E C O N O M I C S   B U L L E T I N

Interest Rate and Inflation in Monetary Models with Exogenous Money Growth Rule

Patrick Fève
University of Toulouse (GREMAQ and IDEI)

Stéphane Auray
University of Nantes (LEN−−C3E) and GREMAQ

Abstract

This paper assesses the joint behavior of the nominal interest rate and the expected inflation in flexible and sticky prices monetary models with exogenous money growth rule and technology shock. We then estimate the relation between the nominal interest rate and the expected inflation implied by each model. Our results first suggest that both models are able to account for the data. Beyond, they also cast doubt on the standard interpretation of the so−−called Taylor rule. It may not necessarily represent a money supply rule describing the behavior of the central bank, but rather describe an equilibrium relation between the nominal interest rate and the expected inflation when money supply is exogenously given.

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Introduction

Over the recent years, there has been a great resurgence of interest on how to conduct monetary policy. In a famous paper, Taylor (1993) showed that US monetary policy after 1986 is well characterized by a simple rule wherein the interest rate – the nominal Federal funds rate – responds positively to the inflation rate and the output gap. This rule presents the attractive empirical feature to be robust across periods, monetary regimes and countries (see Clarida, Galí and Gertler, 1998, 2000 and Taylor, 1999). Despite some substantial differences across periods, the estimated slope parameter between the nominal interest rate and the expected inflation generally has the expected sign and is significant.

Various interpretations may be given to the estimated Taylor rule (see Taylor, 1999). First, it can be used as a way to examine several episodes of monetary history. In this case, the estimated rule does not necessarily represent the "true" central bank behavior, but rather a description of the monetary policy in different historical time periods. Second, the estimated rule can be considered as a guideline – or explicit formula – for the central bank to follow when making monetary policy decisions. In this case, the Taylor rule describes how a central bank sets the nominal interest rate in response to economic variables (the inflation rate and the output gap). The aim of this article is to show that the previous estimations of Taylor rule should not be thought of as an accurate representation of the central bank behavior but may just account for an equilibrium relation between the nominal interest rate and the expected inflation rate in an equilibrium of a monetary economy with exogenous money growth rule.

Any monetary rule must be estimated using aggregate data which are the realization of the economic equilibrium, *i.e* a reduced form that defines a set of endogenous variables in terms of exogenous and predetermined variables. Therefore, the econometrician must use a set of relevant instrumental variables in order to identify and estimate the structural equation that defines the central bank behavior. Empirical studies on the Taylor rule generally use lagged inflation as an instrumental variable (see Clarida, Galí and Gertler, 1998, 2000). Using the same procedure, we estimate the relation between the nominal interest rate and the expected inflation under a cash–in–advance model with an exogenous money growth rule and technology shock. In this monetary model with flexible prices, we show that the estimated relation between the nominal interest rate and the expected inflation is very close to the empirical estimates from actual data spanning the period 1979.3–2001.1 (see Clarida, Galí and Gertler (2000) and Table 1 in the paper). This first result thus questions the previous estimates of the Taylor rule when they are interpreted as the central bank behavior. We check the robustness of this result in a monetary economy with sticky prices (see Table 2 in the sequel). We show that the two models *(i)* do not present significant differences concerning the joint behavior of the nominal interest rate and the expected inflation and *(ii)* are able to match the estimated Taylor type rule. It is worth noting that we estimate the Taylor type rule under the structural models with the same calibration.

The paper is organized as follows. A first section presents a monetary model with flexible prices. Section two characterizes the estimated Taylor type rule under this model with exogenous money growth rule and technology shock. In section three, we check the robustness of this result using a sticky price model. A last section offers some concluding remarks. Proofs are given in appendix.

1 The monetary economy

This section briefly describes the main ingredients of the monetary economy.

Households

The economy is comprised of a unit mass continuum of identical infinitely lived agents. A representative household enters period t with real balances m_t/P_t brought from the previous period and nominal bonds b_t. The household supplies labor at the real wage W_t/P_t. During the period, the household also receives a lump–sum transfer from the monetary authorities in the form of cash equal to N_t and real interest rate payments from bond holdings $((R_{t-1}-1)b_t/P_t)$. All these revenues are then used to purchase a consumption bundle, money balances and nominal bonds for the next period. Therefore, the budget constraint simply writes as

$$b_{t+1} + m_{t+1} + P_t c_t = W_t h_t + R_{t-1} b_t + m_t + N_t$$

Money is held because the household must carry cash — money acquired in the previous period and the money lump sum transfer — in order to purchase goods. She therefore faces a cash–in–advance constraint of the form :

$$P_t c_t \leqslant m_t + N_t + R_{t-1} b_t - b_{t+1}$$

Each household has preferences over consumption and leisure represented by the following intertemporal utility function :

$$E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left[\log(c_\tau) - h_\tau\right]$$

where $\beta \in (0,1)$ is the discount factor, h_t denotes the number of hours supplied by the household. E_t denotes the expectation operator conditional on the information set available in period t. The household determines her optimal consumption/saving, labor supply and money and bond holdings plans maximizing utility subject to the budget and cash–in–advance constraint. Consumption behavior together with labor supply yields

$$\frac{P_t}{W_t} = \beta E_t \frac{P_t}{P_{t+1}} \frac{1}{c_{t+1}}$$

whereas nominal return of bond holdings is given by :

$$R_t = \frac{W_t}{P_t c_t}$$

This last equation together with the CIA constraint determines the money demand where the real balances are a decreasing function of the nominal interest rate for a given real wage.

Firms

The technology is described by the following constant returns to scale production function :

$$Y_t = A_t h_t$$

such that in equilibrium the real wage is $W_t/P_t = A_t$. The technology shock follows an AR(1) process

$$\log(A_t) = \rho_a \log(A_{t-1}) + (1 - \rho_a)\log(\bar{A}) + \sigma_{\varepsilon^a}\varepsilon_t^a$$

where ε_t^a is a white noise with unit variance, σ_{ε^a} and $|\rho_a| < 1$.

Money Supply and Government Budget Constraint

Money is exogenously supplied according to the following money growth rule

$$M_{t+1} = \gamma_t M_t$$

where γ_t follows an AR(1) process :

$$\log(\gamma_t) = \rho_\gamma \log(\gamma_{t-1}) + (1 - \rho_\gamma)\log(\bar{\gamma}) + \sigma_{\varepsilon^\gamma}\varepsilon_t^\gamma$$

ε_t^γ is a white noise with unit variance, $\sigma_{\varepsilon^\gamma} > 0$ and $|\rho_\gamma| < 1$. The government issues nominal bonds B_t to finance open market operations.[1] The government budget constraint is

$$M_{t+1} + B_{t+1} = M_t + R_{t-1}B_t + N_t$$

with M_0 and B_0 given.

Equilibrium

An equilibrium is a sequence of prices and allocations, such that given prices, allocation maximizes profits (when taking technological choice into account) and maximizes utility (subject to the savings behavior), and all markets clear. The equilibrium conditions are approximated using a log-linearization about the deterministic steady state. A nice feature of the approximate economy is that the log-linear solution, *i.e.* aggregate fluctuations in deviation from the deterministic steady–state, depends only on the parameters of the forcing variables $\theta = \{\rho_a, \sigma_{\varepsilon^a}, \rho_\gamma, \sigma_{\varepsilon^\gamma}\}$. In what follows, we will assume that the two shocks are serially and positively correlated ($\rho_a \in (0,1)$ and $\rho_\gamma \in (0,1)$).

[1] These nominal bonds could also be used to finance government consumption. Nevertheless, this issue is beyond the scope of the paper.

2 Nominal interest rate and expected inflation co-movements

We use the model with exogenous money growth rule and technology shock as a Data Generating Process (DGP). This DGP allows to reproduce some features of actual data, which are taken as the realization of an unknown – to the econometrician – stochastic process. The features we are interested in include conditional moments on the nominal interest rate and inflation. More precisely, we specify the following Taylor type rule

$$\widehat{R}_t = \eta E_t \widehat{\pi}_{t+1} \tag{1}$$

where the hat denotes the percentage of deviation from the long run value. This Taylor type rule incorporates only the expected inflation rate and aims at describing the joint behavior of the nominal interest rate and the expected inflation. Previous empirical results suggest that the estimated parameter of (expected) output gap is marginally significant for the Volcker–Greenspan era (see Clarida, Galí and Gertler, 2000). Conversely, the estimates of η are significant and positive in most cases (see Taylor, 1999 and Clarida, Galí and Gertler, 2000). Equation (1) can be expressed in terms of observables:

$$\widehat{R}_t = \eta \widehat{\pi}_{t+1} + \varepsilon_{t+1} \tag{2}$$

where $\varepsilon_{t+1} = -\eta \left(\widehat{\pi}_{t+1} - E_t \widehat{\pi}_{t+1} \right)$. It is worth noting that the nominal interest rate $\widehat{R}_t$ and the inflation rate $\widehat{\pi}_{t+1}$ correspond to the equilibrium conditions of a monetary model with exogenous money growth rule and technology shock (see equations (A.1)-(A.5) in appendix A). Let z_t denote a single instrument known in period t. This instrument verifies the following orthogonality condition

$$E\left(\varepsilon_{t+1} z_t\right) = 0$$

or equivalently

$$E\left(\left(\widehat{R}_t - \eta \widehat{\pi}_{t+1}\right) z_t\right) = 0 \tag{3}$$

Equation (3) is the basis of the GMM estimation of the parameter η. As the number of orthogonality conditions is equal to the number of parameter of interest, it follows that the GMM estimator (or IV estimator in this simple case) is free from any weighting matrix and can be obtained directly from (3). Following previous empirical works (see Clarida, Galí and Gertler, 2000), the instrumental variable is one lag inflation rate.[2] A GMM estimator $\widehat{\eta}_\theta$ of η is thus given by:

$$\widehat{\eta}_\theta = \frac{Cov_\theta\left(\widehat{R}_t, \widehat{\pi}_{t-1}\right)}{Cov_\theta\left(\widehat{\pi}_{t+1}, \widehat{\pi}_{t-1}\right)} \tag{4}$$

[2]Clarida, Galí and Gertler (2000) include lagged inflation rates up to four lags. To keep tractable results, we do not introduce over–identifying conditions and consider only one lag inflation rate as the instrumental variable.

The estimated parameter $\widehat{\eta}_\theta$ depends on the parameters of the forcing variables θ, *i.e.* the parameters that describe the exogenous money growth rule and the technology shock. We now determine the GMM estimator of η under the monetary model.

Proposition 1 *The GMM estimator $\widehat{\eta}_\theta$ of η is given by:*

$$\widehat{\eta}_\theta = \frac{\rho_\gamma^2 \left(1+\rho_\gamma\left(1-\rho_\gamma\right)\right)\left(1-\rho_a^2\right)\sigma_{\varepsilon^\gamma}^2}{\rho_\gamma^3 \left(1+\rho_\gamma\left(1-\rho_\gamma\right)\right)\left(1-\rho_a^2\right)\sigma_{\varepsilon^\gamma}^2 - \rho_a\left(1-\rho_a\right)^2\left(1-\rho_\gamma^2\right)\sigma_{\varepsilon^a}^2}$$

In the absence of technology shock ($\sigma_{\varepsilon^a} = 0$ and/or $\rho_a = 0$), the GMM estimator of η under the monetary model depends only on the exogenous money growth rule parameter :

$$\widehat{\eta}_\theta = \rho_\gamma^{-1}$$

The GMM estimator is strictly positive, provided the growth rate of money supply displays positive serial correlation. Moreover, given some estimates of ρ_γ, it follows that the estimated value of η is greater than one and is quite close to the ones of estimated Taylor rule (see Clarida, Galí and Gertler (2000), tables II and III, p 157 and 160 and Table 1 below). For example, $\rho_\gamma \in (1/2, 2/3)$ – which corresponds to the range of estimates – implies a GMM estimator of the Taylor rule between 1.5 and 2.

Without money supply shock ($\sigma_{\varepsilon^\gamma} = 0$ and/or $\rho_\gamma = 0$), the model implies a zero value for the GMM estimator of η. In this case, the nominal interest rate remains constant. This shows the role played by money shock in order to replicate the observed positive relation between the nominal interest rate and the expected inflation.

Table 1: GMM estimates of η

Data	Model
	Money supply with M_1
1.834	1.469
(0.315)	[24.6%]
	Money supply with M_2
1.834	1.627
(0.315)	[51.0%]

Note : The GMM estimate from actual data is robust to both heteroskedasticity and serial correlation, using a VARHAC(1) estimator. Money supply with M_1: $\rho_\gamma = 0.691$ and $\sigma_{\varepsilon^\gamma} = 0.0104$. Money supply with M_2: $\rho_\gamma = 0.647$ and $\sigma_{\varepsilon^\gamma} = 0.0065$. Technology shock: $\rho_a = 0.95$ and $\sigma_{\varepsilon^a} = 0.007$. Standard–errors in parentheses. P-value of the Wald statistic in brackets.

In order to quantitatively illustrate these findings, we compute the GMM estimator of η with respect to θ. For comparative purposes, the parameters vector θ is calibrated on U.S. data. For the technology shock, we retain the standard values

$\rho_a = 0.95$ and $\sigma_{\varepsilon^a} = 0.007$. Concerning the money shock, we estimate an autoregressive process of order one for the sample period 1979.3–2001.1 on quarterly data. We consider alternatively M_1 and M_2 as measures of the money stock.[3] The estimated values are reported in the bottom of Table 1. In order to compare the model with the actual U.S. data, we estimate the Taylor type rule (1) for the period 1979.3–2001.1. We use as the nominal interest rate the Federal Funds rate. The measure of inflation is the rate of change in the GDP deflator between two subsequent quarters. The GMM estimator is obtained with the lagged inflation rate as the instrumental variable and the weighting matrix is computed using a VARHAC(1) estimator of the long run covariance matrix. The estimated value of η (see the first column of Table 1) is significantly larger than unity and is very close to previous estimates[4].

The GMM estimator η under the model is obtained using the formula of proposition 1. Table 1 shows that this GMM estimator $\widehat{\eta}_\theta$ largely exceeds the unity. Moreover, the change in the parameters of the forcing variable have a sensible effect on $\widehat{\eta}_\theta$ (see the difference between M_1 and M_2). We thus compare this estimator from the structural model to the one obtained from the actual data. The Wald statistic for the null hypothesis $\widehat{\eta}_T = \widehat{\eta}_\theta$ is given by $W = (\widehat{\eta}_T - \widehat{\eta}_\theta)^2 \widehat{\sigma}_T^{-2}$ where $\widehat{\sigma}_T$ denotes a consistent estimate of the standard-error of $\widehat{\eta}_T$. The statistic is thus asymptotically distributed as a chi-square with one degree of freedom. The P-value of the Wald statistics clearly shows that the model is able to match the actual conditional moments on the nominal interest rate and the expected inflation.

3 Interest and inflation in a sticky price model

We now check the robustness of our result using a sticky price model with exogenous money growth rule. We omit any discussion of household behavior as it is symmetric with before. Therefore we only describe the firms' behavior.

In this economy, there is a continuum of firms distributed uniformly on the unit interval. Each firm is indexed by $i \in [0, 1]$ and produces a differentiated good with the previous linear technology $Y_{i,t} = A_t h_{i,t}$. At the end of period $t-1$, *i.e.* before the observation of the realization of the money supply and technology shock in period t, firm i sets the price $P_{i,t}$ at which it will be selling good i during period t, for a given aggregate price P_t. The firm is owned by the household, and pays its profits out to the household at the end of each period. Because of the CIA constraint on

[3] All the data were obtained from the Federal Reserve Data Bases available at `http://www.stls.frb.org/fred/data/`. Money: `m1sl` and `m2sl`. Quarterly data are obtained by quarter average. Federal fund rate: `fedfunds`, monthly frequency, average of daily figures. The quarterly rate is defined as the first month of each quarter. GDP deflator: `gdp` for GDP in current dollars divided by `gdpc1` for the GDP of chained 1996 dollars, S.A.

[4] Clarida, Galí and Gerler (2000) obtained an estimated value of η around 2 over the sample 1979.3–1996.4 (see table II p. 157 and table III p. 160).

the household consumption, the firm discounts its profit using $\Phi_{t+1} = \beta/(P_{t+1}c_{t+1})$. Therefore, the firm i will seek to maximize for a given wage W_t

$$\max_{P_{i,t}} E_{t-1}\left[\Phi_{t+1}\left(P_{i,t}Y_{i,t} - W_t h_{i,t}\right)\right]$$

subject to

$$Y_{i,t} = \left(\frac{P_{i,t}}{P_t}\right)^{-\varepsilon} c_t$$

where $\varepsilon > 1$ is the elasticity of substitution among consumption goods. In a symmetric equilibrium, all firms will set the same price P_t and choose identical output and hours. The equilibrium conditions are approximated using a log-linearization about the deterministic steady state. The log-linear solution depends on the parameters of the forcing variables θ (see equations (A.4),(A.5) and (B.6)-(B.8) in appendix A and B).

Proposition 2 *The GMM estimator $\widehat{\eta}_\theta$ of η under the sticky price model with exogenous money growth rule and technology shock is given by:*

$$\widehat{\eta}_\theta = \frac{\rho_\gamma^3\left(1+\rho_\gamma\left(1-\rho_\gamma^2\right)\right)\left(1-\rho_a^2\right)\sigma_{\varepsilon\gamma}^2}{\rho_\gamma^4\left(1+\rho_\gamma\left(1-\rho_\gamma^2\right)\right)\left(1-\rho_a^2\right)\sigma_{\varepsilon\gamma}^2 - \rho_a^3\left(1-\rho_a\right)^2\left(1-\rho_\gamma^2\right)\sigma_{\varepsilon a}^2}$$

It is worth noting that the GMM estimator under the sticky price model shares some common properties with the GMM estimator under the flexible price model. In the absence of technology shock, the two GMM estimators are identical and positive, provided positive serial correlation in the money growth rule. Without money supply shock, the GMM estimator is zero.

Table 2: GMM estimates of η

Data	Model
	Money supply with M_1
1.834	1.473
(0.315)	[25.1%]
	Money supply with M_2
1.834	1.648
(0.315)	[55.4%]

Note : see table 1.

Table 2 reports the GMM estimator under the sticky price model and from actual data. The GMM estimator η under the model is obtained using the formula of proposition 2. This table points out two mains results. First, the P-value of the Wald statistics shows that this model is able to match the actual conditional moments on the nominal interest rate and the expected inflation. Second, the GMM estimators under the flexible and the sticky prices model are almost identical (see the second column of tables 1 and 2).

4 Concluding remarks

In this paper, we show that monetary models with exogenous money growth rule are able to match the relation between the nominal interest rate and the expected inflation. This result thus questions the previous estimates of the Taylor rule. They do not necessarily represent the central bank behavior. They rather could represent a relation between the nominal interest rate and the expected inflation in an equilibrium of a monetary economy with exogenous money growth rule. From these preliminary results, several issues may be worth considering. First, the Taylor rule must include additional variables (output gap, lagged interest rate). Second, we have to check the robustness of the results with respect to other specifications and market arrangements.

Appendix

A Proof of Proposition 1

The log-linear approximation of the FP economy is given by :

$$\begin{aligned}
\widehat{\pi}_t &= \widehat{\gamma}_t + \widehat{y}_{t-1} - \widehat{y}_t && \text{[inflation] (A.1)}\\
\widehat{y}_t &= \widehat{a}_t - \rho_\gamma \widehat{\gamma}_t && \text{[output] (A.2)}\\
\widehat{R}_t &= \widehat{a}_t - \widehat{y}_t && \text{[interest rate] (A.3)}\\
\widehat{\gamma}_t &= \rho_\gamma \widehat{\gamma}_{t-1} + \varepsilon_t^\gamma && \text{[money supply] (A.4)}\\
\widehat{a}_t &= \rho_a \widehat{a}_{t-1} + \varepsilon_t^a && \text{[technology] (A.5)}
\end{aligned}$$

From equations (A.1)-(A.5), we define nominal interest rate and inflation in terms of the two forcing variables :

$$\begin{aligned}
\widehat{\pi}_t &= (1+\rho_\gamma)\widehat{\gamma}_t - \rho_\gamma \widehat{\gamma}_{t-1} + \widehat{a}_{t-1} - \widehat{a}_t\\
\widehat{R}_t &= \rho_\gamma \widehat{\gamma}_t
\end{aligned}$$

The covariance can be easily deduced :

$$\begin{aligned}
Cov_\theta\left(\widehat{\pi}_{t+1}, \widehat{\pi}_{t-1}\right) &= \rho_\gamma^3(1+\rho_\gamma-\rho_\gamma^2)\sigma_\gamma^2 - \rho_a(1-\rho_a)^2\sigma_a^2\\
Cov_\theta\left(\widehat{R}_t, \widehat{\pi}_{t-1}\right) &= \rho_\gamma^2(1+\rho_\gamma-\rho_\gamma^2)\sigma_\gamma^2
\end{aligned}$$

where $\sigma_\gamma^2 = \sigma_{\varepsilon^\gamma}^2/(1-\rho_\gamma^2)$ and $\sigma_a^2 = \sigma_{\varepsilon^a}^2/(1-\rho_a^2)$.

B Proof of Proposition 2

The log-linear approximation of the SP economy is given by :

$$\begin{aligned}
\widehat{\pi}_t &= \widehat{\gamma}_t + \widehat{y}_{t-1} - \widehat{y}_t && \text{[inflation] (B.6)}\\
\widehat{y}_t &= \rho_a \widehat{a}_{t-1} + \widehat{\gamma}_t - \rho_\gamma(1+\rho_\gamma)\widehat{\gamma}_{t-1} && \text{[output] (B.7)}\\
\widehat{R}_t &= (1+\rho_\gamma)(\widehat{\gamma}_t - \rho_\gamma\widehat{\gamma}_{t-1}) + \rho_a \widehat{a}_{t-1} - \widehat{y}_t && \text{[interest rate] (B.8)}
\end{aligned}$$

From equations (A.4),(A.5) and (B.6)-(B.8), one gets :

$$\begin{aligned}
\widehat{\pi}_t &= (1+\rho_\gamma(1+\rho_\gamma))\widehat{\gamma}_{t-1} - \rho_\gamma(1+\rho_\gamma)\widehat{\gamma}_{t-2} - \rho_a\widehat{a}_{t-1} + \rho_a\widehat{a}_{t-2}\\
\widehat{R}_t &= \rho_\gamma \widehat{\gamma}_t
\end{aligned}$$

and the covariances are :

$$\begin{aligned}
Cov_\theta\left(\widehat{\pi}_{t+1}, \widehat{\pi}_{t-1}\right) &= \rho_\gamma^4\left(1+\rho_\gamma(1-\rho_\gamma^2)\right)\sigma_\gamma^2 - \rho_a^3(1-\rho_a)^2\sigma_a^2\\
Cov_\theta\left(\widehat{R}_t, \widehat{\pi}_{t-1}\right) &= \rho_\gamma^3\left(1+\rho_\gamma(1-\rho_\gamma^2)\right)\sigma_\gamma^2
\end{aligned}$$

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