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Rational Expectation Hypothesis: An Application of the Blanchard and Khan Approach

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Abstract

This paper uses the solution of the linear difference model under rational expectation of Blanchard and Kahn (1980) to test the validity of the inflation stickiness and the Rational Expectation Hypotheses for the Brazilian economy during the period from 06/95 to 09/02. Using the Fuhrer−Moore model and GMM we find evidence favoring both hypothesis.

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1. Introduction

In this paper we focus on the derivation of a model that assumes rigid prices and inflation, based on rational expectations and staggered, multi period overlapping wage contracts. We explore the approaches of Taylor (1980) and Fuhrer and Moore (1995) to characterize the dynamic relationship between interest rates, output and inflation for the Brazilian Economy.

The model we use is derived from the expenditure function of the economy and the Fuhrer and Moore (1995) supply function. The rational expectations hypothesis is intrinsic to the derivation process and leads to a linear model involving output and inflation expectations. The interest rate is treated as exogenous. The resulting system is estimated via GMM and the rational expectation hypothesis is tested by means of the J statistics. Blanchard and Khan (1980) theory is used to provide further support to the J test. As the J test is known to have low potency we present the solution to the corresponding linear system of rational expectation following the Blanchard and Khan approach. This is an original contribution of the article since the Blanchard and Khan approach has never been used for testing purposes before.

The paper proceeds as follows. Section 2 introduces the economic model basing it on an IS dynamic equation and the Fuhrer and Moore(1995) supply function. In Section 3 we derive the linear system of rational expectations. In Section 4 we show the empirical findings for the Brazilian economy data. Section 5 concludes the paper.

2. The Model

The demand equation of the equilibrium model we propose is

$$y_t = a_1 y_{t-1} + a_2 y_{t-2} - a_3 (i_{t-1} - E_{t-1}\pi_t) + \varepsilon_t \qquad (1)$$

where y_t is the output gap, π_t is the rate of inflation, i_t represents the nominal interest rate, $E_{t-1}\pi_t$ is the expected inflation rate with expectations formed in period $t-1$, and ε_t is the error term capturing demand shocks.

In our econometric exercise we use the short-run interest rate as a proxy for long-run interest rates. In general the output gap would depend on long-term interest rates, however a standard practice in the literature is to use short-term interest rates in the output equation. The underlying assumption is that the expectation hypothesis for interest rates is valid, i.e., the long-term interest rate is given by the short-term interest rate plus a constant risk premium for being long. In this case, monetary policy changes would have the same impact on both interest rates.

The view that the inflation rate and not just the price level would exhibit some degree of stickiness has been adopted by Fuhrer and Moore (1995). They assume that wage negotiations are conducted in terms of the wage relative to an average of real contracts wages in effect over the life of a contract. The Fuhrer and Moore (1995) specification is given by

$$\pi_t = a(\pi_{t-1} + E_t\pi_{t+1}) + g(y_t + E_t y_{t+1}) + \eta_t \tag{2}$$

In this expression $E_t y_{t+1}$ is the output gap expectation for instant $t+1$ in instant t and η_t is a supply shock

The Fuhrer-Moore specification is closely related to, but distinct from, Taylor's original work on staggered, multi-period overlapping contracts and aggregate-price adjustment. Taylor's model (1979,1980) of price level adjustment leads to a reduced-form expression for the price level in which p_t depends on p_{t-1} and $E_t(p_{t+1})$. The backward-looking aspect of price behavior causes unanticipated reductions in the money supply to cause real output declines. Only as contracts expire can their real value be reduced to levels consistent with the new, lower money supply. However, the inflation rate depends on $E_t(\pi_{t+1})$, not π_{t-1}, so the inflation process does not display stickiness. As Ball (1994) has shown, price rigidities based on backward-looking behavior in the price-level process need not imply that policies to reduce inflation by reducing the growth rate of money will cause a recession. In the Fuhrer-Moore specification, the backward-looking nature of the inflation process implies that reductions in the growth rate of money will be costly in terms of output.

3. Solving a System of Rational Expectations

The supply and demand equations of the Fuhrer-Moore equations are equivalent to the system of rational expectations

$$\begin{pmatrix} E_t(y_{t+1}) \\ E_t(\pi_{t+1}) \end{pmatrix} = A\begin{pmatrix} y_t \\ \pi_t \end{pmatrix} + C\begin{pmatrix} y_{t-1} \\ \pi_{t-1} \\ i_t \end{pmatrix}$$

where

$$A = \frac{1}{d}\begin{pmatrix} 2a_3 g - a_1 & -2a_3 \\ 2g(a_1+1) & -2 \end{pmatrix} \quad \text{and} \quad C = \frac{1}{d}\begin{pmatrix} -a_2 & a_3 & a_3 \\ 2ga_2 & 1 & -2ga_3 \end{pmatrix}$$

and $d = -2ga_3 - 1$, with a_1 and a_3 being both positive.

This system has the same form as that studied in Blanchard and Khan (1980), that is, it can be written as

$$\begin{pmatrix} y_t \\ \pi_1 \\ E_t(y_{t+1}) \\ E_t(\pi_{t+1}) \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ c_{11} & c_{12} & a_{11} & a_{12} \\ c_{21} & c_{22} & a_{21} & a_{22} \end{pmatrix}\begin{pmatrix} y_{t-1} \\ \pi_{t-1} \\ y_t \\ \pi_t \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ c_{13} \\ c_{23} \end{pmatrix} i_t$$

Let D and F denote the parameter matrices appearing in this representation. We take i_t exogenous. Following the notation in Blanchard and Khan (1980) let $X_t = (y_{t-1}, \pi_{t-1})'$ be the vector of predetermined variables and let $P_t = (y_t, \pi_t)'$ be the vector of nonpredetermined variables. Thus we may write

$$\begin{pmatrix} X_{t+1} \\ E_t(P_{t+1}) \end{pmatrix} = D \begin{pmatrix} X_t \\ P_t \end{pmatrix} + F i_t$$

where E_t in all the above expressions denotes expectation conditional to the information set Ω_t (such sets form an increasing sequence of σ -fields).

According to Blanchard and Khan (1980) the system above has a unique solution for y_t and π_t if and only if the matrix D has two eigenvalues outside the unit circle and two inside. The solution is worked out as follows. Let $D = L^{-1}\Lambda L$ be the Jordan decomposition of D with the eigenvalues in Λ ordered by increasing absolute values. Write

$$\Lambda = \begin{pmatrix} \Lambda_1 & 0 \\ 0 & \Lambda_2 \end{pmatrix}$$

where Λ_1 and Λ_2 are two by two matrices with the roots outside the unit circle in Λ_2. Accordingly, write

$$L = \begin{pmatrix} l_{11} & l_{12} \\ l_{21} & l_{22} \end{pmatrix} \text{ and } F = \begin{pmatrix} f_1 \\ f_2 \end{pmatrix}$$

where the partition in L is in $2x2$ blocks and of F in $2x1$ vectors.

The unique solution to the system of rational expectation is given by

$$P_t = -l_{22}^{-1} l_{21} X_t - l_{22}^{-1} \sum_{k=0}^{+\infty} \Lambda_2^{-k-1} (l_{21} f_1 + l_{22} f_2) E_t(i_{t+k})$$

Assuming that i_t is a martingale with respect to Ω_t then the solution will be

$$P_t = VX_t + Ui_t$$

where

$$V = -l_{22}^{-1} l_{21} \qquad \text{and} \qquad U = l_{22}^{-1}(I - \Lambda_2)^{-1}(l_{21} f_1 + l_{22} f_2)$$

Thus we may write

$$y_t = \upsilon_{11} y_{t-1} + \upsilon_{12} \pi_{t-1} + u_1 i_t$$

$$\pi_t = \upsilon_{21} y_{t-1} + \upsilon_{22} \pi_{t-1} + u_2 i_t \qquad (3)$$

In the next section we present estimates of V and U based on estimates of a_i and g.

4. Empirical Findings

We use logs of monthly observations on interest rate, output gap and inflation rate from 06/95 through 09/02. The data was obtained from the Brazilian Institute for Applied Economic Research[1]. The output gap is the difference between potential and actual real GNP. Potential output was estimated using the Hodrick-Prescott filter.

The unobservables $E_{t-1}(\pi_t)$, $E_t(\pi_{t+1})$, and $E_t(y_{t+1})$ are treated as in MacCallum (1976) and Roberts (2001). Actual future inflation and actual future output are used as proxies for the corresponding expectations. This approach requires the use of instrumental variables in the estimation process. These are government expenditures, lagged values of the output gap and the inflation rate, and a dummy variable indicative of the change in the exchange regime in 01/99.

The method of GMM with Newey-West kernel, applied separately to each equation, lead to the statistics shown in Tables 1 and 2. The rational expectation hypothesis is tested by the J-statistics associated with over-identifying restrictions. The value of J does not indicate evidence against the rational expectation hypothesis.

In the Fuhrer-Moore model $a = 0.5$. A departure from this hypothesis is viewed as evidence against inflation persistence. We see from Table 1 that $(a - 0.5)$ does not differ significantly from zero.

It is seen from the estimates shown in the tables that $a_1 = 0.24651$, $a_2 = 0.34910$, $a_3 = 0.005143$, and $g = 0.00896$. Then

$$A = \begin{pmatrix} 0.99982 & 0.01029 \\ -0.02233 & 1.99982 \end{pmatrix},$$

$$C = \begin{pmatrix} 0.34907 & -0.00514 \\ -0.00625 & -0.99991 \end{pmatrix},\ f_1 = \begin{pmatrix} 0 \\ 0 \end{pmatrix},\ \text{and}\ f_2 = \begin{pmatrix} -0.00514 \\ 0.00009 \end{pmatrix},$$

where $f_2'' = (c_{13}, c_{23})$. With the notation of the previous section these matrices lead to the following Jordan decomposition matrices of D.

$$L = \begin{pmatrix} 0.822298 & 0.00785 & -0.01076 & 0.17993 \\ -0.00007 & -0.57851 & 0.66302 & -0.08444 \\ -0.22552 & 0.00772 & -0.01097 & 0.22877 \\ 0.00002 & -0.56855 & 0.67590 & -0.10736 \end{pmatrix}$$

$$\Lambda = \begin{pmatrix} -0.27403 & 0 & 0 & 0 \\ 0 & 0.98280 & 0 & 0 \\ 0 & 0 & 1.01943 & 0 \\ 0 & 0 & 0 & 1.27144 \end{pmatrix}$$

It follow that

$$V = \begin{pmatrix} 0.15776 & 0.84222 \\ 0.99336 & 0.00664 \end{pmatrix} \quad \text{and} \quad U = \begin{pmatrix} 0.01636 \\ -0.01653 \end{pmatrix}$$

i.e.,

[1] IPEA-BRazil: www.ipeadata.gov.br

$$
\begin{aligned}
y_t &= 0.15776 y_{t-1} + 0.84222 \pi_{t-1} + 0.01636 i_t \\
\pi_t &= 0.99336 y_{t-1} + 0.00664 \pi_{t-1} - 0.01653 i_t
\end{aligned}
\tag{4}
$$

These results generating the solution (4) provide further support to the rational expectations hypothesis in the sense that the Blanchard and Khan (1980) condition is satisfied for the GMM estimates of a_i and g. It implies that this model posits rational behavior in the sense that agents know the model and use all variable information in forming their forecasts.

The positive signs of υ_{12} and υ_{21} can be associated with the comments of Mankiw (2001) on the inexorable and mysterious tradeoff between inflation and unemployment. He argues on the recent experience in the USA and says that "The combination of low inflation and low unemployment enjoyed by the United States in the late 1990s suggests to some people that there is no longer a tradeoff between these two variables, or perhaps that it never existed at all." We find evidence that the interest rate affects the output gap positively and the inflation rate negatively. These results are also present in the Tables 1 and 2 below.

Table 1: Supply Equation. GMM estimation with Newey-West weights. Instruments are government expenditures, a dummy for 01/99, and y_{t-1}.

Variable	Parameter	Estimate	Stand. Error	t	p-value
intercept		-0.00011	0.00028	-0.381	0,704
$E_t(y_{t+1}) + y_t$	g	0.00896	0.00311	2.881	0,000
$E_t(\pi_{t+1}) + \pi_{t-1}$	a	0.50222	0.02450	20.502	0,005
R^2		0.80			
J statistics	Sample		nJ	df	
0.12	88		10.56	8	0,228

Table 2: Demand Equation. GMM estimation with Newey-West weights. . Instruments are government expenditures, a dummy for 01/99, and π_{t-1}.

Variable	Parameter	Estimate	Stand. Error	t	p-value
intercept		0.00608	0.00446	1.36460	0,176
y_{t-1}	a_1	0.24651	0.08114	3.038	0,003
y_{t-2}	a_2	0.34910	0.08187	4.264	0,000
$i_{t-1} - E_{t-1}\pi_t$	a_3	0.00514	0.00259	-1.984	0,051
R^2		0.56			
J statistics	Sample		nJ	df	
0.11	86		9.46	15	0,852

5. Conclusions

In this paper we propose a rational expectations model for the Brazilian economy. Using GMM, the empirical results indicate that one cannot reject the rational expectation and the inflation persistence hypotheses. The Blanchard and Khan(1980) condition is satisfied for the GMM estimates and the solution to the corresponding linear system of rational expectations indicate that the interest rate affects the output gap positively and the inflation rate negatively. The elasticities of the output gap relative to past inflation and of actual inflation relative to past output gap are both positive and therefore consistent with the view of Mankiw (2001)on the tradeoff between inflation and unemployment.

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