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### Efficiency and investment style of European mutual funds

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### Abstract

In this paper, we use Data Envelopment Analysis (DEA) to examine the efficiency of European mutual funds across different investment styles. The DEA methodology goes beyond conventional efficiency to identify the most efficient mutual funds compared to the rest of the sample. Due to its flexibility including inputs and outputs (without a previously established relationship) and the lack of need for a hypothesis about the production function, the DEA allows for building efficient frontiers using the information collected from each fund. The application of the DEA in the sample of European mutual funds has served to show potential investors its usefulness and effectiveness in terms of fund selection. The results obtained in this study allow the identification of mutual funds with the most significant profitability potential and those with lower expectations of profitability.

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# 1. Introduction

To improve the investment decision process, it is convenient to identify the goals for the investment, consider personal risk tolerance, and the desired time horizon for the investment. This implies that selecting a mutual fund becomes a challenge that implies multiple criteria in the evaluation.

The Data Envelopment Analysis (hereafter DEA) tool allows for the identification of mutual funds that efficiently consider all these variables to be evaluated. In addition, it facilitates the action to be taken by reducing the selection problem to an optimization program. This optimization program is oriented to estimate the efficiency of each decision unit based on its inputs and outputs. If the result obtained in the program indicates that the unit is inefficient, the DEA shows how much input and output should be reduced or increased respectively to reach the efficient frontier.

This methodology solves some of the main challenges in portfolio evaluation, as we do not need a benchmark and allows considering mutual fund expenses, the DEA compares each fund to the best available in the same country. In addition, the DEA, unlike regression models, allows portfolio and mutual fund managers to analyze different inputs and outputs without an explicitly defined relationship (see Vidal-García et al. (2018)). As we have mentioned above, this methodology not only determines the efficiency of the decision units but also identifies the reasons that explain the weak performance of those that are not inefficient. This feature enables managers to determine the mutual fund's ability to protect against the risk of certain inputs or outputs (Gregoriou (2006)).

Gregoriou and Zhu (2005) explained that another principal advantage of the DEA concerning regression models was that it does not require an initial hypothesis about the production function. Instead of starting from a function, the DEA builds an efficient frontier for mutual funds based on the information of every fund. Therefore, this methodology allows additional ways to analyze the efficiency in decision-making.

Lozano and Gutiérrez (2008) explain that most DEA approaches overestimate the risk related to the endogenous benchmark portfolio. They point out that this happens because in conventional DEA technology, the risk of the target portfolio is computed as a linear combination of the risk of the evaluated mutual fund and this ignores the relevant effect of portfolio diversification.

The authors combine DEA with stochastic dominance criteria. They create six different DEA-like linear programming (LP) models to estimate relative efficiency scores consistent with second-order stochastic dominance. They show that being second-order stochastic dominance efficient, the new target portfolio would be an optimal benchmark for potential risk-averse investors.

Lin et al. (2017) create a new multi-period network DEA model with diversification and directional distance function. Their new methodology decomposes the overall efficiency of a mutual fund in the whole investment interval into efficiencies at different periods. In this sense, they explain that efficiency decomposition shows the time inefficiency occurs. The authors point out that their new model can provide expected inputs, outputs, and intermediate variables at different interval periods, which are helpful for fund managers to find factors causing the overall fund inefficiency.

Additional insights on DEA models include Galagedera et al. (2018); the authors examine overall and stage-level performance using a data envelopment analysis model. They explain that the stage-level processes are deemed to operate with two different levels of risk exposure. They model operation under different levels of risk exposure through conditions imposed on the intermediate measures. The authors find that a new index proposed to assess linkage performance is demonstrated empirically to improve the discriminatory power of performance.

The rest of the paper is structured as follows. Section 2 describes the data and the sample employed in the research. Section 3 explains the basic methodology models. Section 4 shows the main empirical findings. Finally, Section 5 concludes. Tables are provided in the Appendix.

## 2. Data

Our sample includes daily returns of 1,477 actively managed equity mutual funds. We focus on the six European countries with the largest market capitalization by the end of 2023, as they include over 90% of total assets in Europe. The mutual funds are domiciled in France, Italy, the United Kingdom, Spain, Germany, and the Netherlands. We restrict our sample to open-end domestic equity funds that invest in a single country. All fund returns are expressed in national currency and incorporate any dividend paid. The returns are net of fund operating expenses. We obtain our sample from the Lipper mutual fund database. We employ fund investment style classifications from the Morningstar Direct database. Our period starts on January 1<sup>st</sup>, 1990, and ends on December 31<sup>st</sup>, 2023. We include all mutual funds in our dataset until they are discontinued to account for potential survivorship bias in our results, which is the consequence of incorporating exclusively surviving funds in a sample. For comparison purposes of large samples of daily data, see Vidal, Vidal-García, and Boubaker (2015), Vidal-García et al. (2016) and Vidal et al. (2024) for global samples, or El Ammari, Vidal, and Vidal-García (2023) and Hammouna et al. (2023) for European studies.

In each market, we create a European adaptation of the Carhart 4-factor and Fama-French 3-factor models; we incorporate all stocks included in the Worldscope database (Thomson Financial Company) for each European market. Table I shows the summary statistics of the database for each investment style across countries. Style classifications are based on investment focus (growth, blend, and value) and mutual funds' market capitalization (small, mid, and large-cap). Table I presents the Morningstar investment styles represented in our database.

## 3. Methodology

### 3.1 Models of Mutual Fund Performance

To evaluate fund performance we employ the one-factor CAPM, Fama and French's (1993) three-factor model, and Carhart's (1997) four-factor model:

$$R_{pt} = \mu_{pt} + \beta_{1,pt} (RM_t - RF_t) + e_{1,pt} \quad (1)$$

$$R_{pt} = \mu_{pt} + \beta_{1,pt} (RM_t - RF_t) + \beta_{2,pt} SMB_t + \beta_{3,pt} HML_t + e_{2,pt} \quad (2)$$

$$R_{pt} = \mu_{pt} + \beta_{1,pt} (RM_t - RF_t) + \beta_{2,pt} SMB_t + \beta_{3,pt} HML_t + \beta_{4,pt} MOM_t + e_{3,pt} \quad (3)$$

where  $R_{pt}$  represents the return on portfolio  $p$  for month  $t$ ,  $RF_t$  represents the risk-free rate,  $RM_t$  represents the market return,  $SMB_t$  and  $HML_t$  are the Fama-French (1993) size and book-to-market factors, and  $MOM_t$  is the period  $t$  value of the Carhart (1997) momentum return,  $e_{it}$  represents the residual from the regression model, and  $\mu_{pt}$  represents the mean return over the market benchmark. Regression (1) represents the CAPM model, regression (2) is the Fama-French three-factor model, while regression (3), which includes the  $MOM_t$  factor, represents Carhart's four-factor model. We follow Dimson (1979) and Bollen and Busse (2005) and incorporate lagged values of the four factors to address the influence of infrequent trading of stocks on daily fund performance. Additionally, we use the Newey and West (1987) heteroskedasticity and autocorrelation consistent estimator of the standard deviation.

### 3.2 The Data Envelopment Analysis Model

We estimate the efficiency of national equity funds employing the Data Envelopment Analysis (DEA) non-parametric technique used to solve production functions. This methodology was designed by Charnes, Cooper, and Rhodes (1978) to examine the academic achievements of schools, and it has been used extensively to examine the behavior of decision-making units (DMUs) including a structure with different inputs. It is a useful technique for evaluating performance as it incorporates different inputs and outputs that can be evaluated in separate units. The DEA examines the maximum potential output for a given sum of inputs. It creates an efficiency measure for all decision-making units compared to the best operating unit in a group. The methodology evaluates how efficiently a decision-making unit employs its capability to design the outputs. The performance of the decision-making units is evaluated in DEA using the conception of efficiency as a ratio of total outputs to total inputs. Efficiency values employing DEA are compared to the best performing DMU. The top DMU in terms of efficiency is given an efficiency outcome of unity or 100%, and the remaining DMUs fluctuate between 0 and 100% in comparison to the best one.

The DEA technique has been used to evaluate US mutual fund performance by different authors, for instance, Murthi, Choi, and Desai (1997), Morey and Morey (1999), and Basso and Funari (2001). To the best of our knowledge, our work is the first one to evaluate mutual fund results across different European countries employing DEA. The DEA technique might be employed to explain mutual fund indexes with several inputs such as risk measures and fund expenses. The Data Envelopment Analysis efficiency value for a decision-making unit  $j$  is estimated as a ratio of a weighted sum of outputs to a weighted sum of inputs:

$$\text{Efficiency}_j = \frac{\sum_{r=1}^t \omega_r y_{rj}}{\sum_{i=1}^m \omega_i x_{ij}} \quad (5)$$

where:  $j=1,2,\dots,n$  represents the number of decision-making units,  $r=1,2,\dots,t$  represents the number of outputs and  $i=1,2,\dots,m$  represents the number of inputs. Additionally,  $y_{rj}$  represents the total output  $r$  for unit  $j$ ,  $x_{ij}$  is the amount of input  $i$  for unit  $j$ ,  $\omega_r$  represents the significance of output  $r$ , and  $\omega_i$  is the significance of input  $i$ . If the efficiency is unity, then the DEA methodology leads represent a Pareto efficiency measure and the efficient units lie on the efficiency frontier.

As explained by Charnes et al. (1994), to calculate the DEA efficiency measure for a decision-making unit, we need to estimate the optimal solution to the subsequent fractional linear programming question:

$$\text{Maximize } \theta_j = \frac{\sum_{r=1}^t \omega_r y_{rj}}{\sum_{i=1}^m \omega_i x_{ij}} \quad (6)$$

$$\text{Subject to } \frac{\sum_{r=1}^t \omega_r y_{rj}}{\sum_{i=1}^m \omega_i x_{ij}} \leq 1, \quad j=1,\dots,n \quad (7)$$

$$\omega_r \geq 0, \quad r=1,\dots,t$$

$$\omega_i \geq 0, \quad i=1,\dots,m$$

where  $\epsilon$  represents a small positive number to confirm that the weights are not negative. From equation (6) we get the estimate of the optimal objective function which is the

efficiency measure for unit  $j$ . We can get a similar linear programming question transforming the fractional question presented earlier, we set  $\frac{v_j}{u_j} = 1$ , resulting in the Charnes, Cooper, and Rhodes (CCR) representation:

$$\max \theta \quad (8)$$

Subject to 
$$\sum_{j=1}^n \lambda_j = 1 \quad (9)$$

$$\theta x_j - \sum_{i=1}^m \lambda_i x_{ij} = 0 \quad j=1, \dots, n$$

$$\begin{aligned} & \sum_{r=1}^s \lambda_r y_{rj} - \theta y_j = 0 \quad j=1, \dots, n \\ & \sum_{i=1}^m \lambda_i x_{ij} - \theta x_j = 0 \quad j=1, \dots, n \end{aligned}$$

We calculate the values for the optimization problem of the  $t + m$  variables, which implies the weights  $u_r$  and  $v_i$ , conditional to  $n + t + m + 1$  restrictions.

The advantages of DEA technique for portfolio performance evaluation are the following:

1. DEA does not need any theoretical model as measurement benchmarks. The DEA methodology examines fund performance compared to the best group of funds within the investment style.
2. The model is flexible and can examine fund performance by employing different inputs and outputs at the same time.
3. We can determine the marginal contribution of each input in fund returns.

Fund performance is examined in terms of returns-to-cost ratios. Investors prefer a fund that maximizes returns and minimizes fees simultaneously. In this sense, we consider that DEA is a relevant methodology for portfolio evaluation.

### 3.3 Multiplier Dynamic Data Envelopment Analysis Based on Directional Distance Function

Lin and Liu (2021) extend the multiplier dynamic data envelopment analysis (DEA) by using the directional distance function (DDF), they explain that their empirical evidence shows that the proposed multiplier dynamic model can distinguish performance and good practice value for the current portfolio choice.

We follow their methodology to re-estimate our results and refer the reader to their paper for detailed mathematical explanations. We create the following multiplier dynamic DDF model based upon fractional linear objective function following the Charnes-Cooper transformation (Charnes and Cooper (1962)):

$$\theta_t = \min \left\{ \frac{\lambda_1}{\lambda_1 + \lambda_2} \left( \frac{y_{1t}}{y_{1t}^*} - \beta_1 \right) + \frac{\lambda_2}{\lambda_1 + \lambda_2} \left( \frac{y_{2t}}{y_{2t}^*} - \beta_2 \right) - \frac{\lambda_3}{\lambda_1 + \lambda_2} \left( \frac{y_{3t}}{y_{3t}^*} - \beta_3 \right) + t \right\} \quad (10)$$

$$\text{s.t. } \frac{\lambda_1}{\lambda_1 + \lambda_2} \left( \frac{y_{1t}}{y_{1t}^*} + \beta_1 \right) + \frac{\lambda_2}{\lambda_1 + \lambda_2} \left( \frac{y_{2t}}{y_{2t}^*} + \beta_2 \right) + \frac{\lambda_3}{\lambda_1 + \lambda_2} \left( \frac{y_{3t}}{y_{3t}^*} + \beta_3 \right) = 1$$

Using this model, the efficiency scores at several periods are estimated by different multipliers. In this sense, the efficiency scores in the new period show the performance of the evaluated funds compared to the entire sample of funds in the same interval period.

## 4. Results

We have obtained the inputs and outputs for each mutual fund from the Morningstar Direct database. The inputs selected estimate efficiency for each of the mutual funds, these are the following: expense ratio (%), 3-year standard deviation (%), cash (%), P/E, the beta of each mutual fund, and stocks (%). On the other hand, the outputs to be evaluated are the Sharpe ratio<sup>1</sup>, the average profitability of the fund at 3 years and the average profitability of the fund at 5 years. For comparison purposes, we have used the inputs and outputs most commonly employed in the finance literature about portfolio performance evaluation with the DEA technique (see for instance Murthi, Choi, and

<sup>3</sup>See Vidal-García and Vidal (2024) for performance analysis of the Sharpe ratio around the world.

Desai (1997), Basso and Funari (2001; 2003), Vidal-García et al. (2018) and Peykani et al. (2024)).

The DEA model allows us to examine which funds are efficient, and which are not. Table II shows the efficiency level of our sample of funds across the different investment styles and countries, while Table III presents the number of funds for each efficiency level.

Of our total sample of funds analyzed, 56% are perfectly efficient, which means that 813 funds have a coefficient equal to 1. Within this 56%, value funds have been the predominant investment style. Semi-efficient funds (coefficient between 0.9 and 0.99) account for 19% of the total sample, which is the lowest percentage of funds in the sample. The percentage with a greater domain in this range of efficiency corresponds to the investment style of blended funds. The last group to examine is that of inefficient funds (coefficient below 0.9). This group represents 25% of the sample; the typology with the highest percentage of inefficient funds is again the blend investment style. Thus, the use of DEA presents robust evidence that equity mutual funds in Europe are approximately mean-variance efficient, which implies that they are near the mean-variance efficient frontier. Therefore, the DEA model corroborates the mean-variance efficiency theory. In contrast, the small and mid-capitalization funds are the most representative blend funds for the inefficient and semi-efficiency levels.

Table IV shows the efficiency level of our sample of funds using the multiplier dynamic data envelopment analysis based on the directional distance function as presented by Lin and Liu (2021). The results are very similar across the different investment styles and countries; we have not found any investment style that changes their efficiency category. Thus, we can confirm that the innovative multiplier dynamic model has a strong ability to distinguish performance for the portfolio selection of mutual funds.

Mutual funds classified as efficient, are those with the most favorable values in all inputs and outputs except for beta. Semi-efficient mutual funds present sensitivity to the stock market below the unit. This means that they are less sensitive when the stock market fluctuates. On the other hand, inefficient funds present a beta below 0.9 and are considered more volatile than the market. However, the small differences between

efficient and inefficient funds mean that beta has not had a primary impact on the efficiency analysis. To select outperforming funds, we should look at each fund's alpha and understand the manager's skills and how much value they add to performance.

Our evidence has relevant significance in practice. Firstly, there is a substantial motivation to raise fees among mutual funds to increase fund performance. Therefore, our evidence confirms the agency theory definition. Secondly, our evidence has practical importance for new investors, as they might evaluate some of the fund's variables considered in this study in their investment decisions.

## 5. Conclusion

The results obtained in each of the outputs and inputs support the DEA as a valid methodology to identify efficient mutual funds in Europe across different investment styles. We have appreciated in the analysis that the funds classified as efficient are those with the most favourable estimates in almost all inputs and outputs except for beta (one basis point above semi-efficient funds). Therefore, all the funds that are in the group of efficient ones are more likely to perform above the rest.

In this research, we can observe that the most efficient mutual fund types in terms of the size of assets under management are those funds classified as large ones. Therefore, this category of funds represents both the most efficient and the best results for investors, while in terms of investment style, the best performing is currently the value.

Based on our evidence, we can conclude that the DEA can not only be used to detect the most efficient mutual funds but also to identify trends in sectors, markets, assets, and typologies where efficiency is present for a determined period. We have compared our results using the multiplier dynamic data envelopment analysis based on the directional distance function introduced by Lin and Liu (2021). The results are very similar across the different investment styles and countries. If our sample had been divided by sectors, the DEA would have allowed us to determine in which sectors it is more favorable to invest. Similarly, an international financial markets classification provides information on which geographic markets the investment is more efficient.

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## Appendix: Tables

Table I: Summary Statistics

The table presents descriptive statistics on our sample of mutual funds. The sample period: January 1990 to December 2023. Asset size is expressed in millions of dollars.

	平均 b. 平均	分散 a. 分散	標準偏差 ! 標準偏差
DT 平均	う	れん	れん
[L <sub>ET</sub> , 1/2]	え	れん	えん
[L <sub>ET</sub> 1/2]	え	れん	えん
a. 1/2	ん	れん	えん
a. DIX <sub>ET</sub>	ん	れん	えん
{ I L <sub>ET</sub> 1/2 }	う	れん	えん
{ I L <sub>ET</sub> DIX <sub>ET</sub> }	ん	れん	えん
L <sub>ET</sub>	う	れん	えん
[L <sub>ET</sub> , 1/2]	ん	れん	えん
[L <sub>ET</sub> 1/2]	え	れん	えん
a. 1/2	え	れん	えん
a. 1/2	え	れん	えん
{ I L <sub>ET</sub> 1/2 }	え	れん	えん
{ I L <sub>ET</sub> L <sub>ET</sub> }	ん	れん	えん
{ i L <sub>ET</sub> }	え	れん	えん
[L <sub>ET</sub> , 1/2]	え	れん	えん
[L <sub>ET</sub> DIX <sub>ET</sub> ]	ん	れん	えん
[L <sub>ET</sub> 1/2]	う	れん	えん
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a. 1/2	え	れん	えん
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b. T <sub>ET</sub> 平均	え	れん	えん
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[L <sub>ET</sub> 1/2]	え	れん	えん
a. 1/2	え	れん	えん
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{ I L <sub>ET</sub> L <sub>ET</sub> }	ん	れん	えん
C <sub>ET</sub>	え	れん	えん
[L <sub>ET</sub> , 1/2]	え	れん	えん
[L <sub>ET</sub> DIX <sub>ET</sub> ]	え	れん	えん
[L <sub>ET</sub> 1/2]	え	れん	えん
a. 1/2	え	れん	えん
a. 1/2	え	れん	えん
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- Y	え	れん	えん
[L <sub>ET</sub> , 1/2]	え	れん	えん
[L <sub>ET</sub> DIX <sub>ET</sub> ]	え	れん	えん
[L <sub>ET</sub> 1/2]	え	れん	えん
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Table II: Efficiency Level

The table presents the efficiency level of the different mutual funds across investment styles and countries.

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34000 IT 1 IX 33 C 33 33	34000 IT 1 IX 33 33 33	34000 IT 1 IX 33 {IX 33
34000 IT 1 IX 33 - Y,	34000 IT 1 IX 33 33 33	34000 IT 1 IX 33 bT 33 33 33
34000 IT 1 IX 33 DT 33 33	34000 IT 1 IX 33 C 33 33	34000 IT 1 IX 33 DT 33 33
34000 IT 1 IX 33 {IX 33	34000 IT 1 IX 33 C 33 33	34000 IT 1 IX 33 DT 33 33
34000 IT 1 IX 33 - Y,	34000 IT 1 IX 33 33 33	34000 IT 1 IX 33 {IX 33
34000 IT 1 IX 33 DT 33 33	34000 IT 1 IX 33 C 33 33	34000 IT 1 IX 33 DT 33 33
34000 IT 1 IX 33 bT 33 33 33	34000 IT 1 IX 33 - Y,	34000 IT 1 IX 33 DT 33 33
34000 IT 1 IX 33 - Y,	34000 IT 1 IX 33 DT 33 33 - Y,	34000 IT 1 IX 33 {IX 33
34000 IT 1 IX 33 33 33	34000 IT 1 IX 33 DT 33 33	34000 IT 1 IX 33 - Y,
34000 IT 1 IX 33 {IX 33	34000 IT 1 IX 33 bT 33 33 33	34000 IT 1 IX 33 DT 33 33 - Y,
34000 IT 1 IX 33 - Y,	34000 IT 1 IX 33 bT 33 33 33	34000 IT 1 IX 33 - Y,
34000 IT 1 IX 33 33 33	34000 IT 1 IX 33 C 33 33	
34000 IT 1 IX 33 C 33 33		

Table III: Number of Mutual Funds by Efficiency Level

The table presents the mutual funds for each efficiency level across investment styles and countries. Efficient funds present a coefficient equal to 1, semi-efficient funds have a coefficient between 0.9 and 0.99, and inefficient funds have a coefficient below 0.9.

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ゐ	[ $\text{D}\{\text{ix}\}$ ]	ゑ	[ $\text{U}\{\text{bt}\}^{\text{D}}\text{f}\text{v}\downarrow$ ]	う	[ $\text{U}\{\text{C}\text{v}\}$ 、
わ	[ $\text{D}\{\text{C}\text{v}\}$ 、	え	[ $\text{U}\{\text{x}\}$ ]	わ	[ $\text{U}\{\text{ix}\}$ ]
を	[ $\text{D}\text{-Y}$ 、	ゐ	[ $\text{U}\{\text{x}\}$ ]	ろ	a $\text{U}\{\text{bt}\}^{\text{D}}\text{f}\text{v}\downarrow$
ほ	[ $\text{U}\{\text{Dt}\}\text{I}\text{v}\text{E}$ ]	わ	[ $\text{U}\{\text{C}\text{v}\}$ 、	ん	a $\text{U}\{\text{Dt}\}\text{I}\text{v}\text{E}$
う	[ $\text{U}\{\text{ix}\}$ ]	わ*	a $\text{U}\{\text{C}\text{v}\}$ 、	わ	a $\text{D}\{\text{Dt}\}\text{I}\text{v}\text{E}$
ずん	[ $\text{U}\text{-Y}$ 、	ゑ	a $\text{U}\{\text{x}\}$ ]	ゐ	a $\text{U}\{\text{ix}\}$
ぞわ	[ $\text{U}\{\text{Dt}\}\text{I}\text{v}\text{E}$ ]	ずわ	a $\text{U}\{\text{C}\text{v}\}$ 、	ゐ	{ $\text{D}\{\text{Dt}\}\text{I}\text{v}\text{E}$ }
ろ	[ $\text{U}\{\text{bt}\}^{\text{D}}\text{f}\text{v}\downarrow$ ]	ゐ	a $\text{U}\text{-Y}$ 、	ゐ	{ $\text{U}\{\text{ix}\}$ }
わん	[ $\text{U}\text{-Y}$ 、	わ	a $\text{D}\text{-Y}$ 、	ぜ	{ $\text{U}\text{-Y}$ 、
ぜん	a $\text{U}\{\text{x}\}$ ]	ん	{ $\text{U}\{\text{Dt}\}\text{I}\text{v}\text{E}$ }	わ	{ $\text{D}\text{-Y}$ 、
わ	a $\text{U}\{\text{ix}\}$ ]	わ	{ $\text{U}\{\text{bt}\}^{\text{D}}\text{f}\text{v}\downarrow$ }	そ	{ $\text{U}\text{-Y}$ 、
わ	a $\text{U}\text{-Y}$ 、	ん	{ $\text{U}\{\text{bt}\}^{\text{D}}\text{f}\text{v}\downarrow$ }	そ	{ $\text{U}\{\text{x}\}$ ]
ゐ	{ $\text{U}\{\text{x}\}$ ]	ゐ	{ $\text{U}\{\text{C}\text{v}\}$ 、		
ゐ	{ $\text{U}\{\text{C}\text{v}\}$ 、				
ぞわ		わわ		わわ	

Table IV: Efficiency Level using Multiplier Dynamic Data Envelopment Analysis based on Directional Distance Function

The table presents the efficiency level of mutual funds across investment styles and countries using multiplier dynamic data envelopment analysis based on the directional distance function.

94400 I / IX433	{T I 94400 I / IX433}	LT4400 I / IX433
344 I <sup>ET</sup> DIX <sup>ND</sup> {ix44	444 I <sup>ET</sup> 1 445 bT44 445	444 I <sup>ET</sup> 1 445 C444、
344 I <sup>ET</sup> DIX <sup>ND</sup> C444、	444 I <sup>ET</sup> 1 445 444	444 I <sup>ET</sup> . 445 {ix44
344 I <sup>ET</sup> DIX <sup>ND</sup> - Y、	444 I <sup>ET</sup> . 445 444	444 a 445 bT44 445
344 I <sup>ET</sup> 1 445 DT444	444 I <sup>ET</sup> . 445 C444、	444 a 445 DT444
344 I <sup>ET</sup> 1 445 {ix44	444 a 445 C444、	444 a 445 DIX <sup>ND</sup> DT444
344 I <sup>ET</sup> 1 445 - Y、	444 a 445 . 445 444	444 a 445 {ix44
344 I <sup>ET</sup> . 445 DT444	444 a 445 . 445 C444、	444 I445 445 DT444
344 I <sup>ET</sup> . 445 bT44 445	444 a 445 - Y、	444 I445 DIX <sup>ND</sup> DT444
344 I <sup>ET</sup> . 445 - Y、	444 a 445 DIX <sup>ND</sup> - Y、	444 { I445 445 {ix44
344 a 445 444	444 I445 445 DT444	444 I445 445 - Y、
344 a 445 {ix44	444 I445 445 bT44 445	444 I445 DIX <sup>ND</sup> - Y、
344 a 445 - Y、	444 I445 445 bT44 445	444 I445 445 - Y、
344 I445 445 444	444 I445 445 C444、	
344 I445 445 C444、		