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Rawlsian welfare implications of endogenous transfers in a network of bilateral contests

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Abstract

This paper explores the Rawlsian welfare implications of endogenous transfers in a three- player- system where one player fights the other two bilaterally. Though such transfers can be welfare improving under certain conditions (explicitly mentioned in the paper), Rawlsian welfare maximization cannot be achieved endogenously via transfers. The welfare maximizing distribution is stable i.e immune to endogenous transfers.

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1. Introduction:

In a network of bilateral contests, the utilitarian welfare is fixed (equal to the summation of the valuation of the different prizes which are being contested over) but the post – conflict Rawlsian welfare of the system is a function of the distribution of the conflict-resources (arms) amongst the players (The paper abstracts away from the acquisition of conflict resources and considers a fixed amount of conflict resources distributed amongst the players). Hence it seems that the Rawlsian Welfare improvement needs external intervention in the form of redistribution of conflict-resources. But Kovenock and Roberson (2012) and Rietzke and Roberson (2013) proposed the idea of endogenous transfers of conflict - resources between players engaged in a system of bilateral contests. And why are such transfers interesting in the first place? Because it leverages the economic reasoning of the enemy of one's enemy becoming one's friend. In fact, endogenous transfers as introduced by Kovenock and Roberson (2012) is a refined version of cooperation which does away with the botheration of post – coalition defection by any member of the coalition.

This paper explores the Rawlsian welfare implications of such transfers. Do such transfers improve or exacerbate Rawlsian welfare? Secondly, can such transfers lead to Rawlsian welfare maximization (and thereby render the need for any welfare maximizing external redistributive intervention redundant)? Thirdly is the Rawlsian welfare maximizing conflict resource endowment unstable i.e susceptible to endogenous transfers? These are the three major questions which this paper aims to answer. And what makes these questions interesting? The welfare implications of the endogenous transfers of conflict resources can be thought of as an invisible – hand – effect where the system adjusts itself to improve or exacerbate welfare (in this paper we are consider Rawlsian welfare). The paper delves into the effect and limitations of this 'invisible – hand'. Also, the stability (or the lack of it) of the Rawlsian – welfare maximizing endowment explores the limitation of external redistributive interventions.

2. Model:

Let S = {A, B, C} be the set of players, who are engaged in a network of bilateral contests. Each conflict is modelled according to the Tullock (1980) contest success function. Each player t $\in \{A, B, C\}$ is endowed with $\overline{X}_t$ amount of conflict – resources / arms. Without loss of generality $\overline{X_B} > \overline{X_A}$. C is engaged in a bilateral conflict with both A and B. A and B are not in conflict with each other. C and A (B) are fighting over a prize of common valuation w_{AC} (w_{BC}). Let $\alpha = \sqrt{\frac{w_{Bc}}{w_{Ac}}}$. Let $x_t = \frac{\overline{X_t}}{\overline{X_A} + \overline{X_B} + \overline{X_C}}$ $\forall\, t \in$ {A, B, C}. If $(\overline{X_A}, \overline{X_B}, \overline{X_C})$ is a conflict resource endowment vector, the corresponding normalized conflict resource endowment vector is given by (x_A, x_B, x_C). With this setting the following game takes place.

Stage I: Each player decides whether to transfer conflict - resource (arms) to any other player and simultaneously whether to accept conflict – resource transfer(s) from any other player(s). The transfers & acceptances happen simultaneously. A transfer from player $i\ to\ j$ is denoted by t_{ij}. A transfer $from\ j$ accepted by i is denoted by a_{ij}. The action set of player i in the network is given by ($\{t_{ij}\}_{j=1}^{n}$, $\{a_{ij}\}_{j=1}^{n}$) j ≠ i where $0 \leq t_{ij} \leq X_i, \sum_j t_{ij} \leq \overline{X_i},\ and\ a_{ij} \equiv a_i(t_{ji}) \in \{0, 1\}$. We conclude that that a possible redistribution of conflict - resources has happened if $\exists\, i,j \in S \ni t_{ij} > 0$ and $a_j(t_{ij}) = 1$. The redistributed endowment vector is denoted as $\{\widetilde{X_t}\}_{t \in S}$. If $t_{ij} = a_{ji} = 0\ \forall\, i,j$ then we conclude that $\{\widetilde{X_t}\}_{t \in S} = \{\overline{X_t}\}_{t \in S}$. Since we only consider rational transfers (defined in the next section) which are transfers leading to pareto improvement for both sender and receiver we shall supress the acceptance decisions, for notational simplicity.

Stage II: C decides to allocate his resources in the two conflicts. Let us say that the resource allocated in the contest against k $\in \{A, B\}$ is given by X_{kc}. C's optimization problem is given by: $\max_{X_{Bc},\, X_{Ac}} \pi_C = \frac{X_{Bc}}{X_{Bc}+\overline{X_B}} . w_{Bc} + \frac{X_{Ac}}{X_{Ac}+\overline{X_A}} w_{Ac}$ s.t $X_{Bc} + X_{Ac} = \overline{X_C}$ (1)

Solving the optimization problem will yield the optimal conflict resource allocations by C in both the conflicts i.e. $\{X^*_{kC}\}_{k\in\{A,B\}}$. We solve the two - stage game using backward induction. Players choose transfer of conflict-resources in stage 1, followed by resource allocation (to the two contests against A and B) by C in stage 2. The subgame perfect Nash equilibrium (SPNE), will be denoted by $(\{t_{ij}{}^*\}_{j=1}^{n} \forall\, i,j \in S, j \neq i \text{ and } \{X^*_{Ck}\}_{k\in\{A,B\}})$

Definitions:

Transfer – susceptible endowment vector: A conflict resource endowment vector $\{\overline{X_t}\}_{t \in S}$ is called transfer susceptible if there exists a transfer between two agents i and j that leads to an improvement in payoffs of both the agents or in the payoff of one agent keeping the other's payoff fixed i.e. in the SPNE $\exists\, i,j \in S \ni t_{ij}{}^* > 0$, $a_j^*(t_{ij}^*) = 1$

Transfer – resistant endowment vector: A conflict resource endowment vector $\{\overline{X_t}\}_{t \in S}$ is called transfer resistant if no transfer between any pair of agents is rational i.e in the SPNE $t_{ij} = a_{ji} = 0\ \forall\, i,j \in S$.

Rawlsian Welfare: The Rawlsian social welfare function (henceforth simply referred to as Rawlsian welfare) is a social welfare function that uses as its measure of social welfare the utility of the worst - off member of the society. In the three - player system described in the paper the (post – conflict) Rawlsian welfare of the system is given by $min\{\pi_A, \pi_B, \pi_C\}$.

Remark 1:

a. Since any conflict resource endowment vector can be normalized it makes perfect sense to consider normalized endowment vectors only without loss of generality.
b. Though the acceptance decision has been mentioned in the characterization of the equilibrium it is actually implicit since the paper deals with rational transfers which has the idea of acceptance implicitly embedded.

3. Results and Discussion:

Lemma 1: *C engages in a contest with both A and B if and only if* $\alpha \in \left(\frac{\sqrt{x_A x_B}}{1-x_B}, \frac{1-x_A}{\sqrt{x_A x_B}}\right)$

Proof: It is provided in the Appendix.

Any normalized conflict resource endowment vector (x_A, x_B, x_C) where $\alpha \notin (\frac{\sqrt{x_A x_B}}{1-x_B}, \frac{1-x_A}{\sqrt{x_A x_B}})$ will henceforth be called a 'one – fight endowment vector'. Lemma 1 essentially looks into the participation constraints of C in the two contests. Hence for any given $\alpha \in (0, \infty)$ a conflict – resource endowment vector is either a 'one – fight endowment vector' or a 'two – fight endowment vector'.

Remark 2: The economic intuition behind Lemma 1 is the following. Given a normalized conflict resource endowment vector (x_A, x_B, x_C) if $\alpha = \sqrt{\frac{w_{BC}}{w_{AC}}}$ is large enough (i.e. w_{BC} is significantly larger than w_{AC}) the marginal benefit to C from allocating conflict resources in the conflict against B easily outweighs the marginal benefit from allocating resource in the conflict against A and hence C chooses to fight B alone. An exactly opposite argument holds if $\alpha = \sqrt{\frac{w_{BC}}{w_{AC}}}$ is small enough. Another way to look at the condition is that keeping the geometric mean of x_A and x_B fixed if either one is increased the length of the interval $(\frac{\sqrt{x_A x_B}}{1-x_B}, \frac{1-x_A}{\sqrt{x_A x_B}})$ shrinks i.e. keeping the geometric mean fixed as the difference between x_A and x_B increases the interval in which $\frac{w_{BC}}{w_{AC}}$ must lie in order incentivize C to engage in contests with both A and B, shrinks.

Remark 3: Though it may appear that the condition $\alpha \in (\frac{\sqrt{x_A x_B}}{1-x_B}, \frac{1-x_A}{\sqrt{x_A x_B}})$ that ensures that a normalized conflict resource endowment vector (x_A, x_B, x_C) is a 'two – fight endowment vector' is independent of x_C , that is not true since $\sum_{t\in(A,B,C\}} x_t = 1$ since $x_t = \frac{\overline{X_t}}{\overline{X_A} + \overline{X_B} + \overline{X_C}}$ $\forall\, t \in \{\text{A, B, C}\}$.

Lemma 2: Given any $\alpha \in (0, \infty)$

i. *The one fight endowment vectors are transfer- resistant.*
ii. *No endogenous transfer leads to a transition from a two – fight endowment vector to a one – fight endowment vector.*

Proof: The proof is provided in the Appendix.

Remark 4: Lemma 2 rests on the economic rationale that given a particular $\alpha \in (0, \infty)$ (i.e. for a given relative prize ratio $\frac{w_{BC}}{w_{AC}}$) if the conflict resource endowment vector is such that C only fights with one of the two (i.e. either A or B and not both) then the one C is fighting with, has no incentive to make a transfer to the other because that incentivizes C further to fight the transferer and moreover the transfer reduces the conflict resources of the transferer, thereby reducing his equilibrium payoff. And that renders the 'one fight endowment vectors' transfer resistant.

Since the paper deals with exploring the welfare implications of the endogenous transfers of conflict resources, we will henceforth be more interested in the 'two fight endowment vectors' given any $\alpha \in (0, \infty)$.

Lemma 3: *If* $\alpha \in \left(\frac{\sqrt{x_A x_B}}{1-x_A}, \frac{1-x_B}{\sqrt{x_A x_B}}\right)$ *the equilibrium payoffs are given by:*

$$\pi_B{}^* = \frac{\overline{X_B} w_{Bc} + \sqrt{\overline{X_A}\ \overline{X_B} w_{Bc} w_{Ac}}}{\overline{X_A} + \overline{X_B} + \overline{X_C}} = [\mathrm{x_B}\alpha^2 + \sqrt{\mathrm{x_A x_B}}\ \alpha].\ \mathrm{w_{AC}}$$

$$\pi_A{}^* = \frac{\overline{X_A} w_{Ac} + \sqrt{\overline{X_A}\ \overline{X_B} w_{Bc} w_{Ac}}}{\overline{X_A} + \overline{X_B} + \overline{X_C}} = [x_A + \sqrt{x_A x_B}\ \alpha].w_{AC}$$

$$\pi_C{}^* = (w_{Bc} + w_{Ac}) - \frac{(\sqrt{\overline{X_B} w_{Bc}} + \sqrt{\overline{X_A}\ w_{Ac}})^2}{\overline{X_A} + \overline{X_B} + \overline{X_C}} = [1 + \alpha^2 - (x_B\alpha + x_A)^2].\ w_{AC}$$

Proof: The Proof is provided in the Appendix.

Lemma 4:

If $\alpha \in \left(\frac{\sqrt{x_A x_B}}{1-x_B}, \frac{1-x_A}{\sqrt{x_A x_B}}\right)$ *then*

i. *In SPNE* $t_{Cj} = 0\ \forall\ j \in \{A,B\}$ *and* $t_{iC} = 0\ \forall\ i \in \{A,B\}$.

ii. *Any pre-conflict resource endowment vector* $(\overline{X_A}, \overline{X_B}, \overline{X_C})$ *satisfying the condition* $\overline{X_B} < \theta\overline{X_A}$ *is necessarily transfer – resistant.*

iii. *Any pre-conflict resource endowment vector* $(\overline{X_A}, \overline{X_B}, \overline{X_C})$ *satisfying the condition* $\overline{X_B} > \theta\overline{X_A}$ *is transfer – susceptible. In the SPNE* $t_{BA} = a_{AB} > 0$.

iv. *If* $\overline{X_B} > \theta\overline{X_A}$ *optimal transfer* $\delta^* = \frac{\overline{X_B} - \theta\ \overline{X_A}}{1+\theta}$. *The post transfer conflict resource endowment vector* $(\widetilde{X_A}, \widetilde{X_B}, \widetilde{X_C})$ *satisfies* $\widetilde{X_B} = \theta\widetilde{X_A}$.

where $\theta = 2.\left(\frac{w_{Bc}}{w_{Ac}}\right) + 1 + 2.\sqrt{\frac{w_{Bc}}{w_{Ac}}}.\sqrt{1 + \frac{w_{Bc}}{w_{Ac}}} = 2\alpha^2 + 1 + 2\ \alpha\sqrt{\alpha^2 + 1}$

Proof: The Proof is provided in the Appendix.

Lemma 3 and Lemma 4 point out the conditions under which the conflict resource endowment vector is transfer – susceptible and how much transfer happens in equilibrium and between which pair of players. What remains to be seen is the Rawlsian welfare implications of these transfers in equilibrium. Propositions 1,2 and 3 provide insight into those questions.

Remark 5: It is interesting to note that the endogenous transfers happen from B to A (i.e. between the two corner players) only when the ratio of conflict resource allocations (i.e. $\frac{\overline{X_B}}{\overline{X_A}}$ with $\overline{X_B} > \overline{X_A}$) is beyond a threshold (θ) and the threshold and the threshold is an increasing function of the relative prize $\frac{w_{Bc}}{w_{Ac}}$. In order to gain further economic insight into this result let us first understand why endogenous transfers happen in a SPNE in the first place. When B transfers a unit of conflict resource to A that incentivizes C to divert more resources from the conflict against B to the conflict against A. At the same time now, B has one unit of conflict resource less which he can employ against C. Hence, we have two opposing effects. The first effect increases B's payoff while the second effect brings it down. It turns out that when $\frac{\overline{X_B}}{\overline{X_A}} > \theta$ the marginal benefit of endogenous transfer to B due to the diversion of conflict resource of C to the contest against A outweighs the marginal loss that accrues to B due lowered conflict resource (post transfer). The threshold θ being an increasing function of the relative prize $\frac{w_{Bc}}{w_{Ac}}$ is intuitively obvious. A higher $\frac{w_{Bc}}{w_{Ac}}$ makes the second effect (as described above) far more important than the first. Hence the ratio of conflict resource allocations i.e. $\frac{\overline{X_B}}{\overline{X_A}}$, above which it is rational for B to make a transfer to A becomes higher.

Remark 6: C (who is the central player) has no incentive to make any endogenous transfers to any other player ever. The simple reason is that the first effect (i.e. increased benefit due to diverted resources) is not applicable for C since he is engaged in a conflict with both A and B.

Lemma 5: *There exists* $\bar{\alpha} = 0.58$ *such that* $\theta\alpha^2 > 1 \; \forall \; \alpha > \bar{\alpha}$ *and* $\theta\alpha^2 < 1 \; \forall \; \alpha < \bar{\alpha}$.

Proof: Since $\theta = 2\alpha^2 + 1 + 2\,\alpha\sqrt{\alpha^2+1}$, $\theta\alpha^2$ *is clearly* an increasing function of θ. Hence $\theta\alpha^2 = 1$has a unique root $\bar{\alpha}$. Solving $\bar{\alpha} = 0.58$. ■

Proposition 1:

Given that $\alpha \in (\frac{\sqrt{x_A x_B}}{1-x_B}, \frac{1-x_A}{\sqrt{x_A x_B}})$ *and* $x_B > \theta x_A$, *an endogenous transfer leads to an improvement in Rawlsian welfare if*

$min\,\{\frac{1}{1+\theta}[1-x_C][1+\sqrt{\theta}\,\alpha]\,,\ \frac{1}{1+\theta}[1-x_C][\theta\alpha^2+\sqrt{\theta}\,\alpha]\,,\ [1+\alpha^2-\frac{1}{1+\theta}[1-x_C](1+\sqrt{\theta}\,\alpha)^2\} > min\{x_A+\alpha\sqrt{x_A x_B}\,,\ x_B.\alpha^2+\alpha\sqrt{x_A x_B}\,,\ (1+\alpha^2-(\sqrt{x_A}+\alpha\sqrt{x_B})^2)\}$

Proof: The result follows directly from Lemma 3 and Lemma 4 ■

Proposition 1 provides the condition under which the endogenous conflict – resource transfer will be Rawlsian welfare improving. From policy perspective, it provides insight into when it is useful to impose legal sanctions on transfers. The next two propositions look into the Rawlsian welfare maximizing conflict resource endowment vector – whether it can be achieved via endogenous transfers and whether the welfare - maximizing endowment vector (achieved by external redistributive intervention) is transfer – susceptible.

Proposition 2: *The Rawlsian welfare maximizing conflict resource endowment vector* $(x_A^*\,, x_B^*\,, x_C^*)$ *is given by:*

i. *$x_A^* \geq \frac{2}{\alpha^2+2}$ and $x_B^* = x_C^* = \frac{1-x_A^*}{2}$ if $\alpha \geq \sqrt{2}$. In this case, the maximum Rawlsian welfare = w_{AC}*

ii. *$x_B^* \geq \frac{2\alpha^2}{2\alpha^2+1}$ and $x_A^* = x_C^* = \frac{1-x_B^*}{2}$ if $\alpha \leq \frac{1}{\sqrt{2}}$. In this case, the maximum Rawlsian welfare = $w_{AC}.\alpha^2 = w_{BC}$*

iii. *$x_A^* = \frac{1}{6}(1+\alpha^2)$, $x_B^* = \frac{1}{6}\left(1+\frac{1}{\alpha^2}\right)$ and $x_C = 1-\frac{1}{6}\left(\alpha^2+\frac{1}{\alpha^2}+2\right)$ if $\alpha \in \left(\frac{1}{\sqrt{2}}, \sqrt{2}\right)$. In this case, the maximum Rawlsian welfare = $\frac{w_{AC}+w_{BC}}{3}$*

Proof: $\alpha \geq \sqrt{2}$ implies $w_{BC} \geq 2w_{AC}$. Clearly the Rawlsian welfare is maximized when C does not fight A and has equal amount of conflict resources as B. C does not fight A if $\alpha \geq \frac{1-x_A}{\sqrt{x_A x_B}}$. This leads to (i). (ii) can be proved exactly analogously. If both the 'battlefields' are alive, the Rawlsian welfare maximizing equilibrium will be characterized by $\pi_A{}^* = \pi_B{}^* = \pi_C{}^* = \frac{w_{AC}+w_{BC}}{3}$ which in turn happens only if $x_A = \frac{1}{6}(1+\alpha^2)$, $x_B = \frac{1}{6}\left(1+\frac{1}{\alpha^2}\right)$ and $x_C = 1-\frac{1}{6}\left(\alpha^2+\frac{1}{\alpha^2}+2\right)$. But the necessary condition for both battlefields being alive implies $\alpha \in \left(\frac{\sqrt{x_A x_B}}{1-x_B}, \frac{1-x_A}{\sqrt{x_A x_B}}\right)$ i.e $\alpha \in \left(\frac{\alpha(1+\alpha^2)}{5\alpha^2-1}, \frac{5\alpha-\alpha^3}{1+\alpha^2}\right)$ (in the Rawlsian welfare maximizing equilibrium) which holds true if $\alpha \in \left(\frac{1}{\sqrt{2}}, \sqrt{2}\right)$. Hence the proof for (iii). ■

Proposition 2 clearly characterizes the Rawlsian welfare maximizing outcomes for the three cases i.e. $\frac{w_{BC}}{w_{AC}} \geq 2$, $\frac{w_{BC}}{w_{AC}} \leq \frac{1}{2}$ and $\frac{w_{BC}}{w_{AC}} \in \left(\frac{1}{2}, 2\right)$. In the first two cases the Rawlsian welfare maximizing outcomes consist of 'one fight endowment vectors' while it is a 'two fight endowment vector' in the last case. The economic rationale is obvious from the proof of the proposition.

Proposition 3:

i. *The Rawlsian welfare maximizing conflict resource endowment vector is necessarily transfer – resistant*

ii. *No strictly positive endogenous transfer leads to the Rawlsian welfare maximizing endowment vector.*

Proof: If $\alpha \in \left[0, \frac{1}{\sqrt{2}}\right] \cup \left[\sqrt{2}, \infty\right)$ the Rawlsian welfare maximizing conflict resource allocations are 'one fight endowment vector's and by Lemma 2 they are transfer resistant and no strictly positive endogenous transfer(s) leads to them. If $\alpha \in \left(\frac{1}{\sqrt{2}}, \sqrt{2}\right)$ the Rawlsian welfare conflict resource allocation is given in (iii) of Proposition 2. And $\frac{x_B^*}{x_A^*} = \frac{1}{\alpha^2} = \frac{\theta}{\theta\alpha^2} > \theta$ only if $\alpha < \bar{\alpha} = 0.58$ (using Lemma 5). And $0.58 \in \left(0, \frac{1}{\sqrt{2}}\right)$. Thus $\frac{x_B^*}{x_A^*} < \theta$ for $\alpha \in \left(\frac{1}{\sqrt{2}}, \sqrt{2}\right)$ which implies the transfer resistance of the conflict resource allocation. Also, any endogenous transfer will lead to a post – transfer endowment vector where $\frac{x_B^*}{x_A^*} = \theta$. Since the Rawlsian welfare maximizing endowment vector has $\frac{x_B^*}{x_A^*} < \theta$, it cannot be reached via a a strictly positive rational transfer. ■

Remark 7: If $\alpha \in \left[0, \frac{1}{\sqrt{2}}\right]$ i.e. if $\frac{w_{BC}}{w_{AC}} \leq \frac{1}{2}$ then clearly it is Rawlsian welfare maximizing if C engages in the conflict A alone which is ensured by the conflict resource allocation to B being higher than a threshold and given this A and C should have the same endowment of conflict resources (yielding accrual of equal shares of w_{AC} to A and C) for Rawlsian welfare maximization. Given an endowment vector of this kind, A of course does not have an incentive to transfer any resource to B because that will only increase C's disincentive to fight B (and C is not fighting B already). B too will not transfer any resource to A because he is getting the whole of w_{BC} which is the maximum he can get anyway and a transfer to A might increase C's incentive to fight B (if B becomes sufficiently weak post – transfer). Thus, we see that this 'one-fight endowment vector' is transfer – resistant. An exactly symmetric argument holds if $\alpha \in [\sqrt{2}, \infty)$ i.e. $\frac{w_{BC}}{w_{AC}} \geq 2$. No transfer can lead to a transition from a 'two – fight endowment vector' to a 'one-fight endowment vector' since such a transfer is clearly not rational. Hence the above -mentioned one-fight endowment vectors are not only transfer resistant they are also unattainable via a rational transfer. If $\alpha \in \left(\frac{1}{\sqrt{2}}, \sqrt{2}\right)$ the Rawlsian welfare maximizing endowment vector has $\frac{x_B^*}{x_A^*} = \frac{1}{\alpha^2} < \theta \; \forall \; \alpha \in \left(\frac{1}{\sqrt{2}}, \sqrt{2}\right)$ $(by\ Lemma\ 5)$. Since a rational transfer takes place from B to A if and only if $\overline{X_B} > \theta \overline{X_A}$ $(i.e. x_B > \theta x_A)$ and the optimal transfer is given by $\delta^* = \frac{\overline{X_B} - \theta \overline{X_A}}{1+\theta}$ (which makes $\overline{X_B} = \theta \overline{X_A}$ (i.e. $x_B = \theta x_A$) in the post – transfer endowment) hence the Rawlsian welfare maximizing endowment vector is transfer resistant and also unattainable via rational transfers.

4. Concluding Remarks:

The paper throws light on two key findings. Firstly, though endogenous transfers of conflict – resources between players can lead to Rawlsian welfare improvement, welfare maximization necessarily requires external intervention. Secondly the Rawlsian welfare maximizing conflict resource allocation is transfer – resistant i.e. stable and immune to endogenous transfers. These results have the following economic implications: The Rawlsian welfare changes due to the transfers can be thought of as 'invisible hand' effects since the transfers happen endogenously. The paper shows that the Rawlsian welfare optimum cannot be reached via these 'invisible hand' effects and thereby insinuating towards the need of external intervention. Also, once the optimum conflict resource allocation is achieved it is immune to any endogenous transfer, thereby lending it stability. A generalization of this analysis for all possible networks of bilateral contests is left for future work.

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Appendix

<u>Proof of Lemma 1</u>:

Using (1)

$$\frac{\partial \pi_C}{\partial X_{CA}} = \frac{w_{BC}\,\overline{X_B}}{(X_{CB}+\overline{X_B}\,)^2}\,(-1) + \frac{\overline{X_A}}{(\,\overline{X_A}+X_{CA})^2}\,.\,w_{AC} \tag{2}$$

$$\lim_{X_{CA}\to 0}\frac{\partial \pi_C}{\partial X_{CA}} = \frac{w_{BC}\,\overline{X_B}}{(\overline{X_C}+\overline{X_B}\,)^2}\,(-1) + \frac{w_{AC}}{\overline{X_A}} < 0 \text{ or } \frac{w_{BC}}{w_{AC}} > \frac{(\,\overline{X_C}+\,\overline{X_B})^2}{\overline{X_A}\;\overline{X_B}} = \frac{(x_C+x_B)^2}{x_A x_B} \tag{3}$$

Similarly $\lim_{X_{CB}\to 0}\frac{\partial \pi_C}{\partial X_{CB}} < 0$ implies $\frac{w_{BC}}{w_{AC}} < \frac{\overline{X_A}\;\overline{X_B}}{(\,\overline{X_C}+\,\overline{X_A})^2} = \frac{x_A x_B}{(x_C+x_A)^2}$ (4)

In order to ensure that in equilibrium $(X_{CA}, X_{CB}) > (0,0)$ the necessary conditions are $\lim_{X_{Ci}\to 0}\frac{\partial \pi_C}{\partial X_{Ci}} > 0\ \forall\ i \in \{A, B\}$. From (3) and (4) that happens when $\frac{w_{BC}}{w_{AC}} \in (\frac{x_A x_B}{(x_C+x_A)^2}, \frac{(x_C+x_B)^2}{x_A x_B})$.

or $\alpha^2 = \frac{w_{BC}}{w_{AC}} \in (\frac{x_A x_B}{(1-x_B)^2}, \frac{(1-x_A)^2}{x_A x_B})$ i.e $\alpha \in (\frac{\sqrt{x_A x_B}}{1-x_B}, \frac{1-x_A}{\sqrt{x_A x_B}})$. ■

<u>Proof of Lemma 2</u>:

From (3) C does not fight with A if $\alpha \in \left(\frac{1-x_A}{\sqrt{x_A x_B}}, \infty\right)$. Clearly $\frac{1-x_A}{\sqrt{x_A x_B}}$ is a decreasing function of x_A. Hence $(\frac{1-x_A}{\sqrt{x_A x_B}}, \infty) \subseteq (\frac{1-x_A'}{\sqrt{x_A' x_B}}, \infty)$ if $x_A' > x_A$. Hence A has no incentive to transfer any resource to B and B has no reason to transfer to A. From (4) C does not fight with B if

$\alpha \in \left(0, \frac{\sqrt{x_A x_B}}{1-x_B}\right)$. Clearly $\frac{\sqrt{x_A x_B}}{1-x_B}$ is increasing in x_B. Therefore $(0, \frac{\sqrt{x_A x_B}}{1-x_B}) \subseteq (0, \frac{\sqrt{x_A x_B'}}{1-x_B'})$ if $x_B' > x_B$. Hence B has no incentive to transfer any resource to A and A has no reason to transfer to B. This completes the proof of (i). The proof for (ii) follows directly from (i). ■

<u>Proof of Lemma 3</u>:

By (1) and (2) the first order condition yields

$$X_{CA}^* = \frac{\sqrt{\overline{X_A}\;\overline{w_{Ac}}}}{\sqrt{\overline{X_A}\;\overline{w_{Ac}}}+\sqrt{\overline{X_B w_{Bc}}}}\,(\overline{X_A} + \overline{X_B} + \overline{X_C}) - \overline{X_A} \text{ and } X_{CB}^* = \overline{X_C} - X_{CA}^* \tag{5}$$

Also using (2), $\frac{\partial^2 \pi_C}{\partial X_{CA}{}^2} = w_{AC}.(-2).\frac{\overline{X_A}}{(\overline{X_A}+X_{CA})^3} + w_{BC}.(-1).(-2).\frac{\overline{X_B}}{(\overline{X_B}+X_{CB})^3}.\frac{\partial X_{CB}}{\partial X_{CA}}$

$$= w_{AC}.(-2).\frac{\overline{X_A}}{(\overline{X_A}+X_{CA})^3} + w_{BC}.(-1).(-2).\frac{\overline{X_B}}{(\overline{X_B}+X_{CB})^3}.(-1) < 0 \quad (6)$$

From (5) and (6) the result follows directly. ■

Proof of Lemma 4:

From Lemma 3, in equilibrium $\pi_B{}^* = \frac{\overline{X_B}w_{Bc}+\sqrt{\overline{X_A}\ \overline{X_B}w_{Bc}w_{Ac}}}{\overline{X_A}+\overline{X_B}+\overline{X_C}}$, $\pi_A{}^* = \frac{\overline{X_A}w_{Ac}+\sqrt{\overline{X_A}\ \overline{X_B}w_{Bc}w_{Ac}}}{\overline{X_A}+\overline{X_B}+\overline{X_C}}$ and $\pi_C{}^* = (w_{Bc} + w_{Ac}) - \frac{(\sqrt{\overline{X_B}w_{Bc}}+\sqrt{\overline{X_A}\ w_{Ac}})^2}{\overline{X_A}+\overline{X_B}+\overline{X_C}}$.

Clearly, since $\pi_C{}^* = (w_{Bc} + w_{Ac}) - \frac{(\sqrt{\overline{X_B}w_{Bc}}+\sqrt{\overline{X_A}\ w_{Ac}})^2}{\overline{X_A}+\overline{X_B}+\overline{X_C}}$ in SPNE $t_{Cj} = 0\ \forall\, j \in \{A, B\}$ and $t_{iC} = 0\ \forall\, i \in \{A, B\}$ since any transfer from C to either A or B will necessarily decrease $\pi_C{}^*$.

If $t_{BA} = \delta$ and $a_A(t_{BA}) = 1$, the equilibrium payoffs of B and A are given by $\pi_B{}^*(\delta)$ *and* $\pi_A{}^*(\delta)$ respectively. Clearly,

$$\pi_B{}^*(\delta) = \frac{(\overline{X_B}-\delta).w_{Bc} + \sqrt{w_{Bc}\,w_{Ac}}\sqrt{(\overline{X_A}+\delta)(\overline{X_B}-\delta)}}{\overline{X_A}+\overline{X_B}+\overline{X_C}}$$

$$\pi_A{}^*(\delta) = \frac{(\overline{X_A}+\delta).w_{Ac} + \sqrt{w_{Bc}\,w_{Ac}}\sqrt{(\overline{X_A}+\delta)(\overline{X_B}-\delta)}}{\overline{X_A}+\overline{X_B}+\overline{X_C}}$$

The difference in equilibrium payoffs of A and B when there is a transfer of δ from B to A vis a vis when there is no transfer is given by:

Hence $\pi_B{}^*(\delta) - \pi_B{}^* = \frac{\left[\sqrt{(\overline{X_A}+\delta)(\overline{X_B}-\delta)}-\sqrt{\overline{X_A}\ \overline{X_B}}\right].\sqrt{w_{Bc}\,w_{Ac}} - \delta.w_{Bc}}{\overline{X_A}+\overline{X_B}+\overline{X_C}}$

$$\pi_A{}^*(\delta) - \pi_A{}^* = \frac{\left[\sqrt{(\overline{X_A}+\delta)(\overline{X_B}-\delta)}-\sqrt{\overline{X_A}\ \overline{X_B}}\right].\sqrt{w_{Bc}\,w_{Ac}} + \delta.w_{Ac}}{\overline{X_A}+\overline{X_B}+\overline{X_C}}$$

Define f(δ) = $\left[\sqrt{(\overline{X_A}+\delta)(\overline{X_B}-\delta)} - \sqrt{\overline{X_A}\ \overline{X_B}}\right].\sqrt{w_{Bc}\,w_{Ac}} - \delta.w_{Bc}$

And g(δ) = $\left[\sqrt{(\overline{X_A}+\delta)(\overline{X_B}-\delta)} - \sqrt{\overline{X_A}\ \overline{X_B}}\right].\sqrt{w_{Bc}\,w_{Ac}} + \delta.w_{Ac}$

Therefore $\pi_B{}^*(\delta) - \pi_B{}^* = \frac{f(\delta)}{\overline{X_A}+\overline{X_B}+\overline{X_C}}$ and $\pi_A{}^*(\delta) - \pi_A{}^* = \frac{g(\delta)}{\overline{X_A}+\overline{X_B}+\overline{X_C}}$.

$$g'(\delta) = \frac{(\overline{X_B})-(\overline{X_A})}{2\sqrt{(\overline{X_A}+\delta)(\overline{X_B}-\delta)}}.\sqrt{w_{Bc}\,w_{Ac}} + w_{Ac} > 0 \text{ since } \overline{X_B} > \overline{X_A}. \quad (7)$$

f(0) = 0, and f($\overline{X_B}$) < 0. $f'(\delta) = -w_{Bc} + \frac{1}{2}.\sqrt{w_{Bc}\,w_{Ac}}\,[z - \frac{1}{z}]$ where z = $\sqrt{\frac{\overline{X_B}-\delta}{\overline{X_A}+\delta}}$. (8)

Using (8) $f''(\delta) = \frac{1}{2}.\sqrt{w_{Bc}\,w_{Ac}}\,.\,[1+\frac{1}{z^2}].\frac{\partial z}{\partial \delta} < 0$, since $\frac{\partial z}{\partial \delta} < 0$. (9)

Using (8) $\lim_{\delta \to 0} f'(\delta) > 0$ if $\sqrt{\frac{\overline{X_B}}{\overline{X_A}}} > \alpha + \sqrt{\alpha^2 + 1}$ where $\alpha = \sqrt{\frac{w_{Bc}}{w_{Ac}}}$ i.e. $\lim_{\delta \to 0} f'(\delta) > 0$ if $\overline{X_B} > \theta \overline{X_A}$ $where$ $\theta = (\alpha + \sqrt{\alpha^2 + 1})^2 = 2\alpha^2 + 1 + 2\,\alpha\sqrt{\alpha^2 + 1}$. (10)

Using (7), (9) and (10) it can be inferred that in the SPNE there will be a strictly positive transfer of conflict resources from B to A only if $\overline{X_B} > \theta \overline{X_A}$ (This completes the proof for (ii) and (iii)).

Using (8), $f'(\delta) = 0$ implies $z - \frac{1}{z} = 2\sqrt{\frac{w_{Bc}}{w_{Ac}}}$ i.e z = $\alpha + \sqrt{\alpha^2 + 1}$ where $\alpha = \sqrt{\frac{w_{Bc}}{w_{Ac}}}$ where z = $\sqrt{\frac{\overline{X_B} - \delta}{\overline{X_A} + \delta}}$. Hence the optimal transfer δ^* satitsfies the condition $\frac{\overline{X_B} - \delta}{\overline{X_A} + \delta} = 2\alpha^2 + 1 + 2\,\alpha\sqrt{\alpha^2 + 1} = \theta$. Therefore $\delta^* = \frac{\overline{X_B} - \theta\,\overline{X_A}}{1 + \theta}$. (This completes the proof for (iv)). ■