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Quantity versus price competitions in a vertical relationship with separate downstream markets

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Abstract

Contrary to conventional wisdom, our analysis of vertical relationships involving two independent downstream markets challenges the notion that Bertrand competition yields lower profits than Cournot competition. We show that if one downstream market in which two firms compete on either quantity or price is smaller than the other downstream market, then the input price under Bertrand competition is lower than under Cournot competition. This can lead to higher profits for downstream firms under Bertrand competition.

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1 Introduction

In comparing equilibrium outcomes under Cournot and Bertrand competitions, it is known that Bertrand competition is more competitive (Singh and Vives, 1984). Thus, under Bertrand competition, lower prices are realized, consumer surplus is higher, and profits are smaller. Industrial organization researchers have long debated this relationship. This study provides a new perspective on the Cournot-Bertrand profit difference. We propose a role for an independent downstream market, separate from the market in which the Cournot-Bertrand profit difference is considered.

Considering an independent downstream market is natural from a practical point of view. For example, small ball bearings are an important input for many products (e.g., hard disk drives in personal computers and automobile power steering systems). If we focus on the hard disk drive industry, it is natural to consider the power steering system industry as a separate market, and the downstream firms purchase small ball bearings from an upstream firm (e.g., MinebeaMitsumi, Inc.) with a dominant market position. An example of such a market can be observed when an upstream firm supplies inputs (e.g., sensors and motors) commonly used in many products.

Specifically, we consider a model in which an upstream firm supplies inputs to two downstream markets. We assume that the inverse demand functions in each downstream market are linear, but that the market sizes, as measured by the intercept values, are different. Two downstream firms compete in one downstream market, while a monopolistic downstream firm supplies the other downstream market. We compare profits under Bertrand competition and those under Cournot competition in the market with the two competing downstream firms.

We show that when the monopolistic downstream market is large, Bertrand competition is more profitable for all firms than Cournot competition. The main explanation for this result is the difference in market size in the downstream markets, which leads to a difference in price elasticity. Bertrand competition increases the quantity of inputs used in the downstream market. If the price elasticity in the competitive downstream market is large, the upstream firm has an incentive to lower the input price. Thus, although Bertrand competition lowers the price of the final good, it leads to the possibility of higher profits for downstream firms.

Our study is related to previous research on endogenous downstream marginal costs. There are two approaches to the situation in which input prices differ between Cournot and Bertrand competitions: (i) to consider nonlinear factors in the payoffs and (ii) to consider Nash bargaining with respect to input prices.[1] For (i), the following studies have been conducted. Fanti and Meccheri (2011) considered a market with two unions and two firms where the production technology of the firms exhibits decreasing returns, showing that the input price is lower under Bertrand competition and the profit under Bertrand competition may be larger. Fanti and Meccheri (2012) considered a case with one union and two firms, showing that if the union places more importance on wages, profits under Bertrand competition may be larger. Fanti and Meccheri (2015) considered the cases of firm-specific unions or an industry-wide union for two firms and showed that profit under Bertrand competition is larger when managerial delegation is introduced.

The subsequent studies have been conducted on (ii). Correa-López and Naylor (2004) considered a case where downstream firms engage in Nash bargaining with their unions over wages. The subsequent studies extend their model (Alipranti et al., 2014; Basak and Mukherjee, 2017). Basak (2017) considered a market in which a monopolistic upstream firm and two downstream firms negotiate on input prices through Nash bargaining, showing that the same input prices are realized in the downstream market under Cournot and Bertrand competitions. Thus, the results of Singh and Vives (1984) hold. Basak and Wang (2016) considered a situation where a monopolistic upstream firm and two downstream firms trade under a two-part tariff, which is determined by centralized Nash

[1] Some studies show that the profit under price competition is higher than under quantity competition by considering asymmetries in the downstream market (Fanti and Scrimitore, 2019; Matsuoka, 2023; Mukherjee et al., 2012).

bargaining. They show that the profit margins are larger under Bertrand competition because of the higher fixed fee obtained. Manasakis and Vlassis (2014) focused on a market where two upstream and two downstream firms determine input prices through decentralized Nash bargaining, subject to a renegotiation-proof contract. They demonstrate that the results of Singh and Vives (1984) hold.

The previous studies in (i) and (ii) do not consider a market other than the downstream market in which competition occurs. We include the additional independent downstream market to obtain the input price changes. Therefore, our study complements the previous studies by adding a new factor.

2 Model

We consider a vertical market with one upstream firm (U) and three downstream firms (Di, $i \in \{1, 2, 3\}$). In the downstream sector, two markets exist: market X and market Y. Upstream firm U produces input and sells it to the downstream firms at input price w. To produce one unit of the final product, each downstream firm uses one unit of input. Downstream firms $D1$ and $D2$ supply their products to market X; downstream firm $D3$ supplies its product to market Y. We assume that the production costs of all firms are zero.

We assume that markets X and Y are independent. We denote the output and price of Di by q_i and p_i, respectively. In market X, products produced by $D1$ and $D2$ are differentiated. We assume that consumer surpluses in markets X and market Y are

$$\begin{aligned} CS_X &= a_X(q_1 + q_2) - \frac{1}{2}(q_1^2 + q_2^2) - \gamma q_1 q_2 - p_1 q_1 - p_2 q_2, \\ CS_Y &= a_Y q_3 - \frac{1}{2} q_3^2 - p_3 q_3, \end{aligned}$$

where $a_X, a_Y > 0$ and $\gamma \in (0, 1)$ is the degree of product substitutability. From the consumer surpluses, demand functions are $q_1 = [a_X(1-\gamma) - p_1 + \gamma p_2]/(1-\gamma^2)$, $q_2 = [a_X(1-\gamma) - p_2 + \gamma p_1]/(1-\gamma^2)$, and $q_3 = a_Y - p_3$; inverse demand functions are $p_1 =$

$a_X - q_1 - \gamma q_2$, $p_2 = a_X - q_2 - \gamma q_1$, and $p_3 = a_Y - q_3$. To guarantee positive outputs in equilibrium, we assume that $r_{min} < r < r_{max}$, where $r = a_X/a_Y$, $r_{min} = (2+\gamma)/(8+2\gamma)$, and $r_{max} = (10+\gamma-\gamma^2)/4$.

From the above setting, the profits of upstream and downstream firm Di are as follows.

$$\pi_U = w(q_1 + q_2 + q_3), \quad \pi_{Di} = (p_i - w)q_i.$$

Consumer, producer, and total surpluses are $CS = CS_X + CS_Y$, $PS = \pi_U + \pi_{D1} + \pi_{D2} + \pi_{D3}$, and $TS = CS + PS$, respectively.

The timing of the game is as follows: In the first stage, the upstream firm U sets the input price. In the second stage, each downstream firm chooses its output or price. Using backward induction, we solve this game.

3 Analysis

Quantity competition First, we consider the case of quantity competition in market X. Using the first-order conditions in the first and second stages, respectively, we obtain the equilibrium outcomes as follows.

$$w^C = \frac{4a_X + a_Y(2+\gamma)}{2(6+\gamma)}, \quad \pi_{D1}^C = \pi_{D2}^C = \frac{[2a_X(4+\gamma) - a_Y(2+\gamma)]^2}{4(2+\gamma)^2(6+\gamma)^2},$$
$$\pi_{D3}^C = \frac{[4a_X - a_Y(10+\gamma)]^2}{16(6+\gamma)^2}, \quad \pi_U^C = \frac{[4a_X + a_Y(2+\gamma)]^2}{8(2+\gamma)(6+\gamma)},$$

where the superscript 'C' denotes the case under quantity (Cournot) competition.

Price competition Next, we consider the case of price competition. Using the first-order conditions in the first and second stages, respectively, we obtain the equilibrium outcomes as follows.

$$w^B = \frac{4a_X + a_Y(2+\gamma-\gamma^2)}{2(6+\gamma-\gamma^2)}, \quad \pi_{D1}^B = \pi_{D2}^B = \frac{(1-\gamma)[2a_X(4+\gamma-\gamma^2) - a_Y(2+\gamma-\gamma^2)]^2}{4(2-\gamma)^2(1+\gamma)(6+\gamma-\gamma^2)^2},$$
$$\pi_{D3}^B = \frac{[4a_X - a_Y(10+\gamma-\gamma^2)]^2}{16(6+\gamma-\gamma^2)^2}, \quad \pi_U^B = \frac{[4a_X + a_Y(2+\gamma-\gamma^2)]^2}{8(2-\gamma)(1+\gamma)(6+\gamma-\gamma^2)},$$

where the superscript 'B' denotes the case under price (Bertrand) competition.

Comparison of input prices First, we analyze the difference in input prices between quantity and price competitions. By comparing w^B with w^C, we obtain the following.

$$w^B - w^C = \frac{2(a_X - a_Y)\gamma^2}{(6+\gamma)(6+\gamma-\gamma^2)}.$$

This result directly leads to Lemma 1.

Lemma 1 *Upstream firm U selects a lower input price under price competition than under quantity competition if the size of market X is smaller than that of market Y. Specifically, $w^B < w^C$ if $a_X < a_Y$.*

To provide insight into Lemma 1, we examine the price elasticity of input demand. From the outcomes in the second stage, the price elasticities of input demands in markets X and Y are expressed as $\varepsilon_X = -w/(a_X - w)$ and $\varepsilon_Y = -w/(a_Y - w)$, respectively. Notably, these values remain consistent across both quantity and price competitions. Hence, if the size of market X is smaller than that of market Y, $a_X < a_Y$, the input demand in market X is more elastic than that in market Y.

Next, we analyze the price elasticity of total input demand. Denoting the second-stage outputs of downstream firm Di in quantity and price competitions as $q_i^C(w)$ and $q_i^B(w)$ respectively, we present the price elasticity of total input demand for quantity competitions as follows.

$$\begin{aligned}
\varepsilon^C &= \frac{\partial \sum_{i=1}^3 q_i^C(w)}{\partial w} \cdot \frac{w}{\sum_{i=1}^3 q_i^C(w)} \\
&= \frac{q_1^C(w) + q_2^C(w)}{\sum_{i=1}^3 q_i^C(w)} \cdot \frac{\partial(q_1^C(w) + q_2^C(w))}{\partial w} \cdot \frac{w}{q_1^C(w) + q_2^C(w)} + \frac{q_3^C(w)}{\sum_{i=1}^3 q_i^C(w)} \cdot \frac{\partial q_3^C(w)}{\partial w} \cdot \frac{w}{q_3^C(w)} \\
&= \frac{q_1^C(w) + q_2^C(w)}{\sum_{i=1}^3 q_i^C(w)} \cdot \varepsilon_X + \frac{q_3^C(w)}{\sum_{i=1}^3 q_i^C(w)} \cdot \varepsilon_Y.
\end{aligned}$$

Similarly, that for price competition is

$$\varepsilon^B = \frac{q_1^B(w) + q_2^B(w)}{\sum_{i=1}^3 q_i^B(w)} \cdot \varepsilon_X + \frac{q_3^B(w)}{\sum_{i=1}^3 q_i^B(w)} \cdot \varepsilon_Y.$$

The coefficient of ε_X is larger under price competition than under quantity competition because intense competition results in larger outputs. Thus, if $\varepsilon_X < \varepsilon_Y$, which is equivalent to $a_X < a_Y$, the input demand is more elastic under price competition than under quantity competition: $\varepsilon^B < \varepsilon^C$. Therefore, in this case, the upstream firm U sets a lower input price under price competition than under quantity competition.

Comparison of profits First, we consider the profit of competing downstream firms, $D1$ and $D2$.[2] Using $r = a_X/a_Y$, we obtain the following.

$$\pi_{Di}^B - \pi_{Di}^C = \frac{a_Y^2\gamma^2(\Psi_2 r^2 + \Psi_1 r + \Psi_0)}{2(3-\gamma)^2(2-\gamma)^2(1+\gamma)(2+\gamma)^2(6+\gamma)^2},$$

where $\Psi_0 = -(2-\gamma)^2(12+17\gamma+6\gamma^2+\gamma^3)$, $\Psi_1 = 4(60+40\gamma-33\gamma^2-11\gamma^3+3\gamma^4+\gamma^5)$, and $\Psi_2 = -4(48+116\gamma-52\gamma^2-17\gamma^3+4\gamma^4+\gamma^5)$. Solving $\pi_{Di}^B - \pi_{Di}^C > 0$ for r, we obtain $r < r_D$, where

$$r_D = \frac{60+40\gamma-33\gamma^2-11\gamma^3+3\gamma^4+\gamma^5+(36-24\gamma+\gamma^2+\gamma^3)\sqrt{1-\gamma^2}}{2(48+116\gamma-52\gamma^2-17\gamma^3+4\gamma^4+\gamma^5)}.$$

Next, we compare π_{D3}^B with π_{D3}^C.

$$\pi_{D3}^B - \pi_{D3}^C = \frac{a_Y^2(1-r)\gamma^2[60+16\gamma-7\gamma^2-\gamma^3-r(24+4\gamma-2\gamma^2)]}{2(3-\gamma)^2(2+\gamma)^2(6+\gamma)^2}.$$

Solving $\pi_{D3}^B - \pi_{D3}^C > 0$ for r, we obtain $r < 1$.

Finally, comparing π_U^B with π_U^C, we obtain the following.

$$\pi_U^B - \pi_U^C = \frac{a_Y^2\gamma^2[4r^2(4-\gamma)+r(4+2\gamma-2\gamma^2)-2-\gamma+\gamma^2]}{2(3-\gamma)(2-\gamma)(1+\gamma)(2+\gamma)(6+\gamma)} > 0,$$

where from $r_{min} < r < r_{max}$, the above inequality is satisfied. By summarizing these results, we obtain the following proposition.

Proposition 1 *(i) Downstream firms $D1$ and $D2$ earn larger profits under price than quantity competition if $r < r_D$. (ii) The profits of downstream firm $D3$ under price*

[2] We only make profit comparisons, but we can also make comparisons for consumer and total surpluses. Consumer and total surpluses are larger in the case of price competition than in the case of quantity competition. This is consistent with well-known results.

competition are larger than under quantity competition if $r < 1$. (iii) The profits of upstream firm U under price competition are larger under quantity competition.

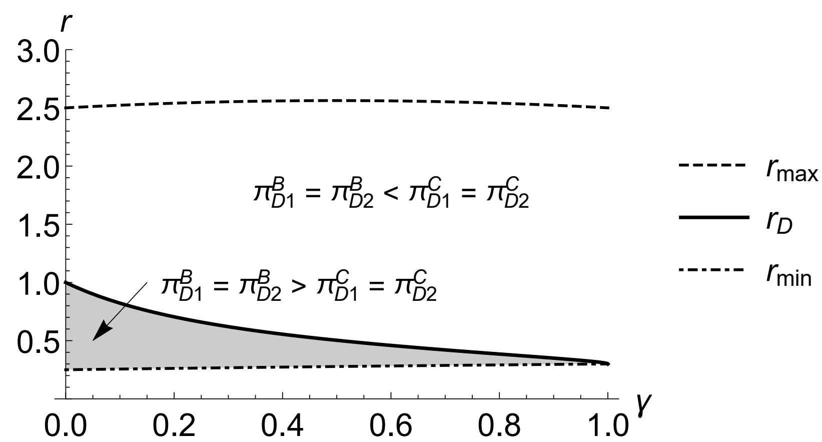

Figure 1: Large market size asymmetry benefiting firms under price competition.

Figure 1 shows the condition for result (i) in Proposition 1. The horizontal axis is product substitutability, γ, and the vertical axis is the ratio of market sizes, $r = a_X/a_Y$. We confirm that the result (i) in Proposition 1 holds if γ and r are small. That is, under price competition, all firms obtain larger profits than under quantity competition if the size of market X is sufficiently smaller than that of market Y and the product substitutability is sufficiently low.

The intuition behind Proposition 1 is as follows. In a price competition between downstream firms $D1$ and $D2$, their profits are small because of greater competition. We call this the *competition effect*. The competition effect weakens when product substitutability, γ, is low. Additionally, as deduced from Lemma 1, in the scenario of $a_X < a_Y$, or equivalently, $r < 1$, the upstream firm opts for a lower input price under price competition than under quantity competition. We refer to this effect as the *input price effect*. The input price effect strengthens when r is small. Thus, when r and γ are sufficiently small, the input price effect dominates the competition effect, leading to higher profits for downstream firms $D1$ and $D2$ under price competition than under quantity competition. Thus, we obtain (i) in Proposition 1. Next, we consider (ii) in Proposition 1. When the input price effect lowers the input price, the profit of downstream firm $D3$ increases because markets X and Y are independent. Finally, we consider (iii) in Proposition 1. Owing to Bertrand competition, greater competition partially resolved the double marginalization problem, increasing the profit of upstream firm U.

4 Conclusions

We consider a vertical market with one upstream and three downstream firms. Two downstream firms compete in one of two downstream markets, and one is a monopolist in the other downstream market. By considering these two distinct downstream markets, we show that an increase in output in a competing downstream market increases the price elasticity of inputs, decreasing input prices. Thus, although price competition increases competition, it can also benefit downstream firms by lowering input prices.

Our study has several limitations. We do not consider a model in which downstream firms choose the type of their strategic variables: price or quantity contracts. Thus, we are unable to discuss whether price competition is realized endogenously and whether it is due to dominant strategies. We also do not know whether price competition results from a prisoner's dilemma, even if price competition yields lower profits than quantity competition. We acknowledge the significance of these considerations and propose them as areas for future research.

References

Aliprantii, M., Milliou, C., and Petrakis, E. (2014) Price vs. quantity competition in a vertically related market. *Economics Letters*, 124(1), 122–126.

Basak, D. (2017) Cournot vs. Bertrand under centralised bargaining. *Economics Letters*, 154, 124–127.

Basak, D. and Mukherjee, A. (2017) Price vs. quantity competition in a vertically related market revisited. *Economics Letters*, 153, 12–14.

Basak, D. and Wang, L.F. (2016) Endogenous choice of price or quantity contract and the implications of two-part-tariff in a vertical structure. *Economics Letters*, 138, 53–56.

Correa-López, M. and Naylor, R.A. (2004) The Cournot-Bertrand profit differential: a reversal result in a differentiated duopoly with wage bargaining. *European Economic Review*, 48(3), 681–696.

Fanti, L. and Meccheri, N. (2011) The Cournot-Bertrand profit differential in a differentiated duopoly with unions and labour decreasing returns. it Economics Bulletin, 31(1), 233–244.

Fanti, L. and Meccheri, N. (2012) Labour decreasing returns, industry-wide union and Cournot-Bertrand profit ranking. A note. *Economics Bulletin*, 32(1), 894–904.

Fanti, L. and Meccheri, N. (2015) On the Cournot-Bertrand Profit Differential and the Structure of Unionisation in a Managerial Duopoly. *Australian Economic Papers*, 54(4), 266–287.

Fanti, L. and Scrimitore, M. (2019) How to compete? Cournot versus Bertrand in a vertical structure with an integrated input supplier. *Southern Economic Journal*, 85(3), 796–820.

Manasakis, C. and Vlassis, M. (2014) Downstream mode of competition with upstream market power. *Research in Economics*, 68(1), 84–93.

Matsuoka, T. (2023) Quantity versus price competition in a vertically related market with sequential contracts. *Applied Economics Letters*, 30(9), 1239–1243.

Mukherjee, A., Broll, U., and Mukherjee, S. (2012). Bertrand versus Cournot competition in a vertical structure: a note. *Manchester School*, 80(5), 545–559.

Singh, N. and Vives, X. (1984) Price and quantity competition in a differentiated duopoly. *RAND Journal of Economics*, 15(4), 546–554.