Economics Bulletin

Volume 45, Issue 1

Growth with human capital accumulation and declining population: an augmented Solow model approach

Hiroaki Sasaki
Kyoto University

Taichi Hori
Kyoto University

Rokuhisa Hasegawa
Kyoto University

Shigehiro Tajiri
Kyoto University

Kaito Nakamura
Kyoto University

Abstract

This study examines how the long-run growth rate of per capita income is determined when population growth is negative. It uses the augmented Solow growth model as a tool for this investigation. The results reveal four distinct types of dynamics, depending on the parameter combinations. In all these dynamics, the long-run growth rate of per capita income remains positive. This finding implies that sustainable growth in per capita income is achievable, even in the context of negative population growth.

The authors declare none.
Citation: Hiroaki Sasaki and Taichi Hori and Rokuhisa Hasegawa and Shigehiro Tajiri and Kaito Nakamura, (2025) "Growth with human capital accumulation and declining population: an augmented Solow model approach", *Economics Bulletin*, Volume 45, Issue 1, pages 63-72
Contact: Hiroaki Sasaki - sasaki.hiroaki.7x@kyoto-u.ac.jp, Taichi Hori - horitaichi1414@gmail.com, Rokuhisa Hasegawa - hasegawa.rokuhisa.55c@st.kyoto-u.ac.jp, Shigehiro Tajiri - tajiri.shigehiro.83v@st.kyoto-u.ac.jp, Kaito Nakamura - nakamura.kaito.78p@st.kyoto-u.ac.jp.
Submitted: March 07, 2024. **Published:** March 30, 2025.

1 Introduction

The phenomenon of population decline is becoming a global issue. Countries such as Germany and Italy have already experienced this decline, and Japan has been witnessing a continuous decrease in population since 2010. The United Nations World Population Prospects 2019 indicates that high-income economies, as classified by the World Bank, are projected to see a population decline post-2050, and middle-income economies are expected to follow suit after 2075. Given these circumstances, there is a growing emergence of economic growth models that take into account population decline.

Christiaans (2011) develops a Solow model that incorporates increasing returns to scale due to a positive externality with capital accumulation, showing that the long-run growth rate of per capita income can remain positive, even if the population growth rate is negative. This result is possible because the effect of capital deepening becomes more powerful when the absolute value of the population decline rate is sufficiently large. Sasaki and Hoshida (2017) apply an R&D growth model, following the approach of Jones (1995), and consider negative population growth. They discover that while R&D activities may stagnate as the population decreases, the effect of capital deepening intensifies, leading to positive growth in per capita income.[1] In these models, when the rate of population decline is high, the capital stock per effective labor continues to rise, meaning capital deepening occurs. Consequently, the balanced growth path (BGP) typically seen in growth models does not exist. However, owing to decreasing returns in relation to capital in the production function, the growth rate of capital stock per effective labor decreases and converges to a positive value. Consequently, the growth rate of per capita income also converges to a positive value. This is a growth path specific to a negative population growth economy.

The aforementioned studies consider the accumulation of physical capital and endogenous technological progress, but do not consider the accumulation of human capital. Elgin and Tumen (2012) and Bucci (2023) incorporate a Lucas (1988) style of human capital accumulation into a continuous time growth model. Both studies conclude that, under certain conditions, the long-run per capita income growth rate can be positive even if the population growth rate is negative..

The two studies mentioned above focus their analysis on the BGP, where the primary variables in models consistently increase at a uniform constant growth rate. Con-

[1] Jones (2022) presents an R&D growth model that endogenizes the population growth rate, but omits capital accumulation. He shows that when population growth is negative, sustained growth of per capita income is unattainable because R&D activities stagnate.

sequently, along the BGP, ratios of variables such as the output-capital ratio or capital stock per effective labor remain constant. In contrast, Christiaans (2011) and Sasaki and Hoshida (2017) direct their analysis toward the Negative Population Growth Path (NPGP), where the output-capital ratio converges to zero and capital stock per effective labor becomes infinite in the long run.

Drawing from the above observations, we apply the augmented Solow growth model by Mankiw, Romer, and Weil (1992), which considers the accumulation of human capital. Similarly to the approaches of Christiaans (2011) and Sasaki and Hoshida (2017), we explore a growth path that is specific to an economy experiencing negative population growth. We then explain the relationship between the rate of population decline and the growth rate of per capita income.

2 Model

The model aligns with the one presented by Mankiw, Romer, and Weil (1992). The production of final goods involves physical capital K, human capital H, and labor L. The production function adopts the Cobb–Douglas form, which exhibits constant returns to scale: $Y = K^{\alpha} H^{\beta} (AL)^{1-\alpha-\beta}, \quad \alpha \in (0,1),\ \beta \in (0,1),\ \alpha+\beta \in (0,1) \Longrightarrow y = k^{\alpha} h^{\beta}$, where Y denotes output; A is the index of labor-augmenting technological progress; α is the output-elasticity of physical capital; and β is the output-elasticity of human capital. All parameters are larger than zero and less than unity. We define $y = Y/(AL)$, $k = K/(AL)$, and $h = H/(AL)$.

Let the population growth rate and labor-augmenting progress rate be n and g, respectively. We assume that $\dot{L}/L = n < 0$ and $\dot{A}/A = g > 0$. Both growth rates are assumed to be constant. The population growth rate is negative.

Let the investment rate of physical capital and that of human capital be $s_k \in (0,1)$ and $s_h \in (0,1)$, respectively. Suppose that s_k and s_h are constant fractions of total output. Then, the dynamical equations of physical capital and human capital are respectively given by $\dot{K} = s_k Y - \delta_k K$ and $\dot{H} = s_h Y - \delta_h H$, where $\delta_k \in (0,1)$ and $\delta_h \in (0,1)$ are the depreciation rates of physical and human capital, respectively.[2]

[2]There are some empirical studies that estimate the depreciation rate of human capital. Using data of U.K. and Netherlands, Groot (1998) estimates it as 11–17% per year. Arrazola, Risueno, and Sanz (2005) consider EU economy and reveal that the depreciation rates of human capital differ for those who are unemployed and for those who are employed during the coverage period. The depreciation rate for the unemployed is 2.3% per year while that for the employed is 1.4% per year. Dinerstein, Megalokonomou, and Yannelis (2022) find that the depreciation rate of skill in Greece is 4.3% per year. From these studies, the depreciation rate of human capital is 1.4–17% per year. By contrast, the depreciation rate of physical capital is usually 3–7% per year. Accordingly, we cannot

Summarizing the above equations, the dynamical equations of k and h are as follows: $\dot{k} = s_k k^\alpha h^\beta - (n+g+\delta_k)k$ and $\dot{h} = s_h k^\alpha h^\beta - (n+g+\delta_h)h$. When $n+g+\delta_k < 0$ or $n+g+\delta_h < 0$ holds, for $k > 0$ and $h > 0$, we have $\dot{k} > 0$ or $\dot{h} > 0$, which suggests that k or h continues to increase. In this case, the usual steady states of k and h do not exist, because $\dot{k} = 0$ or $\dot{h} = 0$ is never obtained, and we obtain the growth path specific to an NPGP.

The growth rates of k and h are given by

$$\frac{\dot{k}}{k} = s_k \frac{h^\beta}{k^{1-\alpha}} - (n+g+\delta_k), \tag{1}$$

$$\frac{\dot{h}}{h} = s_h \frac{k^\alpha}{h^{1-\beta}} - (n+g+\delta_h). \tag{2}$$

The growth rate of per capita income $g_{Y/L}$ is the sum of the growth rate of y and that of A, given by

$$g_{Y/L} = g + \underbrace{\alpha \left[s_k \frac{h^\beta}{k^{1-\alpha}} - (n+g+\delta_k) \right] + \beta \left[s_h \frac{k^\alpha}{h^{1-\beta}} - (n+g+\delta_h) \right]}_{\equiv g_y}. \tag{3}$$

When $n+g+\delta_k < 0$ or $n+g+\delta_h < 0$, we cannot use the usual phase diagram analysis, as employed in Mankiw, Romer, and Weil (1992), because we cannot obtain $\dot{k} = 0$ or $\dot{h} = 0$. Therefore, considering equations (1) and (2), we introduce the following new state variables: $x \equiv h^\beta / k^{1-\alpha}$ and $z \equiv k^\alpha / h^{1-\beta}$. The differential equations of the newly introduced state variables are given by

$$\dot{x} = x[-(1-\alpha)s_k x + \beta s_h z + C_1], \quad C_1 = (1-\alpha)(n+g+\delta_k) - \beta(n+g+\delta_h), \tag{4}$$

$$\dot{z} = z[\alpha s_k x - (1-\beta)s_h z + C_2], \quad C_2 = (1-\beta)(n+g+\delta_h) - \alpha(n+g+\delta_k). \tag{5}$$

The parameters C_1 and C_2 can be positive or negative, and the size relationship between them is ambiguous. Substituting x and z into equation (3), we obtain

$$g_{Y/L} = g + \alpha [s_k x - (n+g+\delta_k)] + \beta [s_h z - (n+g+\delta_h)]. \tag{6}$$

To draw the phase diagram of (x, z), we find the loci of $\dot{x} = 0$ and $\dot{z} = 0$:

$$\dot{x} = 0 \Longrightarrow z = \frac{s_k}{s_h} \cdot \frac{1-\alpha}{\beta} x - \frac{C_1}{\beta s_h}, \tag{7}$$

say which is larger, δ_k or δ_h.

$$\dot{z} = 0 \Longrightarrow z = \frac{s_k}{s_h} \cdot \frac{\alpha}{1-\beta} x + \frac{C_2}{(1-\beta)s_h}. \tag{8}$$

The slopes of $\dot{x} = 0$ and $\dot{z} = 0$ are positive. For the size relationship between them, we obtain

$$\frac{s_k}{s_h} \cdot \frac{1-\alpha}{\beta} - \frac{s_k}{s_h} \cdot \frac{\alpha}{1-\beta} = \frac{s_k}{s_h} \cdot \frac{1-\alpha-\beta}{\beta(1-\beta)} > 0. \tag{9}$$

Hence, the slope of $\dot{x} = 0$ is steeper than that of $\dot{z} = 0$.

The intercepts of $\dot{x} = 0$ and $\dot{z} = 0$ can be positive or negative. For the size relationship between them, we obtain

$$\begin{aligned} -\frac{C_1}{\beta s_h} - \frac{C_2}{(1-\beta)s_h} &= -\frac{1}{s_h} \cdot \frac{(1-\beta)C_1 + \beta C_2}{\beta(1-\beta)} \\ &= -\frac{1}{s_h} \cdot \frac{(1-\alpha-\beta)(n+g+\delta_k)}{\beta(1-\beta)}. \end{aligned} \tag{10}$$

Hence, this sign depends on whether $n + g + \delta_k > 0$ or $n + g + \delta_k < 0$.

3 Analysis

We obtain four outcomes, depending on the combination of the signs of $n+g+\delta_k$ and $n + g + \delta_h$.[3]

3.1 Case 1: $n + g + \delta_k > 0$ and $n + g + \delta_h > 0$

We define Case 1 as a case in which $n < 0$ but its absolute value is relatively small; hence, both $n+g+\delta_k > 0$ and $n+g+\delta_h > 0$ hold. Case 1 is compatible with $\delta_k < \delta_h$ or $\delta_k > \delta_h$. Case 1 is the same as the case examined by Mankiw, Romer, and Weil (1992).

In Case 1, C_1 and C_2 can be positive or negative. Possible combinations are (a) $C_1 > 0$, $C_2 > 0$, and $C_1 < C_2$ ($\delta_k < \delta_h$), (b) $C_1 < 0$ and $C_2 > 0$ ($\delta_k < \delta_h$), (c) $C_1 > 0$, $C_2 > 0$, and $C_1 > C_2$ ($\delta_k > \delta_h$), and (d) $C_1 > 0$ and $C_2 < 0$ ($\delta_k > \delta_h$).[4] Phase diagrames are shown in Figures 1–3.

[3]If we assume $\delta_k = \delta_h$ as Mankiw, Romer, and Weil (1992), we obtain $n + g + \delta_k = n + g + \delta_h$, which leads to $C_1 = C_2$. When $C_1 = C_2 > 0$, we obtain Cases 1-(a) and 1-(c). When $C_1 = C_2 < 0$, we obtain Cases 4-(b) and 4-(d). When $\delta_k = \delta_h$, we cannot obtain Cases 2 and 3.

[4]We cannot have $C_1 < 0$ and $C_2 < 0$ because $\alpha + \beta < 1$.

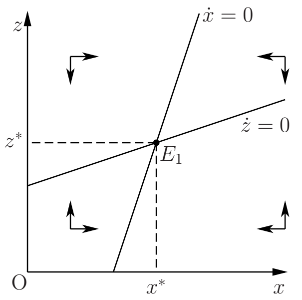

Figure 1: Phase diagram in Cases 1-(a) and 1-(c)

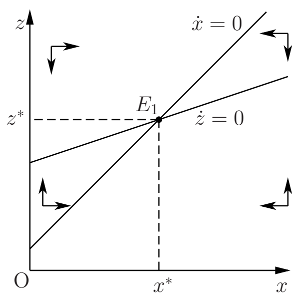

Figure 2: Phase diagram in Case 1-(b)

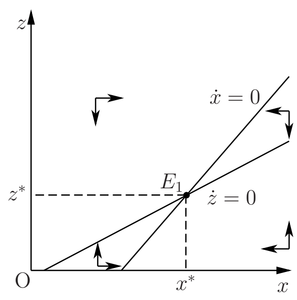

Figure 3: Phase diagram in Case 1-(d)

In all cases, $\dot{x} = 0$ and $\dot{z} = 0$ have an intersection, which gives the steady state in Case 1, E_1:

$$x^* = \frac{n + g + \delta_k}{s_k} > 0, \quad z^* = \frac{n + g + \delta_h}{s_h} > 0. \tag{11}$$

From Figures 1–3, the steady state is stable. The long-run growth rate of per capita income $g^*_{Y/L}$ is equal to the labor augmenting technological progress rate:

$$g^*_{Y/L} = g > 0. \tag{12}$$

3.2 Case 2: $n + g + \delta_k < 0$ and $n + g + \delta_h > 0$

We define Case 2 as a case in which $n < 0$ and its absolute value is relatively large; hence, both $n+g+\delta_k < 0$ and $n+g+\delta_h > 0$ hold.[5] Case 2 is compatible with $\delta_k < \delta_h$.

In Case 2, we have $C_1 < 0$ and $C_2 > 0$. Hence, the intercept of $\dot{x} = 0$ and that of $\dot{z} = 0$ are positive. Since $n + g + \delta_k < 0$ in Case 2, the sign of the RHS of equation (10) is positive. This means that the intercept of $\dot{x} = 0$ is larger than that of $\dot{z} = 0$. Hence, the phase diagram is shown in Figure 4.

[5] If $g = 0.01$ and $\delta_k = 0.03$, n must be smaller than -4% to satisfy the condition $n + g + \delta_k < 0$.

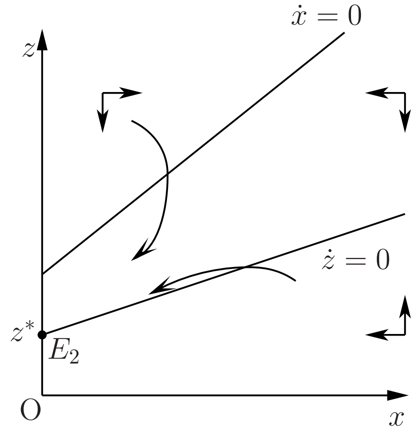

Figure 4: Phase diagram in Case 2

From Figure 4, the economy converges to the corner solution E_2, and the long-run situations are as follows:

$$x^* = 0, \quad z^* = \frac{\overbrace{(1-\beta)(n+g+\delta_h) - \alpha(n+g+\delta_k)}^{\equiv C_2 > 0}}{(1-\beta)s_h} > 0. \tag{13}$$

From equation (6), the long-run growth rate of per capita output is given by

$$g^*_{Y/L} = g - \frac{\alpha}{1-\beta}\underbrace{(n+g+\delta_k)}_{-} > 0. \tag{14}$$

3.3 Case 3: $n + g + \delta_k > 0$ and $n + g + \delta_h < 0$

We define Case 3 as a case in which $n < 0$ and its absolute value is relatively large; hence, both $n+g+\delta_k > 0$ and $n+g+\delta_h < 0$ hold. Case 3 is compatible with $\delta_k > \delta_h$.

In Case 3, we have $C_1 > 0$ and $C_2 < 0$. Hence, the intercept of $\dot{x} = 0$ and that of $\dot{z} = 0$ are negative. The difference of the intercepts is given by equation (10). Since $n+g+\delta_k > 0$ in Case 3, the sign of the RHS of equation (10) is negative. This means that the intercept of $\dot{x} = 0$ is smaller than that of $\dot{z} = 0$. Hence, the phase diagram is shown in Figure 5.

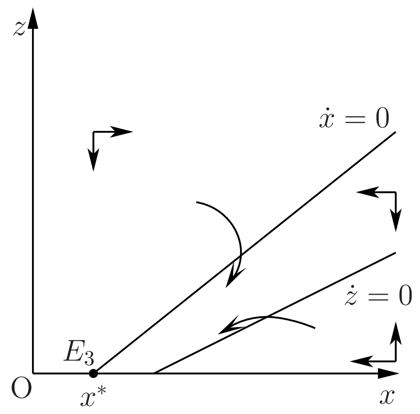

Figure 5: Phase diagram in Case 3

From Figure 5, the economy converges to the corner solution E_3, and the long-run situations are as follows:

$$x^* = \frac{\overbrace{(1-\alpha)(n+g+\delta_k) - \beta(n+g+\delta_h)}^{\equiv C_1 > 0}}{(1-\alpha)s_k}, \quad z^* = 0. \tag{15}$$

From equation (6), the long-run growth rate of per capita income is given by

$$g^*_{Y/L} = g - \frac{\beta}{1-\alpha}\underbrace{(n+g+\delta_h)}_{-} > 0. \tag{16}$$

3.4 Case 4: $n + g + \delta_k < 0$ and $n + g + \delta_h < 0$

We define Case 4 as a case in which $n < 0$ and its absolute value is extremely large; hence, both $n+g+\delta_k < 0$ and $n+g+\delta_h < 0$ hold. Case 4 is compatible with $\delta_k < \delta_h$ or $\delta_k > \delta_h$. In Case 4, C_1 and C_2 can be positive or negative. Possible combinations are (a) $C_1 < 0$, $C_2 > 0$ ($\delta_k < \delta_h$), (b) $C_1 < 0$, $C_2 < 0$, and $C_1 < C_2$ ($\delta_k < \delta_h$), (c) $C_1 > 0$ and $C_2 < 0$ ($\delta_k > \delta_h$), and (d) $C_1 < 0$, $C_2 < 0$, and $C_1 > C_2$ ($\delta_k > \delta_h$).[6]

In Cases 4-(b) and 4-(d), we obtain Figure 6.

[6]We cannot have $C_1 > 0$, $C_2 > 0$, and $C_1 < C_2$ ($\delta_k < \delta_h$) or $C_1 > 0$, $C_2 > 0$, and $C_1 > C_2$ ($\delta_k > \delta_h$) because $\alpha + \beta < 1$.

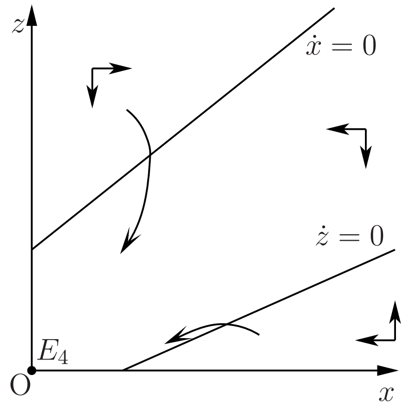

Figure 6: Phase diagram for Cases 4-(b) and 4-(d)

From Figure 6, we find that the economy converges to the origin, E_4, and the long-run situations are as follows:

$$x^* = z^* = 0. \tag{17}$$

From equation (6), the long-run growth rate of per capita output is given by

$$g^*_{Y/L} = g - \alpha \underbrace{(n + g + \delta_k)}_{-} - \beta \underbrace{(n + g + \delta_h)}_{-} > 0. \tag{18}$$

In Case 4-(a), we obtain Figure 4. This case is essentially identical with Case 2. Hence, the long-run growth rate of per capita income $g^*_{Y/L}$ in Case 4-(a) is given by equation (14), which is positive.

In Case 4-(c), we obtain Figure 5. This case is essentially identical with Case 3. Hence, the long-run growth rate of per capita income $g^*_{Y/L}$ in Case 4-(c) is given by equation (16), which is positive.

4 Conclusion

This study examines the issue of a decreasing population within the context of the augmented Solow growth model by Mankiw, Romer, and Weil (1992), which incorporates human capital accumulation. The study investigates whether the long-run growth rate of per capita income remains positive when the population growth rate is negative. The analysis reveals four potential scenarios based on the parameter sizes. In each scenario, the long-run growth rate of per capita income is positive.

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