences are characterized by the lifetime utility function:[5]

$$E_t \sum_{\tau=0}^{\infty} \beta^\tau U\left(c_{t+\tau} - \theta c_{t+\tau-1}, \frac{M_{t+\tau+1}}{P_{t+\tau}}, h_{t+\tau}\right) \tag{12}$$

where $0 < \beta < 1$ is a constant discount factor, c denotes the domestic consumption bundle, M/P is real balances and h is the quantity of hours supplied by the representative household. The utility function,$U\left(C, \frac{M}{P}, h\right) : \mathbb{R}_+ \times \mathbb{R}_+ \times [0,1] \longrightarrow \mathbb{R}$ is increasing and concave in c and M/P and basically takes the form

$$U\left(c_t - \theta c_{t-1}, \frac{M_{t+1}}{P_t}, h_t\right) = \frac{1}{1-\sigma}\left[\left((c_t - \theta c_{t-1})^\eta + a\left(\frac{M_{t+1}}{P_t}\right)^\eta\right)^{\frac{1-\sigma}{\eta}} - 1\right] - h_t$$

with $a > 0$, $\eta < 1$ and $\sigma \in \mathbb{R}^+ \backslash \{1\}$.

In each and every period, the representative household faces a budget constraint of the form

$$c_t + \frac{M_{t+1}}{P_t} \leqslant w_t h_t + \frac{M_t}{P_t} + \frac{N_t}{P_t} \tag{13}$$

[5] $E_t(.)$ denotes mathematical conditional expectations. Expectations are conditional on information available at the beginning of period t.

Hereafter, we will note $m_{t+1} = M_{t+1}/P_t$, $\psi_t = \left(s_t^\eta + a m_{t+1}^\eta\right)^{\frac{1}{\eta}}$ and $s_t = c_t - \theta c_{t-1}$. The first order conditions for the problem then write

$$z_t - \beta\theta E_t z_{t+1} = \lambda_t \tag{14}$$
$$1 = \lambda_t w_t \tag{15}$$
$$a z_t \left(\frac{m_{t+1}}{s_t}\right)^{\eta-1} = \lambda_t - \beta E_t \frac{\lambda_{t+1}}{\pi_{t+1}} \tag{16}$$

where we noted $z_t = s_t^{\eta-1}\psi_t^{1-\sigma-\eta}$. In equilibrium, we further have $c_t = y_t = h_t$, $w_t = 1$ and $m_{t+1} = \gamma m_t/\pi_t$.

First of all, note that (29) rewrites

$$z_t - \beta\theta E_t z_{t+1} = 1$$

in equilibrium, implying that $z_t = (1-\beta\theta)^{-1}$ along the whole path.

Steady state:

$$z = (1-\beta\theta)^{-1} \tag{17}$$
$$s = (1-\theta)c \tag{18}$$
$$az\left(\frac{m}{s}\right)^{\eta-1} = \frac{\pi-\beta}{\pi} \tag{19}$$
$$\pi = \gamma \tag{20}$$
$$\left(\frac{\psi}{s}\right)^{\eta} = \left(1 + \frac{\pi-\beta}{\pi}\frac{1-\beta\theta}{1-\theta}\zeta\right) \text{ where } \zeta = m/c \tag{21}$$

Log–linear approximation The log–linear approximation of the previous system is given by

$$\widehat{\psi}_t = \omega\widehat{s}_t + (1-\omega)\widehat{m}_{t+1} \tag{22}$$
$$(\eta-1)\widehat{s}_t + (1-\sigma-\eta)\widehat{\psi}_t = 0 \tag{23}$$
$$\widehat{s}_t = \frac{\widehat{c}_t}{1-\theta} - \frac{\theta}{1-\theta}\widehat{c}_{t-1} \tag{24}$$
$$\widehat{m}_t = \widehat{m}_{t-1} - \pi_t \tag{25}$$
$$(\eta-1)(\widehat{m}_{t+1} - \widehat{s}_t) = \frac{\beta}{\gamma-\beta}E_t(\widehat{\pi}_{t+1}) \tag{26}$$

where $\omega = \frac{\gamma(1-\theta)}{(\gamma(1-\theta)+(\gamma-\beta)(1-\beta\theta)\zeta)}$. Note that $\omega \in (0,1)$ as long as the average growth rate of money supply is greater or equal to zero ($\gamma > 1$) and for all $\theta \in (0,1)$. Plugging (22) and (23) in (24), we obtain the law of motion of consumption

$$\widehat{c}_t = \theta\widehat{c}_{t-1} + \frac{(1-\omega)(1-\eta-\sigma)}{\omega\sigma + (1-\eta)(1-\omega)}\widehat{m}_{t+1} \tag{27}$$

Using the latter equation and the law of motion of money in (26), we obtain the dynamic behavior of the inflation rate

$$E_t(\widehat{\pi}_{t+1}) = \frac{\sigma(\gamma-\beta)(\eta-1)}{\beta(\omega\sigma + (1-\omega)(1-\eta))}\left(\widehat{m_t} - \widehat{\pi}_t\right) \tag{28}$$

It should be clear to the reader that the system formed by equations (25) and (28) is autonomous with regard equation (27). Further, this system is determinate iff the number of eigenvalues lying outside the unit circle is equal to the number of free variables; in this case the sole free variable is the inflation rate. Therefore determinacy of the system occurs iff its two eigenvalues lie on each side of the unit circle. The system may be rewritten

$$E_t x_{t+1} = J x_t$$

where $x_t = \{\widehat{m}_t, \widehat{\pi}_t\}$ and

$$J = \begin{pmatrix} 1 & -1 \\ \nu & -\nu \end{pmatrix} \text{ with } \nu = \frac{\sigma(\gamma-\beta)(\eta-1)}{\beta(\omega\sigma+(1-\omega)(1-\eta))}$$

The two eigenvalues of the system are then 0 and $1-\nu$. Note that we have shown that $\omega \in (0,1)$ for all $\theta \in (0,1)$, and by assumption, we have $\eta < 1$, $\beta \in (0,1)$ and $\sigma \in \mathbb{R}^+ \backslash \{1\}$, therefore $\nu < 0$. Hence whatever the value for $\theta \in (0,1)$ the system displays saddle path.

B Cash–in–advance timing

In this case, the household values the money balances carried over the previous period rather than those she acquires. Hence, the instantaneous utility function function rewrites

$$U\left(c_t - \theta c_{t-1}, \frac{M_t}{P_t}, \ell_t\right) = \frac{1}{1-\sigma}\left[\left((c_t-\theta c_{t-1})^\eta + a\left(\frac{M_t}{P_t}\right)^\eta\right)^{\frac{1-\sigma}{\eta}} - 1\right] - h_t$$

with $\eta < 1$ and $\sigma \in \mathbb{R}^+ \backslash \{1\}$. The first order conditions for the household's problem then write

$$z_t - \beta\theta E_t z_{t+1} = \lambda_t \tag{29}$$

$$1 = \lambda_t w_t \tag{30}$$

$$\lambda_t = \beta E_t a \frac{z_{t+1}}{\pi_{t+1}}\left(\frac{m_{t+1}}{\pi_{t+1}s_{t+1}}\right)^{\eta-1} + \beta E_t \frac{\lambda_{t+1}}{\pi_{t+1}} \tag{31}$$

where we adopted the same notational conventions as in the previous section and $\psi_t = \left(s_t^\eta + a\left(\frac{m_t}{\pi_t}\right)^\eta\right)^{\frac{1}{\eta}}$. In equilibrium, $c_t = y_t = h_t$, $w_t = 1$, such that the preceding system reduces to

$$z_t - \beta\theta E_t z_{t+1} = 1 \tag{32}$$

$$1 = \beta E_t a \frac{z_{t+1}}{\pi_{t+1}}\left(\frac{m_{t+1}}{\pi_{t+1}s_{t+1}}\right)^{\eta-1} + \beta E_t \frac{1}{\pi_{t+1}} \tag{33}$$

We therefore retrieve the fact that z_t is constant ($z_t = (1-\beta\theta)^{-1}$) along the dynamic path.

Steady state:

$$z = (1 - \beta\theta)^{-1} \tag{34}$$
$$s = (1 - \theta)c \tag{35}$$
$$\beta a \frac{z}{\pi}\left(\frac{m}{\pi s}\right)^{\eta-1} = \frac{\pi - \beta}{\pi} \tag{36}$$
$$\pi = \gamma \tag{37}$$
$$\left(\frac{\psi}{s}\right)^{\eta} = \left(1 + \frac{\pi - \beta}{\beta}\frac{1 - \beta\theta}{1 - \theta}\zeta\right) \text{ where } \zeta = m/c \tag{38}$$

Log–linear approximation The log–linear approximation of the previous system is given by

$$\widehat{\psi}_t = \omega\widehat{s}_t + (1 - \omega)(\widehat{m}_t - \widehat{\pi}_t) \tag{39}$$
$$(\eta - 1)\widehat{s}_t + (1 - \sigma - \eta)\widehat{\psi}_t = 0 \tag{40}$$
$$\widehat{s}_t = \frac{\widehat{c}_t}{1 - \theta} - \frac{\theta}{1 - \theta}\widehat{c}_{t-1} \tag{41}$$
$$\widehat{m}_{t+1} = \widehat{m}_t - \widehat{\pi}_t \tag{42}$$
$$(\gamma + (\gamma - \beta)(\eta - 1))E_t\widehat{\pi}_{t+1} = (\gamma - \beta)(\eta - 1)E_t(\widehat{m}_{t+1} - \widehat{s}_{t+1}) \tag{43}$$

where $\omega = \frac{\beta(1-\theta)}{(\beta(1-\theta)+(\gamma-\beta)(1-\beta\theta)\zeta)}$. Note that $\omega \in (0, 1)$ as long as the average growth rate of money supply is greater or equal to zero ($\gamma > 1$) and for all $\theta \in (0, 1)$. Plugging (39) and (40) in (41), we obtain the law of motion of consumption

$$\widehat{c}_t = \theta\widehat{c}_{t-1} + \frac{(1 - \omega)(1 - \eta - \sigma)}{\omega\sigma + (1 - \eta)(1 - \omega)}(\widehat{m}_t - \widehat{\pi}_t) \tag{44}$$

Using the latter equation and the law of motion of money in (43), we obtain the dynamic behavior of the inflation rate

$$E_t(\widehat{\pi}_{t+1}) = \frac{\gamma(\omega\sigma + (1 - \omega)(1 - \eta))}{\gamma(\omega\sigma + (1 - \omega)(1 - \eta)) + \sigma(\gamma - \beta)(\eta - 1)}(\widehat{m_t} - \widehat{\pi}_t) \tag{45}$$

Like in the previous case, we can rewrite the system in the following matricial form

$$E_t x_{t+1} = J x_t$$

where $x_t = \{\widehat{m}_t, \widehat{\pi}_t\}$ and

$$J = \begin{pmatrix} 1 & -1 \\ \nu & -\nu \end{pmatrix} \text{ with } \nu = \frac{\gamma(\omega\sigma + (1 - \omega)(1 - \eta))}{\gamma(\omega\sigma + (1 - \omega)(1 - \eta)) + \sigma(\gamma - \beta)(\eta - 1)}$$

Like in the previous case, the two eigenvalues of the system are then 0 and $1 - \nu$. Further, $1 - \nu$ may be written as

$$1 - \nu = \frac{1}{1 - \frac{\sigma(1-\eta)(\gamma-\beta)}{\gamma(\omega\sigma+(1-\omega)(1-\eta))}} > 1 \iff \eta < 1,\ \beta \in (0, 1),\ \omega \in (0, 1), \gamma \geqslant 1, \sigma \in \mathbb{R}^+ \backslash \{1\}$$

such that the model will always exhibit saddle path.