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Endogenous fluctuations and public services in a simple OLG economy

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BETA−Theme

Abstract

In a simple OLG model, we introduce public services which work as a source of utility. Assuming constant returns to scale in the production, we show that public services promote the occurrence of local indeterminacy. In contrast to a lot of existing contributions which have exploited productive externalities, this paper establishes that the existence of externalities in the utility function can explain the emergence of endogenous fluctuations.

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1 Introduction

Since the seminal contribution of Barro (1990), it is well-known that the existence of a public sector can explain endogenous growth. In this paper, we are interested to a related question: can public services also explain endogenous fluctuations? In this work, we want to answer to this question and then, study the relationship between endogenous fluctuations and public services. We show that local indeterminacy and cycles can occur when public goods enter the utility function, without assuming increasing returns in the production sector.

In order to do that, we consider an overlapping generations model with endogenous labor supply and productive capital. Firms produce the final good under a constant returns to scale technology and we assume that a government provides public goods and services. The public sector finances its expenditures by levying taxes on wage income, the tax rate being constant and the budget balanced at each period. Moreover, public expenditures are a source of utility. Indeed, we refer to the idea that public goods and services can be interpreted as infrastructure services, schools or roads and provide welfare to consumers.

In this framework, we study local dynamics. We show that endogenous fluctuations can emerge because public expenditures create an externality in the utility. In particular, the steady state can be locally indeterminate and then stochastic endogenous fluctuations due to self-fulfilling expectations can occur. Deterministic cycles can also appear through the occurrence of Hopf bifurcations. In this work, in contrast to a lot of existing contributions,[1] the emergence of endogenous fluctuations is not due to increasing returns in the production sector, but rather because an externality enters the utility function. Since Benhabib and Farmer (1994), the conditions for the emergence of endogenous fluctuations are often related to labor market features. In this work, labor demand is decreasing because returns to scale are constant. The local indeterminacy requires that the labor supply is decreasing with respect to the real wage, at the steady state. It is also interesting to notice that the tax rate does not influence local dynamics, because for simplicity, we use an exponent utility function and a Cobb-Douglas technology, and the tax rate is constant.[2]

[1]See among others Benhabib and Farmer (1994), Cazzavillan (2001) and Cazzavillan, Lloyd-Braga, and Pintus (1998).

[2]Considering one-sector optimal growth models, Schmitt-Grohé and Uribe (1997) and Guo and Lansing (1998) analyze local indeterminacy when the tax rates are pro-cyclical or counter-cyclical, whereas Guo and Lansing (2001) study the case where the tax rates are constant.

We can finally remark that this paper is closely related to the contribution by Cazzavillan (1996). However, we consider an overlapping generations model with endogenous labor supply, whereas this author considers an optimal growth model with inelastic labor supply. Moreover, in contrast to Cazzavillan (1996), we assume constant returns to scale in the production sector.

This paper is organized as follows. We present the model in section 2. In section 3, we study local dynamics and discuss the results.

2 The Model

We consider a perfectly competitive overlapping generations model with discret time $t = 1, 2, ...\infty$ and perfect foresight. There are three types of agents in the economy: a government, consumers and firms.

In each period, the government provides a flow of public goods and services g_t by levying a proportional tax $\tau \in (0, 1)$ on consumers income $w_t l_t$ according to the balanced budget rule:

$$g_t = \tau w_t l_t \tag{1}$$

Furthermore in period t, a continuum of consumers of mass one is born and lives two periods. When young, each consumer supplies labor l_t, is remunerated at the real wage w_t and must pay taxes on his income. He entirely saves the rest of his wage earning through the purchase of productive capital k_t. Capital totally depreciates after one period of use. When old, he rents to firms the capital at the expected real interest rate r_{t+1} and consumes the final good (c_{t+1}). Moreover, public services g_{t+1} enter the utility function. Indeed, we assume that government spending directly affects consumer preferences, because consumer derives utility from public goods and services (public roads, schools, nurseries...). The problem solved by a young consumer is the following:

$$\text{Max } \frac{c_{t+1}^{\alpha} g_{t+1}^{\beta}}{\alpha} - \frac{l_t^{\gamma}}{\gamma} \tag{2}$$

$$\begin{aligned} \text{s.t. } k_t &= (1 - \tau) w_t l_t \\ c_{t+1} &= r_{t+1} k_t \end{aligned} \tag{3}$$

where $\alpha \in (0, 1)$, $\beta > 0$ and $\gamma > 1$. Since the consumer takes as given the level of public expenditures, we obtain:

$$c_{t+1}^{\alpha} g_{t+1}^{\beta} = l_t^{\gamma} \tag{4}$$

$$c_{t+1} = r_{t+1}(1-\tau) w_t l_t \tag{5}$$

Finally, a continuum of mass one of identical firms produces the final good using the constant returns to scale Cobb-Douglas technology $y_t = k_{t-1}^{a} l_t^{1-a}$, with $a \in (0,1)$. Since all markets are perfectly competitive, the first order conditions of the profit maximization are given by:

$$r_t = a k_{t-1}^{a-1} l_t^{1-a} \text{ and } w_t = (1-a) k_{t-1}^{a} l_t^{-a} \tag{6}$$

Substituting these two expressions and equations (1) and (5) into (3) and (4), we have that:

$$k_t = (1-\tau)(1-a) k_{t-1}^{a} l_t^{1-a} \tag{7}$$

$$a^{\alpha}(1-a)^{\alpha+\beta} \tau^{\beta} (1-\tau)^{\alpha} k_t^{-\alpha(1-a)+\beta a} l_{t+1}^{(1-a)(\alpha+\beta)} = k_{t-1}^{-\alpha a} l_t^{\gamma-\alpha(1-a)} \tag{8}$$

Then, we can define an intertemporal equilibrium as follows:

Definition 1 *An intertemporal equilibrium with perfect foresight is a sequence* $(k_{t-1}, l_t) \in \mathbb{R}_{++}^2$, $t = 1, 2, ...\infty$, *such that (7) and (8) are satisfied.*

Equations (7) and (8) govern the dynamics of the model. Given the initial values for (k_{t-1}, l_t), we are able to determine (k_t, l_{t+1}). Moreover, in each period, the capital is predetermined by the savings of the previous young generation.

3 Local Dynamics

In this section, we analyze the occurrence of endogenous fluctuations in the neighborhood of a steady state. In order to do that, we first prove that it exists an unique steady state (k, l), which is a solution of (7)-(8) such that $k_{t-1} = k_t = k$ and $l_t = l_{t+1} = l$. It is defined by:

$$l = \left[a^{\alpha}(1-a)^{(\alpha a+\beta)/(1-a)} \tau^{\beta} (1-\tau)^{a(\alpha+\beta)/(1-a)}\right]^{1/(\gamma-\alpha-\beta)} \tag{9}$$

$$k = \left[a^{\alpha}(1-a)^{\gamma/(1-a)-\alpha} \tau^{\beta} (1-\tau)^{\gamma/(1-a)-\alpha-\beta}\right]^{1/(\gamma-\alpha-\beta)} \tag{10}$$

We can now study local dynamics around this steady state and hence analyze the emergence of endogenous fluctuations.

Proposition 1 *Assume that* $\alpha+\beta>\gamma$. *Then, the steady state can never be a saddle. Furthermore, the steady state is a sink (locally indeterminate) when* $(\alpha+\beta)/\gamma > a/(1-a)$, *a Hopf bifurcation can occur when* $(\alpha+\beta)/\gamma = a/(1-a)$ *and the steady state is a source when* $(\alpha+\beta)/\gamma < a/(1-a)$.

Proof. Consider the dynamic system (7)-(8). In order to analyze local dynamics, we log-linearize equations (7) and (8) around the steady state and determine the trace T and the determinant D of the associated Jacobian matrix:

$$T = \frac{\gamma}{(1-a)(\alpha+\beta)} \tag{11}$$

$$D = \frac{\gamma a}{(1-a)(\alpha+\beta)} \tag{12}$$

The trace T and the determinant D are both strictly positive. Then, the two eigenvalues are complex conjugates or real and positive. Moreover,

$$1-T+D = \frac{\alpha+\beta-\gamma}{\alpha+\beta} > 0 \quad \text{if } \alpha+\beta>\gamma \tag{13}$$

This last condition excludes saddle point, because it ensures that the two eigenvalues are both greater or smaller than 1. Then, the steady state is a sink if $D<1$, a Hopf bifurcation can occur if $D=1$ and the steady state is a source if $D>1$. We conclude that the steady state is a sink if $(\alpha+\beta)/\gamma > a/(1-a)$, a Hopf bifurcation can occur if $(\alpha+\beta)/\gamma = a/(1-a)$ and the steady state is a source if $(\alpha+\beta)/\gamma < a/(1-a)$.

■

This proposition shows that endogenous deterministic and stochastic fluctuations can emerge if the public good externality in the utility function is sufficiently high. Indeed, when a Hopf bifurcation occurs, an invariant closed curve appears around the steady state. Moreover, endogenous stochastic fluctuations emerge in the neighborhood of the steady state if it a sink and in the neighborhood of the cycle if the bifurcation is supercritical.

We can notice that a represents the capital share in income. In one-sector macroeconomic models, one often considers that it is smaller than $1/2$, which means that $a/(1-a)$ is smaller than 1. Under this added assumption, the steady state is always locally indeterminate when $\alpha+\beta>\gamma$.

Furthermore, we can connect our results to the contribution by Benhabib and Farmer (1994). As it is well-known, they have shown that the occurrence of indeterminacy requires that the slope of the increasing labor demand is greater than the slope of the labor supply. In this paper, we obtain quite different conditions. Since firms use a constant returns to scale technology, the labor demand is decreasing.[3] Moreover, using equations (1), (4) and (5), we can determine the elasticity of labor supply at the steady state. It is equal to $(\alpha+\beta)/(\gamma-\alpha-\beta)$. So the condition $\alpha+\beta>\gamma$ means that the labor supply is decreasing.

Finally, we can notice that the tax rate does not influence local dynamics, but only affects the steady state. We obtain such a result because, in contrast to Schmitt-Grohé and Uribe (1997) or Guo and Lansing (1998), the tax rate is constant. Moreover, our model being characterized by an exponent utility function and a Cobb-Douglas technology, the level of the steady state does not influence local dynamics.

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[3] Using equation (6), it is not difficult to see that the elasticity of labor demand is equal to $-1/a$.

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