E C O N O M I C S   B U L L E T I N

Effect of small−sample adjustments for Cox test under non−nested linear regression models

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Abstract

We consider the effect of Godfrey and Pesaran's (1983) two small−sample adjustments for the Cox (1961, 1962) non−nested test statistic under linear regression models. Based on convenient representations of the test statistics in terms of the correlation coefficients, we compare the confidence contours of the test statistics.

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1 Introduction

In this note, we consider the effect of Godfrey and Pesaran's (1983) small-sample adjustments for the Cox (1961, 1962) non-nested test statistic under linear regression models. For simple normal linear non-nested regression models, we represent the adjusted-Cox tests in terms of sample correlation coefficients and illustrate the small-sample confidence contours of the tests. We find that the acceptance regions of the adjusted-Cox tests are broader than that of the unadjusted-Cox test, and the size distortion of the unadjusted-Cox test can be corrected by the small-sample adjustments. However, due to the implicit null hypothesis (Mizon and Richard (1986)), the adjusted-Cox tests have powerless regions against the alternative hypothesis as well as the unadjusted-Cox test.

2 Main results

Let y_t be a regressand, and (x_t, z_t) be conditioning variables for $t = 1, \dots, n$. Consider the following non-nested testing problem for simple normal linear regression models,

$$H_0 : \ y_t | x_t, z_t \sim IN(a_0 + b_0 x_t, \sigma_0^2), \quad H_1 : \ y_t | x_t, z_t \sim IN(a_1 + b_1 z_t, \sigma_1^2),$$

where $a_0, b_0, \sigma_0, a_1, b_1$, and σ_1 are unknown parameters. These hypotheses can be equivalently written in terms of the population correlation coefficients $\rho \equiv (\rho_{xy}, \rho_{yz}, \rho_{xz})$, i.e.,

$$H_0' : \ \rho_{yz} = \rho_{xy}\rho_{xz}, \quad H_1' : \ \rho_{xy} = \rho_{yz}\rho_{xz}.$$

Hall (1983) showed that the Cox (1961, 1962) test statistic for testing model H_0 against model H_1 can be written as

$$N_0 \equiv \frac{\sqrt{n}(1 - r_{xy}^2 r_{xz}^2) \log\{(1 - r_{yz}^2)/(1 - r_{xy}^2 r_{xz}^2)\}}{2[r_{xy}^2 r_{xz}^2 (1 - r_{xy}^2)(1 - r_{xz}^2)]^{\frac{1}{2}}},$$

where $r \equiv (r_{xy}, r_{yz}, r_{xz})$ are the sample correlation coefficients for x_t, y_t, and z_t. Cox (1961, 1962) and Pesaran (1974) showed that N_0 follows the standard normal limiting distribution under H_0, and N_0 can be used as a specification test statistic for H_0 against H_1.

Let $N_0^* \equiv \text{plim}_0(N_0/\sqrt{n})$, where plim_0 is the stochastic limit under H_0. The implicit null hypothesis for N_0, proposed by Mizon and Richard (1986), is defined as a region for the parameters that satisfy $N_0^* = 0$, i.e.,

$$H_0^N : \ \rho_{yz}^2 = \rho_{xy}^2 \rho_{xz}^2.$$

Note that H_0^N has a broader acceptance region than H_0', i.e., $\{\rho : \rho \text{ satisfies } H_0'\} \subseteq \{\rho : \rho \text{ satisfies } H_0^N\}$. In particular, if $\rho_{yz} = -\rho_{xy}\rho_{xz}$, then N_0 wrongly accepts H_0. In other words, N_0 has a powerless region for H_0. Hall (1983) illustrated confidence contours of N_0 on the $r_{xy} - r_{yz}$ plane for given values of r_{xz}, and showed the existence of the regions where N_0 wrongly accepts model H_0 by the implicit null hypothesis. This note extends Hall's (1983) analysis to two small-sample adjusted versions of the Cox test, which are proposed by Godfrey and Pesaran (1983), and then illustrate the effect of the small-sample adjustments to the acceptance regions for H_0.

Godfrey and Pesaran (1983, p.138) proposed two small-sample adjusted-Cox tests (denoted W_0 and $\tilde{N}_0$) based on linear regression models. After some lengthy mathematical manipulations, the small-sample adjusted test statistics are written as

$$W_0 = \frac{\sqrt{n-2}\{r_{xy}^2 r_{xz}^2 - r_{yz}^2 + \frac{1}{n-2}(1-r_{xy}^2)(1-r_{xz}^2)\}}{[\frac{2}{n-2}(1-r_{xy}^2)^2\{1-r_{xz}^4 - \frac{1}{n-2}(1-r_{xz}^2)^2\} + 4(1-r_{xy}^2)(r_{xy}^2 r_{xz}^2 - r_{xy}^2 r_{xz}^4)]^{\frac{1}{2}}},$$

$$\tilde{N}_0 = \frac{\sqrt{n-2}\{1-r_{xy}^2 r_{xz}^2 - \frac{1}{n-2}(1-r_{xy}^2)(1-r_{xz}^2)\}\log\{\frac{1-r_{yz}^2}{1-r_{xy}^2 r_{xz}^2 - \frac{1}{n-2}(1-r_{xy}^2)(1-r_{xz}^2)}\}}{2[(1-r_{xy}^2)\{r_{xy}^2 r_{xz}^2 - r_{xy}^2 r_{xz}^4 + \frac{1}{2(n-2)}(1-r_{xy}^2)(1-r_{xz}^4 - \frac{1}{n-2}(1-r_{xz}^2)^2)\}]^{\frac{1}{2}}}.$$

Similarly to N_0, W_0 and $\tilde{N}_0$ are also represented in terms of r. Note that from $\text{plim}_0(W_0/\sqrt{n})$ and $\text{plim}_0(\tilde{N}_0/\sqrt{n})$, the implicit null hypothesis of W_0 and $\tilde{N}_0$ is also H_0^N. However, although the adjusted- and unadjusted-Cox test statistics are asymptotically equivalent, their small sample properties may differ considerably.

In order to compare the small-sample properties of the unadjusted-Cox test, N_0, and the adjusted-Cox test, W_0, we consider the example case of $n = 20$, $r_{xz} = 0.5$, and $\alpha = 0.05$, where α is the nominal size of the tests (i.e., the critical value of the tests is 1.96). Figure 1 illustrates the confidence contours for N_0 and W_0.[1] We omit the results for $\tilde{N}_0$, which are quite similar to those for W_0. We find the following facts. First, since $\{r : |N_0| \leq 1.96\} \subseteq \{r : |W_0| \leq 1.96\}$, the actual size of W_0 is smaller than that of N_0. This result is compatible with the simulation results of Godfrey and Pesaran (1983), where N_0 tends to over-reject the true model. Second, as evidenced by the region where W_0 accepts H_0 but N_0 rejects (i.e., $|W_0| \leq 1.96$ and $|N_0| > 1.96$), W_0 corrects the size distortion mainly along the r_{yz}-axis. Third, since the powerless region due to the implicit null hypothesis (i.e., the region along the line $r_{yz} = -0.5r_{xy}$) still remains for W_0, the problem of implicit null hypothesis remains unresolved.

In summary, for smaller values of $|r_{xy}|$, the small-sample adjustment by W_0 or $\tilde{N}_0$ guards against over-rejection of H_0. However, for larger values of $|r_{xy}|$, the small-sample adjustment by W_0 or $\tilde{N}_0$ is sensitive to the problem of the implicit null hypothesis H_0^N.

[1] Since the sample correlation matrix for x, y and z is positive definite, all acceptance and rejection regions are within an ellipse.

References

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Figure 1: Confidence contours for N_0 and W_0 : $n = 20,\ r_{xz} = 0.5,\ \alpha = 0.05$ (horizontal axis: r_{xy}, vertical axis: r_{yz})