Economics Bulletin

Volume 0, Issue 0

The impacts of cryptocurrencies in the performance of Brazilian stocks' portfolios

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Abstract

This study analyses the impact of including cryptocurrencies in Brazilian stocks' portfolios performances from September 2014 to April 2020. The comparisons were made between stocks' only portfolios against portfolios that allowed stocks and cryptocurrencies. Three portfolios served as benchmarks: the naïve but relevant equally weighted portfolio, the tangency and the MVP portfolios built from the Markowitz mean-variance theory. Performances were compared through out-of-sample returns, volatilities, Sharpe, Sortino and Omega ratios. Our results indicate positive statistically significant return and risk-adjusted improvements after the inclusion of cryptocurrencies, although also increasing the volatility. The equally weighted portfolios with cryptocurrencies often outperformed the tangency and minimum variance models, which only exhibited better results when more data was used as input to the models. Moreover, the portfolios that included cryptocurrencies consistently outperformed the IBrX-100 in the period studied. The results of this study are important for investors and fund managers, especially because cryptocurrencies are yet not considered by most of them.

Mateus Portelinha would like to thank CNPq (National Council for Scientific and Technological Development) for financial support to his research. Carlos Heitor Campani would like to thank the following Brazilian institutions for financial support to his research: Brasilprev Research Chair, ENS (Escola Nacional de Seguros), CNPq (National Council for Scientific and Technological Development), FAPERJ (Fundação de Amparo à Pesquisa do Estado do Rio de Janeiro), and Quantum Finance (Data Provider).
Citation: Mateus Portelinha and Carlos Heitor Campani and Raphael Roquete, (2021) "The impacts of cryptocurrencies in the performance of Brazilian stocks' portfolios", *Economics Bulletin*, Vol. 0 No. 0 p.A163.
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Submitted: April 15, 2021. **Published:** September 17, 2021.

Economics Bulletin

Volume 41, Issue 3

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Citation: Mateus Portelinha and Carlos Heitor Campani and Raphael Roquete, (2021) "The impacts of cryptocurrencies in the performance of Brazilian stocks' portfolios", *Economics Bulletin*, Vol. 41 No. 3 pp. 1919-1931
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1. Introduction

Nakamoto (2008) created the most popular cryptocurrency so far, Bitcoin, a peer-to-peer digital currency that would allow online payments directly from one party to another, without the need to go through a financial institution. One of the main features is its independency from central authorities that cannot control the supply, which is predetermined and finite, being ultimately deflationary, what he argues would make it a better store of value. On the other hand, this feature also raises concern about the long-term survival capacity of the cryptocurrency.

The aim of this study is to explore the performance of stock portfolios with the inclusion of cryptocurrencies in Brazil. There are still very few studies of cryptocurrencies portfolios in emerging markets and therefore this paper seeks to contribute to this literature. The hypothesis is that the inclusion of cryptocurrencies in stock portfolios will improve the out-of-sample performances.

Brazil is a country with its own peculiarities. Government bonds had higher returns than stocks for the period of 2004 to 2016 (Andrino & Leal, 2018), which go against what is expected by the theory of higher risks and higher returns. On the other hand, Brazil is right now going through an unprecedented reduction of interest rates, with the target that was 14.25% per year in October 2016 being 2.00% in December 2020 (Brazilian Central Bank, 2020). The real interest rates have also reduced during the period and many investors started looking for investments other than the fixed income market. The Brazilian stock market boosted during 2020, with the entrance of millions of new investors. Cryptocurrencies are getting more and more attention from investors and many of them do consider this asset class in their portfolios. As a natural consequence, we want to understand if cryptocurrencies should be considered as a good option for a diversified stock portfolio.

Bitcoin gained more attention from the public and the media when its prices started skyrocketing. The related literature for cryptocurrencies started then to develop. Currently there are more than 5,000 cryptocurrencies with 150 billion dollars of total market capitalization, with Bitcoin, Ethereum and Ripple being the three biggest ones (CoinMarketCap, 2020).

This new financial instruments have exhibited high returns, but also very high volatility (Elendner et al., 2017; Hu et al., 2019), leading many investors to not enter the market due to higher risks and also because of uncertainties about the long term sustainability. Cheah and Fry (2015) tested its fundamental value to not being significantly different than zero in the long term, while Kim (2017) showed that cryptocurrencies are already reducing transaction costs when used as an intermediary for exchange, which argue in favour of the creation of value by the use of cryptocurrencies.

In order to understand cryptocurrencies as an asset class, several studies aimed at testing its potential as a diversifier. The correlations with other established asset classes like global stocks, bonds, gold, commodities, currencies and real estate are very low, close to zero in many cases (Brière et al., 2015; Elendner et al., 2017; Hu et al., 2019). These finding caught the attention for their potential to be included in portfolios as a good diversifier.

At the same time, cryptocurrencies exhibit high correlations among themselves, especially with Bitcoin, since many of them are traded against it instead of against fiat currencies. However, there is still evidence of idiosyncratic risks across cryptocurrencies according to the recent literature (Elendner et al., 2017; Hu et al., 2019; Mensi et al., 2019).

Cryptocurrencies showed significant results at increasing the performance of portfolios of stocks and bonds (Platanakis & Urquhart, 2019) and an emerging market currencies basket (Carrick, 2016). The results are also consistent for three different regions: US, China and Europe (Kajtazi & Moro, 2019). In Brazil, this is the first academic study to analyse this issue.

Three portfolio benchmarks were considered: the tangency and the minimum variance portfolios, built from the Markowitz mean-variance theory (Markowitz, 1952), and the equally weighted portfolio, as in DeMiguel et al. (2009) study. The performance was compared against the Brazilian stock market index IBrX-100 and against CRIX (Trimborn & Härdle, 2018), an index representing the cryptocurrencies market.

The results show that the inclusion of cryptocurrencies in Brazilian stocks' portfolios was successful to increase the out-of-sample returns and the overall risk-adjusted performances, as measured by Sharpe, Sortino and Omega ratios. However, it often increased the volatility, augmenting the risk exposure of investors. Among the portfolio benchmarks, the equally weighted ones were often better than the minimum variance and tangency portfolios, which only exhibited better results when more data were used as input to the model. Furthermore, the portfolios that included cryptocurrencies were consistent to outperform the IBrX-100 returns and risk-adjusted measures, while they underperformed against the CRIX in the period. However, the CRIX returns exhibited exceedingly high volatilities, being impeditive for almost all investors in practical matters. The paper continues with data and methodology, followed by the results and the conclusions.

2. Data and methodology

2.1. Data

The data used in the study consist of daily prices from Brazilian stocks and cryptocurrencies during the period of September 2014 until April 2020. September 2014 was chosen as the beginning of the sample because it represents a period that we started to see other cryptocurrencies (besides Bitcoin) becoming more relevant, like XRP and Litecoin.

The cryptocurrencies' daily prices are from coinmarketcap.com. Our universe consists of all the ones that have figured as the top five most capitalized coins anytime during the period analysed. Only five cryptocurrencies at each time were picked because it is still a market with very few big players, so the more liquid assets were prioritized. The cryptocurrencies' prices are given in US Dollar, so, in order to convert it to Brazilian currency, we collected the daily exchange rates between the US Dollar and the Brazilian Real. This set of data was collected from Economatica database.

As for the Brazilian stocks daily prices, they were collected from Economatica database, adjusted to cash and stock dividends, stock splits and all related events, and it consists of the ones that have figured in the IBrX-100 index during the period. The index is one of the most important for the performance of stocks in the Brazilian market. It consists of the 100 assets with highest liquidity and which are most representative in the market. The index is rebalanced every four months. We preferred the IBrX-100 instead of the main Brazilian index (Ibovespa) because it has more stocks in its composition, increasing our universe of analysis, and also because the Ibovespa methodology in the past did not fully correspond to market value indexation (Roquete et al., 2018). Despite the differences, we could observe that the correlation between the IBrX-100 and the Ibovespa is remarkably high (more than 0.97 across the period analysed) and the results would not be qualitatively different.

Some of the calculations in this paper required a risk-free rate. Therefore, we considered the CDI rates, a quite common benchmark for investments in Brazil serving as the opportunity cost with no risk. We collected the daily values of the CDI rate from the Economatica database.

2.2. Estimation procedure

We used two portfolio strategies in our analysis: equally weighted portfolios (1/N), for its simplicity and popularity, while performing better than more complex models in several situations (Brauneis & Mestel, 2019; Liu, 2019), and mean-variance portfolios (Markowitz, 1952), for its relevance, being a common first step for investors who want to get into more complex calculations to optimize their portfolios.

The universe of assets consists of stocks listed in the IBrX-100 index in the date of each rebalance, along with the five cryptocurrencies with highest market capitalization at that moment, with short positions not allowed. For each strategy, we selected two portfolios, one picking from all the universe of and the other being restricted to the stocks. The portfolios that allow stocks and cryptocurrencies were not obligated to include a minimum weight of cryptocurrencies, leading to situations in which both portfolios were identical until the next rebalancing date.

To avoid diversification issues, only one stock of the same company was allowed in the final portfolio: the most liquid one. The frequency of the rebalances was every four months, matching the schedule of the IBrX-100 index.

We considered four months of data as input for the rebalances. After selecting the portfolios, the analysis followed an out-of-sample performance estimation process. An in-sample process would not be realistic for investors and would assume no estimation errors of the parameters.

After computing the daily results, we compared how the portfolios with cryptocurrencies perform against the ones without them, how the models performed against each other, and how they performed against the IBrX-100 and the CRIX.

2.3. Equally weighted portfolios

The weights calculation of equally weighted portfolios is a simple division of 1/N for each of the N assets. It therefore does not require any optimization. DeMiguel et al. (2009) consider it the benchmark for any other portfolio selection strategy, due to its simplicity to calculate and execute, while models that are more complex fail to outperform it consistently in different situations. They indicate diversification gains compensate for the estimation error.

We had to limit the number of assets in the selected portfolio in order to keep complexity from getting too high. Similar to Leal and Campani (2016), in every rebalancing date the assets were ranked by performance in the previous four months. We computed their Sortino ratio (Sortino & Van der Meer, 1991) in the period (more on this below), and the twenty better performers were selected to the portfolio, each one with a five per cent weight. Leal and Campani (2016) argue that the amount of twenty assets displayed a great trade-off between diversification and rebalancing costs.

2.4. Markowitz portfolios

In the mean-variance portfolio selection model proposed by Markowitz (1952), investors optimize the trade-off between risk and return of a portfolio and select the portfolio with highest expected return for a given level of risk.

We calculated two different: the minimum variance portfolio (MVP), which is the portfolio in the efficient frontier with the lowest variance; and the tangency portfolio, which is the portfolio that maximizes the Sharpe ratio.

Each asset from the selected portfolio cannot have a weight of less than 1% or more than 30%. Thomé Neto et al. (2011) argue that the minimum weight restriction avoids the rebalancing

costs getting too high and the maximum weight restriction helps to keep an efficient diversification in the portfolio.

2.5. Performance assessment

To compare the models, we first computed the daily return R_t of the portfolios, based on the daily returns of the individual assets and their weights in the portfolios. The cryptocurrencies are traded every day, while the stocks follow a 252 calendar, only being traded in working days. To unify the data, we adjusted the cryptocurrencies data to the stocks calendar, by aggregating the returns in non-working days to the following working day.
To present the results, we calculated the annualized geometric returns RG as in equation 1, using the 1,315 days of the sample period, as well as the annualized volatility (proxied by the standard deviation of daily returns), following equation 2.

$$RG = \prod_{t=1}^{1,315}(1 + R_t)^{252/1,315} \quad (1)$$

$$Vol = \sqrt{252 * \left[\frac{1}{1,314}\sum_{t=1}^{1,315}(R_t - \bar{R})^2\right]} \quad (2)$$

Although it might give us a good first assessment, the returns alone are not sufficient to compare portfolios due to different risk levels. Therefore, we used the Sharpe ratio (Sharpe, 1966) (equation 3), the Sortino ratio (equation 4) and the Omega ratio (Keating & Shadwick, 2002), pictured in equation 5. R_p is the average return of the portfolio, R_f is the average risk-free rate, σ_p is the standard deviation of the portfolio. σ_d is the standard deviation of the downside portfolio returns (and it considers all other returns as zero), F is the cumulative probability distribution function of the returns and Θ is the target return threshold that will define what is a gain versus a loss. The reference return to define a downside and the threshold is the risk-free rate.

$$Sharpe\ Ratio = \frac{R_p - R_f}{\sigma_p} \quad (3)$$

$$Sortino\ Ratio = \frac{R_p - R_f}{\sigma_d} \quad (4)$$

$$Omega\ Ratio = \frac{\int_{\theta}^{\infty}[1 - F(r)]dr}{\int_{-\infty}^{\theta}F(r)dr} \quad (5)$$

3. Results

3.1. Main results

Table 1 presents the descriptive statistics for the portfolios. For all models, the annualized geometric returns of the stocks+CCs portfolios (i.e., the portfolios that allow cryptocurrencies) were higher than the stocks-only portfolios. Following the bootstrap method (Efron, 1992), we tested the null hypothesis that the difference between the stocks+CCs and stocks-only portfolios returns were equal to zero against the alternative hypothesis that these returns were different than zero. We found statistical significance to reject the null hypothesis for the equally weighted portfolios and for the tangency portfolio. However, it is important to highlight the poor performance of the stocks-only tangency portfolio, presenting lower returns than the IBrX-100 index in the period studied.

Alongside the higher returns, the stocks+CCs portfolios also exhibited higher volatilities, expressed as the standard deviation. In addition, for the equally weighted portfolio, the volatility of the stocks+CCs portfolio was considerably higher than the stocks-only portfolio. The higher volatilities of cryptocurrencies against stocks in the period of analysis explain this

effect, as shown in the comparison of the monthly volatilities of the daily returns of the IBrX-100 and CRIX indexes (Figure 1).

Table 1

Descriptive statistics of the portfolios

Model /Index	Portfolio	Annual geom. return (%)	Annual vol. (%)	Sharpe ratio	Sortino ratio	Omega ratio	Daily return (%)			
							Avg.	Med.	Min.	Max.
Equally Weighted	Stocks-only	12.37	26.29	0.01	0.02	1.12	0.06	0.12	-16.81	11.37
	Stocks+CCs	42.77**	31.82	0.06**	0.09***	1.29**	0.16	0.13	-16.81	15.03
MVP	Stocks-only	10.85	19.35	0.01	0.01	1.14	0.05	0.11	-13.14	10.95
	Stocks+CCs	12.38	19.40	0.01	0.02	1.16	0.05	0.11	-13.14	10.95
Tangency	Stocks-only	4.45	24.98	0.00	-0.01	1.06	0.03	0.10	-17.47	12.87
	Stocks+CCs	13.36*	26.02	0.02*	0.02*	1.13	0.06	0.11	-17.47	12.87
IBrX-100		10.58	26.81	0.01	0.01	1.10	0.05	0.08	-14.89	13.75
CRIX (BRL)		132.32	76.95	0.09	0.13	1.33	0.45	0.40	-37.95	23.99

Note: The statistics refer to the 1,315 daily returns between January 2015 and April 2020.
* denotes 10% significance, ** denotes 5% significance and *** denotes 1% significance measured by a bootstrap test. For the portfolios that included cryptocurrencies, the test was performed for the returns and the Sharpe, Sortino and Omega ratios, under the null hypothesis that the difference to the stocks-only portfolio was equal to zero and alternative hypothesis that it was different than zero.

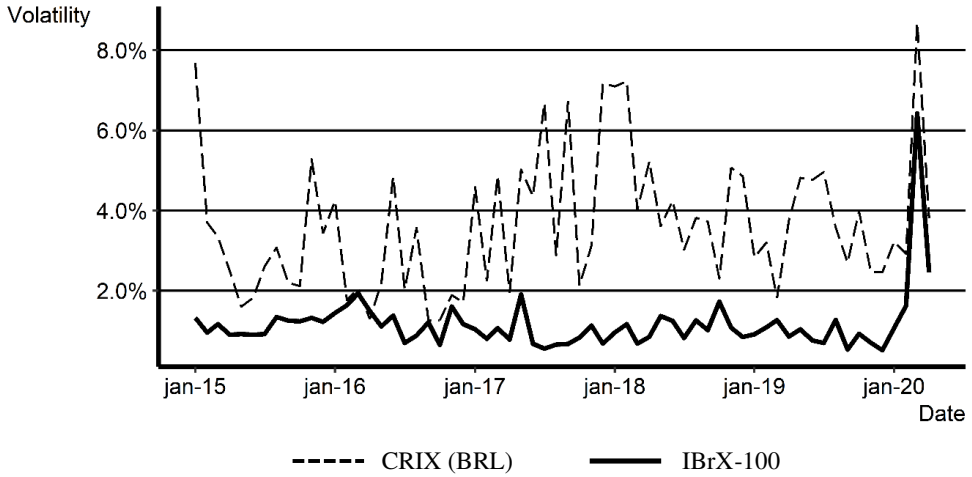

Figure 1. Daily returns monthly volatility (standard deviation) of the IBrX-100 against the CRIX (BRL)

For all models, the stocks+CCs portfolios performed better against the stocks-only portfolios in the Sharpe, Sortino and Omega ratios in the period of analysis. The results from the three

ratios were statistically significant for the equally weighted portfolio, while for the tangency portfolio only the Sharpe and Sortino ratios were statistically significant and none of them were for the MVP portfolio.

Observing the information presented in Table 1 about the minimum and maximum daily returns, in some cases the stocks+CCs and stocks-only portfolios were equal, suggesting there were periods that the two portfolios were identical and, therefore, the weight of the cryptocurrencies was zero. It happened four times for the equally weighted and for the MVP models and three times for the tangency, out of sixteen rebalancing dates.

Therefore, we decided for an additional test that generated new portfolios with a forced weight for cryptocurrencies different from zero. These portfolios were calculated using the equally weighted portfolio selection model, where the W_c is the weight of cryptocurrencies and W_s is the weight of stocks, where $W_s = 100\% - W_c$. Six new portfolios were calculated, with W_c ranging between 5%, 10%, 15%, 20%, 25% and 30%. The assets selections were made in two parts: the stocks and the cryptocurrencies. For the stocks, we selected 20 assets according to the same methodology applied before, which resulted in the stocks-only portfolio, each stock with weight equal to $W_s/20$. For the cryptocurrencies, we selected all five available in each rebalancing date, each one with weight $W_c/5$. Table 2 presents the descriptive statistics for the new portfolios.

Table 2

Descriptive statistics of the equally weighted portfolios that include cryptocurrencies

Portfolio	Annual geom. return (%)	Annual vol. (%)	Sharpe ratio	Sortino ratio	Omega ratio	Daily return (%)			
						Avg.	Med.	Min.	Max.
Stocks+CCs	42.77***	31.82	0.06**	0.09***	1.29**	0.16	0.13	-16.81	15.03
Stocks 95% + CCs 5%	23.87***	25.95	0.04***	0.05***	1.20***	0.10	0.16	-18.39	11.47
Stocks 90% + CCs 10%	34.86***	27.11	0.06**	0.08***	1.27***	0.13	0.16	-19.86	11.57
Stocks 85% + CCs 15%	45.38***	29.24	0.07***	0.10***	1.32***	0.17	0.14	-21.24	11.67
Stocks 80% + CCs 20%	55.44***	31.96	0.08***	0.12***	1.35***	0.20	0.13	-22.53	13.20
Stocks 75% + CCs 25%	65.06***	35.02	0.08***	0.13***	1.37***	0.22	0.13	-23.74	15.21
Stocks 70% + CCs 30%	74.24***	38.28	0.09***	0.14***	1.38***	0.25	0.13	-24.88	17.00

Note: The statistics refer to the 1,315 daily returns between January 2015 and April 2020.
** denotes 5% significance and *** denotes 1% significance measured by a bootstrap test. The test was performed for the returns and the Sharpe, Sortino and Omega ratios, under the null hypothesis that the difference to the stocks-only portfolio was equal to zero and alternative hypothesis that it was different than zero.

All portfolios exhibited significant results for the returns, Sharpe, Sortino and Omega ratios, when compared to the equally weighted stocks-only portfolios, suggesting that these strategies had superior performances in the period of analysis. In addition, some of them had more controlled volatilities than the stocks+CCs portfolio, with the stocks 95% + CCs 5%, the stocks 90% + CCs 10% and the stocks 85% + CCs 15% volatilities values being in a similar level to the IBrX-100 index. In this analysis, when compared to the stocks+CCs equally weighted

portfolio, the options that included at least 15% of cryptocurrencies performed better in the returns and in the Sharpe, Sortino and Omega ratios.

3.2. Comparison of the models

All seven equally weighted portfolios showed higher returns than the MVP and tangency portfolios. Additionally, these results were statistically significant in all the comparisons, except for the equally weighted stocks 95% + CCs 5% against the tangency portfolio. From a volatility standpoint, the MVP portfolio presented the lowest value, as expected by construction, followed by the stocks 95% + CCs 5% and the tangency portfolios. The equally weighted stocks+CCs portfolio presented higher volatility in the period, in a similar level to the stocks 80% + CCs 20% portfolio. There was no significant difference between the MVP and tangency portfolios in the returns.

Comparing the Sharpe, Sortino and Omega ratios, all equally weighted portfolios performed better than the other models, despite the stocks 95% + CCs 5% not being significant at a 10% level, as well as the Omega of the stocks+CCs and the stocks 90% + CCs 10% against the MVP portfolio. The tangency portfolio presented higher Sharpe and Sortino values than the MVP, but a lower Omega, despite not having a statistically significant difference between them.

The hypothesis for the higher returns and volatility from the equally weighted stocks+CCs portfolio than the tangency and MVP portfolios is due to higher weighting of cryptocurrencies in the period of 2017 to 2018, in which the cryptocurrencies showcased remarkably high returns when compared to the Brazilian stocks (Figure 2), which was followed by also higher volatilities (Figure 1).

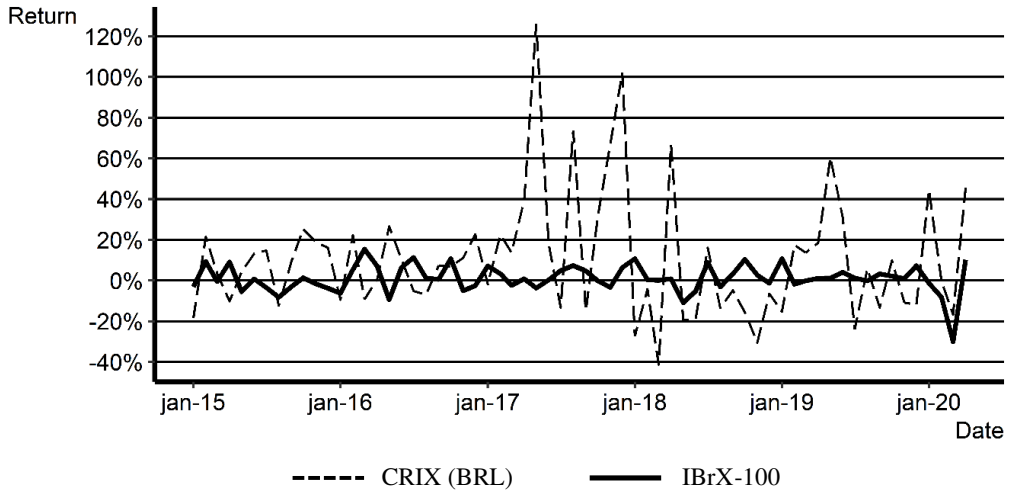

Figure 2. Monthly returns of the IBrX-100 against the CRIX (BRL)

3.3. Comparison with indexes

All portfolios that include cryptocurrencies presented higher returns than the IBrX-100 for the period studied, although only the equally weighted portfolios presented a significant result at a 10% significance level. On the other hand, the returns from CRIX were much higher than all the models.

When comparing the volatility, the tangency portfolio and the equally weighted with pre-defined weight for cryptocurrencies until 15% presented a volatility of similar level to the

IBrX-100, while it was lower for the MVP portfolio and higher for the other portfolios. As for the returns, the volatility of the CRIX was much higher than all models.
For the Sharpe, Sortino and Omega ratios, all models performed better than the IBrX-100 in the studied period, although the results were only significant for the equally weighted portfolios. On the other hand, the CRIX performed better in these metrics than almost all models, except against some of the equally weighted portfolios that forced higher weights for cryptocurrencies.

3.4. Testing for one-year inputs

The results so far exhibited a performance below expectations for the Markowitz models, especially comparing to the equally weighted portfolios. Our hypothesis is that the small period used as sample to run the models generates weak estimators. Therefore, increasing this period would improve the optimization. We decided for an additional test, using the previous twelve months as input to the models. This change will also make the portfolios less reactive to short terms events. Table 3 exhibits the results.

Table 3

Descriptive statistics of the portfolios using yearly data as inputs

Model /Index	Portfolio	Annual geom. return (%)	Annual vol. (%)	Sharpe ratio	Sortino ratio	Omega ratio	Daily return (%)			
							Avg	Med	Min	Max
Equally Weighted	Stocks-only	19.51	27.46	0.03	0.04	1.17	0.09	0.15	-16.21	12.49
	Stocks+ CCs	58.56***	36.62	0.08**	0.12**	1.35**	0.21	0.19	-16.21	22.22
	Stock 95% + CCs 5%	32.05***	27.35	0.05***	0.07***	1.26***	0.13	0.17	-17.79	12.51
	Stock 90% + CCs 10%	43.72***	28.86	0.07***	0.10***	1.20***	0.16	0.20	-19.26	12.54
	Stock 85% + CCs 15%	54.62***	31.30	0.08**	0.12***	1.37**	0.19	0.17	-20.65	13.81
	Stock 80% + CCs 20%	64.83***	34.26	0.09**	0.13***	1.40**	0.22	0.17	-21.96	17.98
	Stock 75% + CCs 25%	74.40***	37.51	0.09**	0.14**	1.42**	0.25	0.16	-23.20	22.02
	Stock 70% + CCs 30%	83.36***	40.90	0.09**	0.15**	1.43**	0.27	0.15	-24.36	25.92
MVP	Stocks-only	8.74	17.85	0.00	0.01	1.13	0.04	0.07	-11.70	9.43
	Stocks+ CCs	17.98***	18.33	0.03***	0.04***	1.24***	0.07	0.08	-13.50	9.50

Tangency	Stocks-only	10.50	24.00	0.01	0.01	1.12	0.05	0.10	-14.14	12.32
	Stocks+CCs	45.47***	28.49	0.07***	0.11***	1.35***	0.17	0.15	-16.31	16.08
IBrX-100		13.06	27.58	0.02	0.02	1.12	0.06	0.09	-14.89	13.75
CRIX (BRL)		153.40	78.40	0.09	0.14	1.35	0.49	0.43	-37.95	23.99

Note: The statistics refer to the 1,147 daily returns between September 2015 and April 2020.
** denotes 5% significance and *** denotes 1% significance measured by a bootstrap test. For the portfolios that included cryptocurrencies, the test was performed for the returns and the Sharpe, Sortino and Omega ratios, under the null hypothesis that the difference to the stocks-only portfolio was equal to zero and alternative hypothesis that it was different than zero.

For the equally weighted portfolios, all options that include cryptocurrencies remain with statistically significant higher returns than the stocks-only portfolio, as well as statistically significant higher risk-adjusted performance. Compared to the basis scenario, the portfolios in this test present slightly better returns and higher volatilities, but the risk-adjusted performances are also better.

However, the major impacts occur in the mean-variance portfolios. Both the MVP and tangency stocks+CCs portfolios exhibit strong improvements in the returns and risk-adjusted performance, without relevant impacts in their volatilities. In both cases, the returns, Sharpe, Sortino and Omega are significantly better than the stocks-only portfolio.

The comparison between the models indicates that the equally weighted portfolios are only consistently better in the returns against the tangency portfolio when the weight of cryptocurrencies is at least 25%, and even those cases were not significantly better in the risk-adjusted performance. When compared to the MVP portfolio, only the portfolio with 5% of cryptocurrencies did not show statistically significant difference in the risk-adjusted performance.

Compared to the IBrX-100 index, the differences between the returns and the Sharpe, Sortino and Omega ratios became statistically significant for the tangency portfolio, while it remained not significant for the MVP portfolio.

4. Conclusions

This paper analysed portfolios using a dataset from September of 2014 until April of 2020, with the objective of understanding the impact of including cryptocurrencies in Brazilian stocks' portfolios. We compared the changes in the returns, volatilities, and Sharpe, Sortino and Omega ratios. We worked with three different portfolio selection models: the equally weighted portfolio, the minimum variance portfolio, and the tangency Markowitz portfolio, each one with and without cryptocurrencies.

The portfolios with cryptocurrencies presented higher total returns for the period analysed than the stock-only portfolios, being statistically significant at a 10% level for the equally weighted portfolios and the tangency portfolio. Notwithstanding, this result comes with an increase in the portfolio volatilities, due to the high-risk characteristic of the cryptocurrencies (highly volatile in the period). Despite the increased volatility, the portfolios with cryptocurrencies performed better in the Sharpe, Sortino and Omega ratios. Therefore, the analysis carried out by this study suggests that cryptocurrencies could be included in stocks' portfolios by Brazilian investors, although with some caution, especially by investors less inclined to riskier investments.

The equally weighted portfolios performed better than the Markowitz models, with higher returns and higher Sharpe, Sortino and Omega ratios. The differences were more relevant in the portfolios with a pre-defined cryptocurrencies weight, which gave investors more options to choose from depending on their risk-tolerance: more cryptocurrencies in the portfolio will tend to result in higher expected returns and higher volatilities, with an overall higher risk-adjusted performance. The tangency and minimum variance portfolios only become good alternatives if we extend the data used as inputs for the portfolio selection, since they provided better significant out-of-sample returns, Sharpe, Sortino and Omega in the test using one year of data.

Overall, the strategy of including cryptocurrencies in the portfolios presented consistent results against the IBrX-100 for both returns and risk-adjusted performances in the period studied. Even regarding the volatility, the investors have options that improve the returns with similar risk to the index.

We must consider two important limitations in this study. One is the short period of data available for cryptocurrencies, which should be addressed as this asset class becomes more mature. Nonetheless, due to the strong gains these assets had in the period studied, the results leaned in the direction that the more cryptocurrencies investors include in their portfolios the better, which might not hold true in the future. Second, we did not consider transaction costs and income taxes in this work: we opted to do so for clarity purposes and because the Brazilian market is evolving very fast with declining transaction costs. The choice of how to quantify transaction costs could impact our results and quickly make them out-dated.

As suggestion for future studies, first is an application of different portfolio selection models, with more complex calculations, which could present better returns than the ones analysed in this study, although being only suited for investors with more knowledge. In addition, the use of more data as inputs for the models, which was a limitation due to the small database the cryptocurrencies have in the time of this study. Another suggestion would be to analyse the inclusion of cryptocurrencies in portfolios with more asset classes, like fixed income, real estate, currencies, global stocks, and even commodities.

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