E C O N O M I C S   B U L L E T I N

Risk−adjusted measures of wage inequality and safety nets

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Abstract

Income variablity is likely to increase wage inequality if poorer households are more vulnerable to shocks. Using a simple method to estimate risk−adjusted measures of wage inequality and data from Mexico, this note shows that safety nets could offset a good part of the impact of risk aversion on wage inequality.

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1 Introduction

When performing distributional analysis, economists usually rely on cross-sectional data which give information on income at one point in time (e.g., Atkinson and Bourguignon, 2000 and Lambert, 2002). However, it has long been recognized that incomes are subject to variability and that this may affect welfare if individuals are risk averse. Of course, part of the wage variation may be "certain", or known a priori. In such case, well functioning credit markets should allow individuals to manage the variations. Furthermore, if insurance markets are complete, individuals may also be able to offset the impact of the "uncertain" part of income variability on consumption. Blundell and Preston (1998) suggest that this is the case in Britain. But the impact of shocks may be more difficult to offset in developing countries where safety nets and credit markets are less well developed, and this may have implications for inequality.

Atkinson (1970), Yitzhaki and Slemrod (1991) and Davies and Hoy (1995) argue that there are links between the literatures on risk and inequality, because the mathematical structures used to discuss both concepts are similar. In this note, we show that the concepts of risk and inequality can be used jointly to estimate risk-adjusted measures of inequality. We propose a simple method to do so and we use this method to show how safety nets can help offset the impact of risk on wage inequality. The idea consists in replacing the mean income observed over a period of time in panel data by the certainty-equivalent income. This is done using various parameters for risk aversion among the population in order to test for the sensitivity of the results to the level of risk aversion. We can then assess whether safety nets such as unemployment benefits for workers who lose their jobs would be able to offset the negative impact of risk aversion[1]. The rest of the notes runs as follow. Section 2 presents the approach, which is then illustrated in section

[1] As pointed out by Ravallion (1988), when assessing income variability from conventional data sets, one may have difficulties in distinguishing between two sources of variability: social mobility, which is linked to opportunities for self-advancement, and income risk. In this note, we focus on the risk component of income variability. Because we use a panel with quarterly interviews over a relatively short period of time, we assume that the income variability observed in the data is entirely due to risk.

3 using Mexican wage data.

2 Framework

Suppose that there are S possible states of nature. Consider an individual i with income x_{is} in state s. The individual is exposed to variability in income over the states of nature. As defined in the risk theory literature, the certainty equivalent of the individual's income, y_i, is

$$u(y_i) = \frac{1}{S}\sum_{s=1}^{S} u(x_{is}). \tag{1}$$

This certainty equivalent is the amount that, if received for sure in each state of the world, would generate the same utility as the state-contingent x_{is}.Within a social welfare context, $u(\cdot)$ does not represent the Bernoulli utility function of the individual, but rather a social judgement on the welfare value of the random variable x, as suggested by the argument on the impartial evaluation of welfare behind a veil of ignorance (see Vickrey, 1945 and Harsanyi, 1953, 1955). We assume that $u(\cdot)$ is continuous with $u'(\cdot) \geq 0$ and $u''(\cdot) \leq 0$ for all y. The assumption on the first derivative implies non-satiation. The assumption on the second derivative implies that the social welfare evaluation function is risk averse. One such function is the Constant Relative Risk Aversion (CRRA) utility function

$$u(x) = \begin{cases} \frac{x^{1-\rho}}{1-\rho} & \text{if } \rho \neq 1 \\ \log x & \text{if } \rho = 1 \end{cases}, \tag{2}$$

where ρ is the constant Arrow-Pratt relative risk aversion coefficient. The CRRA function is interesting because it is similar in structure to the Atkinson indices of inequality (and social welfare.) We could of course use another formulation for $u(\cdot)$. For example, relative risk aversion may be declining in income. While a more general formulation such as the Hyperbolic Absolute Risk Aversion (HARA) utility function would enable us to capture such phenomenon, using the simpler CRRA will be sufficient here to highlight the likely impact of risk on inequality measurement.

Using (2), the certainty equivalent income is

$$y_i = \begin{cases} \left[\frac{1}{S}\sum_{s=1}^{S} x_{is}^{1-\rho}\right]^{\frac{1}{1-\rho}} & \text{if } \rho \neq 1 \\ y_i = \prod_{s=1}^{S} x_{is}^{1/S} & \text{if } \rho = 1 \end{cases} . \tag{3}$$

Given the vector of certainty equivalent incomes $y = (y_1, y_2, ..., y_n)$ where n is the number of individuals, wage inequality can be measured by using any inequality index such as the Gini coefficient, the Theil index or the Atkinson index among others. Here, we use the Atkinson (1970) index

$$I_A = \begin{cases} 1 - \left[\frac{1}{n}\sum_{i=1}^{n} \left(\frac{y_i}{\mu_y}\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}} & \text{if } \varepsilon \neq 1 \\ 1 - \frac{\prod_{i=1}^{n} y_i^{1/n}}{\mu_y} & \text{if } \varepsilon = 1 \end{cases} . \tag{4}$$

where ε is an inequality aversion parameter.

In theory, inequality may be increased or reduced when risk aversion is taken into account. If those who suffer from variability in income are ranked near the top of the distribution, risk-adjusted inequality measures may be lower than measures not taking into account risk. In practice however, since much of the variability in income is likely to come from losses in employment, and since poorer households tend to be more exposed to such losses, we would expect risk-adjusted measures of inequality to be larger than inequality measures estimated on mean expected income. Note that the same idea for estimating risk-adjusted measures of inequality could easily be applied to social welfare and poverty measurement, since we could use the certainty-equivalent income of individuals to compute standard poverty measures.

3 Illustration

The above framework is illustrated using data from Mexico's urban employment survey (ENEU) for 1996. Information on wage income is given on a quarterly basis, and we have five observations for each individual. Since in practice, we do not know about potential states of the world, we assume that those income observations are drawn from the five point distribution given in the panel. We restrict the sample to adult males who have a positive income in at least one of the five periods.

To deal with zero observations ($x_{is} = 0$) and to simulate the potential impact of safety nets on inequality, we assume that instead of having no resources at all, individuals actually have a minimum level of resources. For baseline estimates of inequality, we set this minimum at $1/20^{th}$ of the mean wage observed in the data. We then test for the sensitivity of the impact of risk aversion to increases in this subsistence level wage. The simulations can be interpreted as a test of whether various levels of unemployment benefits (which are currently not existing in Mexico) for individuals with zero income would be able to offset the impact of risk aversion. For example, if with a transfer of $1/5^{th}$ of the mean wage to all those with zero income, the level of risk-adjusted inequality is not higher than the level of inequality observed with the mean wage over the five periods (which assumes no risk aversion with $\rho = 0$) and a subsistence wage of $1/20^{th}$ of the mean wage, we will say that the transfer is successful in offsetting the impact of risk on inequality. Said differently, the empirical question is what level of transfers might be necessary to offset the impact of risk. A higher level of risk aversion will require a higher transfer to compensate for the impact of risk aversion on inequality.

Table 1 provides the estimates of the Atkinson indices of inequalilty for different values of ρ and ε. We compute estimates for values of $\rho \in [0, 8]$, because these are the values typically used in the literature. For instance, Mehra and Prescott (1985) use $\rho \in [0, 10]$. Arrow (1971) has argued on theoretical grounds that ρ should be around 1, but Friend and Blume (1975) have presented empirical evidence based on portfolio holdings that the coefficient may be around 2, and Hildreth and Knowles (1982) have obtained estimates between 1 and 2. By considering a larger interval, as done by Mehra and Prescott (1985), we can be confident that we cover most possible values for this coefficient.

Consider the first row of Table 1 as the base case (minimum income support and no risk aversion). For $\varepsilon = 0.5$, the Atkinson index of inequality is 0.17. This increases to 0.18 with $\rho = 2$ and 0.19 with $\rho = 4$. The question is: what level of transfer for those with zero income would be enough to offset the impact of risk aversion, namely the increase in inequality from 0.17 to 0.18 or 0.19. Table 1 shows that a transfer equal to $1/10^{th}$ of the mean wage would be sufficient to offset the impact of risk aversion, since for both $\rho = 2$ and $\rho = 4$ and that level of transfer, we remain at a level of inequality of 0.17.

The impact of risk aversion is illustrated in Figures 1 and 2. Figure 1 shows the impact of risk aversion (as represented by the curves for different values of ρ) and aversion to inequality (on the horizontal axis) on inequality under the baseline scenario (subsistence wage equal to $1/20^{th}$ of the mean wage.) Higher aversion to risk and higher aversion to inequality both increase inequality measures. Figure 2 shows how inequality measures behave with a transfer of $1/3^{rd}$ of mean wage. The transfer is able to offset a good part of the impact of risk aversion on inequality.

4 Conclusion

Analysts often rely on cross-sectional statistics on income to assess inequality, social welfare and poverty. But the results do not reflect the risk component associated with income variability. In this note, we have provided a simple method for ensuring that risk is taken into account. The method is based on the concept of the certainty-equivalent income standard in the risk literature. An illustration for Mexico suggests that transfers to those who lose their income due to unemployment and other factors would have the potential to offset a good part of the impact of risk aversion on inequality.

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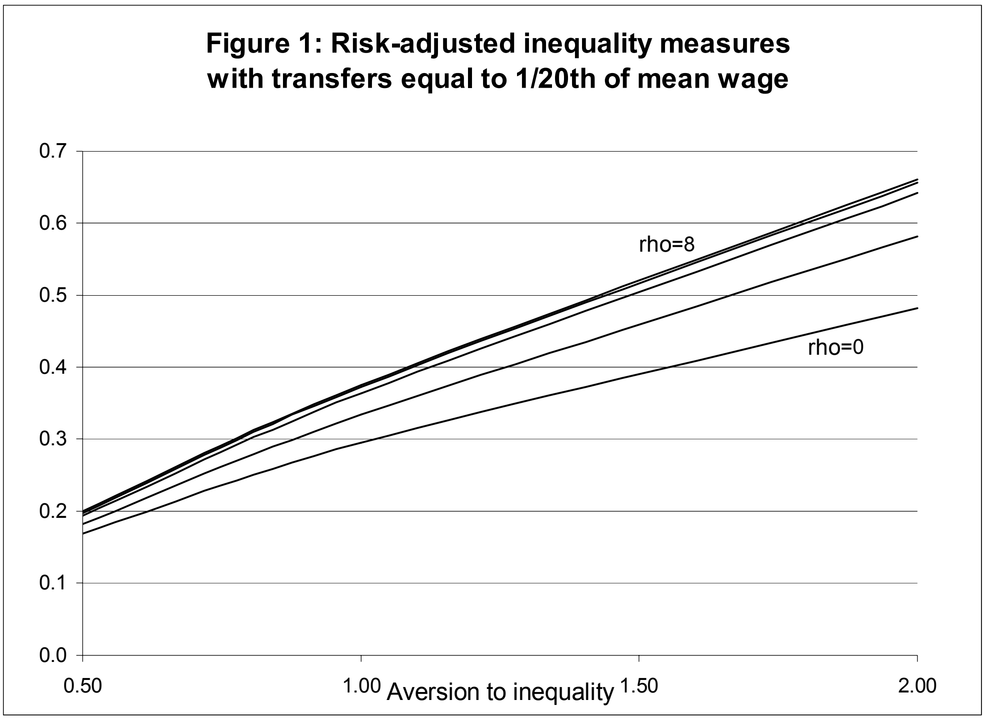
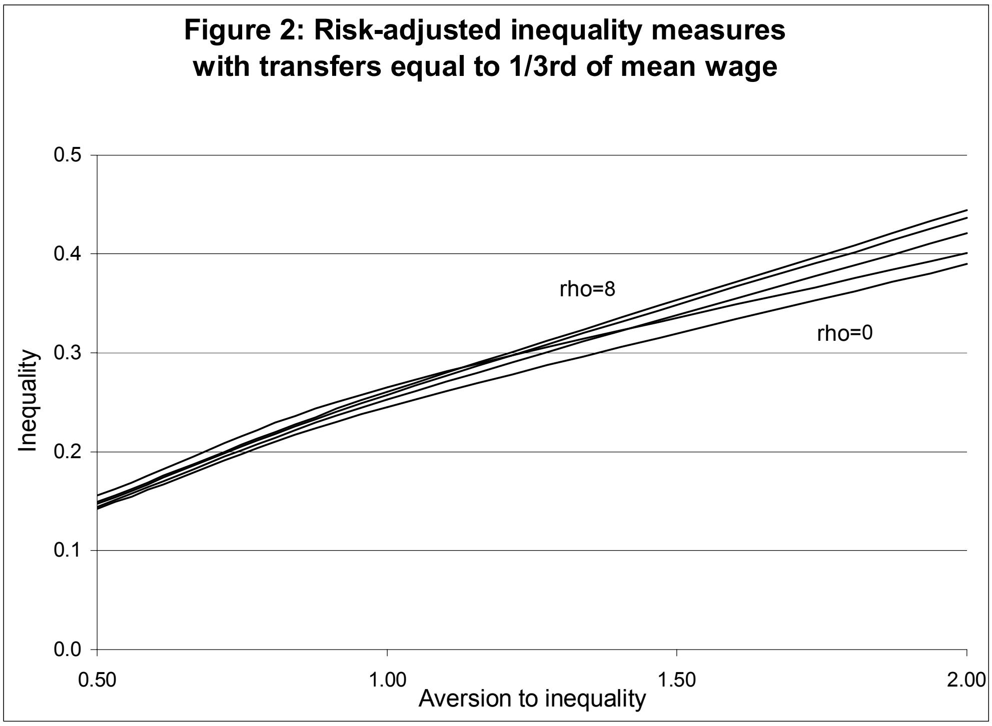
Table 1: Inequality under risk aversion and alternative safety nets

Aversion to Risk (ρ)	Inequality (Atkinson measure) A(0.5)	A(1)	A(2)
	Transfer equal to 1/20th of mean wage for those with zero wage		
0	0.17	0.29	0.48
2	0.18	0.33	0.58
4	0.19	0.36	0.64
6	0.20	0.37	0.66
8	0.20	0.38	0.66
	Transfer equal to 1/10th of mean wage for those with zero wage		
0	0.17	0.29	0.45
2	0.17	0.29	0.48
4	0.17	0.31	0.53
6	0.18	0.32	0.54
8	0.18	0.32	0.55
	Transfer equal to 1/5th of mean wage for those with zero wage		
0	0.16	0.27	0.42
2	0.15	0.26	0.42
4	0.16	0.27	0.45
6	0.16	0.28	0.46
8	0.16	0.28	0.47
	Transfer equal to 1/3rd of mean wage for those with zero wage		
0	0.16	0.26	0.40
2	0.14	0.25	0.39
4	0.14	0.25	0.42
6	0.15	0.26	0.44
8	0.15	0.26	0.44
	Transfer equal to half of mean wage for those with zero wage		
0	0.15	0.26	0.39
2	0.14	0.24	0.38
4	0.14	0.24	0.41
6	0.14	0.25	0.43
8	0.14	0.25	0.44

Source: Authorsí estimation using Mexicoís ENEU for 1996.





Source: Authorsí estimation using Mexicoís ENEU for 1996.