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THE WELFARE COST OF INFLATION AND THE REGULATIONS OF MONEY SUBSTITUTES

Benjamin Eden
Vanderbilt University

Maya Eden
World Bank

Abstract

This paper studies the possibility of using financial regulation that prohibits the use of money substitutes as a tool for mitigating the adverse effects of deviations from the Friedman rule. We establish that when inflation is not too high regulation aimed at eliminating money substitutes improves welfare by economizing on transaction costs. The gains from regulation depend on the distribution of income and on the level of direct taxation. The area under the demand for money curve is equal to the welfare cost of inflation only when there are no direct taxes and no proportional intermediation costs: otherwise, the area under the demand curve overstates the welfare cost of inflation when money substitutes are not important and understates the welfare cost when money substitutes are important.

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Contact: Benjamin Eden - ben.eden@vanderbilt.edu, Maya Eden - mayeden@gmail.com

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1. INTRODUCTION

In his seminal contribution, Friedman (1969) established the optimality of the Friedman rule: the social expenditure on transaction costs is minimized when the rate of return on money is the real interest rate. Despite the relative robustness of this theoretical result, the Friedman rule is rarely implemented. This paper studies the possibility of using financial regulation as a tool for mitigating the adverse effects of deviations from the Friedman rule.

We adopt a public finance approach and model inflation tax as a consumption tax that is not perfectly enforced and can be evaded by using bonds. The model incorporates elements from Baumol (1952), Tobin (1956), Lagos and Wright (2005) and Williamson (2012). Risk neutral agents with heterogeneous labor productivities can bridge the gap between receipts and expenditures either by using money or by using bonds. Bonds have a higher rate of return but there is a fixed cost for using them and therefore agents will use bonds only when their income is sufficiently high. Inflation affects the labor supply of agents who use money but does not affect the labor supply of agents who use bonds.

The welfare cost of inflation has two components: Transaction costs that are paid by bond users and the distortion to the labor supply of money users. The area under the demand for money curve is equal to the welfare cost of inflation only in the special case in which there are no direct taxes and no proportional intermediation costs. Otherwise, the area under the demand for money curve overstates the welfare cost if money substitutes are not important and understates the welfare cost if money substitutes are important. The importance of money substitutes declines with the nominal interest rate and therefore for low nominal interest rate the area under the demand for money overstates the welfare cost of inflation.

A "weak" social planner who can tell agents how to invest their wage payment (but cannot tell them how much to work) faces a tradeoff between two distortions: the labor-leisure choice and the cost of using bonds. The social cost of the labor-leisure distortion is higher for more productive workers: consequently the "weak" social planner will have sufficiently productive workers save in bonds, and sufficiently unproductive workers save in money, consistent with their privately optimal choice. However, for agents with intermediate productivity levels, the privately optimal saving choice is different from the choice of the "weak" planner because agents ignore the effect of their choice on seigniorage revenues (and the lump sum that they get). From the social point of view there is excessive use of bonds in the decentralized equilibrium.¹

The gains from financial regulation depend crucially on the income distribution. In particular, the gains from regulation depend on the density of agents with intermediate productivity levels who choose to save in bonds, but would increase social welfare by saving in money.

An important feature of our model is that the gains from financial regulation depend on the *aggregate* expenditure on transaction costs. One might wonder whether, given relatively low transaction costs associated with bonds, the distortion that we highlight is quantitatively relevant. Our model suggests that it is, because lower transaction costs may increase aggregate expenditure through the extensive margin: when transaction costs are low, more savers rely on bonds, and thus the aggregate expenditure on transaction costs may be higher.

Section 2 is the basic model. Section 3 considers the case of endogenous inflation that is determined by the need to raise a certain amount of seigniorage revenues. Section 4 extends the basic model to allow for income tax and proportional intermediation costs.

¹ This is a corollary of the following well-known result in monetary economics. Money is not super-neutral when the transfer payment is in a lump sum form, but it is super-neutral when it is proportional to the amount of money held.

Calibration is in section 5. The bias in measuring the welfare cost of inflation by the area under the demand curve is in section 6 and concluding remarks are in section 7.

Related literature.

The regulation of "money substitutes" is an old question that is not fully resolved. Simons (1948) argued for the prohibition of short-term bonds but not of long-term bonds (consols). Friedman and Schwartz (1986) see a role for government in money but are against financial regulations.² Bryan and Wallace (1979) assume that the use of bonds requires costly intermediation and therefore financing government spending with bonds is inefficient. We find ourselves with a broad agreement with their conclusion and focus here on inflation that occurs for reasons that are not related to seigniorage revenues. Kocherlakota (2003) and Andolfatto (2011) argue that illiquid nominal bonds are essential for intertemporal trade. Here we focus on the "transaction motive" for holding money. We think that an extension of the model to many periods will not change our main results. See, Eden (2012). Our paper is more closely related to Schreft (1992) and Gillman (2000) who use a cash-in-advance model with credit and cash goods to argue that a restriction on buying non-durable goods with credit may improve matters. Here we model the transaction cost explicitly and take into account the effect of inflation on labor supply. We argue that eliminating money substitutes will improve matters if the rate of inflation is not too high. There is also a difference in the measurement of the welfare cost of inflation.

Aiyagari, Braun and Eckstein (1998) use a cash-in-advance economy with credit and cash goods. In their model and in Lucas (2000) the welfare cost of inflation is equal

² They seem to struggle with the Austrian school that advocates no role of government in money (see for example, Hayek [1970]) and the empirical evidence they accumulated about the importance of controlling the money supply. Their definition of money is empirically based (they use M2 because it works well empirically and there is better data on this particular aggregate).

to the area under the demand for money curve. Here this result holds only in the special case in which there are no direct taxes and no proportional intermediation costs.²⁰

Bryant and Wallace (1984) use a legal restriction model of money. Their analysis points to the direction of many types of bonds. Here we advocate a reduction in the number of available assets. Rather than having a restriction on the size of bonds, we assume that the cost of verifying balances in checking accounts is less than the cost of verifying balances in savings accounts. As a result balances in checking accounts can be used to buy goods everywhere while balances in savings accounts can be used to buy goods only after paying a fixed verification cost. One way to verify balances in the savings account is to execute the transaction at a bank. Another way is to pay an intermediary as in Ireland (1994). The going to the bank interpretation is in the spirit of Baumol (1952) and Tobin (1956) and is similar to Alvarez, Atkeson and Kehoe (2002) who assume that for a fixed payment it is possible to relax the cash-in-advance constraint by transferring money from the asset market to the goods market.

The paper is also related to the old debate about 100% reserve requirement and the role of banks. Lucas and Stokey (2011) describe the role of banks in economizing on cash using the analysis in Diamond and Dybvig (1983). They consider an economy in which bills for purchases arrive with unpredictable lags and bills must be paid exactly when due. They think of a bank “as an institution that pools payment risks, making all of its client better off than they would be acting on their own”. The pooling of payment risks and economizing on cash is beneficial to the clients of a particular bank but has no social benefit. Here we argue that economizing on cash may reduce social welfare if real resources are required for this operation.

Lucas (2013) sees the benefit of 100% reserve requirement in terms of eliminating the risk of bank runs without creating a moral hazard problem. The cost is in terms of trips to the banks. Here we argue for regulations that eliminate bonds and the trips to the bank. Under these regulations, a 100% reserve requirement will eliminate the risk of bank

runs at no cost of trip to the bank (but with a small distortion cost). The paper also supports regulations aimed at eliminating the "shadow banking" sector and in this respect goes one step further than Gorton and Metrick (2010) who propose to add regulations to the Dodd-Frank Act.

2. THE MODEL

The economy lasts for two periods. It consists of two types of agents: a continuum of buyers with mass one and a continuum of sellers with mass one. There are two goods: X and Y . Buyers produce Y in the first period and consume X in the second period. All buyers have the same utility function:

$$(1) \quad U^b(x^b, L^b) = x^b - v(L^b)$$

where the superscript "b" is for buyer, x^b is the consumption of X , L^b is the labor supply and $v(L^b)$ is the utility cost of labor. We assume $v(L) = (\frac{1}{2})L^2$. Buyers are not equally productive. The production function of a buyer with productivity θ is:

$$(2) \quad y = \theta L$$

The distribution function of θ , $F(\theta)$, is continuous and differentiable and is defined on the interval $[\theta^{\min}, \theta^{\max}]$, where $\theta^{\min} > 0$.

Sellers produce X in the second period and consume Y in the first period. The representative seller's utility function is:

$$(3) \quad U^s(y^s, L^s) = y^s - L^s$$

The production function is the same for all sellers and is given by:

$$(4) \quad x = L$$

There is a government bank. The bank lends money to sellers and takes deposits from buyers. It offers two types of deposit accounts: checking (money) and savings (bonds). The gross real interest on the government loan to sellers is 1. The gross real interest on savings is $R \leq 1$ and the gross interest on checking is $R_m \leq R$. We start by assuming

$R=1$.

The market structure is similar to Lagos and Wright (2005) and Williamson (2012). In the first period, everyone meets in a centralized Walrasian market (CM). In this market sellers use the money they got as a loan from the government to buy Y from the buyers. The market in the second period is decentralized (DM). During the DM buyers and sellers can meet either at a bank or at a non-bank location. The ability to pay out of a savings account can be verified only at the bank. The ability to pay out of a checking account can be verified in all locations. As a result, balances in the savings account can be exchanged for goods only at a bank while balances in the checking account can be exchanged for goods everywhere. There is a cost of meeting at the bank. The cost is α units of X and is paid by the buyer. There are no costs for operating the bank. Proportional intermediation costs will be added later.

We choose units so that the dollar price of X is the same as the dollar price of Y . In this sense the model is "real" but all payments are made in money. The sellers accept money from the buyers (money that is withdrawn either from the checking account or from the savings account) because they have to repay their debt to the government. The government uses lump sum taxes (transfers when $R_m \leq R \leq 1$) to finance the interest payments to buyers who hold deposits (savings and checking) in the government bank and to insure market clearing. The sequence of events is summarized in Table 1.

Table 1: Sequence of events.

	Period 1	Period 2
Buyers	Produce $y = \theta L$ units and sell them for money. They save revenues either in bonds at the gross interest R or in money at the gross interest $R_m \leq R$.	Use savings and lump sum transfers to consume X . Pay α units of X to cash bonds.
Sellers	Borrow to consume Y at the gross interest of 1.	Produce $x = L$ units and sell them for money. Use the revenues to repay their debt.
Government	Lends to sellers	Makes lump sum payments and collects debt from sellers.

In the Appendix we consider a monetary version of the model in which bonds are private IOUs. In the monetary version $R_m = (1 + \pi)^{-1}$ where π is the rate of inflation. The rate of inflation may not be optimal for a variety of reasons like time inconsistency problems of the type discussed by Kydland and Prescott (1977) and downward wage rigidities. Later we consider the case in which the government chooses the rate of inflation to raise a certain amount of revenues.

The "strong" planner's problem.

It may be useful to consider the problem of a "strong" planner who can tell agents how much to work and can distribute output without cost. Later we introduce a "weak" planner who can tell agents which assets to use but not how much to work. We assume that the "strong" planner maximizes total surplus by solving the following problem:

$$(5) \quad \max_{y, x, l^s, l^b(\theta) \geq 0} y + x - l^s - \int v(l^b(\theta)) dF(\theta) \quad \text{s.t.} \quad y = \int \theta l^b(\theta) dF(\theta) \quad \text{and} \quad x = l^s$$

Here (y, x) are the aggregate consumption of (Y, X) , l^s is the aggregate supply of labor by sellers and $l^b(\theta)$ is the supply of labor by a buyer with productivity θ . Any amount supplied by sellers l^s solves (5). The first order conditions that determine the amount of labor supplied by buyers requires that the marginal cost should equal the marginal benefit:

$$(6) \quad v'(l^b(\theta)) = \theta$$

Unlike the planner, a market economy uses assets to transfer output from producers to consumers. It will be argued that the first best (6) can be achieved if the government chooses $R_m = R = 1$ but will not be achieved if it chooses $R_m < 1$. We focus on the "second best" that takes $R_m < 1$ as given.

The portfolio choice.

Once the transaction cost is paid, it is optimal to invest the entire portfolio in bonds that yield a higher rate of return. Buyers thus specialize in either bonds or money. The utility from using money is:

$$V_m(\theta, R_m) = \max_L R_m \theta L - v(L)$$

and the utility from using bonds is:

$$V_b(\theta, R) = \max_L R \theta L - v(L) - \alpha$$

To solve for these functions, let:

$$(7) \quad L(w) = \arg \max_L wL - v(L)$$

denote the labor supply of a buyer who faces the wage w . The first order condition for the problem in (7) is: $v'(L) = L = w$ and therefore the labor supply of a money user is

$L(R_m \theta) = R_m \theta$ and the labor supply of a bond user is $L(R\theta) = R\theta$. The maximum utilities are therefore: $V_m(\theta, R_m) = \frac{1}{2}(\theta R_m)^2$ and $V_b(\theta, 1) = \frac{1}{2}(\theta)^2 - \alpha$. The buyer will choose bonds if:

$$(8) \quad \frac{1}{2}\theta^2 - \alpha \geq \frac{1}{2}(\theta R_m)^2$$

Or,

$$(9) \quad \theta \geq \sqrt{\frac{2\alpha}{1 - [R_m]^2}}$$

Let $\theta^*(R_m) = (2\alpha)^{\frac{1}{2}}(1 - [R_m]^2)^{-\frac{1}{2}}$ denote the cutoff point. Then buyers with productivity $\theta \geq \theta^*(R_m)$ use bonds and buyers with productivity $\theta < \theta^*(R_m)$ use money. Since $\theta^*(R_m)$ is an increasing function, we can state the following claim.

Claim 1: An increase in R_m increases the measure of money users.

This is of course, not surprising.

Welfare.

Social welfare is the sum of the utilities of all buyers as sellers' utilities are zero. As was said above, the real wage of a buyer with productivity θ is $w = R_m \theta$ if he uses money

and $w = R\theta$ if he uses bonds. His contribution to social welfare is therefore:

$S(\theta, R_m) = \theta L(R_m \theta) - v(L(R_m \theta))$, if he uses money and $S(\theta, R) - \alpha$ if he uses bonds, where $S(\theta, R) = \theta L(R\theta) - v(L(R\theta))$. Since buyers with $\theta \geq \theta^*$ use bonds, social welfare is:

$$(10) \quad W(R_m, R) = \int_{\theta^{\min}}^{\theta^*(R_m)} S(\theta, R_m) dF(\theta) + \int_{\theta^*(R_m)}^{\theta^{\max}} (S(\theta, R) - \alpha) dF(\theta)$$

When $R = 1$, the contribution of a buyer with productivity θ to social welfare can be expressed in terms of the areas A, B and C in Figure 1. Since these areas are a function of (θ, R_m) , we use the notation $[A(\theta, R_m), B(\theta, R_m), C(\theta, R_m)]$. A bond user with productivity θ supplies $L(\theta)$ units of labor and makes $\theta L(\theta)$ units as income. The cost of production is the area under the supply curve and therefore his contribution to social welfare is:

$$(11) \quad A(\theta, R_m) + B(\theta, R_m) + C(\theta, R_m) - \alpha$$

units. A money user with productivity θ supplies $L(R_m \theta)$ units of labor and makes $\theta L(R_m \theta)$ units as income. Subtracting the area under the supply curve we get his contribution to social welfare:

$$(12) \quad A(\theta, R_m) + C(\theta, R_m)$$

From the social point of view a buyer should use bonds if (11) is greater than (12):

$$(13) \quad A(\theta, R_m) + B(\theta, R_m) + C(\theta, R_m) - \alpha \geq A(\theta, R_m) + C(\theta, R_m)$$

From the private point of view a money-user gets the wage $R_m \theta$ and his income is $R_m \theta L(R_m \theta)$. Subtracting the area under the supply curve from his income leads to the surplus: $C(\theta, R_m)$. The difference between the social and the private point of view for a money-user is the area $A(\theta, R_m) = \theta L(R_m \theta) - R_m \theta L(R_m \theta)$ that is equal to the seigniorage that he pays to the government. There is no difference between the private and the social point of view for bond users because they do not pay seigniorage. It follows that from the private point of view the bond option is preferred if:

$$(14) \quad A(\theta, R_m) + B(\theta, R_m) + C(\theta, R_m) - \alpha \geq C(\theta, R_m)$$

The inequalities (13) and (14) imply that from the social point of view a buyer should use bonds if:

$$(15) \quad B(\theta, R_m) \geq \alpha$$

and from the private point of view he should use bonds if:

$$(16) \quad A(\theta, R_m) + B(\theta, R_m) \geq \alpha$$

The difference between the social and the private points of view arises because the private calculation does not take into account the seigniorage (the area A) that is paid by money users.

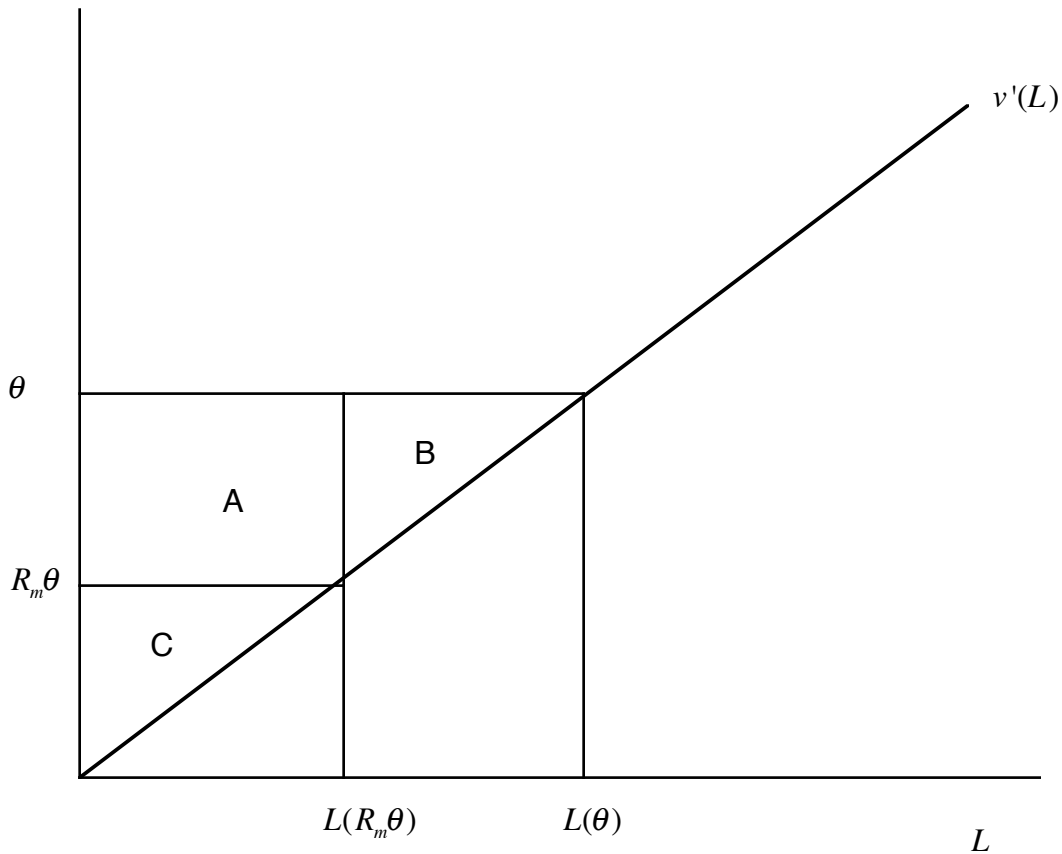


Figure 1

Since $L(w) = w$ we can compute the areas in Figure 1 and the following derivatives.

Lemma: (a) $B = \frac{1}{2}\theta^2(1-R_m)^2$ is increasing in θ and decreasing in R_m ;
 (b) $A = \theta^2(1-R_m)R_m$ and $C = \frac{1}{2}(\theta R_m)^2$; (c) $A+B = \frac{1}{2}\theta^2(1-[R_m]^2)$ is increasing in θ and decreasing in R_m and (d) $A+C = \theta^2 R_m(1-\frac{1}{2}R_m)$ is increasing in θ and in R_m .

We start by establishing the standard result about the relationship between inflation and welfare in the steady state.

Claim 2: An increase in R_m improves welfare.

Proof: When R_m increases, some bond users will not change their behavior and some will switch to money. Assuming that the increase in R_m is small, the buyers who switch to money are initially close to being indifferent between money and bonds and for them we may use the approximation that (16) holds with equality. Initially, the contribution of the bond users who are indifferent between the two options to social welfare is C . (To see this, substitute $\alpha = A+B$ in [11]). After switching to money their contribution goes up to $A+C$. It follows that the switchers increase social welfare by the area A . Money users will increase labor and increase their contribution to social welfare (because the area $A+C$ increases with R_m). It follows that the contribution of all buyers to social welfare either strictly increase or stay the same and therefore welfare goes up when R_m increases. \square

Maximum welfare is attained at the Friedman rule when $R_m = R = 1$. This is a special form of the Friedman rule because there is no discounting. At the optimal policy, buyers will choose to invest in checking only and will produce the optimal amount. Thus, at the optimal policy money completely crowds out bonds.

We now turn to a second best world in which $0 < R_m^{\min} \leq R_m \leq 1$, where the minimum rate of return on money R_m^{\min} satisfies $\frac{1}{2}(\theta^{\min} R_m^{\min})^2 > \frac{1}{2}(\theta^{\min})^2 - \alpha$. This

assumption insures that money will always be held. We also assume that θ^{\max} is large and therefore bonds will be used when $R_m = R_m^{\min}$.

The buyer's choice as a function of R_m holding θ (and $R = 1$) constant.

Since the area $A+B$ decreases with R_m and goes to zero when $R_m \rightarrow 1$, (14) is not satisfied when R_m is close to 1. In the intermediate range in which (16) is satisfied and (15) is not, the buyer will use bonds but his choice is not efficient from the social point of view. When we reduce R_m further so that (15) is satisfied, the buyer will use bonds and his choice is efficient. Figure 2 illustrates.

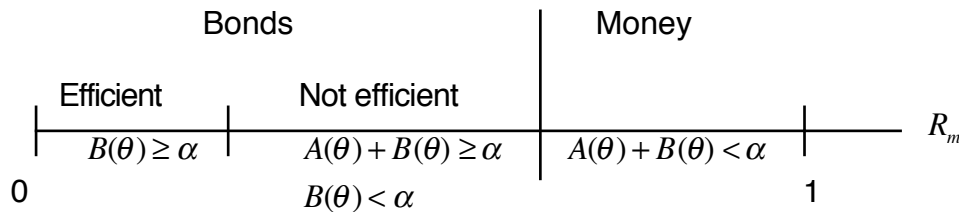


Figure 2: The buyer's choice as a function of R_m holding θ constant at a sufficiently high level (R_m increases from left to right)

We have thus shown the following claim:

Claim 3: A buyer with a sufficiently high θ will make an efficient portfolio choice when R_m is very low or very high. But in the intermediate case there is an excessive use of bonds.

The intuition is as follows. There may be an excessive use of bonds because private agents ignore the adverse effect of using bonds on seigniorage revenues. But this is not always the case. When R_m is very low the labor market distortion is larger than the cost of the trip to the bank and therefore it is efficient to make the trip. And when R_m is high bonds are not used.

The buyer's choice as a function of θ holding R_m constant.

Using the Lemma we can write condition (15) as: $B = \frac{1}{2}\theta^2(1-R_m)^2 \geq \alpha$. Therefore the choice of bonds is optimal from the social point of view if:

$$(17) \quad \theta \geq \sqrt{\frac{2\alpha}{(1-R_m)^2}}$$

This is different from (9) that characterizes the private point of view. Let $\theta^{**}(R_m) = (2\alpha)^{\frac{1}{2}}(1-R_m)^{-1} > \theta^*(R_m)$ denote the cutoff point. Buyers with $\theta_{\min} \leq \theta < \theta^*(R_m)$ use money. Buyers with $\theta^*(R_m) \leq \theta < \theta^{**}(R_m)$ use bonds but this choice is not efficient from the social point of view. Buyers with $\theta \geq \theta^{**}(R_m)$ use bonds and this choice is efficient from the social point of view. Figure 3 illustrates.

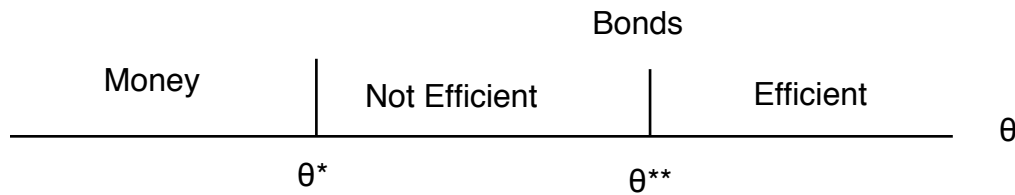


Figure 3: The buyer's choice as a function of θ holding $R_m < R = 1$ constant (θ increases from left to right)

We have thus shown the following claim:

Claim 4: When R_m is given, low productivity buyers use money and high productivity buyers use bonds. Only the choice of the very high productivity bond users is efficient.

The intuition here is as follows. For the very high productivity buyers overcoming the labor distortion has a large social benefit and therefore the trip to the bank is justified. For buyers with intermediate level of productivity the trip is not justified but they still do it because they ignore the adverse effect of holding bonds on seigniorage revenues.

Since $\theta^*(R_m)$ is an increasing function we also have the following claim:

Claim 5: An increase in R_m reduces the measure of bond users whose choice is optimal from the social point of view.

Figure 4 uses claims 1 and 5 (and Figures 2 and 3) to characterize the choice of buyers. Buyers who face the parameter pair (R_m, θ) in areas C+ F+H+I use money. Buyers who face the parameter pair in the area B+E+G+A+D use bonds but from the social point of view the use of bonds is efficient only for the buyers with the parameter pair in the area A+D.

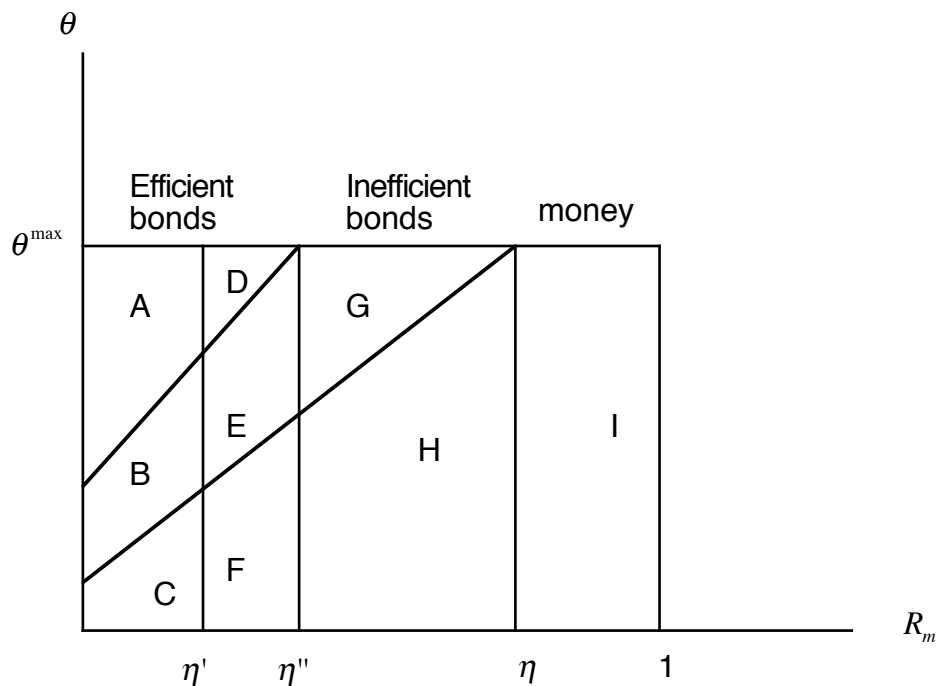


Figure 4: The buyer's choice as a function of (θ, R_m) where $R_m^{\min} \leq R_m \leq 1$.

He will choose money if the pair is in the area CFHI. His choice of bonds is not efficient in the area BEG and efficient in the area AD.

The effect of Simons' type regulations.

Simons (1948) argued for regulations aimed at discouraging money substitutes. Here bonds are the only money substitute and we therefore consider the effect of regulations that effectively prohibit the use of bonds. (In our setup this can be accomplished simply by not offering savings accounts, but in the case of private bonds this requires some additional regulations).

The effect of the regulation depends on R_m . Using Figure 4, the regulation will have no effect when $\eta \leq R_m \leq 1$. Welfare goes up as a result of the regulation when $\eta'' \leq R_m \leq \eta$ because in this range all bonds users increase social welfare when they switch to money. Welfare also goes up if R_m is slightly below η'' because the welfare gain from switching bond users in E is larger than the welfare loss from switching bond users in D. The welfare gains from imposing the restrictions are thus maximized at $R_m = \eta' < \eta''$ as in Figure 5. When R_m is very low the regulations reduce welfare because the distortion in the labor choice is very large and from the social point of view, it is worth paying the cost of the trip to the bank to eliminate this distortion for high productivity buyers.

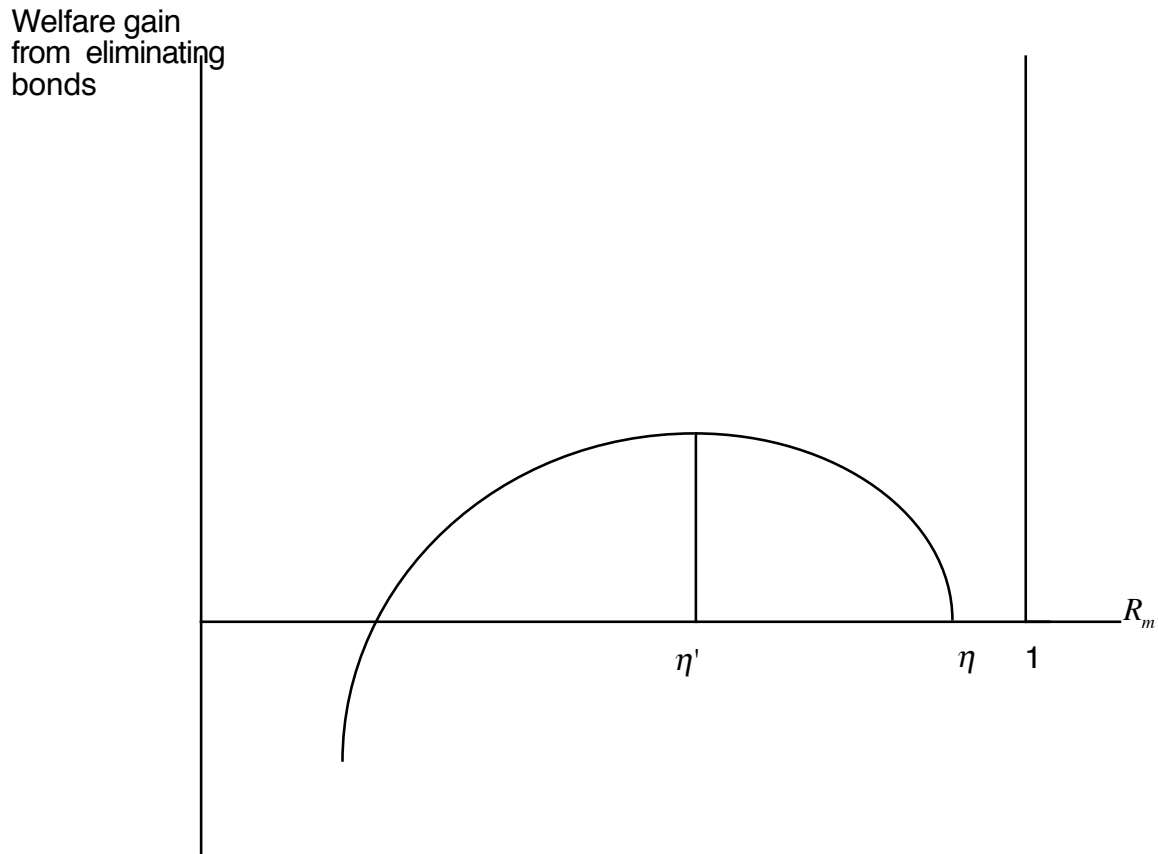


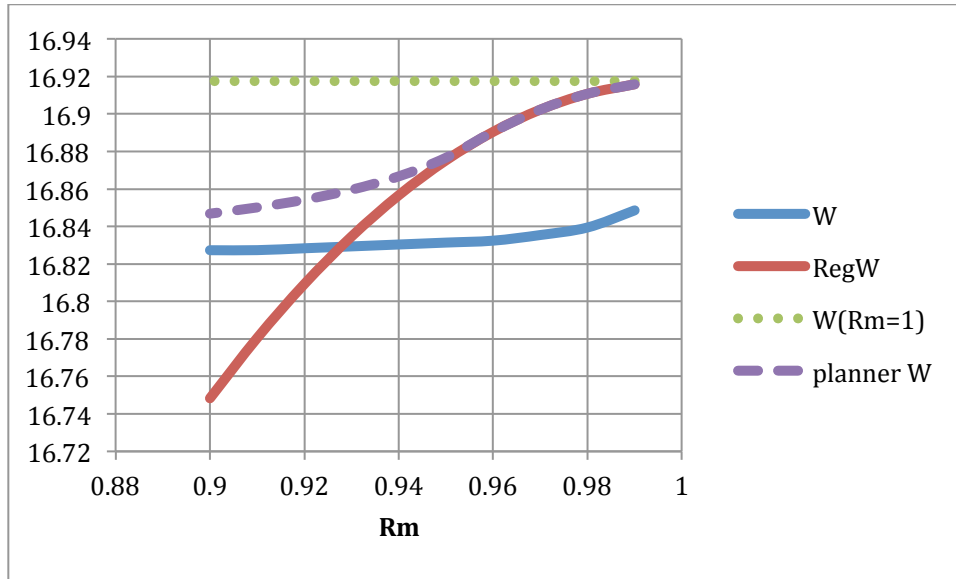
Figure 5: Welfare gain from eliminating bonds as a function of the real rate of return on money ($R_m \leq R=1$).

Numerical example.

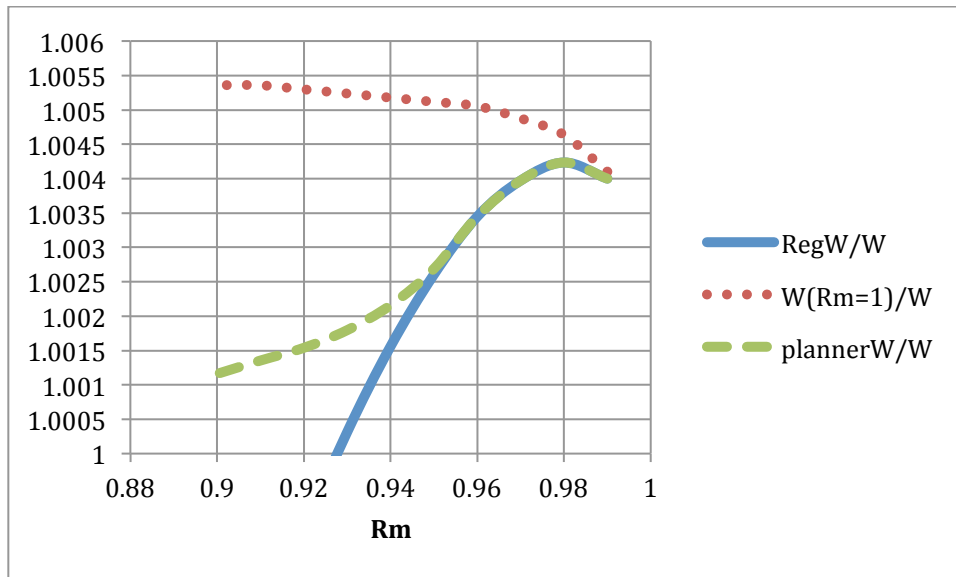
To illustrate, we assume that θ is uniformly distributed in the range $0.01 \leq \theta \leq 1$ and $\alpha = 0.001$. Later we use parameters that are consistent with recent US data. There are large differences between the two. In recent US data the top 1 percent makes about 15 percent of total income while here they make about 3 percent of income. But this example allows for a smooth variation in the number of bond users and it is useful for illustrating the different concepts of welfare used.

In Figure 6A welfare (W) is computed under the assumption that agents with $\theta \geq \theta^*$ use bonds. Regulation welfare (RegW) assumes that all agents use money and the "weak" planner welfare (PlannerW) assumes that agents with $\theta \geq \theta^{**}$ use bonds. (Note that our "weak" planner can only tell agents what asset to use but unlike the "strong" planner he cannot tell them how much labor to supply). The line denoted by $W(R_m = 1)$ is

the first best obtained by setting $R_m = 1$. The graph in Figure 6B is the ratios of the three welfare measures to welfare: $RegW/W$ is the ratio of regulation welfare to welfare, $W(R_m = 1)/W$ is the ratio of the first best to welfare and $PlannerW/W$ is the ratio of the planner welfare to welfare. The Figure illustrates that regulation welfare is the same as the planner's welfare when $R_m \geq 0.95$. In this range, the planner will tell all agents to use money. When $R_m < 0.95$ the planner will use some bonds and therefore the planner's welfare is greater than the regulation welfare. The regulation welfare is higher than welfare when $R_m \geq 0.93$. This says that the regulation improves welfare only when R_m is not too low while the planner always improve matters.



A. Welfare measures



B. Ratios

Figure 6: Welfare (W), Planner welfare (Planner W), Regulation welfare (Reg W) and the first best ($W[Rm=1]$) when θ is uniformly distributed and $\alpha = 0.001$

3. SEIGNIORAGE

We have assumed that lump sum payments are possible and inflation arises for reasons that are not related to raising revenues, like time inconsistency problems and downward wage rigidities. Here we consider the case in which inflation arises as a result of the inability of the government to collect direct taxes in an amount that is sufficient to finance its expenditures.³ From the point of view of governments that use inflation to raise revenues, bonds allow agents to evade the tax on holding money for a fixed cost. Does this "loophole in the tax system" improve matters?

To answer this question we consider the problem of maximizing welfare under the constraint that the seigniorage tax revenues must be greater than k , where k does not exceed the maximum seigniorage possible. We consider this problem under two regimes: The Friedman regime that allows the use of bonds and the Simons regime that does not allow the use of bonds.

Under the Friedman regime, the problem of the policy maker is

$$(18) \quad H(k) = \max_{R_m} W(R_m, 1)$$

s.t.

$$(19) \quad SE(R_m) = \int_{\theta^{\min}}^{\theta^*(R_m)} \theta(1 - R_m)L(\theta R_m) dF(\theta) \geq k$$

The constraint says that the seigniorage tax revenues $SE(R_m)$ are greater than k .

Under the Simons regime, the problem of the policy maker is:

$$(20) \quad h(k) = \max_{R_m} \bar{W}(R_m) = \int_{\theta^{\min}}^{\theta^{\max}} \left\{ \theta L(R_m \theta) - v(L(R_m \theta)) \right\} dF(\theta) \\ = \int_{\theta^{\min}}^{\theta^{\max}} \left\{ \theta(R_m \theta) - \frac{1}{2}(R_m \theta)^2 \right\} dF(\theta)$$

s.t.

$$(21) \quad se(R_m) = \int_{\theta^{\min}}^{\theta^{\max}} \theta(1 - R_m)L(\theta R_m) dF(\theta) \geq k$$

³ Our reading of the literature is that consumption tax usually dominates seigniorage taxation and therefore the Friedman rule is optimal when a consumption tax can be perfectly enforced. See, for example, Chari, Christiano and Kehoe (1996), Correia and Pedro (1996) and da Costa and Werning (2008).

Since by Claim 2, welfare is increasing in $R_m \leq 1$ the solution to both problems is to choose the highest possible R_m . Since more agents pay taxes under (21), the rate of return on money is higher under the Simons regime.

The case for bonds is weaker than before because now the elimination of bonds allows for an increase in R_m . To elaborate, it may be helpful to consider the problems of two planners. Planner 1 takes R_m as given while planner 2 is allowed to change R_m subject to the constraint that revenues do not change. Both planners can tell buyers which asset to use: Money or bonds. Below, we argue that planner 2 will use money more than planner 1.

Planner 1's choice was already discussed. He will tell buyers with productivity θ to use bonds if $\theta \geq \theta^{**}(R_m)$. Planner 2 will tell buyers to use bonds if $\theta \geq \theta^{***}(R_m)$.

Claim 6: $\theta^{***}(R_m) \geq \theta^{**}(R_m)$

The proof is as follows. Welfare is increasing in R_m (Claim 2) and therefore (21) is binding. Planner 1 is indifferent between switching a buyer with $\theta = \theta^{**}(R_m)$ from bonds to money. Planner 2 will strictly prefer to switch this buyer to money because by switching him he will get some seigniorage revenues that will allow him to increase R_m and improve welfare.

To build intuition it may be useful to go over the calculations in (11)-(16) from the point of view of planner 2. The contribution of a buyer to social welfare is defined by the reduction in social welfare that will occur if the individual labor supply is set to zero. Using Figure 1, the contribution of a bond user is: $A+B+C-\alpha$. The contribution of a money user is: $(1+\lambda)A+C$, where $\lambda > 0$ is the dead weight loss from a unit raised as tax revenues. To motivate λ note that if the money user's labor supply is reduced to zero, the government will have to raise additional A units in revenues. To do that it will have to reduce R_m and therefore the loss of social welfare from setting the labor supply of a

money user to zero is larger than A . It follows that planner 2 will tell a buyer to use bonds only if: $A+B+C-\alpha > (1+\lambda)A+C$, or $B > \alpha + \lambda A$. Comparing this to (13) reveals that he will tend to use bonds less than planner 1. Figure 4' illustrates.

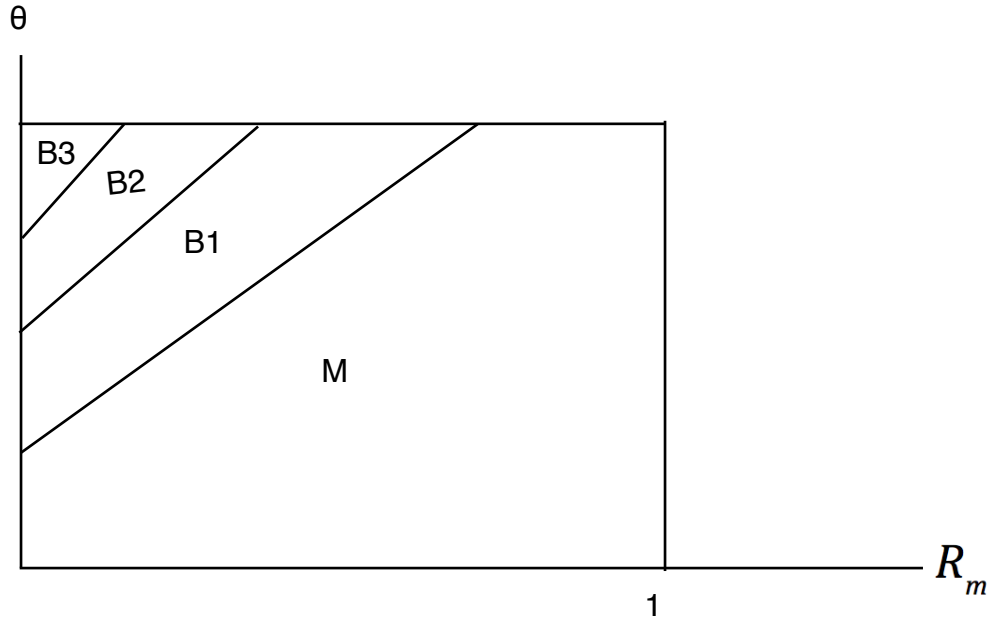


Figure 4': A buyer in the area M will use money. A buyer in the area B will use bonds but his choice is not optimal from the social point of view if he is in B1. His choice of bonds is optimal from the social point of view if inflation is exogenous and he is in B2+B3. When inflation is endogenous and is determined by the need to raise seigniorage revenues his choice of bonds is optimal only if he is in B3.

An Example.

We consider a special case in which θ has two possible realizations: half of the buyers get the realization θ_1 and the remaining half get the realization $\theta_2 < \theta_1$.

In the Friedman regime we can have a solution in which all buyers use money. In this case, welfare is the same across the two regimes. We consider now the case in which the high productivity buyers use bonds; the supply of labor is $L_1 = \theta$ for the high productivity buyers and $L_2 = \theta R_m$ for the low productivity buyers. The choice of R_m must satisfy the following equation:

$$(22) \quad 2k = R_m(1 - R_m)(\theta_2)^2$$

In the Simons regime R_m must satisfy:

$$(23) \quad 2k = R_m(1 - R_m)((\theta_1)^2 + (\theta_2)^2)$$

There are 2 solutions for each of the quadratic equations (22) - (23). We pick the high solution that maximizes welfare. In Figure 7, \hat{R}_m is the optimal rate of return on money under the Friedman regime and \bar{R}_m is the optimal rate under the Simons regime.

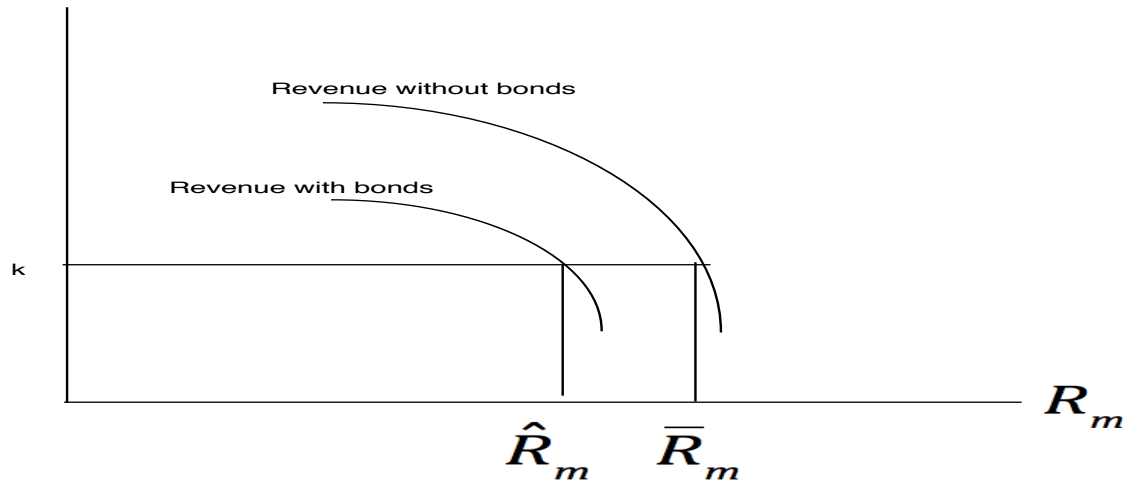


Figure 7

We establish the following claim⁴.

Claim 7: $h(k) \geq H(k)$ for all feasible k .

Proof: We show that by imposing Simons' regulations it is possible to improve on the optimal policy under the Friedman regime. Starting from the Friedman regime with

$R_m = \hat{R}_m$, we impose the regulations and increase R_m from \hat{R}_m to:

$$\bar{R}_m = \frac{\theta_1 + \theta_2 \hat{R}_m}{\theta_1 + \theta_2} > \hat{R}_m.$$

⁴ The Claim can be generalized to the case in which the fraction of agents with the high productivity is different from half. The proof should go through as long as the two groups of buyers can each be represented by a single agent. Adding concavity to the utility from consumption will only strengthen the case for Simons' type regulations.

As a result of the increase in R_m , the real wage of the high productivity buyers drops by $\theta_1(1 - \bar{R}_m)$ and the real wage of the low productivity buyers increases by $\theta_2(\bar{R}_m - \hat{R}_m)$. The choice of \bar{R}_m insures that the absolute value of the change in real wage is the same for both groups:

$$(24) \quad \theta_2(\bar{R}_m - \hat{R}_m) = \theta_1(1 - \bar{R}_m)$$

The per-buyer contribution of the high productivity buyers to social welfare went down from $(\frac{1}{2})(\theta_1)^2 - \alpha$ to $(\theta_1)(\theta_1\bar{R}_m) - (\frac{1}{2})(\theta_1\bar{R}_m)^2$. In terms of the areas in Figure 8 the drop in the per-buyer contribution is $g - \alpha$.

The per-buyer contribution of the low productivity buyers went up from $(\theta_2)(\theta_2\hat{R}_m) - (\frac{1}{2})(\theta_2\hat{R}_m)^2$ to $(\theta_2)(\theta_2\bar{R}_m) - (\frac{1}{2})(\theta_2\bar{R}_m)^2$. This is the area m in Figure 8. Since (24) implies $m = g$, it follows that welfare increases by $(\frac{1}{2})\alpha$ as a result of imposing the regulations.

We now turn to examine the change in revenues. Under the Friedman regime the revenues in terms of the areas in Figure 8 were $(\frac{1}{2})(h + k)$. After imposing the regulations the revenues are: $(\frac{1}{2})(h + i) + (\frac{1}{2})(d + e + f)$. Since (24) insures that $k = d$, it follows that as a result of the regulation revenues went up by: $(\frac{1}{2})(i + e + f)$.

Since by imposing the regulation we can improve welfare and increase revenues, it must be the case that the maximum welfare that can be obtained with the regulation is higher than without the regulations. \square

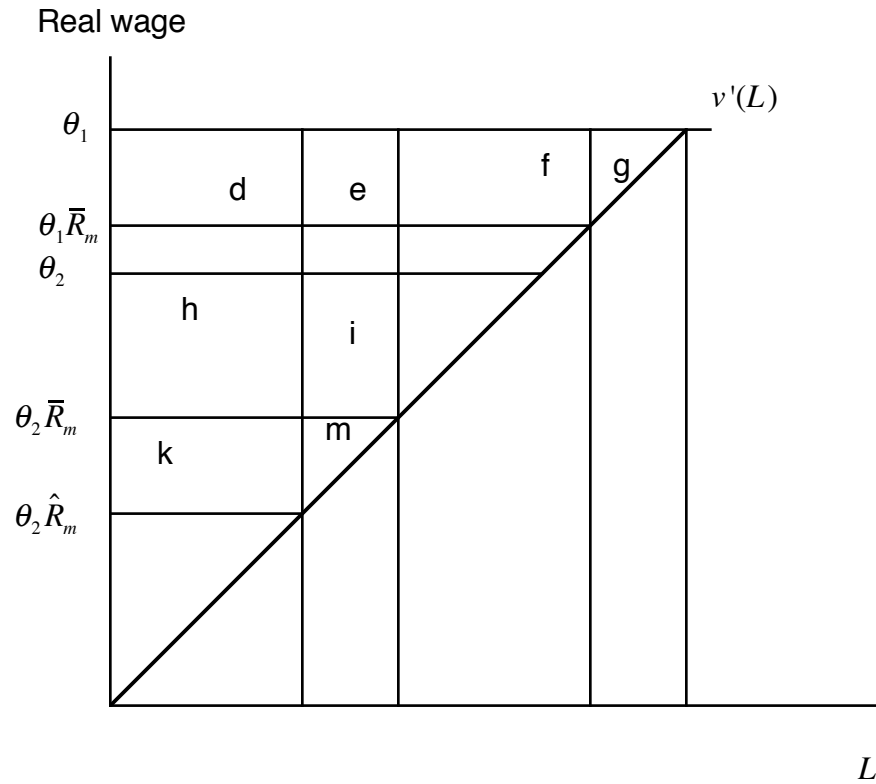


Figure 8

4. ALLOWING FOR $R < 1$, INCOME TAX AND PROPORTIONAL COSTS

This section discusses several extensions to the basic model. First, we allow for $R < 1$. Note that private contracts may limit the ability of the government to set $R < 1$: A seller can offer a contract that promises to deliver at the DM a unit of X for every unit of Y that he receives at the CM. If private contracts can be executed at the bank they will limit the government's ability to sell bonds that promise $R < 1$. The government has however an advantage in the ability to commit and enforce contracts. We therefore assume that the government can sell bonds at $R \geq R^{\min}$ where $\frac{1}{2} < R^{\min} < 1$ is a lower bound on the interest rate.

Second, we allow for an income tax at the flat rate of τ . Figure 1' is a generalization of Figure 1 and can be used to illustrate the difference between the social and the private point of view when $R < 1$ and $\tau > 0$.

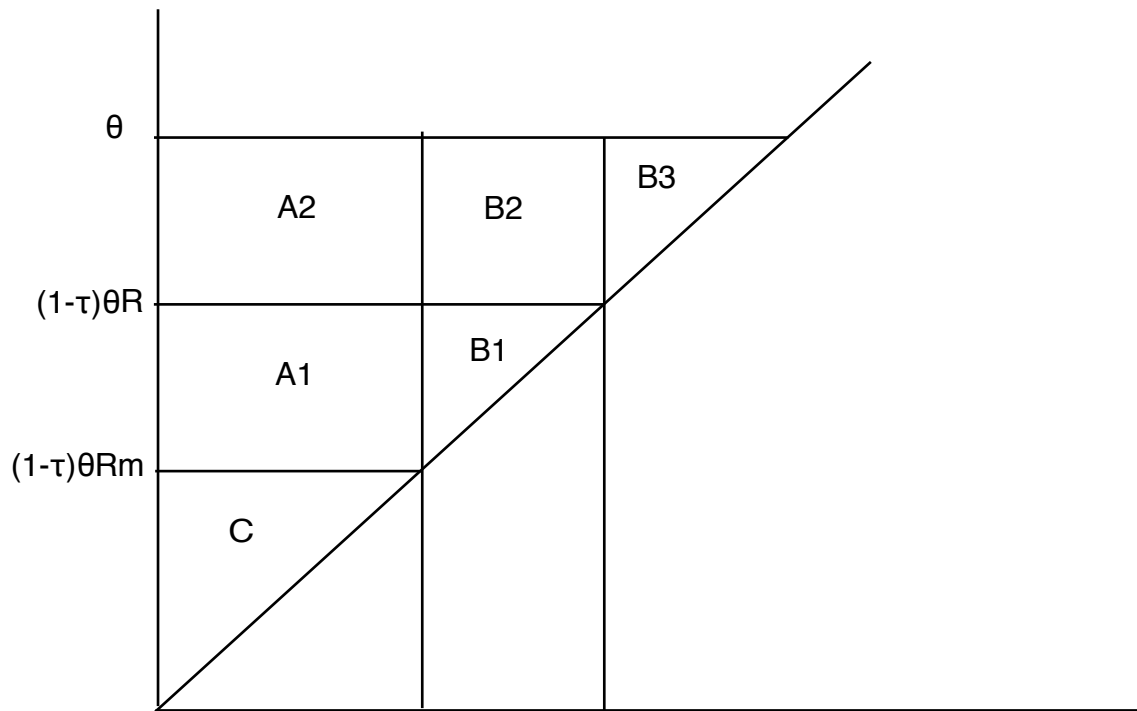


Figure 1'

From the social point of view, a bond user contributes:

$$(11') \quad A1 + A2 + B1 + B2 + C - \alpha$$

A money user contributes:

$$(12') \quad A1 + A2 + C$$

It follows that from the social point of view the buyer should use bonds if:

$$(15') \quad B1 + B2 \geq \alpha$$

From the private point of view the buyer should hold bond if:

$$(14') \quad A1 + B1 + C - \alpha \geq C$$

or

$$(16') \quad A1 + B1 \geq \alpha$$

We now turn to use (15') and (16') to generalize (17) and (9) under the assumption that $\tau = 0$. The case in which $\tau > 0$ will be discussed shortly. From the social point of view, the buyer should use bonds if $\theta \geq \theta^{**}(R, R_m)$ where the cutoff point is:

$$(17') \quad \theta^{**}(R, R_m) = \sqrt{\frac{2\alpha}{2(R - R_m) - (R^2 - (R_m)^2)}}$$

From the private point of view he should hold bonds if $\theta \geq \theta^*(R, R_m)$ where the cutoff point is:

$$(9') \quad \theta^*(R, R_m) = \sqrt{\frac{2\alpha}{R^2 - (R_m)^2}}$$

Note that θ^{**} is decreasing in R and in R_m and that θ^* is decreasing in R and increasing in R_m . Note also that $\theta^{**}(R, R_m) \geq \theta^*(R, R_m)$ for all $1 \geq R > R_m > \frac{1}{2}$.

Finally, we augment the model by allowing for proportional intermediation costs. Philippon (2015) examined 130 years of US data and found that intermediation technology has constant returns to scale, and the cost of intermediation is 1.5 to 2 percent of assets. In our model, the government is doing the intermediation. It lends to the seller at the gross interest of 1 and "borrows" from the buyers at two different rates: R for savings and R_m for checking. To capture the proportional cost of intermediation, we assume that each dollar deposited in the savings account costs ψ dollars to manage and the government subtracts the proportional cost of running the savings account from the lump sum transfer that it gives to the agents. The buyer make the portfolio choice on the basis of the lending rate that is now less than the borrowing rate.

A private bank will charge a borrowing rate (R^b) that is higher than the lending rate (R) by the proportional cost: $R^b - R = \psi$. Also in this case, the lender does not internalize the proportional cost and make his choice on the basis of the lending rate R .

4.1 Direct Taxes

Figure 1' makes it clear that a proportional tax has an important effect on the welfare calculations. Inflation will affect the labor supply of a money user in the same way as a consumption tax of the same rate. To see this, note that the budget constraint of a money-user is: $C = \theta LR_m$. Suppose that instead of inflation the government imposes a consumption tax at the rate σ . In this case the budget constraint is: $(1 + \sigma)C = \theta L$. The two budget constraints are the same when $R_m = \frac{1}{1 + \sigma}$. Since $R_m = \frac{1}{1 + \pi}$, it follows that inflation at the rate π is equivalent to a consumption tax at the same rate.

Similar reasoning may be used to show that the effect of inflation on money users is the same as the effect of an income tax at the rate τ that satisfies: $1 - \tau = \frac{1}{1 + \pi}$.

We may therefore view inflation as a form of a consumption tax (or an income tax) that is levied on top of the income tax. Since in general the marginal welfare cost of a tax is greater than the average cost, we may expect that the welfare cost of inflation will be higher when the income tax is higher.

In what follows we assume that there is no consumption tax and there is a proportional or flat income tax at the rate of τ . We start with the individual decision of whether to use bonds or money. The individual welfare when using money is:

$\frac{1}{2}((1 - \tau)\theta R_m)^2$. His welfare when using bonds is: $\frac{1}{2}((1 - \tau)R\theta)^2 - \alpha$. The individual will choose bonds if: $\frac{1}{2}((1 - \tau)R\theta)^2 - \alpha \geq \frac{1}{2}((1 - \tau)R_m\theta)^2$. The cutoff point is:

$$(25) \quad \theta^* = \sqrt{\frac{2\alpha}{(1 - \tau)^2(R^2 - (R_m)^2)}}$$

A money user supplies $L = w = (1 - \tau)\theta R_m$ units of labor and his contribution to welfare is:

$$(26) \quad \theta L - \frac{1}{2}L^2 = \theta^2(1 - \tau)R_m - \frac{1}{2}((1 - \tau)\theta R_m)^2$$

A bond user supplies $L = w = (1 - \tau)\theta R$ units of labor and his contribution to welfare is:

$$(27) \quad (1 - \psi)\theta L - \frac{1}{2}L^2 - \alpha = (1 - \psi)\theta^2(1 - \tau)R - \frac{1}{2}((1 - \tau)\theta R)^2 - \alpha$$

At the Friedman rule (when $R = R_m = 1$) everyone uses money and supplies

$L = w = (1 - \tau)\theta$ units of labor. The contribution to welfare at the Friedman rule is:

$$(28) \quad \theta L - \frac{1}{2}L^2 = (1 - \tau)\theta^2 - \frac{1}{2}((1 - \tau)\theta)^2$$

In general a "weak" social planner will tell agents to use bonds if (27) is greater than (26).

It is possible that there is no $\theta \geq 0$ such that (26) is equal (27). But when

$2(1 - \tau)[(1 - \psi)R - R_m] - (1 - \tau)^2[R^2 - (R_m)^2] > 0$ a solution exists and is given by:

$$(29) \quad \theta^{**} = \sqrt{\frac{2\alpha}{2(1 - \tau)[(1 - \psi)R - R_m] - (1 - \tau)^2[R^2 - (R_m)^2]}}$$

In this case the "weak" planner will tell agents with $\theta \geq \theta^{**}$ to use bonds. Note that the individual cutoff point (26) is intuitive. It says that the individual will make his choice on the basis of after tax income. The social cutoff point is more complicated to describe.

Figure 9 plots the social cutoff points ($\theta^{**} = \theta^{**}$) for various levels of income tax ($\tau = \tau$). As can be seen θ^{**} is decreasing in τ . The intuition is in viewing the inflation tax as a form of consumption or income tax. It is well known that the marginal cost of raising a dollar in income tax is increasing in the tax rate. See Browning (1987) for example.

Therefore when the income tax rate is relatively high the welfare cost of the inflation tax is higher and the "weak" social planner is more tolerant to "inflation tax avoidance" and tells more agents to use bonds. Note that $R = 0.985$ in Figure 9A, and $R = 1$ in Figure 9B.

The increase in the rate of return on bonds lowers the social cutoff points.

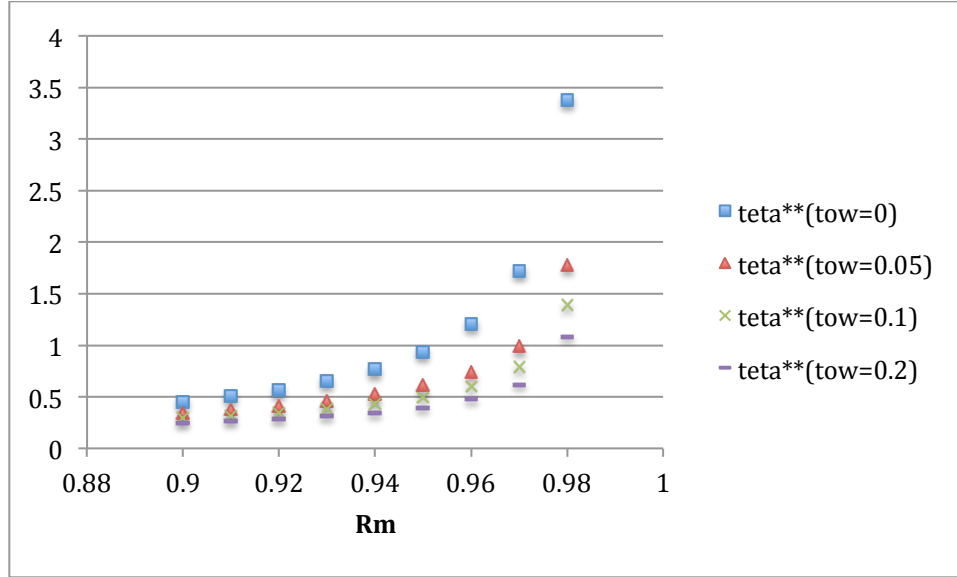
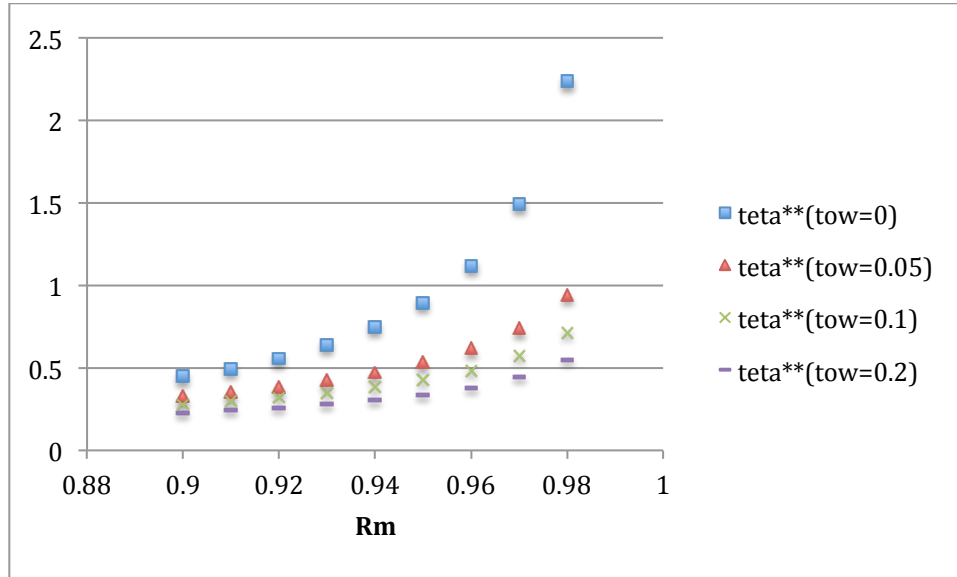
A. $R=0.985, \alpha=0.001, \psi=0$ B. $R=1, \alpha=0.001, \psi=0$

Figure 9: The effect of income tax on the social cutoff point

4.2 Welfare Cost Calculations

Welfare is given by:

$$(30) \quad W(R, R_m, \alpha, \psi, \tau) = \int_{\theta^{\min}}^{\theta^*} \left\{ (1-\tau)\theta^2 R_m - \frac{1}{2}((1-\tau)\theta R_m)^2 \right\} dF(\theta) \\ + \int_{\theta^*}^{\theta^{\max}} \left\{ (1-\tau)(1-\psi)\theta^2 R - \frac{1}{2}((1-\tau)\theta R)^2 - \alpha \right\} dF(\theta)$$

Welfare at the Friedman rule is:

$$(31) \quad W(1,1,\alpha,\psi,\tau) = \int_{\theta^{\min}}^{\theta^{\max}} \left\{ (1-\tau)\theta^2 - \frac{1}{2}((1-\tau)\theta)^2 \right\} dF(\theta)$$

And the welfare cost is:

$$(32) \quad WC(R,R_m,\alpha,\psi,\tau) = \frac{W(1,1,\alpha,\psi,\tau)}{W(R,R_m,\alpha,\psi,\tau)} - 1$$

Note that in this model the area under the demand for money curve is not equal to the welfare cost. For one reason, the proportional cost is not internalized and as a result the demand for money does not depend on ψ . Therefore variation in ψ will lead to variation in the welfare cost but not in the area under the demand for money curve. A second reason is that the fixed cost introduces "jumps" and therefore the area may not equal the welfare cost even when $\psi = 0$.

5. CALIBRATION

We use (25)-(32) for welfare calculations under the assumption that lump sum taxes are possible. This assumption understates the benefits from regulation because eliminating bonds increases seigniorage revenues and allows for the reduction in distortive taxes.

To perform welfare calculations we need to choose the length of the period. If the length of the period is one month, an annual rate of inflation of 12% is equivalent to a consumption tax on money-users of 1%. If the length of the period is one year it is equivalent to a consumption tax of 12%. The length of the period depends on the velocity of money. The quarterly velocity of M2 has fluctuated in the post war era from 2.2 to 1.5. Therefore if we adopt M2 as our definition of money, the length of the period is between 1.3 to 2 months. It would be shorter for higher interest rates and for narrower definitions of money. The results are set in terms of the nominal interest per period and

the reader can easily translate them to the length of the period that he or she think is appropriate.

The compensated labor elasticity is a key parameter in the calculations of the deadweight loss of income tax. Browning (1987) uses the compensated labor elasticity of 0.2-0.4. Feldstein (1999) uses an elasticity of 1.04. Here we use a compensated elasticity of 1.

Income distribution matters for our calculations. The Congressional Budget Office provides data about income distribution in the US.⁵ We use the income distribution for 2011 described in Table 1. The share of income of the top 1 percent is 14.6 percent while the share of income of the lowest quintile is only 5.3 percent. We use these data to create 100 individuals indexed 0.01, 0.02,...,1. The income of the highest earning individual (indexed 1) is 14.6. The income of each of the four individuals indexed 0.96-0.99 is $12.7/4=0.03175$ and so on. In the absence of distortions the income of each individual is θ^2 . We use 14.6 as a proxy for the θ^2 of the individual indexed 1 and 0.03175 as a proxy for each of the individuals indexed 0.96-0.99 and so on. The last row in Table 2 is the imputed θ^2 per individual in each income bracket. Note the difference from Figure 6 that assumes a uniform distribution.

Table 2: The distribution of income in the US (2011)

Percentile	1-20	21-40	41-60	61-80	81-90	91-95	96-99	100
Share	5.3	9.6	14.1	20.4	14.6	10.0	12.7	14.6
θ^2	0.00265	0.0048	0.00705	0.0102	0.0146	0.02	0.03175	0.146

We adopt the lower estimate of Philippon (2015) for the proportional cost and assume $\psi = 0.015$.

When α is "too high" no one uses bonds and when α is "too low" no one uses money. We choose α subject to the constraint that when the quarterly nominal interest is

⁵ <https://www.cbo.gov/publication/49440>

between 1.5% and 8.5% both assets are used. When $\tau = 0$, the acceptable range is: $0.0002 \leq \alpha \leq 0.0021$. When $\tau = 0.2$, the acceptable range is: $0.0002 \leq \alpha \leq 0.0013$.

It may be useful to describe the implications of choosing $\alpha = 0.001$ and $\tau = 0$. To appreciate the implication of this choice we look at the ratio of the fixed cost to potential income, α / θ^2 . When $0.96 \leq R_m \leq 0.98$ and the nominal lending rate, $R - R_m$, is in the range of 0.5 to 2.5 percent, only the top 1% uses bonds and they spend 0.7% of their potential income on bond transactions. When R_m drops to 0.95, the top 5% uses bonds and the group that joins in spends 3% of their potential income on bonds transactions. When it drops further to 0.93 the top 10% uses bonds and the group that joined in spends 5% of their potential income on bond transactions.

Figure 10 reports welfare calculations for various parameters holding $R = 0.985$ constant.

Unlike Figure 6, the ratios do not vary continuously because of the jumps in the productivity estimates. The Figure plots the ratio of regulation welfare (welfare when no bonds are used) to welfare and the ratio of first best welfare (welfare when $R_m = 1$) to welfare as a function of the nominal rate $R - R_m$. Figure 10A assumes $\alpha = 0.0015$, no income tax and no proportional cost. When the nominal interest is 0.5% the welfare cost of inflation is small and there are no benefits from regulations because everyone uses money. When the nominal interest increases to 1.5% the top 1% of the income distribution uses bonds. The welfare cost of inflation jumps to $\frac{W(R_m = 1)}{W} - 1 = 0.0038$ or 0.38% and the benefits from regulation are $\frac{RegW}{W} - 1 = 0.0029$ or 0.29%. When we

increase the nominal interest further to 2.5%, the welfare cost of inflation is 0.44% and the benefits from regulation are 0.28%. Note that the benefits from regulation went down slightly. The fall in the gains from regulation occurs because of the fall in the welfare contribution of money users. Since more agents use money under regulations, regulation welfare (RegW) declines by more than (unregulated) welfare (W). This effect dominates when there is no change in the number of bond users. When we raise the nominal interest

from 4.5% to 5.5% an additional group switches to bonds (the percentiles 96-99 of the income distribution) and as a result the welfare cost of inflation jumps to 1.88% and the benefits from regulation to 1.38%. The number of bond users does not change until the nominal interest rate is 8.5%. In this case an additional group (the percentiles 91-95) switches to bonds, the welfare cost jumps to 3.74% and the benefits from regulation to 2.7%.

Figure 10B reports the calculation for $\alpha = 0.001$. The top 1% uses bonds when the nominal interest is greater than 1.5%. The second group (percentile 96-99) switches to bonds when the interest rate increases from 2.5 to 3.5. This leads to a jump in the welfare cost of inflation from 0.34% to 1.19%. After the second group switches to bonds the gains from regulation jump from 0.18% to 0.94%. The third group (percentile 91-95) switches to bonds when the interest rate increases from 4.5% to 5.5% and a fourth group (percentile 81-90) switches to bonds when the interest rate increases from 6.5% to 7.5%. Note that the reduction in α leads initially to a reduction in the welfare cost of inflation but when the interest rate is greater than 3.5% it leads to an increase in the welfare cost. The reason is that in this range there are more bond users when $\alpha = 0.001$ and the cost of using bonds dominates the welfare calculations.

Figure 10C uses $\alpha = 0.0005$. Here the top 1% uses bonds when the nominal interest is greater than 0.5% and other groups switch to bonds in almost a continuous fashion. The welfare cost of inflation and the gains from regulations tend to be higher than for the higher α cases. Thus, technological changes that reduce the fixed cost tend to reduce welfare because they lead to an increase in bond usage.

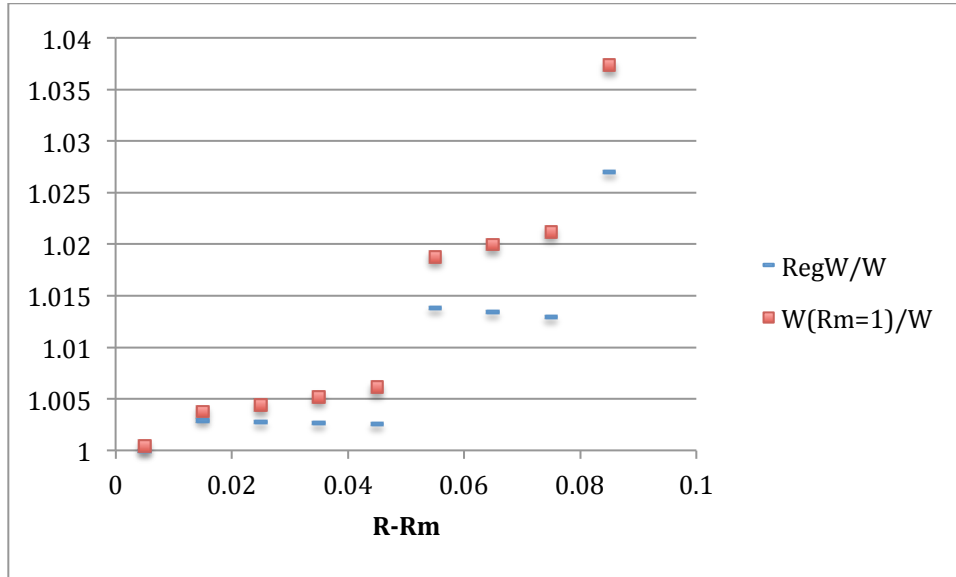
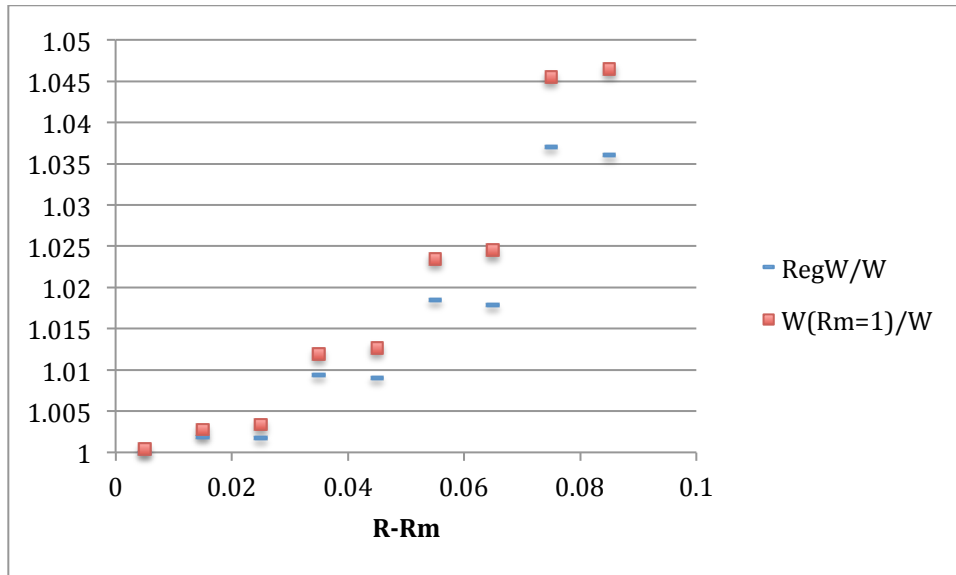
Figures 10D-10F repeat the calculations under the assumption that the proportional cost is 1.5%. The behavior of the agents is the same as before but the welfare cost of inflation and the benefits from regulations are now higher. When $\alpha = 0.001$ (Figure 10E) and the nominal interest is 1.5% the welfare cost of inflation is 0.71% and the gains from regulation is 0.62%. Comparing to Figure 10B, these numbers

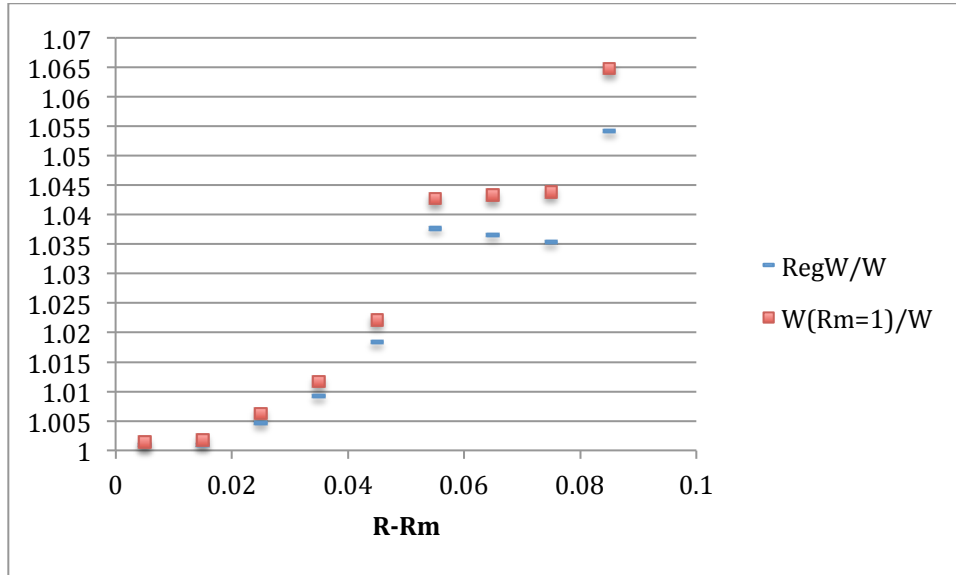
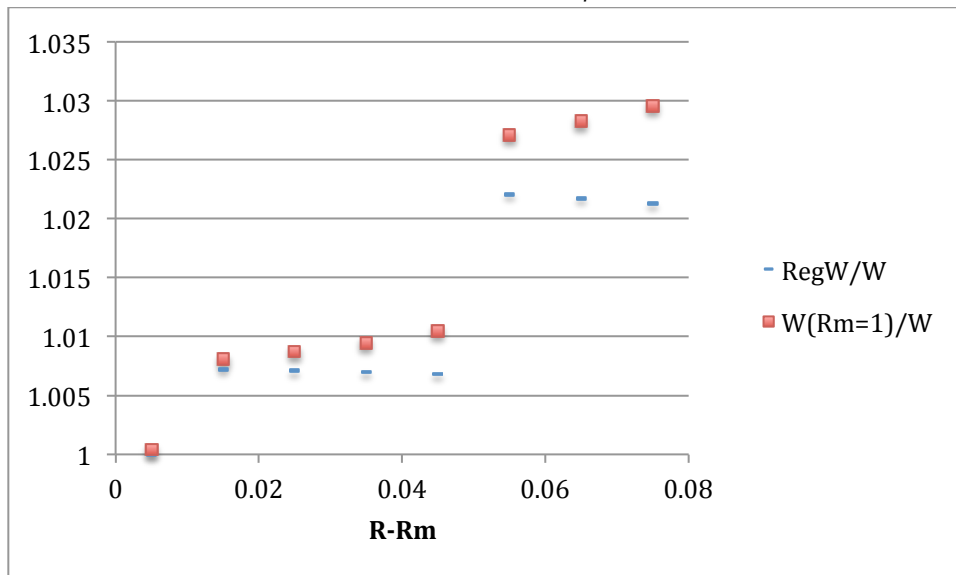
are higher by a factor of 2.5 and 3.3 respectively. On average the numbers in Figure 10E are larger than the numbers in Figure 10B by a factor of 1.7 for the welfare cost and 2.1 for the gains from regulation, suggesting that the proportional cost plays a dominant role in the calculations.

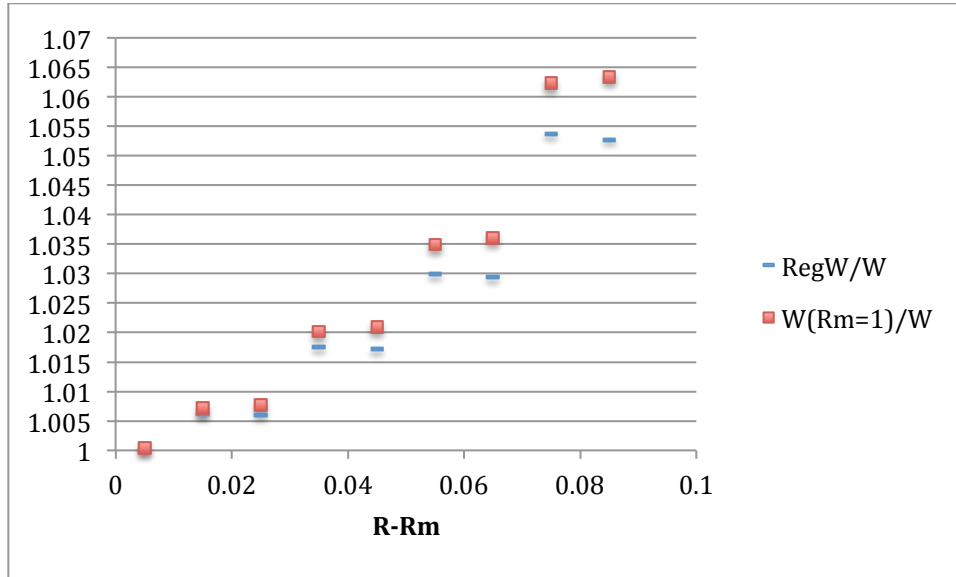
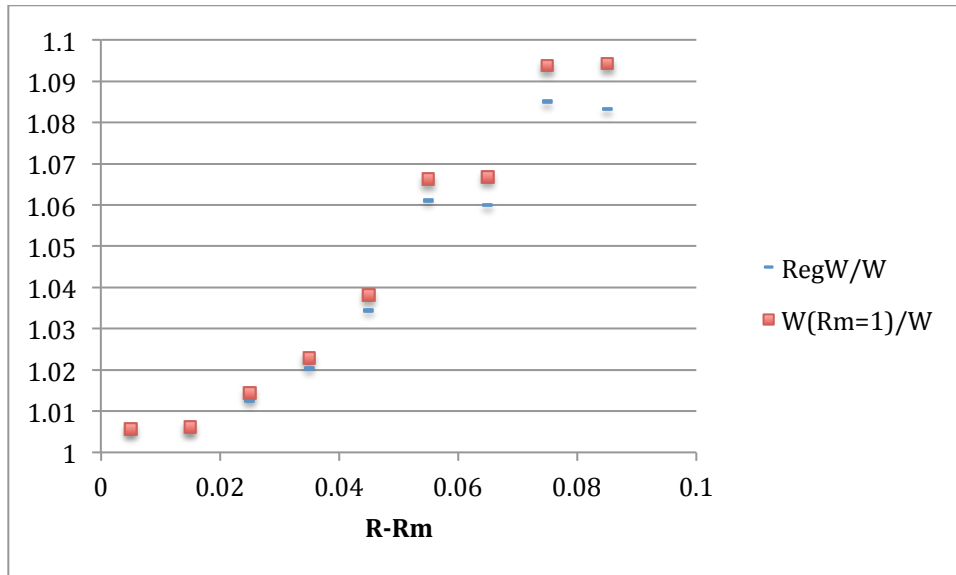
Figures 10G-10I add income tax at the flat rate of $\tau = 0.2$. This tends to increase the welfare cost of inflation and decrease the gains from regulation. As can be seen from (25), an increase in τ increases θ^* and reduces bond usage. There is also an effect on the welfare cost of inflation when the nominal interest is 0.5% and no one uses bonds. To understand this somewhat puzzling observation, let r_m be the gross interest on money that yields the same consumption per unit of labor when $\tau = 0$ so that: $(1 - \tau)\theta R_m = \theta r_m$ or $r_m = (1 - \tau)R_m$. This says that $R_m = 1$ with $\tau = 0.2$ is equivalent to $R_m = 0.8$ when $\tau = 0$ and $R_m = 0.98$ with $\tau = 0.2$ is equivalent to $R_m = 0.784$ with $\tau = 0$. When $R = 0.985$ and $R_m = 0.98$ no one uses bonds and therefore:

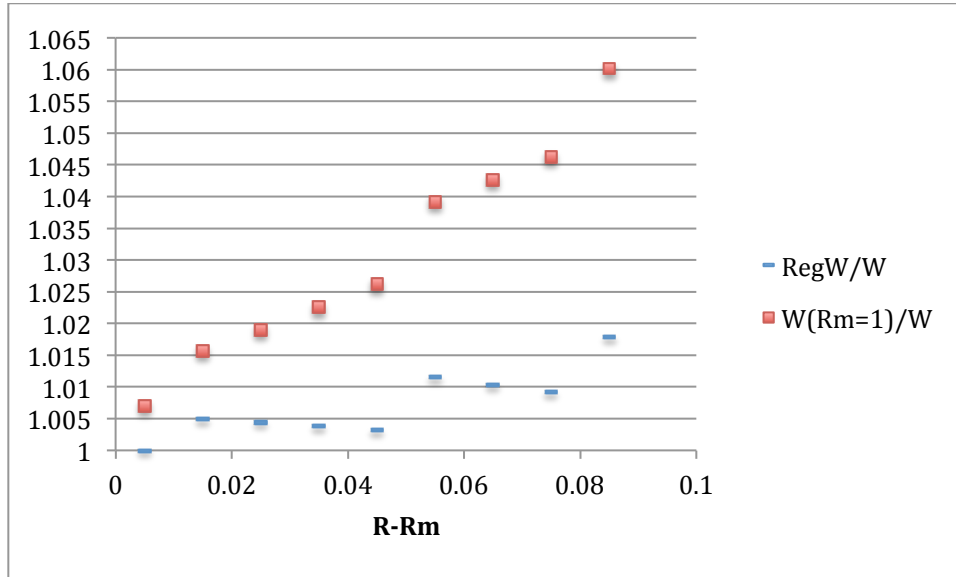
$$\frac{W(R_m = 1, \tau = 0.2)}{W(R_m = 0.98, \tau = 0.2)} = \frac{W(R_m = 0.8, \tau = 0)}{W(R_m = 0.784, \tau = 0)}$$

Thus, reducing the nominal interest from 0.5% to zero when $\tau = 0.2$ is comparable to reducing the nominal interest from 20.1% to 18.5% when $\tau = 0$. The welfare cost is higher when $\tau = 0.2$ for two reasons. First the reduction from 20.1% to 18.5% is 1.4 percentage points while the reduction from 0.5 to zero is only 0.5 percentage points. Second the marginal tax distortion is increasing in the level of the tax.

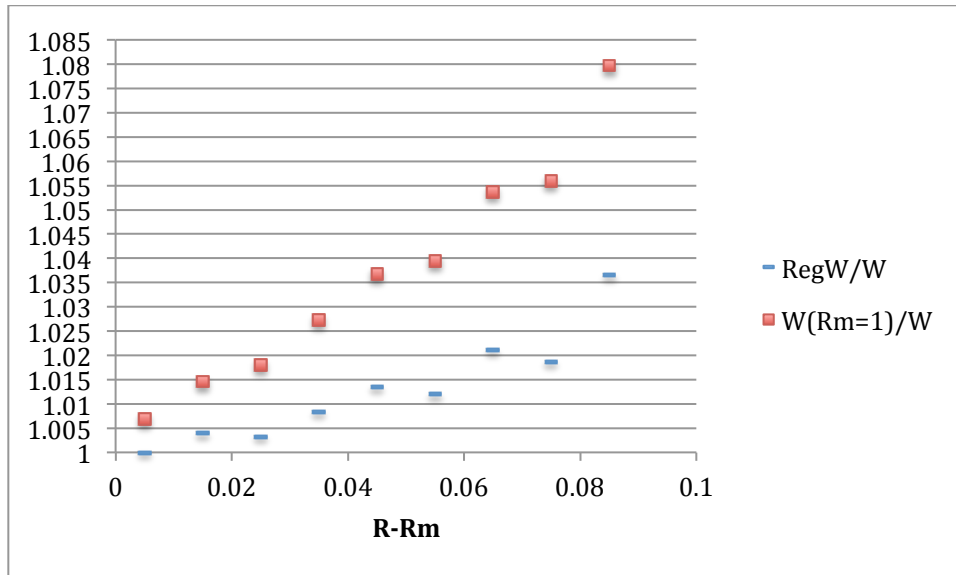
A. $\alpha = 0.0015, \tau = \psi = 0$ B. $\alpha = 0.001, \tau = \psi = 0$

c. $\alpha = 0.0005, \tau = \psi = 0$ d. $\alpha = 0.0015, \tau = 0, \psi = 0.015$

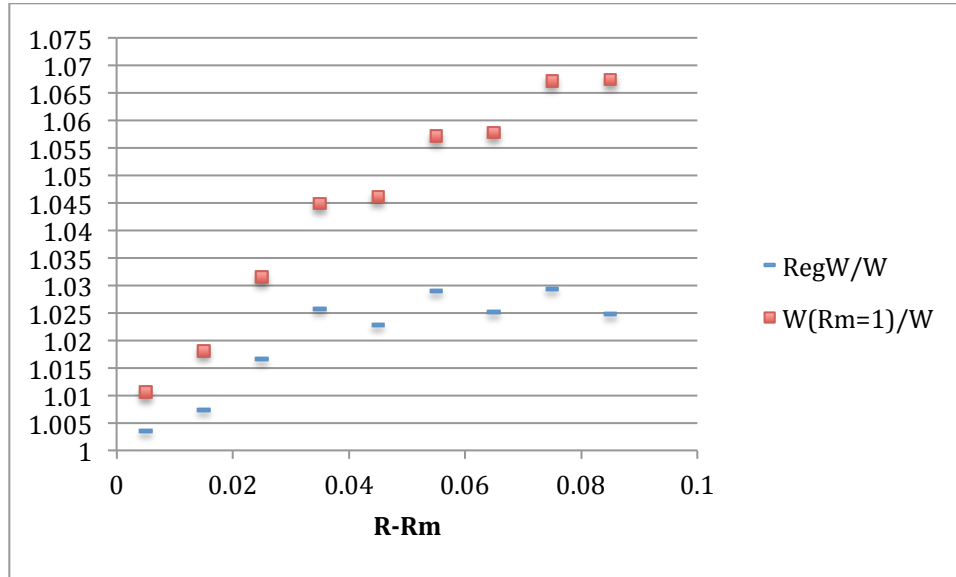
E. $\alpha = 0.001, \tau = 0, \psi = 0.015$ F. $\alpha = 0.0005, \tau = 0, \psi = 0.015$



Г. $\alpha = 0.001, \tau = 0.2, \psi = 0.015$



Н. $\alpha = 0.0005, \tau = 0.2, \psi = 0.015$



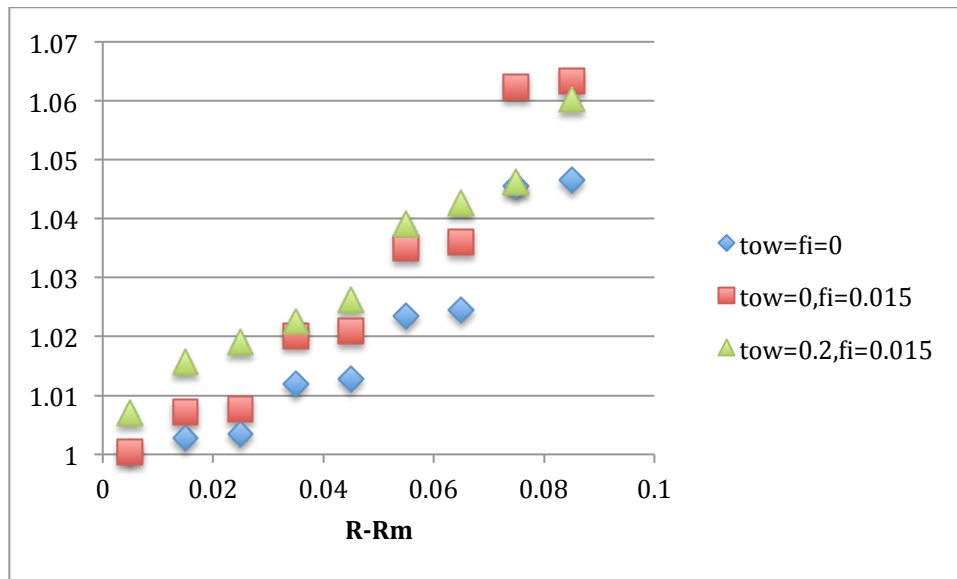
1. $\alpha = 0.0002, \tau = 0.2, \psi = 0.015$

Figure 10: The ratio of welfare under regulations to welfare with no regulation and the welfare cost of inflation for various parameters holding $R=0.985$ constant. The nominal interest rate per period is on the horizontal axis.

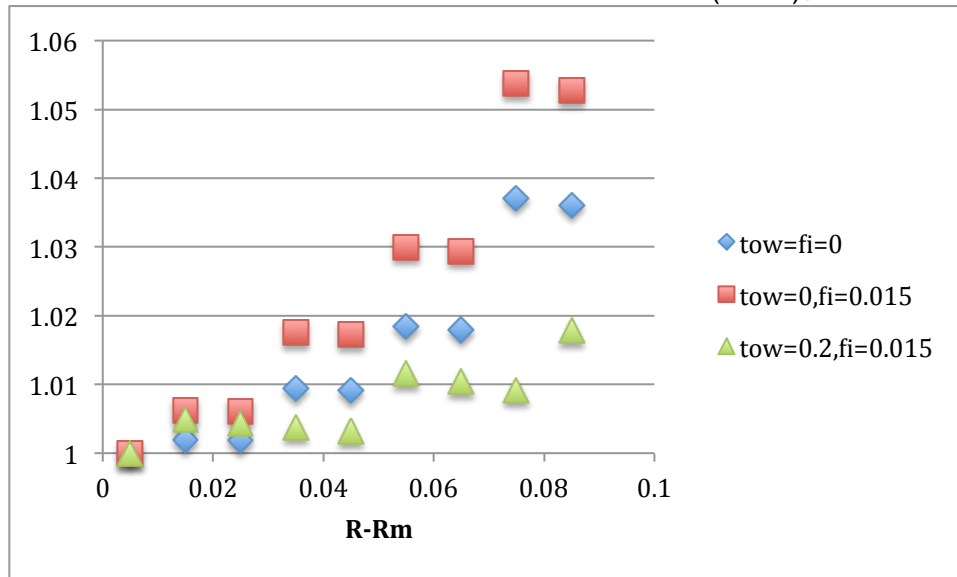
Figure 11A compares the welfare cost of inflation across different ψ and τ holding $\alpha = 0.001$ constant. The lowest welfare cost occurs when $\tau = \psi = 0$. Increasing ψ from zero to 0.015 increases the welfare cost. When the nominal interest is 1.5% per period the welfare cost increases from 0.28% to 0.71%. The welfare cost tends to increase when we add an income tax of 0.2. When the nominal interest is 1.5% it increases to 1.6%. In general, an increase in the income tax has two effects. It discourages the use of bonds because agents choose the asset on the basis of the after tax income. But since income tax increases the distortive effect of inflation on the labor supply of money users the effect is not straightforward. When the nominal interest is relatively low the distortive effect on money user dominates and adding an income tax increases the welfare cost. But when the nominal interest is high (7.5-8.5 percent in Figure 11A) the effect on the asset choice dominates.

Figure 11B studies the effect of ψ and τ on the benefits from regulations.

Increasing ψ increases the benefits. When the nominal interest is 1.5% increasing ψ from zero to 1.5% increases the benefits from 0.19% to 0.61%. But increasing τ reduces the benefits from 0.61% to 0.49%. This occurs for all interest rates because income tax discourages the use of bonds.



A. The welfare cost of inflation: $W(R_m=1)/W$



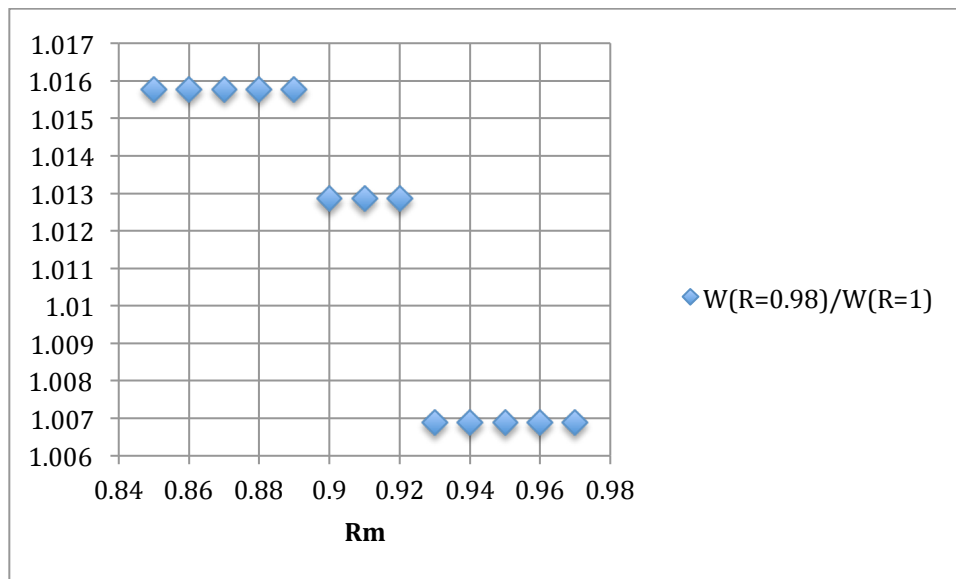
B. The gains from regulations: $RegW/W$

Figure 11: Comparing the results for different choices of τ and ψ while holding $\alpha=0.001$ constant

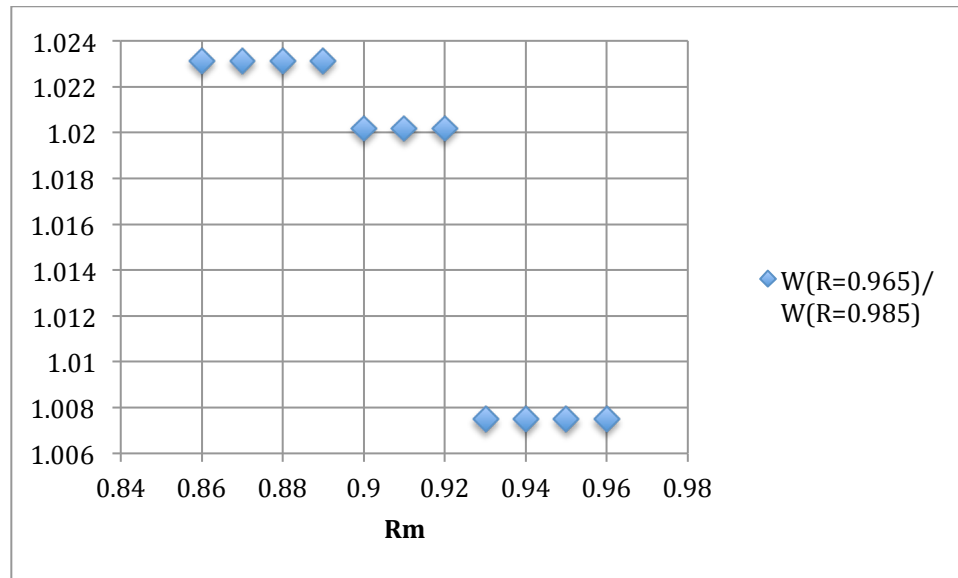
5.1 Repression

It is possible to discourage bond usage by lowering R . Will this improve matters?

Figure 12A assumes that there is no proportional cost and no income tax (as in Figure 10A). The Figure plots the ratio of welfare when $R=0.98$ to welfare when $R=1$. Figure 12B plots the ratio of welfare when $R=0.965$ to welfare when $R=0.985$ allowing for proportional cost of $\psi = 0.015$. The ratios are above 1 because low R discourages bond usage. This is different from Roubini and Sala-i-Martin (1995) who argue that financial repression may be optimal when seigniorage revenues are required.



A. Average ratios of welfare when $R=0.98$ to welfare when $R=1$, assuming $\alpha=0.001$ and no proportional cost.



B. Average ratios of welfare when $R=0.965$ to welfare when $R=0.985$, assuming $\alpha=0.001$ and a proportional cost of 1.5%

Figure 12: Average ratios of welfare under repression to welfare with no repression.

5.2 Changing the distribution of potential income

In the above calculations we used the income distribution in the US at 2011. But the income distribution changes over time and across countries. How will changes in the income distribution affect our measures of the welfare cost and the benefits from regulations? In an attempt to get a partial answer to this question we try two additional distributions. The first, denoted by *teta1*, is obtained from the income distribution in Table 2 (*teta*) in the following way. We assume that the potential income (θ^2) of the most productive agent (the agent indexed 1) is 0.046 instead of 0.146. The remaining 10% of aggregate potential output are distributed evenly over the rest of the population. In Figure 13, *teta1* is in red and is very close to *teta* (in blue) except that the agent index 1 has now $\theta=0.21$ rather than $\theta=0.38$ that he had before the change. The second income distribution, denoted by *teta2*, is constructed as follows. We take the index of the agent i and compute $\theta_i = i/5.77$. This is similar to Figure 6 that uses $\theta_i = i$. Here we

divide by 5.77 to insure that total potential output is 1.⁶ Average potential output (the mean of θ^2) is 0.01 for all the three distributions but the standard deviations are different. The standard deviation of θ^2 is 0.0152 for *teta*, 0.0075 for *teta1* and 0.0091 for *teta2*.

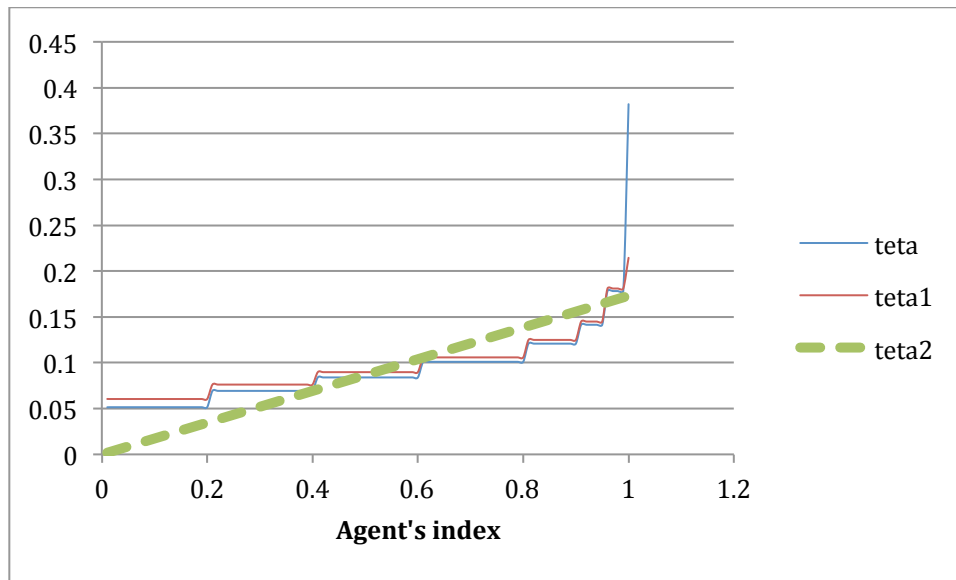
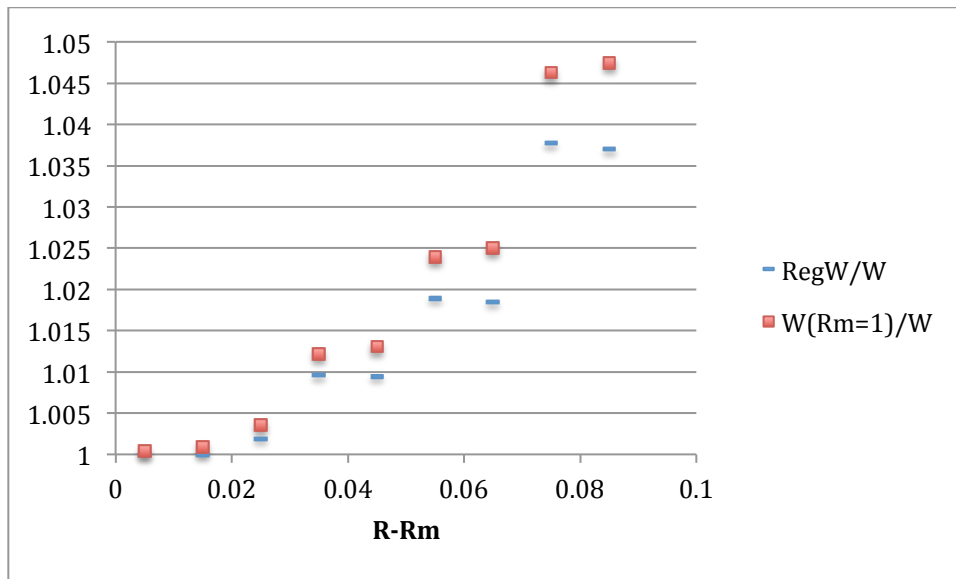


Figure 13: Alternative productivity distributions

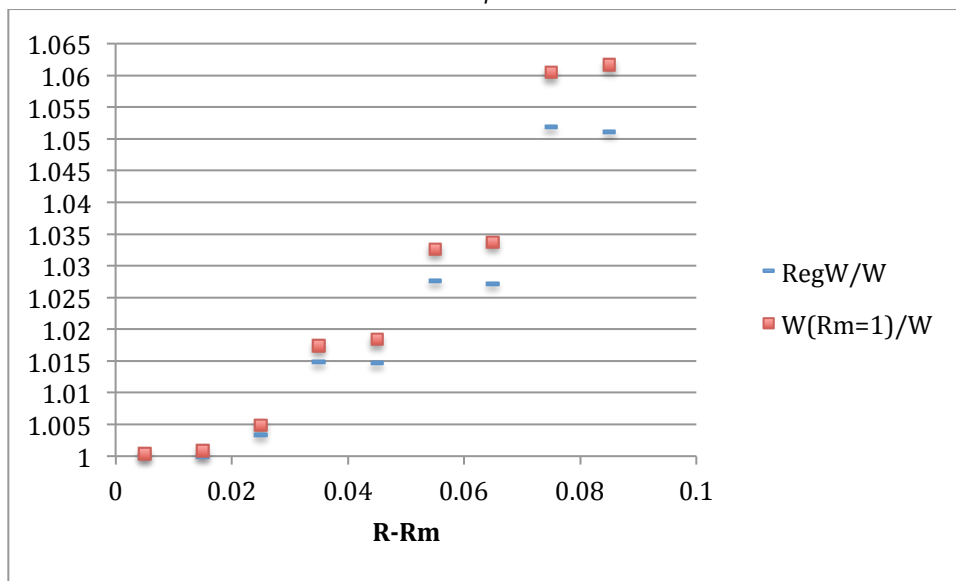
Figure 14 replicates some of the graphs in Figure 10. Figure 14A uses the distribution *teta1*. It assumes no proportional cost and no income tax and is thus comparable to Figure 10B. The main difference between the figures occurs when the nominal interest rate is low. In Figure 10B the top 1% (the agent with the highest potential income) uses bonds when the interest rate is greater than 1.5%. In Figure 14A the top 1% uses bonds when the interest is greater than 2.5%. But for interest rates that are greater than 2.5% the results are similar. Figure 14B assumes proportional cost of intermediation and is comparable to Figure 10E. Also here the main difference is in relatively low nominal interest rates. Figure 14C and 14D use the distribution *teta2*. Under *teta2* the potential income of the top 1% is lower than under *teta* and *teta1* and as a

⁶ As a result the fixed cost here is 10% of average potential output while in Figure 6 it is only 3% of average potential output.

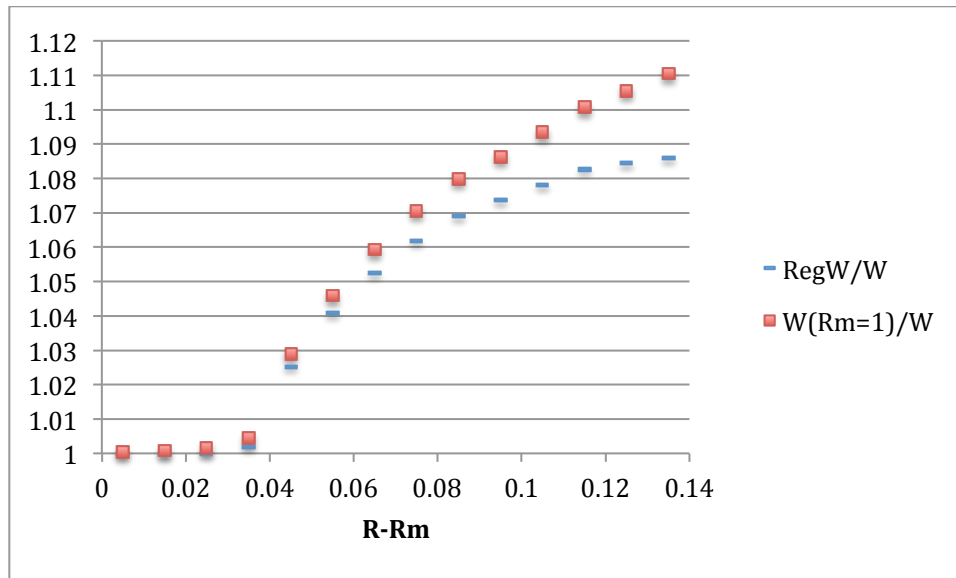
result he uses bonds only when the nominal interest rate is relatively high (greater than 3.5%). The welfare cost of inflation is lower than the welfare cost under *teta* (Figure 10B and 10E) when the interest is low but is higher when the interest rate is high. Note that for low interest rates most of the welfare cost can be eliminated by regulations.



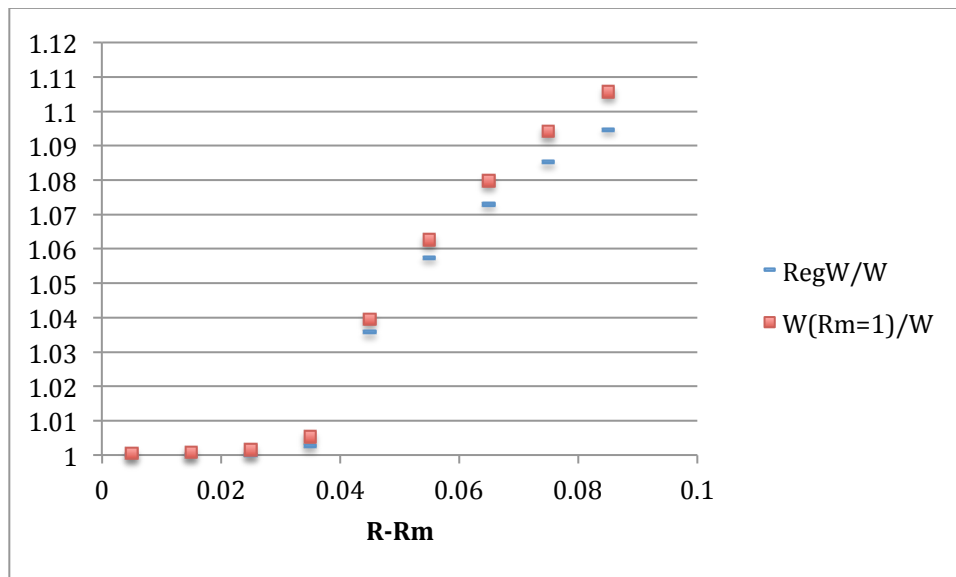
A. Potential income distribution is *teta* and $\tau = \psi = 0$



B. Potential income distribution is *teta* and $\tau = 0, \psi = 0.015$



C. Potential income distribution is $teta2$ and $\tau = \psi = 0$



D. Potential income distribution is $teta2$ and $\tau = 0, \psi = 0.015$

Figure 14: The ratio of welfare under regulations to welfare with no regulations and the welfare cost of inflation for different potential income distributions, holding $R=0.985$ and $\alpha=0.001$ constant.

6. THE AREA UNDER THE DEMAND FOR MONEY CURVE

In the standard "money in the utility function" approach the representative agent holds both bonds and money and we may consider the following small deviation from the optimal portfolio choice: Take a unit from the saving account (bonds) and put it in the checking account (money). The cost of doing that is the nominal interest rate. The benefit is in the "liquidity services" provided by money. Since a small change in the optimal portfolio choice does not change utility, the nominal interest rate is equal to the "liquidity services" provided by the additional unit of real balances. This leads to measuring the "liquidity services" by the area under the demand for money curve. This line of reasoning does not hold in our model because agents are risk neutral and adopt a corner solution: They use either money or bonds.

We start from money-users. We consider the following small deviation from the optimal program: The money-user increases his labor supply by a unit and deposits the additional revenue in his checking account. The social revenue from a unit increase in labor is θ . The social cost is: $v'(L) = (1 - \tau)\theta R_m$. The social net benefit is therefore: $\Delta W = \theta - (1 - \tau)\theta R_m = \theta(1 - (1 - \tau)R_m)$. The change in money holding that result from the unit increase in labor is: $\Delta m = (1 - \tau)\theta$. The nominal interest rate is: $i = R - R_m$. It follows that as a result of the unit increase in labor the area under the demand curve grew by: $i(\Delta m) = (R - R_m)(1 - \tau)\theta$.

Claim 8: When $R \leq 1$, $\Delta W \geq i(\Delta m)$ with strict inequality when $R(1 - \tau) < 1$.

Note that when $R = 1$ and $\tau = 0$, $\Delta W = i(\Delta m)$ and the area under the demand for money curve is equal to the welfare gain. Otherwise, if $\tau > 0$ or $R < 1$, the area under the demand for money curve understate the welfare gain.

The ratio of the change in welfare to the change in the area under the demand for money curve is:

$$(33) \quad \frac{\Delta W}{i(\Delta m)} = \frac{1 - (1 - \tau)R_m}{(R - R_m)(1 - \tau)}$$

Figure 15 computes (33) for different levels of τ and $i = R - R_m$ assuming $R = 0.985$. The ratio of the change in welfare to the change in the area under the demand for money curve can be large. When $\tau = 0$ this ratio is: 4 when $i = 0.5\%$, 2 when $i = 1.5\%$ and 1.6 when $i = 2.5\%$. These numbers are large. When $i = 1.5\%$ and there are no direct taxes, the change in the area under the demand for money curve is equal only to half of the change in welfare. The ratio is much larger when $\tau = 0.2$. In this case, the ratio is: 35 when $i = 0.5\%$, 12 when $i = 1.5\%$ and 7 when $i = 2.5\%$.

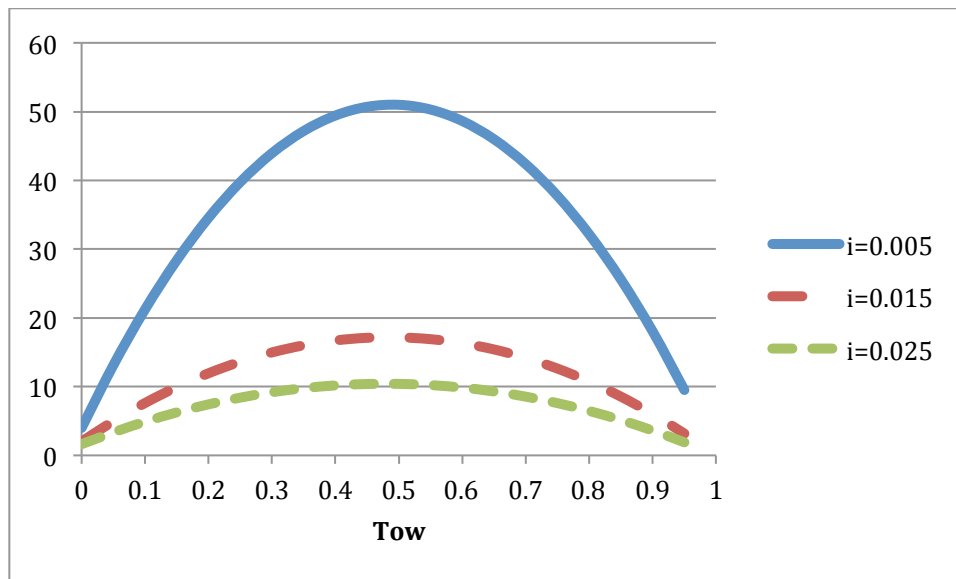


Figure 15: The ratio of the change in welfare to the change in the area under the demand for money curve (33) when $R = 0.985$

Claim 8 is about the measurement of the welfare gain from reducing the nominal interest by an infinitesimal amount. The welfare cost of inflation is measured by the welfare gain from reducing the nominal interest rate to zero. We now turn to perform this

computation. The labor supply of a buyer with productivity θ is: $L = (1 - \tau)\theta R_m$. His demand for money is: $m = \theta L = (1 - \tau)\theta^2 R_m$. We can therefore write:

$$R_m = \frac{m}{(1 - \tau)\theta^2} \text{ and } i = R - R_m = R - \frac{m}{(1 - \tau)\theta^2}. \text{ Figure 16A illustrates. Note that the}$$

demand for money (not the inverse) is $m = (R - i)(1 - \tau)\theta^2$ and the intersection point on the horizontal axis is therefore: $m = R(1 - \tau)\theta^2$. The area under the demand for money curve A is:

$$(34) \quad A = 0.5i^2(1 - \tau)\theta^2$$

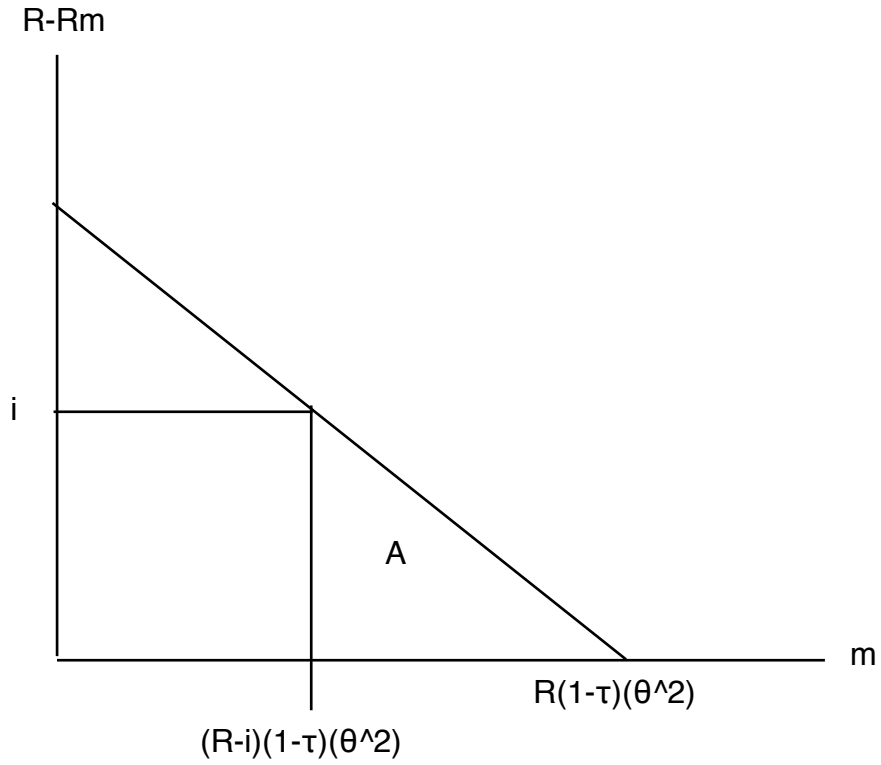
In Figure 16B the welfare cost of inflation is measured by the area above the supply curve: $a + b$. Since $a = 0.5(1 - \tau)^2\theta^2(R - R_m)^2 = 0.5i^2(1 - \tau)^2\theta^2$ and

$b = \theta^2(1 - R + \tau R)(1 - \tau)i$ we have:

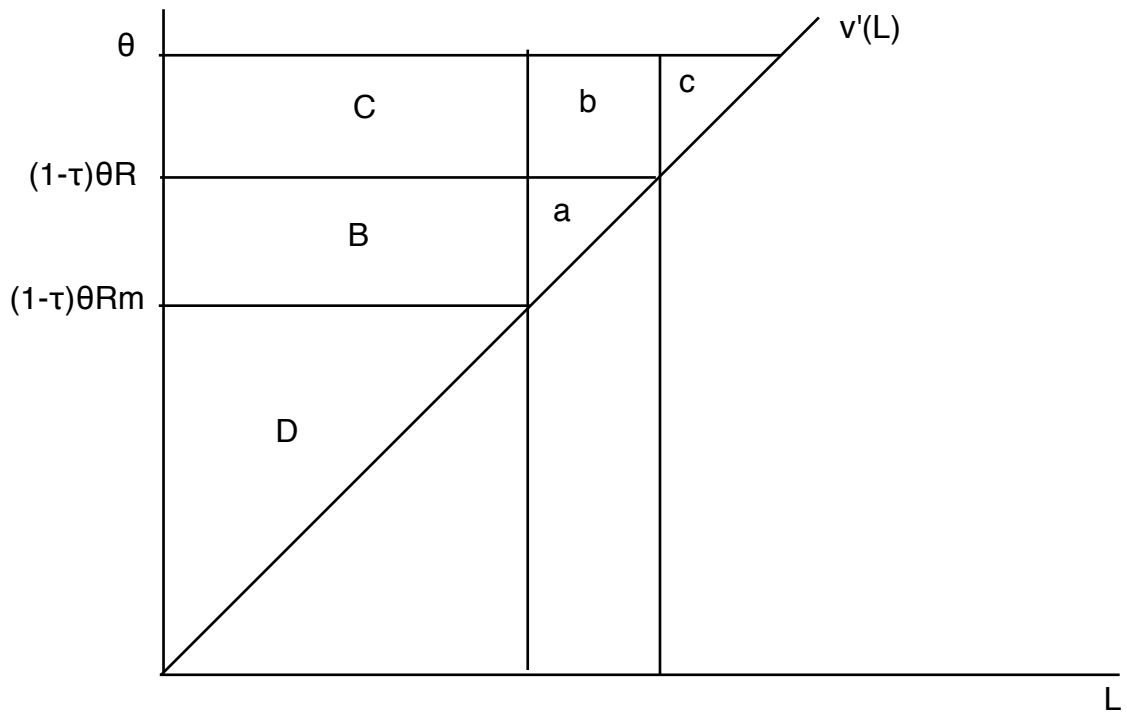
$$(35) \quad \begin{aligned} a + b &= (1 - \tau)\left(0.5i^2(1 - \tau)\theta^2 + \theta^2(1 - R + \tau R)i\right) \\ &= A + \theta^2(1 - \tau)(1 - R + \tau R)i - 0.5i^2(1 - \tau)\tau\theta^2 \end{aligned}$$

We can now show the following Claim:

Claim 8': $a + b \geq A$ with strict inequality when $\tau > 0$.



A, The area under the demand for money curve



B. The area above the supply curve

Figure 16: The welfare cost of inflation

The difference between the area under the demand curve and the area above the supply curve may be large. Figure 17 illustrates. When reducing the nominal interest rate from 2% to zero, there is almost no benefit if we look at the area under the demand curve but there is a large benefit of 4% of the additional output when we look at the correct measure of the area above the supply curve.

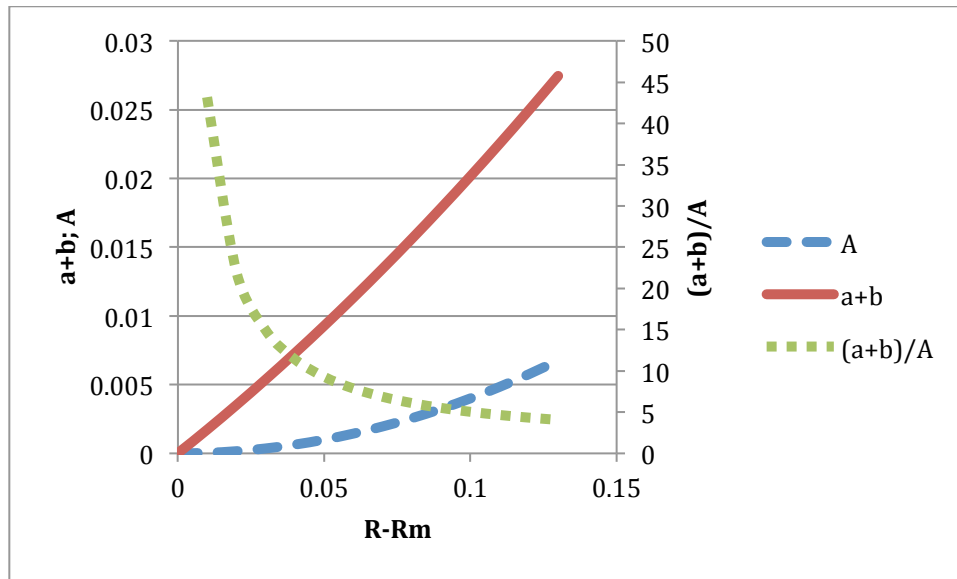


Figure 17: The area under the demand curve (A - left axis), the area above the supply curve ($a+b$ - left axis), and the ratio $(a+b)/A$ (right axis) when $R=0.985$ $\tau=0.2$ and $\theta=1$

The extensive margin.

We now turn to a bond user who is just indifferent between using money and bonds and as a result of a small change in the nominal interest rate switches from bonds to money. The labor supply of a bond user is: $L = (1 - \tau)\theta R$. His contribution to welfare in terms of Figure 16B, is the area above the supply curve minus the transaction cost:

$W^b = C + B + D + a + b - \alpha$. The contribution of a money user to welfare is:

$W^m = C + B + D$. Using the same notation to denote different things, the change in welfare that occurs when a bond user switches to money is:

$$(36) \quad \Delta W = W^m - W^b = \alpha - (a + b).$$

From the private point of view using bonds yield $U^b = D + B + a - \alpha$ utils while using money yields $U^m = D$ utils. For "switchers" who are indifferent between using bonds to using money: $U^m = U^b$ and $\alpha = a + B$. Substituting in (36) leads to: $\Delta W = B - b$.

We use $\psi = 1 - R$ and compute:

$$B = \theta^2(1 - \tau)^2(R - R_m)R_m = \theta^2(1 - \tau)^2iR_m$$

$$b = \theta(1 - R + \tau R)\theta((1 - \tau)R - (1 - \tau)R_m) = \theta^2(1 - \tau)(\psi + \tau - \psi\tau)i$$

This leads to:

$$(37) \quad \Delta W = B - b = \theta^2(1 - \tau)i((1 - \tau)R_m - \psi - \tau + \psi\tau)$$

A switcher also changes the area under the demand for money curve. After switching to money his labor supply is: $L = (1 - \tau)\theta R_m$. The change in his money holdings is therefore:

$\Delta m = \theta L = \theta^2(1 - \tau)R_m$. The change in the area under the demand curve is:

$$(38) \quad i(\Delta m) = \theta^2(1 - \tau)iR_m.$$

Substituting (38) in (37) leads to:

$$(39) \quad \Delta W = i(\Delta m) - \tau\theta^2(1 - \tau)iR_m - \theta^2(1 - \tau)i(\psi + (1 - \psi)\tau)$$

We can now show the following Claim.

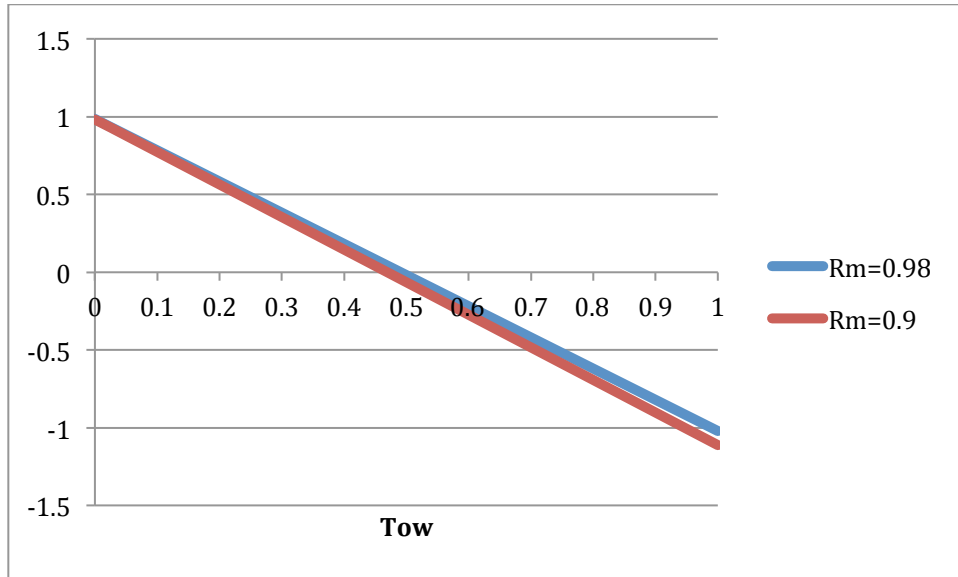
Claim 9: $\Delta W \leq i(\Delta m)$ with strict inequality when either $\tau > 0$ or $\psi > 0$.

Claim 9 says that when $\tau > 0$ or $\psi > 0$, the change in the area under the demand for money curve that results from a "switch" overstates the change in welfare. The ratio of welfare to the area under the demand for money curve is:

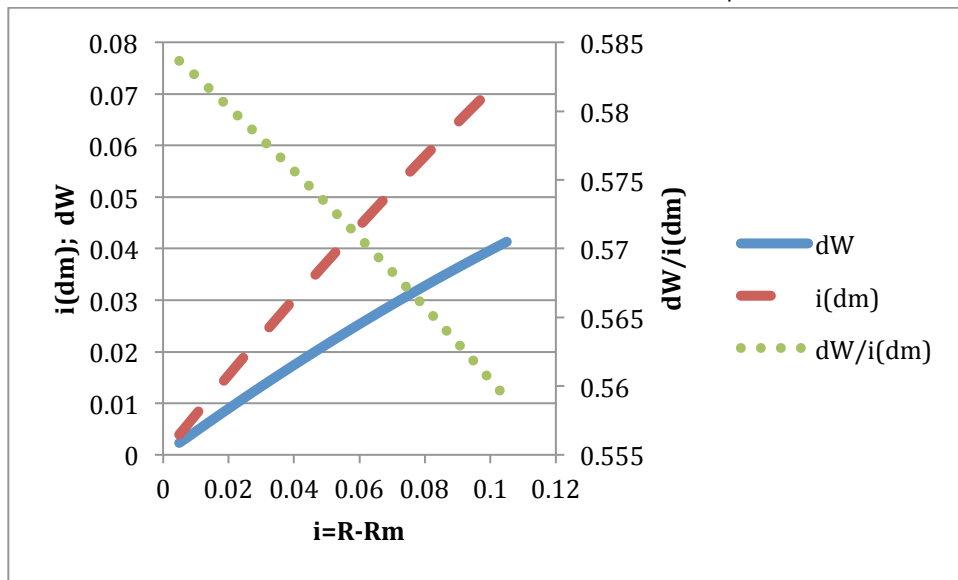
$$(40) \quad \frac{\Delta W}{i(\Delta m)} = \frac{(1 - \tau)(R_m - \psi) - \tau}{R_m} \leq 1$$

This ratio can be negative because $\Delta W < 0$ when τ and ψ are large. Figure 18a illustrates. The ratio (40) is not sensitive to R_m and reaches zero when τ is close to 0.5.

Figure 18B describes the effect of a "switch" on welfare and the area under the demand curve when $\tau = 0.2$ and $\psi = 0.015$. The effect is larger for a larger interest rate but the ratio (40) is not very sensitive to changes in the interest rate. It ranges from 0.55 to 0.58.



A. The ratio of the change in welfare to the change in the area under the demand for money curve (40) when $\psi = 0.015$



B. The area under the demand curve ($i(dm)$ - left axis), the welfare gain (dW -left axis) and the ratio $dW/i(dm)$ when $\psi = 0.015$, $\tau = 0.2$ and $\theta = 1$

Figure 18: The effect of a "switch" on welfare and the area under the demand for money curve.

Since in general there is action both on the extensive and the intensive margins, we cannot establish the direction of the bias. Claim 8 suggests that the area under the demand for money curve understates the welfare cost when the intensive margin dominates while Claim 9 suggests that the opposite occurs when the extensive margin dominates.

The direction of the bias is unambiguous only when bonds are not used. This may occur when the nominal interest rate is low or when there are regulation that prohibits the use of bonds.⁷

7. CONCLUDING REMARKS

Inflation tax can be evaded by using bonds and is similar to a consumption tax that is not perfectly enforced. It is therefore a "bad" tax for countries that can enforce a consumption tax.⁸ But inflation may arise for reasons that are not related to seigniorage revenues, like time inconsistency problems. In most of the paper we assume that inflation is exogenously given and that lump sum taxes are possible. We ask whether the government should encourage the avoidance of the "bad" tax by using bonds or whether it should attempt to increase enforcement by regulations that are aimed at eliminating money substitutes.

Since inflation is a "bad" tax, the case for tax evasion by the use of bonds, can be made when the cost of evasion is relatively small. But in practice the cost of tax evasion is not too small because money is used and some people pay the tax. We argue that for

⁷ According to Deaton (1991), 20% of total consumption is accounted for by households who do not have checking or saving accounts. These relatively poor households will benefit from reducing the inflation rate by more than the area under their demand for money curve. The benefit for this group should be estimated directly as the benefit from reducing the implied consumption tax.

⁸ It is a "bad" tax because it encourages agents to use resources to legally evade the tax and because it is more regressive than a consumption tax: In our model only relatively productive agents use bonds.

realistic cost parameters the increase in enforcement by imposing restrictions on the use of bonds actually improves matters.

The benefits from regulations may be substantial even when inflation tax is relatively unimportant in terms of revenues. For example, when the inflation tax is equivalent to a 4% consumption tax and most agents spend 2% of their income on tax evasion, the benefit from regulation is close to 2% of GDP but the revenues from the inflation tax are small. This type of example can also explain why a steady state inflation of say 12% is considered a disaster while a consumption tax of 4% (that is equivalent to an inflation of 12% when the length of the period is a quarter) seems acceptable. To understand the difference in the attitude towards the two seemingly similar taxes, we may want to think of the case in which labor supply is inelastic. Under the consumption tax regime agents do not spend resources on tax evasion and get services or lump sum payments from the government in an amount that is equal to the tax they pay. The inflation tax regime is akin to a situation in which the consumption tax is not perfectly enforced and as a result agents spend resources on tax evasion and get a relatively small transfer payment or services.

In our model the welfare cost of inflation is equal to the area under the demand for money curve only in the special case in which there are no direct taxes and no proportional intermediation costs. To estimate the welfare cost and the gains from regulation, we use the 2011 income distribution in the US. We focus on the range of the fixed cost α that leads to the use of both assets (bonds and money) when the interest rate is between 1.5% and 8.5%. The choice $\alpha = 0.001$ is in the middle of this range.

When $\alpha = 0.001$ and the nominal interest is 1.5% per period (or 4.5% annual rate if the length of the period is 3 months), the welfare cost of inflation is 0.28% if the fixed cost is the only cost of using bonds, it is 0.71% if we add a proportional cost of 1.5% of the assets intermediated and it is 1.57% if we add income tax at the flat rate of 20%.

The gains from regulation are: 0.19% if the fixed cost is the only cost and 0.62% if we add the proportional cost. The benefits from regulation go down to 0.49% if income tax is added. This is because income tax reduces the incentive to use bonds which depends on the after tax income.

There are many unresolved issues. The length of the period is a major issue that is related to the relevant definition of money. It seems that assets will have different "liquidity" and the division between money and bonds is rather arbitrary. We have not found a solution to this problem and therefore presented various estimates.⁹

The argument for not using inflation to raise revenues is rather straightforward in a closed economy in which the government can reasonably enforce consumption tax. But the issue is more complicated in an open economy. The US for example raises substantial seigniorage revenues from the rest of the world. Should it give up these revenues?

⁹ Using the area under the demand form money approach and assuming a narrow definition of money, Lucas (2000) estimate that reducing the annual nominal interest rate from 14% to 3% will increase welfare by 0.8%. An annual rate of 14% is a quarterly rate of 3.3% and an annual rate of 3% is a quarterly rate of 0.75%. Reducing the nominal interest rate from 3.3% to 0.75% in our model with $\alpha = 0.001$ will increase welfare by 0.92% if $\psi = 0$ and by 1.31% if $\psi = 0.015$. If we add income tax of 20% the welfare cost increases to 1.38%. Thus, if the length of the period is one quarter, our estimates of the welfare cost are higher than Lucas' estimates.

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APPENDIX: A MONETARY VERSION WITH PRIVATE LENDING AND
BORROWING

In the paper the alternative to money is government bond. Here the alternative is private bond and there is no explicit interest on money. Instead the price level varies over time. We assume here that agents live forever and adopt the two sub-periods structure in Lagos and Wright (2005) and some elements from Williamson (2012).

Each period is divided into two sub-periods: In the first there is a centralized Walrasian market (CM) and in the second there is a decentralized market (DM) in which bilateral meetings take place. Buyers produce Y in the first sub-period (when the CM is active) and sellers produce X in the second sub-period (when the DM is active). Buyers derive utility from X while sellers derive utility from Y .

Both buyers and sellers maximize expected utility. The utility function of the typical buyer is:

$$(A1) \quad \sum_{t=1}^{\infty} \beta^t (x_t^b - v(L_t^b))$$

where x_t^b is the quantity of X consumed by the buyer, L_t^b is the amount of labor supplied, $0 < \beta < 1$ is a discount factor and $v(L) = (\frac{1}{2})L^2$ is the utility cost of supplying labor. Using the superscript s for seller, the utility function of the seller is:

$$(A2) \quad \sum_{t=1}^{\infty} \beta^t (y_t^s - L_t^s)$$

In the CM the buyer produces θ units of Y per unit of labor where the productivity parameter θ is an *iid* random variable with a continuous and differentiable distribution function $F(\theta)$ defined for $0 < \theta^{\min} \leq \theta \leq \theta^{\max}$. At the DM the seller produces 1 unit of X per unit of labor.

At the CM all agents are in the same geographical location and buyers can sell Y for money or for a contract (IOU). At the DM agents are distributed over many locations in a random manner. Money is generally accepted and buyers can always find a seller in

their DM location that will sell them X for money. But contracts are between a specific buyer and a specific seller. During the DM, the buyer who holds a contract on the output of a specific seller is likely to be in a different location than the specific seller and he must travel to the location of the specific seller to execute the contract. Alternatively, we may assume that the seller deliver the good to the location of the buyer and charge a delivery cost.

We assume that the typical contract offers to deliver $y - \alpha$ units of X to the buyer's location at the DM for y units of Y received at the CM. Thus the seller borrows from the buyer at the gross real interest $R = 1$ and the buyer pays the delivery fee that covers on average the cost of transportation.

At the beginning of the period t CM the seller holds M_t dollars. The seller gets a lump sum transfer of μM_t at the end of the CM and an additional transfer payment of $\mu(1 + \mu)M_t$ at the end of the DM sub-period. The money supply at the beginning of period $t + 1$ is: $M_{t+1} = M_t(1 + \mu)^2$.

The government pays interest on money at the beginning of the CM. The nominal interest is i and the interest is financed out of lump sum taxes imposed on the sellers.

The dollar price of the two goods at time t is proportional to the beginning of sub-period (pre-transfer) money supply and is given by:

$$(A3) \quad P_{xt} = p_x(1 + \mu)M_t \text{ and } P_{yt} = p_y M_t$$

Unlike the dollar prices, the normalized prices (p_y, p_x) do not change over time.

The problem of a buyer with productivity θ who use money is:

$$(A4) \quad \max_{x,L} x - v(L) \text{ s.t. } P_{xt}x = P_{yt}\theta L.$$

Using (A3) we can write the constraint in (A4) as:

$$(A5) \quad x = R_m \theta L$$

where $R_m = \frac{p_y}{p_x(1 + \mu)}$. We can write the problem (A4) as:

$$(A6) \quad V_m(\theta, R_m) = \max_L R_m \theta L - v(L)$$

The real wage for a money user is thus θR_m and his labor supply is: $L(\theta R_m)$.

The problem of a buyer who sells for a contract is:

$$(A7) \quad V_b(\theta, R) = \max_L R\theta L - v(L) - \alpha$$

where $R=1$. As before, we define the cutoff point by the solution $\theta^*(R_m)$ to:

$$V_b(\theta, 1) = V_m(\theta, R_m). \text{ The buyer will use bonds if } \theta \geq \theta^*(R_m).$$

The money market clearing condition at the CM is:

$$(A8) \quad 1 + \mu = p_y \int_{\theta_{\min}}^{\theta^*(R_m)} \theta L(\theta R_m) dF(\theta)$$

On the left hand side of (A8) is the amount of money held by the sellers (in terms of normalized dollars) at the end of the CM. On the right hand side of (A8) is the nominal supply of money users (again in normalized dollars).

A seller that sells a unit of X for money in the DM will get:

$$(A9) \quad \frac{(1+i)P_{xt}}{P_{yt+1}} = \frac{(1+i)p_x(1+\mu)}{p_y} = \frac{1+i}{R_m}$$

units of Y in the next period. Since the seller must be indifferent between producing and not producing the nominal interest rate must satisfy:

$$(A10) \quad \beta \frac{1+i}{R_m} = 1 \text{ or } 1+i = \frac{R_m}{\beta}$$

A steady state equilibrium is a vector (μ, R_m, i, p_y, p_x) such that

$$(A11) \quad R_m = \frac{p_y}{p_x(1+\mu)}$$

$$(A12) \quad 1+i = \frac{R_m}{\beta}$$

$$(A13) \quad 1+\mu = p_y \int_{\theta_{\min}}^{\theta^*(R_m)} \theta L(\theta R_m) dF(\theta)$$

Solving for the steady state.

Choose $R_m \leq 1$ arbitrarily and choose i that satisfies (A12). Substituting (A13) in (A11)

leads to: $p_x = \frac{1}{R_m \int_{\theta_{\min}}^{\theta^*(R_m)} \theta L(\theta R_m) dF(\theta)}$. There are many pairs (p_y, μ) that satisfy (A13).

We can set $p_y = p_x$ and let μ be determined by (A13). This leads to $\frac{1}{1+\mu} = R_m$ which is

the standard expression for the real rate of return on money. Thus, as in standard models, an increase in μ leads to a lower R_m .