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Modeling Brazilian federal government fiscal reaction in the time-frequency domain

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Abstract

We use unconditional wavelet to address fiscal reaction in Brazil over the period from 2001 to 2022. We identify medium-term insolvency of net debt only in the period 2003-2004. We find antiphase (short and medium term) co-movements led by the primary balance in the subprime crisis, during the 2016-2017 fiscal crisis and in the recent pandemic. There is this same evidence with low frequency between 2013 and 2019. Mainly after the change of fiscal regime in 2017, government has issued public bonds as a solution to balance public accounts. Although the current level of debt to GDP is not insolvent, the forecast of 60% at the end of 2022 and the high annual inflation level (12.13% in April 2022) require a lot of attention.

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1. Introduction

There is a strand of literature on public finance studying the drivers and consequences of indebtedness as collateral for issuing bonds or accessing borrowing to finance investments or even current expenditure. Some of the pioneers in this theoretical literature are Modigliani (1961), Mishan (1963) and Diamond (1965). In the 70’s, the discussion promoted by Barro (1974) was important for the literature to recognize that public debt plays an important role in theoretical analysis of monetary and fiscal effects. We also highlight the theoretical approach proposed by Barro (1979), seen as the usual starting point of any discussion on sustainability of a public debt issue by a large national government.

Until then, one could expect a nonstationary debt-income ratio implied by some models of optimal government finance. However, a high and growing debt-GDP ratio was viewed as worrisome by macroeconomic models with limited taxation (Blanchard, 1984). Moreover, given the worrying debt levels in several countries from the 80’s, the concept of solvency of public debt has become more explored in empirical studies, such as Hamilton and Flavin (1986), and Bohn (1998), for example. For instance, total public debt as a ratio of GDP in U.S. rose from 31.16% in December 1980 to 56.04% a decade later.

Following this literature, we define the debt solvency concept, by assuming that debt is sustainable so long as the probability of a debt explosion, and thus debt default, remains very low. Addressing this discussion is relevant given the longer-term macroeconomic implications of much higher public debt. In this context, Reinhart and Rogoff (2010) find that whereas the link between growth and debt seems weak at normal debt levels, median growth rates for countries with public debt over roughly 90% of GDP are about one percent lower than otherwise. Herndon et al. (2014) propose revisiting this issue and they argue – contrary to Reinhart and Rogoff’s (2010) main conclusions – that the relationship between public debt and GDP growth varies significantly by period and country.

Despite empirical divergences on the macroeconomic role of high debt, the nonlinear effect of debt on growth can be related to a response of market interest rates as economies reach debt tolerance limits (Reinhart et al., 2003). This pass-through related to interest rates is also useful to understand the role played by inflation in GDP – a classic research agenda of monetary economics. [1] Specifically in the post-pandemic period, most of economies are registering high and unprecedented levels of indebtedness together with high inflation: a dangerous combination, and perhaps in some economies irreversible in the short term.

This scenario has motivated us to revisit the solvency of public debt in one of the world's largest emerging economies with a very interesting history of fiscal rules: Brazil.

Brazilian net debt to GDP used to range between 30% and 40% between 2001 and 2006. From 2007 it registered values between 20% and 30% until the end of 2016. During the fiscal crisis, and due to the change in the fiscal regime (control of real spending evolution instead of primary surplus target) this indebtedness started to grow again in the years of 2017 and 2019, reaching again the level of 40%. However, our main concern is in the unprecedented behavior due to the pandemic in the years 2020 and 2021. Net debt to GDP rose from 31.23% (Mar/20) to 48.34% (Apr/22). More specifically, to compensate in 2020 for the drop in revenue (R$ 112 billion with revenue not managed by the Federal Revenue) and the increase in extraordinary credit (R$ 426 billion with aid to the most vulnerable and in transfers to combat the pandemic), net debt grew by R$ 600 billion in 2020. The forecast of 60% at the end of 2022 by Focus report (Central Bank of Brazil) is also worrying.[2]

[1] Matos et al. (2022) are studying the effects of high levels of inflation and debt on GDP in Brazil.

[2] The Focus Market Report provides weekly mean market expectations for inflation over following month, 12 months, and following year as well as expectations for SELIC target rate, real GDP growth, net public sector debt/GDP, industrial production growth, current account, and trade balance, collected from over 130 banks, brokers, and fund’s managers.

According to Berenguer-Rico and Carrion-i-Silvestre (2011), one can summarize the empirical contributions proposed until then, considering the role of non-stationarity time series analysis and observing that two different approximations can be found in the literature: a univariate approach that has focused on the stochastic properties of the stock of debt and, a multivariate one that has focused on the long-run properties of the flows of expenditures and revenues, i.e., in the stochastic properties of the deficit. Their main contribution is unifying these approaches considering the stock–flow system that fiscal variables configure.

Regarding this literature applied to Brazil, Rocha (1997) finds an intertemporal budget balance only until 1990 when financial assets are frozen. Issler and Lima (1998) and Cavalcanti and Silva (2010) find evidence of stationarity in the public deficit with adjustment always via taxes, with expenditures being exogenously determined, while Lima and Simonassi (2005) find that seigniorage affects deficit management and a fiscal reaction is taken when the deficit becomes greater than 2.2%. Simonassi (2007) finds a structural change in fiscal policy from 1995 onwards able to modify sustainable behavior in a low public sector response to deficit increases. Mendonça et al. (2009) find a post-2000 regime change and corroborate the previous results of low fiscal reaction to deficits.

More recently, Campos and Cysne (2019) use Kalman filter, spline smoothing, and co-integration and they find a reduction in the variation of the primary deficit in response to the debt to GDP.

A first problem is that this broad discussion does not capture the most recent effects of the pandemic. Moreover, this literature has not measured yet the relation between primary balance and debt in the time frequency domain for Brazil.

Nonetheless, the most recent empirical literature on the relationship between fiscal, macroeconomic, and financial variables (Lo Cascio, 2015 and Matos et al., 2021, for instance) has employed continuous wavelet transformation tools to address empirical issues in U.S. This is a classical mathematical methodology, that has not yet been so explored in this literature for emerging markets, although the practical advantages. This mathematical framework enables us to infer on which primary balance cycle has been leading or lagging each debt cycle.

In other words, this technique offers us greater freedom in the analysis of co-movements, by allowing different responses, in intensity, and in the direction of the leadership of the cycles between the variables involved, over time and with different frequencies. This freedom in terms of time-varying results is relevant in a long-time horizon in a country with complex problems, as well as the freedom to respond with different frequencies is equally important when working with monthly series, even knowing that changes in the economy occur not in the short term, but after semesters or years.

We are conceptually and methodologically aligned to Lo Cascio (2015), which suggests the wavelet approach to verify fiscal sustainability in the U.S. from the annual time series of fiscal variables during the period between. She finds fiscal sustainability in the U.S. long term debt only until 1995, and she also shows that governments tend to respond more vigorously to budget deficits when the level of debt is high rather than low.

We add to this empirical literature on the sustainability in indebtedness, by analyzing for the first time the co-movements between primary balance and debt cycles in Brazil. Monthly data on the primary balance and net debt (both in current R$ and as a ratio of GDP) were collected for Brazilian federal government, including Brazilian Central Bank (BACEN), over the longest period available – December 2001 to April 2022 – in our source data: BACEN.

This paper is structured as follows. In Section 2 we have a theoretical discussion, and in Section 3 we show the wavelet framework. Section 4 reports our main findings, while Section 5 is devoted to the final discussion.

2. Debt Theory

The usual starting point of any discussion on sustainability of a public debt issue by a large national government is the model proposed by Barro (1979). The author assumes that the government can finance its expenditure through current taxation and public debt issue.[3] The government's budget in each period is

$$G_t + rb_{t-1} = \tau_t + (b_t - b_{t-1}) \tag{1}$$

In this model, real government expenditure in t, aside from interest rate payment on debt, is given by G_t, real tax revenue in t is denoted by τ_t, real stock of public debt outstanding ate the end of period t is denoted by b_t, and r means the rate of return on public and private debt. It is usual analyzing this same relationship powered by GDP. Solving the equation forward in time, this implies that the debt ratio in the future depends on the initial debt ratio, current and future interest and growth rates, and current and future primary balances. If the debt ratio is to remain constant, it is straightforward deriving a steady state relation between debt and the primary balance ratio. However, if the purpose is discussing on what debt level might be sustainable, we need to know more about the behavior of the primary balance. We follow Blanchard et al. (2021) by believing that a reassuring theoretical and empirical answer was given in an influential paper by Bohn (1998).

Bohn (1998) proposes an empirical approach trying to address two relevant questions: How do governments react to the accumulation of debt? Do they take corrective measures when the debt-GDP ratio starts rising, or do they let the debt grow? In a very didactic way, the author suggests that one can find direct evidence for corrective actions by examining the response of the primary (noninterest) budget surplus to changes in the debt-income ratio. A positive response shows that the government is taking actions to counteract the changes in debt, as reducing noninterest expenditures or raising tax. The idea is to search for a systematic relationship between debt to GDP (or debt-income ratio) and primary balance to GDP (or primary balance-income ratio), called fiscal reaction, by estimating the following regression:

$$s_t = \rho b_t + \alpha \boldsymbol{Z}_t + \varepsilon_t \tag{2}$$

where s_t means the primary balance to GDP, i.e., and $\boldsymbol{Z}_t$ is a set of other determinants and ε_t an error term. In this context, we propose to revisit this model of fiscal reaction, following methodologically Lo Cascio (2015), which applied unconditional wavelet to U.S. debt solvency. This technique makes it possible to infer, over time and with different frequencies, whether there are co-movements (in the same direction or not) and whether there is leadership of any of the two variables: primary surplus and net debt (as a ratio of GDP).

3. Methodology

The Fourier analysis can be considered one of the most important bases for the wavelet transform development. This analysis is a powerful tool to modelling time series on frequency domain. The function is reversible, which allow back-and-forth between the original and transformed signals, and it gives an effective localization in frequency. So, we can access the power spectra of the signal, which describe the power distribution on different frequency bands.

Given a time series $x(t)$, the continuous wavelet transform (CWT) is defined as:

$$W_x(\tau, s) = \int_{-\infty}^{+\infty} x(t)\, \psi_{\tau,s}^{*}(t)dt \tag{3}$$

where * denotes the complex conjugate, τ determines the position, s is the scaling factor and $\psi_{\tau,s}$ is the basis function suited to scale and shift the original signal, which allows the

[3] He does not deal with currency issue, although this type of finance could be included as one form of current taxation.

decomposition of the time series in space and scale. To capture different frequencies of the signal, we use a mother wavelet that is stretched and shifted:

$$\psi_{\tau,s}(t) = \frac{1}{\sqrt{s}}\psi\left(\frac{t-\tau}{s}\right) \tag{4}$$

The factor $1/\sqrt{s}$ is added to guarantee preservation of the unit energy ($\|\psi_{\tau,s}\| = 1$). Low scales are captured rapidly changing detail generating a compressed wavelet ($|s| < 1$), capturing high frequencies movements, and high scales capture slowly changing features ($|s| > 1$), or low frequencies movements. So, the CWT can be defined by:

$$W_x(\tau, s) = \int_{-\infty}^{+\infty} x(t)\frac{1}{\sqrt{s}}\psi\left(\frac{t-\tau}{s}\right)dt \tag{5}$$

The first wavelet measure that we will present it's the Wavelet Power Spectrum (WPS), which reports the variance distribution of the original time series $x(t)$ around the time-scale (or time-frequency) plane. Following Torrence and Compo (1998) we define the WPS by:

$$WPS_x(\tau, s) = |W_x(\tau, s)|^2 \tag{7}$$

To compare the oscillation in energy among a range of bands (or frequency) we define the Global Wavelet Power Spectrum (GPWS), which takes the average of wavelet power spectrum over all times:

$$GWPS_x(\tau, s) = \int_{-\infty}^{+\infty} |W_x(\tau, s)|^2 d\tau \tag{8}$$

To study the dependencies between two time series $x(t)$ and $y(t)$ in time-scale/frequency plane, primary balance to GDP and debt to GDP in our exercise, we use wavelet coherency, a measure that is associated to the cross-wavelet spectrum (XWT), which in turn can be derived by:

$$W_{xy}(\tau, s) = W_x(\tau, s)W_y^*(\tau, s) \tag{9}$$

where $W_x(.)$ and $W_y(.)$ are continuous wavelet transform of $x(t)$ and $y(t)$, respectively, and * denotes the conjugates complex. As the cross-wavelet transform is complex, we can express the XWT as $|W_{x,y}(\tau, s)|$. It computes the local covariance between two signals at each scale. The squared wavelet coherence is given by the squared of the wavelet cross-spectrum normalized by the individual power spectra. Following Torrence and Webster (1999) the squared wavelet coherence is denoted as:

$$R^2(\tau, s) = \frac{\left|S\left(s^{-1}W_{x,y}(\tau, s)\right)\right|^2}{S(s^{-1}W_x(\tau, s)^2)S(\, s^{-1}W_y(\tau, s)^2)} \tag{10}$$

where $S(.)$ expresses a smoothing operator in both time and scale, s^{-1} is a normalization fator ensuring the conversion to an energy density. Torrence and Webster (1999) note that in numerator of the squared wavelet coherence, both the real and imaginary parts of the cross-wavelet transform are smoothed separately before taking the absolute value, while the smoothing operator is taking on square of the wavelet power spectra in denominator. By these definitions, it's ensured that $0 \leq R^2 \leq 1$.

Hence, the main advantage of the wavelet coherence on XWT is the common measure unit to examine several combinations of signals. Torrence and Compo (1999) reveal that once the wavelet transforms conserves variance, the wavelet coherence is a good representation of the normalized covariance between two-time series, where the closer to zero (one) the coherence, the weaker (stronger) the local correlation between the time-series. The wavelet coherence has not theoretical distribution known; hence we follow the approach of Aguiar-Conraria and Soares (2011) deriving the confidence interval using Monte Carlo methods.

Although the wavelet coherence computes the degree of local linear correlation between two signals, it does not reveal patterns of lead-lag relationship neither if the movements are positives or negatives. To deal with these limitations, the phase-difference is commonly used to examine the delays in the fluctuations between the two time-series. Following Torrence and Webster (1999) we define the phase difference as:

$$\phi_{xy}(\tau,s) = \tan^{-1}\left(\frac{\Im\left\{S\left(s^{-1}W_{x,y}(\tau,s)\right)\right\}}{\Re\left\{S\left(s^{-1}W_{x,y}(\tau,s)\right)\right\}}\right) \tag{11}$$

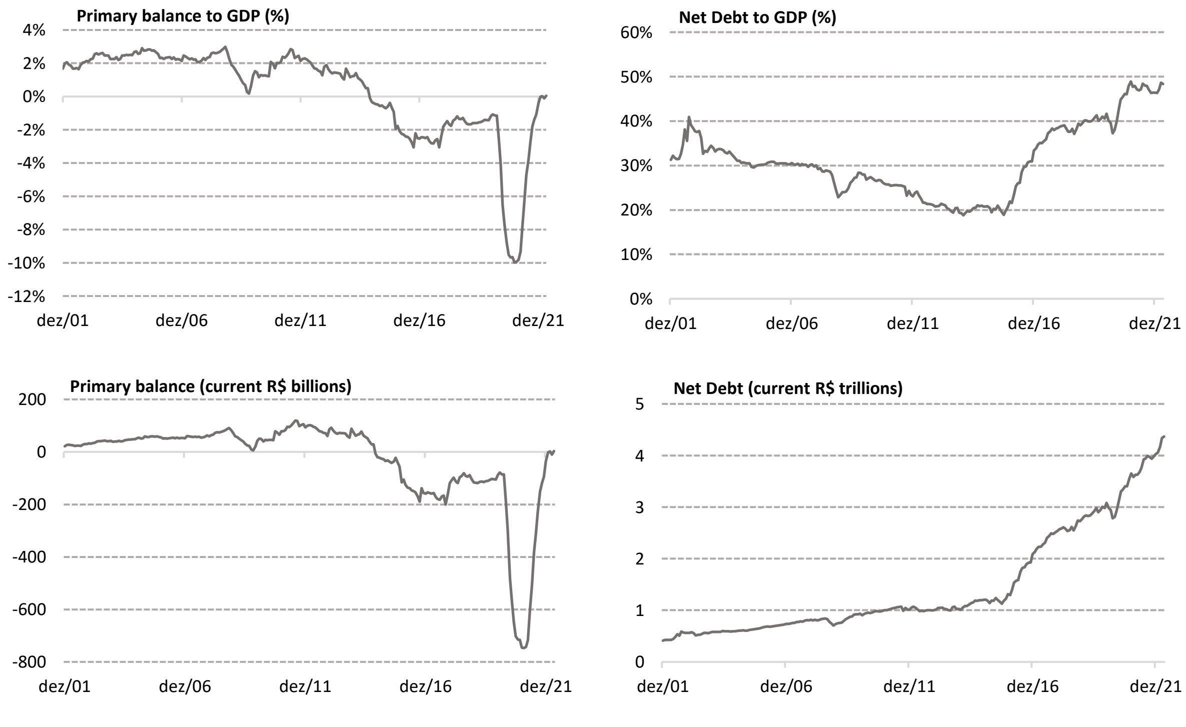
The smoothed real ($\Re$) and imaginary ($\Im$) parts should already be calculated in the wavelet coherence function. Both $R^2(\tau,s)$ and $\phi_{xy}(\tau,s)$ are functions of the position index (τ) and scale (s). Finally, we need the information on the signs of each part to determine the value of $\phi_{xy}\epsilon[-\pi,\pi]$. Next, we summarize the discussion of possible outcomes, depending on the region indicated in the phase difference:

- A phase-difference of zero indicates that the time-series move together at the specified frequency, while a phase-difference of $\phi_{xy} = \pm\pi$ indicates an anti-phase relation, and in both cases, there is no leadership,
- If $\phi_{xy}\epsilon\left(0,\frac{\pi}{2}\right)$ the series move in phase, but the time-series y leads x, while if $\phi_{xy}\epsilon\left(-\frac{\pi}{2},0\right)$ then it is x that is leading,
- If $\phi_{xy}\epsilon\left(\frac{\pi}{2},\pi\right)$, then x is leading and time-series y is leading if $\phi_{xy}\epsilon\left(-\pi,-\frac{\pi}{2}\right)$, and in both cases, we find an out-of-phase relation.

4. Empirical Exercise

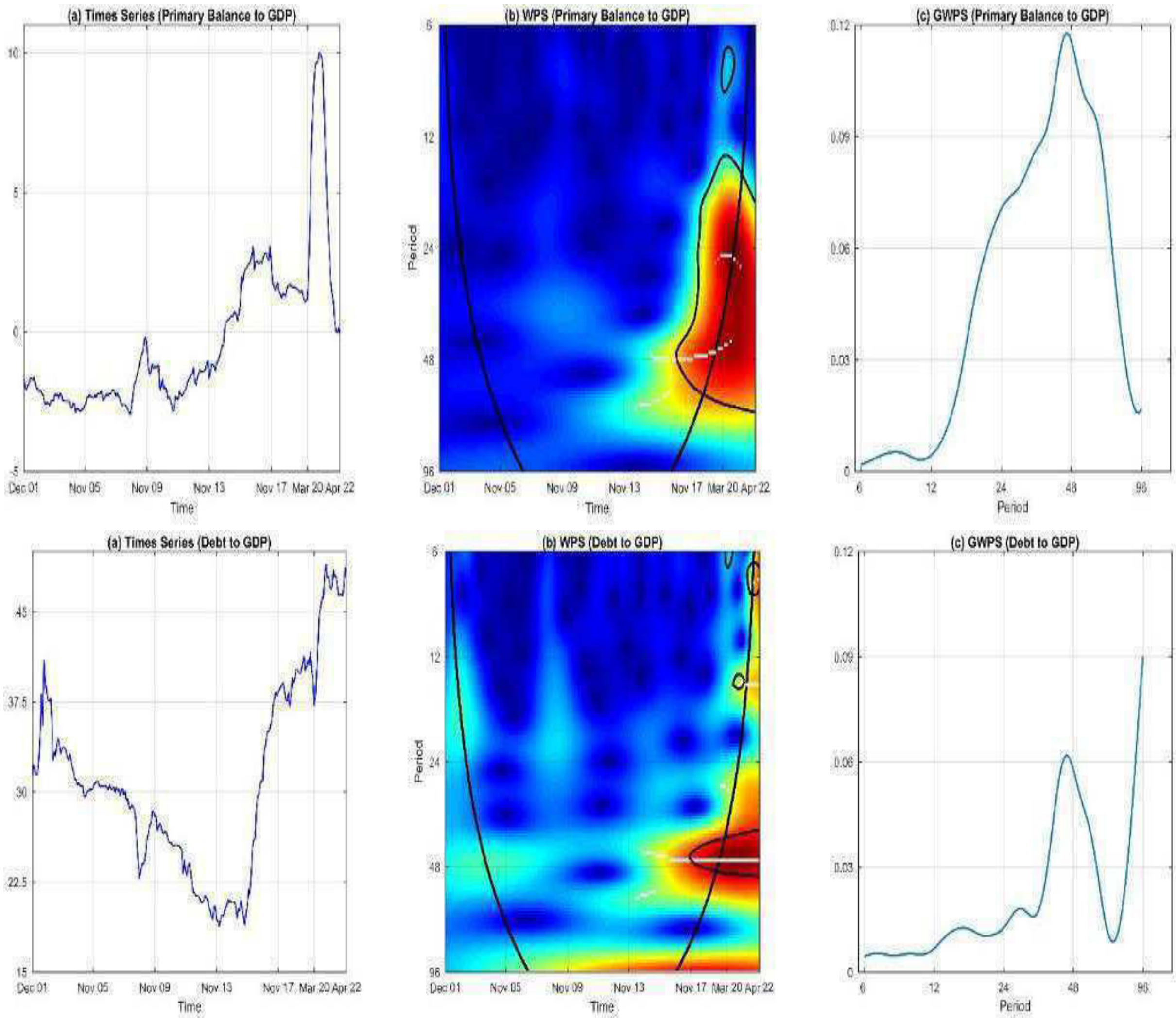
Monthly data on the primary balance and net debt (both in current R$ and as a ratio of GDP) were collected for Brazilian federal government, including Brazilian Central Bank (BACEN), over the longest period available – December 2001 to April 2022 – in our source data: BACEN. In Fig. 1, we report the related time series, while Fig. 2 shows WPS and GWPS for all series used here, and in Fig. 3 we report the results on wavelet coherency, phase-difference and gain.

Fig. 1. Primary Balance and Net Debt.



Source: Data can be found in the Brazilian Central Bank's Time Series Management System (SGS). Variable codes are: 4468 - Public Sector Net Debt (R$); 4503 - Public Sector Net Debt (% GDP); 5068 - NFSP Flow accumulated in 12 months without currency devaluation (R$); 5783 - NFSP without currency devaluation (% GDP).

Fig. 2. Debt to GDP and primary balance series (a), respective WPS (b) and Global WPS (c).



Notes: The black contours on the Wavelet Power Spectrum (WPS) plot refers to 5% significance and is theoretically obtained considering an AR (1) as the null hypotheses. In the heatmap, colder colors represent lower power while warmer colors represent higher power. The shaded area outside the Cone of Influence is subject to edge effects.

The analysis of indebtedness in nominal values suggests three periods with very clear characteristics. Between December 2001 and late 2015, there is an apparently linear growth trend, and from 2016 to March 2020 another trend with greater slope. In the last 2 years of the sample, there is a third phase of evolution, with more accentuated growth and with an additional element: high inflation. In this third moment, this net debt rose from R$ 2.8 trillion in March 2020 to R$ 4.4 trillion in April 2022. When weighted by GDP, the behavior presents different characteristics. After a strong increase of almost 10% at the beginning of the time window, the debt-GDP ratio reduces from 40.91% (September 2002) to around 19% at the end of 2013. In this period, there was just one unusual swing during the subprime crisis. We also find a stability around 20% in 2014 and 2015, and a subsequent growth trend, breaking the 40% level at the end of 2019. Once more, the concern is in the unprecedented behavior due to the pandemic: debt to GDP rose from 31.23% (March 2020) to 48.34% (April 2022).

The primary balance is the inverse of the public sector financing need (NFSP). Observing its accumulated value (12 months), the federal government used to record a surplus until October 2014. Even during the subprime crisis, the primary result was compromised, but there was no deficit. The first deficit was recorded in November 2014, followed by new deficits, with emphasis on the deficit value exceeding 200 billion in October 2017, during the fiscal crisis in the country. The recovery that began on that date was compromised by the pandemic.

We highlight the record deficit of almost R$ 750 billion in January 2021. The recovery was fast, and we see a deficit in December 2021 lower than the registered before the pandemic. The behavior of the GDP-weighted series is visually very similar. It is important to highlight the value of approximately 10% of GDP associated with the record deficit at the beginning of 2021.

Regarding the analysis based on Wavelet Power Spectrum (WPS) and Global Wavelet Power Spectrum (GWPS), we know that WPS reports the variance distribution of the original time series $x(t)$ around the time-scale (or time-frequency) plane, while GPWS takes the average of wavelet power spectrum over all times. According to Fig. 2 (upside), WPS for primary balance shows a great power intensity between 2017 and 2020 with medium and low frequency, which is not recorded in the other periods. Regarding debt to GDP (Fig. 2. bottom), we observe that the WPS has high power at lower frequencies only between 2016 and 2017 (fiscal crisis), and in the other periods the variance is low. Still according to WPS analysis, we find that the spectrum varies so much along the time, indicating the non-stationarity of data and suitability of the method employed. The GWPS shows peaks only at long frequency, close to 48 months, between 6% and 9% for primary balance, and between 9% and 12% for debt.

Our main findings are based on the wavelet analysis, and they are reported in Fig. 3.

The first step in this analysis is to look at the heat map (left side of Fig. 3). The coherency ranges from low (blue) to high (red) values and the respective cone of influence is shown with a black line, designating the 5% significance level. The color and the significance in the heat map measure the degree of adjustment in the time-frequency domain. Once we identify a red and significant region in each period, we need to identify the respective frequency (vertical axis of the heat map), and then proceeded to analyze the graph in the phase difference.

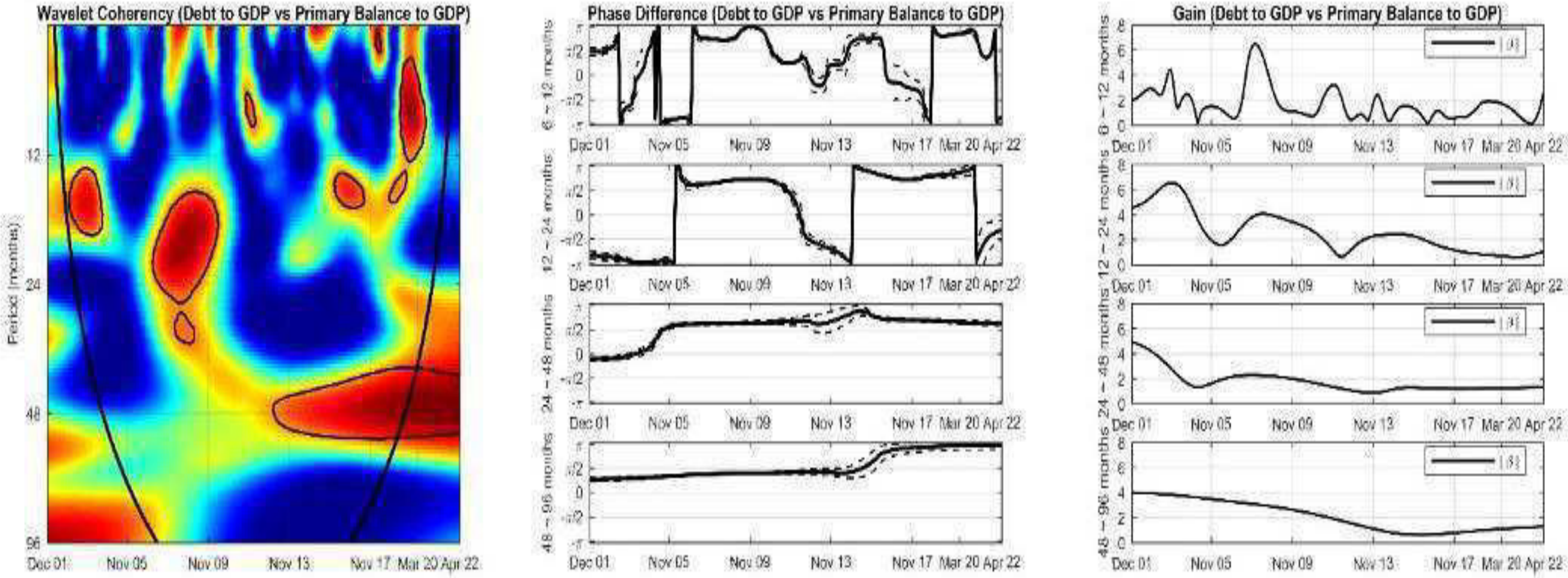
There are 4 phase difference graphs (center of Fig. 3). The first serves to identify the leadership of high frequency cycles (6 to 12 months), while the second is useful to analyze the cycles with frequency between 12 and 24 months, and so on, until the fourth charts show the cycles of low frequency, between 48 and 96 months. The analogy is the same for using the 4 gain graphs (right of Fig. 3).

Observing the heat map, we can highlight five periods in which the fiscal cycles show strong interaction. Analyzing these moments in chronological order, first we identify insolvency of Brazilian government net debt in the period 2003-2004 (period characterized by electoral turmoil followed by a new government administration) with frequency ranging from 1 year to 2 years period. This conclusion is associated with the presence of a red area with frequency between 12 and 24 months in the years 2003 and 2004, according to the heat map. Observing, the phase difference (graph with the same frequency), in this period the line is in the region between $(-\pi, -\pi/2)$, so we find an anti-phase relationy with y (debt) leading x (primary balance). We can measure this insolvent behavior (increase in debt implying a reduction in the primary balance approximately 1 to 2 years ahead) with an elasticity ranging between 4 and 7, according to the respective gain graph. This is the reasoning for the next analysis.

Next, we find co-movements (frequency ranging from 1 year to 2 years period) with opposite direction led by the primary balance in the subprime crisis (2007 – 2008) and during the 2016-2017 fiscal crisis. There is this same evidence – antiphase leadership by primary balance – with low frequency (2 to 4 years) between 2013 and 2019. This finding on debt cycles anticipated by primary balance cycles can be measured or characterized by elasticities lower than 4.

More recently, once more we find that primary balance cycles leading debt cycles in the opposite direction with high frequency (less than 1-year period). All of our evidence are robust when using gross debt to GDP. We do not report these results, to save space, but we can disclose them if requested.

Fig. 3. Wavelet coherency: net debt to GDP vs primary balance to GDP



Notes: The coherency ranges from low (blue) to high (red) values and the respective cone of influence is shown with a black line, designating the 5% significance level. The color and the significance in the heat map measure the degree of adjustment in the time-frequency domain of the explanatory variable, primary balance.

5. Conclusion

Since the Tax Liability Law (LRF) in 2000, Brazilian federal government has chosen for a primary surplus target. From 2017, we observe a new rule controlling real spending evolution. Mainly after this change of tax regime, government has not chosen for a fiscal adjustment through tax increases or expenditure reductions. Given that the interest payment on debt was not included in the previous rule (primary surplus target), nor in the new rule of spending ceiling, the consequence is that government has issued public bonds in local currency with the longest possible term as a solution to balance public accounts, instead of issuing currency. [4]

In this historical fiscal context, on the one hand, it is comforting finding here in our paper that the current debt is not insolvent in the current pandemic period, for instance, and that only in 2003 and 2004 there was a significant sign of insolvency. It is also important the preliminary evidence reported in Matos et al. (2022) that the level of indebtedness has not compromised the growth of real GDP per capita in recent decades, and that the villain of growth seems to be the inflation. Still according to them, the cycles of GDP per capita in the last two decades have determined the debt cycles in Brazil in an antiphase direction. Here, we find that the cycles of the primary result in periods of fiscal imbalances are also capable of driving in the opposite direction the cycles of indebtedness.

On the other hand, the debt of almost 50% of GDP in April 2022, and the Focus forecast of 60% for December 2022 draw attention and this needs to be monitored, mainly in a context of very high current inflation (12.13% accumulated in 12 months until April 2022) and above the target of 3.5% and the ceiling of 5.0% in 2022. Moreover, Focus forecast suggests that only in 2024, official inflation should converge to the target of 3.25%.

Our set of evidence enables us to add to this discussion, and we claim that is essential monitoring and understanding the drivers and possible impacts of Brazilian federal government net debt. In this context of high levels of debt and inflation in Brazil, we invite researchers to revisit this issue by using instruments, other techniques, other fiscal variables, or even other theoretical relationship useful to discuss on debt solvency.

Conflict of Interest

The authors declare that they have no conflict of interest.

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[4] The inflation target regime was adopted in Brazil in 1999. Although high inflation is undesirable in a democracy that experienced hyperinflation in the 1990s, inflation was outside the tolerance range in 2001, 2002, 2003, 2015, 2016 and 2021.

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