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Childcare Support and Public Capital in an Ultra-Declining Birthrate Society

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Abstract

This paper analyzes whether public capital investment or childcare support maximize the growth rate in an ultra-declining birth rate society using a labor-augmented model with public capital. We clarify the global stability of the private capital-public capital ratio in the steady state. In addition, we analyze the effect of increasing the expenditure share of tax revenue on economic growth. The result of this analysis shows that an increased share of public capital investment brings higher economic growth. This means that if all tax revenue is allocated to public capital investment, the growth rate will be maximized.

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1. Introduction

The number of children born in Japan continues to decrease. The total fertility rate was 1.36[2] in 2019, the lowest level to date, as indicated by the Japanese Ministry of Health, Labor and Welfare (MHLW). The Cabinet Office continues to insist that Japan has been in a state of declining birth rates for many years, resulting in what is referred to as an "ultra-declining birth rate society." The demographic trends are such that, by 2050, one in 2.5 people will be elderly (aged 65 or older).[3] Viewing life in the long term, workers should determine their spending based on their estimated lifetime income. According to the overlapping generations (OLG) model proposed by Diamond (1965), the lifetime income of an individual is assumed to consist of earnings received in two periods: their working period and their later life. Individuals make decisions from a lifetime perspective while adhering to budgetary constraints. Becker (1981) and Becker and Lewis (1973) showed that the number of children in developed countries will decline; at first glance, this is seemingly a contradiction, considering that children are positive to societies, however, results from the fact that the cost of childcare is proportional in scale to its quantity multiplied by its quality. In this study, models are established based on a neoclassical theory that suggests that growth in capital boosts gross domestic product (GDP) and leads to a greater growth rate for the whole nation. The main portion of this study utilizes Romer's endogenous growth model (1986) to introduce the public capital models proposed by Barro (1990), Barro and Sala-i-Martin (1992), Futagami et al. (1993), Turnovsky (1997), Yakita (2008), and Maebayashi (2013). These models indicate that public capital stock boosts labor productivity. Investment in public capital is financed through the levying of income taxes (on labor income and capital income). Yakita (2008) used a birth rate internalization model that considers two public expenditures: public capital investment and public capital maintenance. Maebayashi (2013) showed the dynamics of the private-public capital ratio and confirmed the existence of a steady state and global stability. Furthermore, the author analyzed the optimal allocation of tax revenue between expenditure on public capital investment and public pension subsidies under a pay-as-you-go pension system. The study concluded that it was clear that the best policy for growth is to allocate all financial resources to public capital investment;

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[2] "Current population survey," MHLW website (https://www.mhlw.go.jp/toukei/list/81-1a.html) (accessed on September 20, 2020)

[3] "Situations of Aging" (Japanese), Cabinet Office website (https://www8.cao.go.jp/kourei/whitepaper/w-2012/zenbun/pdf/1s1s_1.pdf) (accessed on June 15, 2020)

However, from a social-welfare perspective, the optimal tax revenue allocation rate depends on the magnitude of the social discount rate.

In this study, we analyze the policy trade-off between public capital investment and childcare support and the effects on the growth rate under government budget constraints, where the government sources revenue only from income taxes. First, we prove the existence of a steady state, and confirm that the economy converges to the steady state globally and stably. We show that all variables: public capital, private capital, and GDP, grow at the same rate on the balanced growth path (BGP). Second, we analyze the effect of increasing the share of public capital investment on growth under constant tax revenue, and using a numerical example, we find that this growth is positive though the relative value of capitals will decline by effect of rising childcare cost exceeds the effect of pushing up income. The model is constructed using the Diamond model (1965), a two-period OLG model. We introduce public capital stock to construct a model that has labor-augmented production technology.

The remainder of this paper is organized as follows. The next section presents the model and its dynamics in terms of (private and public) capital. The global stability of the dynamics in the steady state is then confirmed. The effects of governmental increases in public capital investment shares in the steady state are analyzed. The final section concludes the paper.

2. Model

2.1 Individuals

The two-period OLG model presented by Diamond (1965), with fully competitive markets, is considered. A homogeneous individual is assumed, who obtains utility from consumption in the working and later periods of life, and selects the number of children that they have. We consider a child to be a consumer good rather than a capital good, and there is no public pension. Individuals supply labor inelastic in only the first period, and it is assumed that every individual has one unit of labor to supply to the labor market. Individuals allocate income for consumption, saving, and childcare costs in the first period. The individual consumes all income, including saving and interest, in the first period, with no bequests in the second period. A logarithmic linear utility function and lifetime budget constraint, which must hold in order for the economy to be sustainable in the long term, are specified as follows:

$$max. \quad u_t = logc_t + \rho logd_{t+1} + \varepsilon logn_t \tag{1}$$

$$s.t \quad w_t\,(1-\tau)[1-n_t(z-h_t)] = c_t + \frac{d_{t+1}}{r_{t+1}(1-\tau)} \tag{2}$$

$$c_t^* = \frac{(1-\tau)w_t}{(1+\varepsilon+\rho)} \tag{3}$$

$$n_t^* = \frac{\varepsilon}{(1+\varepsilon+\rho)(z-h_t)} \tag{4}$$

$$d_{t+1}^* = \frac{\rho r_{t+1} w_t (1-\tau)^2}{(1+\varepsilon+\rho)} \tag{5}$$

$$s_t^* = \frac{\rho}{(1+\varepsilon+\rho)}(1-\tau)w_t \tag{6}$$

Where time preference, child preference, childcare cost, childcare support, and income tax are denoted as $\rho \in (0,1)$, $\varepsilon > 0$, $z \in (0,1)$, $h_t \in (0,1)$, $z > h_t$, and $n_t \geq 1$, respectively.

2.2 Production

A Cobb-Douglas production technology in which labor increases with public capital investment, as in Romer (1986), is used. It is assumed that there are many firms in a goods market, and these firms have access to the same technology. The inputs are the private capital stock and labor. The production function of firm i is specified as follows:

$$Y_{it} = K_{it}^{\alpha}(A_t L_{it})^{1-\alpha} \tag{7}$$

$$A_t = \frac{G_t}{L_t} \tag{8}$$

$$Y_t = K_t^{\alpha} {G_t}^{1-\alpha} = \left(\frac{K_t}{G_t}\right)^{\alpha} G_t = x_t^{\alpha} G_t \tag{9}$$

The equilibrium condition in labor market is shown as follows:

$$L_t = N_t[1-n_t(z-h_t)] \tag{10}$$

Where N_t is the number of households in period t. We assume a perfectly competitive market and solve the profit maximization problem as follows:

$$(1-\alpha)\left(\frac{K_{it}}{L_{it}}\right)^{\alpha-1} {A_t}^{1-\alpha} = w_t \tag{11}$$

$$\alpha\left(\frac{K_{it}}{L_{it}}\right)^{\alpha-1}{A_t}^{1-\alpha}=r_t \tag{12}$$

From (11) and (12), the private capital-labor ratio will become the same value as in $K_{ti}/L_{ti}=K_t/L_t$.Also, $\sum_{i=1}^{\infty}L_{it}=L_t,$ $\sum_{i=1}^{\infty}K_{it}=K_t$ can be derived, where L_t and K_t denote the total labor supply and total private capital, respectively. By defining a new variable, $x=\frac{K}{G}$, to be the ratio of private and public capital, (11) and (12) can be rewritten as the following equations:

$$(1-\alpha)\left(\frac{K_t}{G_t}\right)^{\alpha}\frac{G_t}{L_t}=(1-\alpha)x_t^{\alpha}\frac{G_t}{L_t}=w_t \tag{13}$$

$$\alpha\left(\frac{K_t}{G_t}\right)^{\alpha-1}=\alpha x_t^{\alpha-1}=r_t \tag{14}$$

2.3 Government

The government taxes income and divides tax revenues between public capital investment, $E>0$, and childcare support, $H>0$. The share of spending on public capital investment and the income tax rate are respectively denoted $\varphi\in[0,1]$, $\tau\in[0,1]$. The depreciation rate of public and private capital is 1. The government budget constraint is shown in the following equations:

$$E_t+H_t=\tau Y_t=\tau x_t^{\alpha}G_t \tag{15}$$

$$E_t=G_{t+1}-G_t=\varphi\tau Y_t=\varphi\tau x_t^{\alpha}G_t \tag{16}$$

$$w_t h_t n_t N=(1-\varphi)\tau Y_t=(1-\varphi)\tau x_t^{\alpha}G_t \tag{17}$$

The per-capita childcare support is determined using (17), and is indicated by the following equation (the value of which will be constant):

$$h_t=\frac{(1-\varphi)(1-\varepsilon z)\tau}{\varepsilon[1-(1-\varphi)\tau]} \tag{18}$$

$$L_t=N_t\left(\frac{1+\rho}{1+\varepsilon+\rho}\right) \tag{19}$$

The number of households in period t+1 is denoted by $N_{t+1} = N_t n_t$, and the number of children is constant in the steady state. Therefore, the growth of labor force can be written in the following form:

$$g_L = \frac{L_{t+1}}{L_t} = \frac{n_t N_t}{N_t} = n_t \tag{20}$$

3. Equilibrium

There are three markets, and we consider only the capital market by Walras' law. The equilibrium condition is as follows:

$$s_t N_t = K_{t+1} \tag{21}$$

We substitute the optimal savings (6) for the equilibrium condition (21), and substitute for the wage rate (13). These allow us to rewrite condition (21) as the next equation:

$$K_{t+1} = \frac{\rho}{(1+\varepsilon+\rho)}(1-\tau)w_t = \frac{\rho}{(1+\varepsilon+\rho)}(1-\tau)(1-\alpha)x_t^{\alpha} N_t \frac{G_t}{L_t} \tag{22}$$

And we can get equation (23) by dividing both sides of equation (22) by K_t:

$$g_K = \frac{K_{t+1}}{K_t} = \frac{\rho(1-\tau)(1-\alpha)}{(1+\rho)} x_t^{\alpha-1} \tag{23}$$

The dynamics of private capital are obtained in the following section.

4. Dynamics

The dynamics of public capital are indicated by equation (24):

$$g_G = \frac{G_{t+1}}{G_t} = \varphi\alpha\tau x_t^{\alpha} + 1 \tag{24}$$

The growth of x is indicated by the following equation, which combines the capital dynamic equations (23) and (24).

$$g_x = \frac{x_{t+1}}{x_t} = \frac{\frac{K_{t+1}}{K_t}}{\frac{G_{t+1}}{G_t}} = \frac{\rho(1-\tau)(1-\alpha)x_t^{\alpha-1}}{(\varphi\alpha\tau x_t^{\alpha}+1)(1+\rho)} \tag{25}$$

$$\frac{dx_{t+1}}{dx_t} = \frac{A(x_t, x_{t+1})}{B(x_t\ )} = f(x_t, x_{t+1}) \tag{26}$$

$$A(x_t, x_{t+1}) = \rho(1-\tau)(1-\alpha)x_t^{\alpha-1} - \emptyset\varphi\alpha\tau x_t^{\alpha-1}x_{t+1} \quad (27)$$

$$B(x_t\ ) = (\varphi\alpha\tau x_t^{\alpha} + 1)(1+\rho) > 0 \quad (28)$$

In order to analyze the signs of "A" and "B" in equation (26) , the parameters in equations (27) and (28) are quantified concretely as Table 1. Next, we derive the second derivative of equation (25) (Table.2.) When x_t approaches 0, the growth of x is zero in equation (25) $\left(\lim_{x_t\to 0}\frac{x_{t+1}}{x_t} = 0\right)$. In other words, the curve in the curve passes through the origin.

$$\frac{\partial^2 x_{t+1}}{(\partial x_t)^2} = \frac{\partial f(x_t, x_{t+1})}{\partial x_t} = \frac{A'B - AB'}{B^2} \quad (29)$$

$$A' = \frac{\partial A(x_t, x_{t+1})}{\partial x_t} = -\alpha\rho(1-\tau)(1-\alpha)^2 x_t^{\alpha-2} + (1-\alpha)(1+\rho)\varphi\alpha^2\tau x_t^{\alpha-2}x_{t+1} \quad (30)$$

$$B' = \frac{dB(x_t\ )}{dx_t} = (1+\rho)\varphi\alpha^2\tau x_t^{\alpha-1} \quad (31)$$

$$\lim_{x_t\to 0}\frac{dx_{t+1}}{dx_t} = indifinitly, \quad \lim_{x_t\to\infty}\frac{dx_{t+1}}{dx_t} = 0 \quad (32)$$

x	1	2	3	4	5
α	0.4	0.4	0.4	0.4	0.4
ε	0.7	0.7	0.7	0.7	0.7
ρ	0.7	0.7	0.7	0.7	0.7
φ	0.83	0.83	0.83	0.83	0.83
τ	0.3	0.3	0.3	0.3	0.3
A	0.0499	-0.0574	-0.097	-0.115	-0.097
B	1.87	3.53	2.81	2.82	2.81
C	0.0267	-0.0179	-0.035	-0.041	-0.0626

Table.1.

x	1	2	3	4	5
α	0.4	0.4	0.4	0.4	0.4
ε	0.7	0.7	0.7	0.7	0.7
ρ	0.7	0.7	0.7	0.7	0.7
φ	0.83	0.83	0.83	0.83	0.83
τ	0.3	0.3	0.3	0.3	0.3
A'	-0.034	-0.029	-0.032	0.01	0.015
B'	0.068	0.045	0.352	0.035	0.03
A	-0.0266	-0.0574	-0.097	-0.115	-0.097
B	1.075	3.53	2.81	2.82	2.81
D	-0.0347	-0.0998	-0.0558	0.03223	0.04506

Table.2.

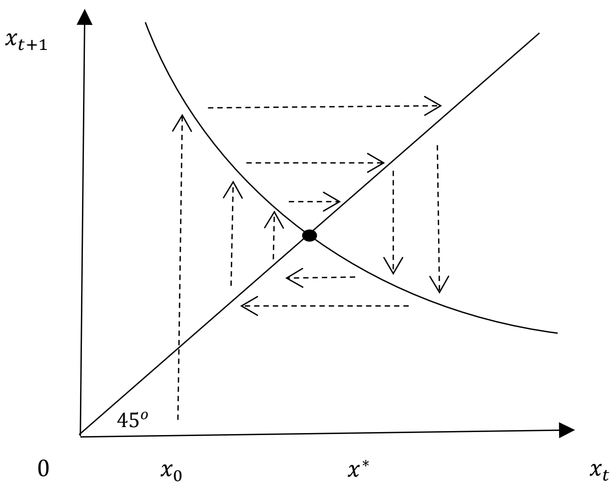

Fig. 1. Dynamics of x

The private-public capital ratio will increase, and the steady state of x is shown as x^*. If equation (33) is satisfied with x^* the growth rate of GDP, the private capital and public capital will be the same:

$$\rho(1-\tau)(1-\alpha)(x^*)^{\alpha-1} = [\varphi\alpha\tau(x^*)^{\alpha} + 1](1+\rho) \tag{33}$$

Proposition 1. There is a unique value that shows the public-private capital ratio in the steady state. If the equation (33) is satisfied, public capital, private capital and GDP will grow at the same rate. That is, the growth path is balanced and globally stable.

$$\frac{\partial g}{\partial \varphi} = \tau\alpha^2(x^*)^{\alpha}\left[\frac{1}{\alpha} + \frac{\varphi}{x^*}\frac{dx^*}{d\varphi}\right] \tag{34}$$

$$\frac{dx^*}{d\varphi} = \frac{A}{B} < 0 \tag{35}$$

$$\mathrm{A} = \varphi^2\alpha\tau(x^*)^{\alpha} > 0 \tag{36}$$

$$B = -[\varphi\,\alpha^2\tau(x^*)^{\alpha-1} + \rho(1-\tau)(1-\alpha)^2(x^*)^{\alpha}] < 0 \tag{37}$$

Where the second term in brackets indicates the elasticity of the share for the relative capital value, and the sign is negative. That is, the effect of rising cost to raise children exceeds the effect of boosting income. Next, we analyze the effect of rising share on growth and Parameters in equations (38) are quantified concretely as $(\alpha, \varepsilon, \rho, \tau, x, z, \varphi) = (0.4, 0.7, 0.7, 0.3, 3, 0.06, 0.83)$.

$$\frac{\partial g_G}{\partial \varphi} = \tau\alpha^2(x^*)^{\alpha}\left[\frac{1}{\alpha} + \frac{\varphi}{x^*}\frac{dx^*}{d\varphi}\right] = \tau\alpha^2(x^*)^{\alpha} \times \tag{38}$$

$$\left\{\frac{[\varphi\,\alpha^2\tau(x^*)^{\alpha-1} + (\varepsilon+\rho)(1-\tau)(1-\alpha)^2(x^*)^{\alpha}] - \varphi^2\alpha^2\tau(x^*)^{\alpha}}{[\varphi\,\alpha^2\tau(x^*)^{\alpha-1} + (\varepsilon+\rho)(1-\tau)(1-\alpha)^2(x^*)^{\alpha}]}\right\} = 0.911 > 0$$

Proposition 2. A policy in which all tax revenue is spent on public capital investment is better policy in terms of growth.

5. Concluding remarks

This study focused on the relative value of private-public capital in the presence of a childcare support policy. First, the global stability of economic growth and the unique steady state to which the economy converges was clarified. In the steady state, the economy is on a balanced growth path in which private capital, public capital, and GDP grow at the same rate. Second, the effect of increasing the share of public capital investment on the steady-state growth rate is analyzed, and we found that the growth rate will increase though the relative value of capitals decline in the steady state. This indicates that dividing all tax revenue for public capital investment will be optimal policy in terms of growth maximization.

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