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Measuring consumer surplus in the case of addiction: A re-examination of the rational benchmark algebra

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Abstract

Measuring consumer surplus for addicted consumers is challenging because the presence of internalities makes the observed demand schedule a biased basis to estimate the actual welfare experienced by consumers. A common practice in literature consists of using non-addicted consumers as a rational benchmark. This short contribution points out some limitations of existing approaches and provides a revised measure that satisfies desirable properties in this rational benchmark framework. Comparative estimates based on data from the Australian Productivity Commission 1999 report on gambling indicate that existing approaches lead to largely overestimate the net consumer surplus. The new measure we propose is easy to implement and could be a useful tool when it comes to assessing welfare in addictive contexts.

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1. Introduction

Consumer surplus, as a measure of aggregate welfare of individuals who consume a given commodity, constitutes a key element of cost-benefit analyses for public policies. It is traditionally estimated by subtracting expenditures from the willingness to pay of consumers obtained from the demand schedule. To the extent that agents are rational and fully informed, the observed demand schedule is assumed to provide an accurate estimate of the actual welfare experienced by consumers. This standard approach raises concerns when consumption involves addictive behaviours. There is currently little doubt that most consumers of addictive goods are aware of the health and social risks of their behaviour (see, e.g., Levy et al. 2018 for tobacco). There is however a widespread questioning of the assumption of rationality. The well-known rational addiction model developed by Becker and Murphy (1988) has been widely criticized (see, e.g., Rogeberg 2004) and a large literature explicitly describes addictive behaviours as departing from the rational choice framework (see, e.g., Gruber and Köszegi 2001; O'Donoghue and Rabin 2002; Bernheim and Rangel 2004). In such models, addicted consumers face internalities, i.e. costs that they impose on themselves against their interest, due to self-control problems or imperfect consideration of future damage related to their consumption. As a consequence, the observed demand schedule constitutes a biased basis to estimate consumer welfare and the measure of surplus must be amended.

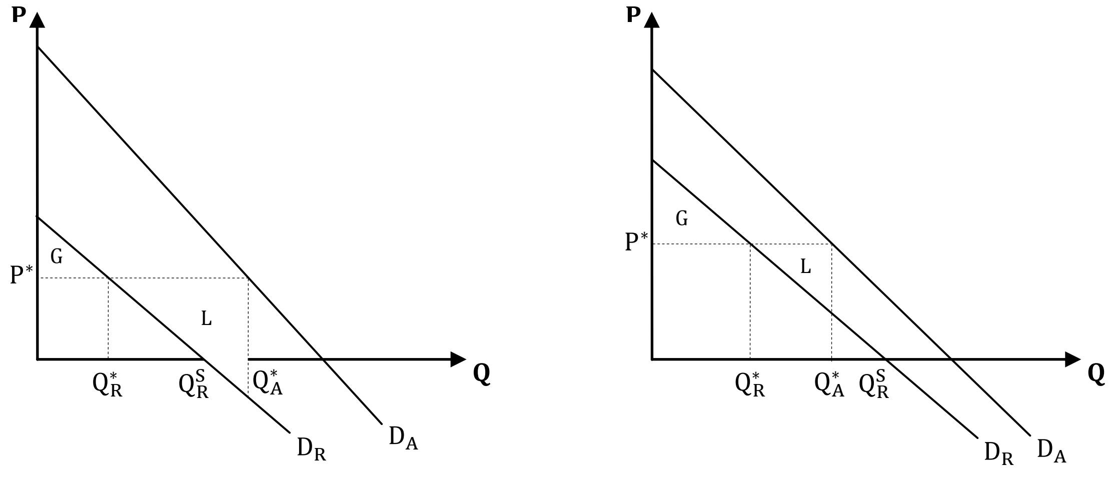
Several approaches have been developed to measure consumer welfare in addictive contexts (see Cutler et al. 2016), relying for instance on surveys measuring willingness to pay for cessation or surveys directly measuring subjective well-being. One approach, identified by Cutler et al. (2016, p. S25) as "currently most tractable, given available models and data", is referred to as "the rational benchmark approach". It was independently developed by the Australian Productivity Commission (1999) (hereafter APC 1999) and Laux (2000), respectively for gambling and tobacco. It relies on a binary conception of consumption, with a group of rational consumers and a group of addicted consumers. The underlying consideration is that the decision making of rational consumers is expected to be unbiased, unlike that of addicted consumers. Thus, the behaviour of rational consumers can be used as a benchmark to measure the excess of consumption and the loss of welfare of addicted consumers, who are assumed to suffer their high level of consumption. The welfare of addicted consumers is then measured as a net surplus combining a welfare gain on the rational part of consumption and a welfare loss on the excessive part of consumption. This rational benchmark approach has since been widely adopted in literature (see, e.g., Weimer et al. 2009; Ashley et al. 2015; Cutler et al. 2015; Jin et al. 2015; Levy et al. 2018).

An explicit analytical formulation of the net consumer surplus is not always required, for instance, when the goal is to measure welfare changes induced by regulation changes, as in Laux (2000), Ashley et al. (2015), or Jin et al. (2015). That is why such formulations are rare. To date, the initial measure of the APC (1999) remains the standard in benefit-cost analysis (see, e.g., Boardman 2018, p. 137-139). More recently, several authors have developed analyses using a parallelism assumption of addicted and rational demand schedules (see, e.g., Weimer et al. 2009, Cutler et al. 2015, and Levy et al. 2018). In this methodological note, we show that both APC and parallelism-based approaches suffer from important biases due to the algebra that is used and specific assumptions that are made. We also provide a revised measure that satisfies desirable properties to estimate consumer surplus in the case of addiction using the rational benchmark approach in its most general form.

The rest of this note is organized as follows. We present the general philosophy of the rational benchmark approach in the next section. We focus on the standard measure of the APC and its limitations in section 3. In section 4, we propose a revised measure of the net consumer surplus and examine its properties. In section 5, we assess the impact of the parallelism assumption using our revised framework. Section 6 provides a comparison of estimates obtained with the different approaches using the original data on gambling from the APC. Section 7 concludes.

2. The rational benchmark approach

The rational benchmark approach has been developed to measure the aggregate welfare of a group of addicted consumers, i.e. agents assumed to overconsume a commodity compared to what would maximize their welfare. It derives the net consumer surplus for this group by imagining a counterfactual where its unbiased (or "true") preferences would be the same as those of a benchmark group of rational consumers. Up to a certain quantity, consumption of the good is assumed to be the result of rational decision making and provides a welfare gain. Beyond this quantity, agents are supposed to overconsume and consumption provides welfare loss. The net surplus is obtained by subtracting the welfare loss from the welfare gain. This approach is depicted in Figures 1.a and 1.b. Figure 1.a shows a situation in which the optimal consumption of addicts is so high that rational consumers would ask to be paid to consume so much (low satiation of rational consumers). Figure 1.b illustrates the opposite situation where rational consumers are willing to pay for the optimal level of consumption of addicts (high satiation of rational consumers).



a. Low satiation ($Q_R^S < Q_A^*$)   b. High satiation ($Q_R^S > Q_A^*$)

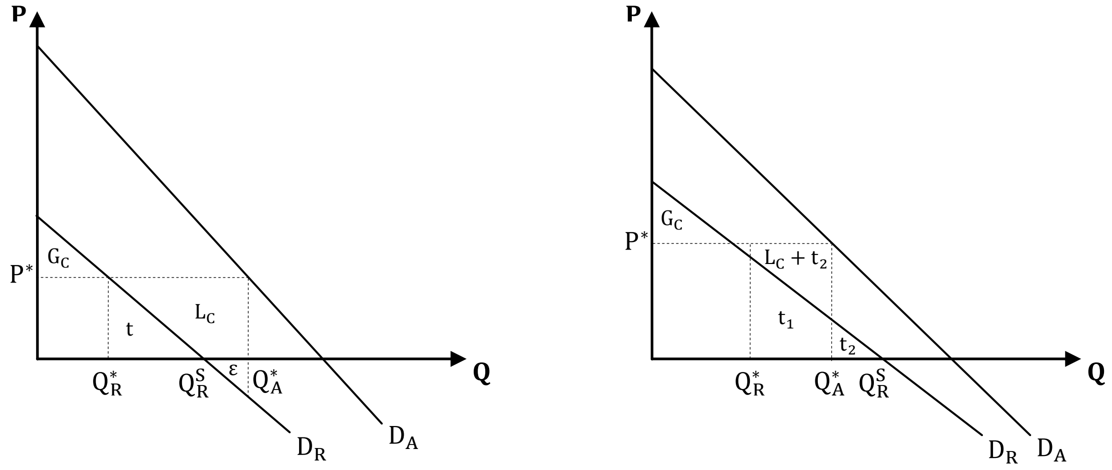
Figure 1. Net consumer surplus using a rational benchmark

We note P^* the equilibrium price. Line D_A represents the demand schedule of addicted consumers for which the optimal consumption is Q_A^*. Line D_R symbolises the hypothetical demand schedule of rational consumers, which is associated with the optimal consumption Q_R^* and the satiation point Q_R^S. The only difference between Figures 1.a and 1.b is the position of Q_R^S relative to Q_A^*.

The rational benchmark approach consists of deriving the surplus for the addictive consumption Q_A^* with respect to the rational demand schedule D_R. This approach leads to distinguishing two welfare components. On the one hand, willingness to pay for rational consumers is higher than the price equilibrium when $Q < Q_R^*$. Addicted consumers then receive a welfare gain G on the rational consumption Q_R^*. On the other hand, willingness to pay for rational consumers is lower than the price equilibrium when $Q > Q_R^*$. The willingness to pay for rational consumers is even negative for the quantity exceeding the satiation point Q_R^S. Addicted consumers then bear a welfare loss L on the excess consumption $Q_A^* - Q_R^*$. Finally, the net consumer surplus is obtained by subtracting the welfare loss from the welfare gain: $CS = G - L$.

3. The APC measure (APCM)

The APC (1999) provides the most common operational measure of the net consumer surplus using a rational benchmark approach. We summarise it in this section by focusing on the most relevant features given our purpose, i.e., ignoring subsidiary aspects, such as the role of taxes and adjustment for the effect on income of having to pay for the consumer surplus. The components of the measure are depicted in Figures 2.a and 2.b.



a. Low satiation ($Q_R^S < Q_A^*$)

b. High satiation ($Q_R^S > Q_A^*$)

Figure 2. The measure of the APC (1999)

The APC provides a suitable measure for welfare gain (vol. 3, p. C.17, Eq. 1):[1]

$$G_C = \frac{R}{2|\eta_R^*|}, \tag{1}$$

with $R = P^* Q_R^*$, the rational spending, and η_R^*, the price elasticity of the rational demand at equilibrium.

[1] This formula is commonly used to measure consumer surplus in the standard non-addictive context (see, e.g., Peck et al. 2000). Technical details regarding the derivation of this measure are usually omitted. We suggest a simple proof in Appendix A.

However, the proposed measure for welfare loss is most questionable. The APC derives the loss measure by subtracting the triangle t from the excess spending E (vol. 3, p. C.21-C.23):[2]

$$L_C = E - \frac{|\eta_R^*|}{2} R, \tag{2}$$

where $E = P^*(Q_A^* - Q_R^*)$. This approach is only relevant when Q_R^S and Q_A^* are arbitrarily close. In the general case, the loss measure suffers from two serious biases:

(1) The measure ignores the area ε when $Q_R^S < Q_A^*$ (Figure 2.a).[3] The omission of this area is surprising since the APC provides an explicit argument to justify its relevance in the case of gambling: “For recreational gamblers this can be seen as a situation where you would need to pay them to spend as much time and effort on gambling as problem gamblers, in effect a negative price. (…) There is therefore, potentially an area below the zero price line which could be added to our estimate of lack of value for money for problem gamblers.” (vol. 3, p. C.23). This omission has a significant impact regarding the philosophy of the measure. By construction, the welfare loss is necessarily lower than excess spending: $L_C < E = P^*(Q_A^* - Q_R^*)$. This means, in particular, that internalities due to addictive consumption cannot exceed consumption spending for addictive good, which is highly questionable.[4]

(2) The triangle t, which is independent from D_A, includes an irrelevant area t_2 when $Q_R^S > Q_A^*$ (Figure 2.b). The APC approach then leads to an underestimation of the loss by an amount corresponding to the area of the triangle t_2 ($L_C = L - t_2$). The loss even becomes a gain when excess spending is sufficiently small: $lim_{E\to 0} L_C = -R|\eta_N^*|/2 = -t$. This especially implies that consumer surplus is significantly higher than rational welfare gain when the measure is applied to rational consumers: $lim_{E\to 0} CS_C = lim_{E\to 0}(G_C - L_C) = G_C + t > G_C$.

Finally, it appears that, except in the case where Q_R^S and Q_A^* are close (which amounts in particular to assuming that rational consumers are presumed to be willing to bear the level of consumption of addicted consumers and its consequences without monetary compensation), the APCM is likely to significantly underestimate welfare loss and, consequently, overestimate the net consumer surplus for addicted consumers. Section 6 will provide an estimate of this bias using gambling data from the APC. The magnitude of the estimated bias can be seen as evidence that the assumption that Q_R^S and Q_A^* are close is not satisfied.

4. The revised measure (RM)

These previous considerations lead to a revision of the APCM. In this spirit, we retain the original relevant measure for welfare gain (Eq. 1):

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[2] The triangle t is divided in two parts in Figure 2.b: $t_1 + t_2 = t$.

[3] Note that the area ε may be much greater than the depicted area in Figure 2.a. It simply requires that Q_R^S is small compared to Q_A^* and that D_R is steeper.

[4] For instance, Gruber and Köszegi (2001) estimate that internalities are on the order of \$30 and Chaloupka et al. (2019) about \$80 per pack of cigarettes.

$$G = G_C = \frac{R}{2|\eta_R^*|}. \quad (3)$$

Our contribution is to provide a suitable measure for welfare loss L. We derive an unbiased measure for welfare loss using a point-elasticity approach and an integral calculus method (see Appendix B):

$$L = \frac{E^2}{2R|\eta_R^*|}. \quad (4)$$

This unbiased measure logically satisfies the desirable properties that are not satisfied by the original measure. Welfare loss is positive when excess spending is positive and null when excess spending is null. It can also be higher than the excess spending.

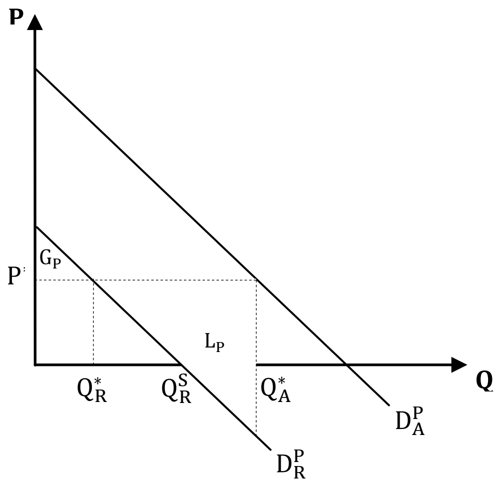
The relevant surplus for addicted consumers is then given by:

$$CS = G - L = \frac{R^2 - E^2}{2R|\eta_R^*|}. \quad (5)$$

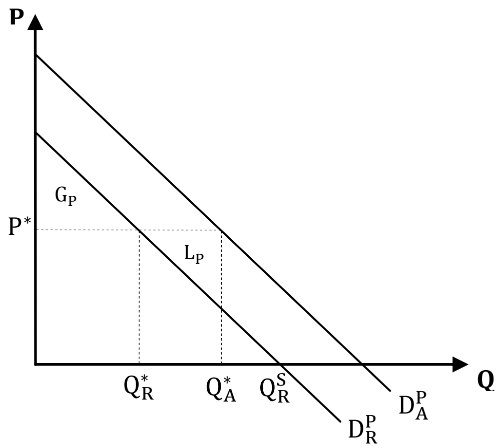
In addition to correctly measuring the net consumer surplus, the revision ensures that it is consistent with the standard approach of consumer surplus. Thus, the measure gives the rational welfare gain when it is applied to rational consumers: $\lim_{E \to 0} CS = G$.

5. The parallelism-based measure (PBM)

Several authors use a parallelism assumption to study the effects of regulation on consumer welfare in ad hoc models (see, e.g., Weimer et al. 2009, Cutler et al. 2015, Levy et al. 2018). They assume that demand schedules of both addicted and rational consumers are parallel. The philosophy of the measure remains unchanged. Both the welfare gain and loss generated by addicted consumption are determined using the rational demand schedule as a reference. The only difference is that rational and addicted demand lines are assumed to be parallel. This approach is depicted in Figure 3. We note D_R^P et D_A^P the demand lines under the parallelism assumption, and G_P and L_P the corresponding welfare measures.



a. Low satiation ($Q_R^S < Q_A^*$)



b. High satiation ($Q_R^S > Q_A^*$)

Figure 3. The assumption of parallelism

The authors who use this assumption do not provide a tractable measure of surplus in the general case. Nonetheless, we can assess the impact of this assumption starting from the previous results on the RM.

The parallelism assumption is not problematic when both rational and addicted demand lines are actually parallel. The derived measure then trivially corresponds to the unbiased measure (the PBM is equivalent to the RM). However, the parallelism assumption is very strong. Both demand lines likely have different actual slopes. To assess the potential bias when the parallelism assumption is not satisfied, it is then necessary to determine what is the common slope of the two demand lines that are assumed to be parallel. There exist two polar cases: either we consider that both demand lines have the slope of the rational demand or we consider that both demand lines have the slope of the addicted demand. In the first case, the PBM is trivially equivalent to the RM. The PBM is hence unbiased, but the parallelism assumption is superfluous since there is no need to assume parallelism to obtain the measure. In the second case, the PBM is different from the RM (see Appendix C):

$$\mathrm{G_P} = \frac{\mathrm{R}^2}{2\mathrm{A}|\eta_\mathrm{A}^*|}, \tag{6}$$

$$\mathrm{L_P} = \frac{\mathrm{E}^2}{2\mathrm{A}|\eta_\mathrm{A}^*|} \tag{7}$$

and

$$\mathrm{CS_P} = \mathrm{G_p} - \mathrm{L_p} = \frac{\mathrm{R}^2 - \mathrm{E}^2}{2\mathrm{A}|\eta_\mathrm{A}^*|}, \tag{8}$$

with $A = P^* Q_A^*$, the total spending of addicted consumers, and η_A^*, the price elasticity of the addicted demand at equilibrium.

It thus appears that the PBM is biased relative to the RM when the parallelism assumption is neither superfluous nor satisfied.[5] More generally, there is a bias as soon as the selected slope is different from the slope of the addicted demand line. The magnitude of the bias simply decreases as the selected slope approaches the slope of the rational demand line. Assuming that rational consumers are more price-sensitive than addicted consumers and that their demand schedule is less steep than that of addicted consumers, the parallelism assumption then leads to overestimating the consumer surplus.[6]

6. A comparison of estimates

A comparison of estimates provides a helpful insight into the differences between the APCM, the RM, and the PBM. For this purpose, we use the gambling data from the APC (1999, vol. 3, p. C.25, Table C.7).[7] The APC data distinguishes between recreational and problem

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[5] Let $b_R = -P^*/\eta_R^* Q_R^*$, the slope of rational demand schedule, and $b_A = -P^*/\eta_A^* Q_A^*$, the slope of addicted demand schedule (both slopes in absolute value). If $b_R \neq b_A$, then $\eta_R^* P^* Q_R^* \neq \eta_A^* P^* Q_A^*$, $|\eta_R^*| R \neq |\eta_A^*| A$ and $CS = (R^2 - E^2)/2R|\eta_R^*| \neq CS_P = (R^2 - E^2)/2A|\eta_A^*|$.

[6] If $b_R < b_A$, then $\eta_R^* P^* Q_R^* < \eta_A^* P^* Q_A^*$, $|\eta_R^*| R > |\eta_A^*| A$ and $CS = (R^2 - E^2)/2R|\eta_R^*| < CS_P = (R^2 - E^2)/2A|\eta_A^*|$.

[7] The dataset is quite old. This is justified by the fact that the main goal of this note is to highlight the measurement bias in the APC approach. It is thus legitimate to rely on the original dataset of the APC. Similar results would most likely be obtained with more recent data.

gamblers using a standardized assessment instrument called the South Oaks Gambling Screen (SOGS). In this context, the recreational gamblers correspond to rational consumers, and the problem gamblers to addicted consumers. We recompute the net consumer surplus for problem gamblers with the three measures using this original data. We rename the components according to our presentation. The results are given in Table 1.

Most calculations (G, L and CS) are based on the price elasticity of demand. Since the price elasticity of demand for gambling is difficult to estimate and can lead to various results (see, e.g., Gallet 2015), the APC uses a range of values rather than a single value for the price elasticity of demand. More precisely, it uses the following ranges of values: [-1,30 ; -0,80] for recreational gamblers, [-1,00 ; -0,60] for moderate problem gamblers and [-1.00 ; -0.30] for severe problem gamblers. Both APCM and RM are based on the slope of the rational demand schedule. The elasticity range used to calculations is then that of recreational gamblers. The PBM is based on the slope of the addicted demand schedule. The elasticity range used to calculations is that of severe problem gamblers (which includes that of moderate problem gamblers).

Table I. Net consumer surplus for addicted gamblers in Australia (1997–98, $ million)

	APCM	RM	PBM
Reference price elasticity (η_R^* or η_A^*)	*[-1,30 ; -0,80]*	*[-1,30 ; -0,80]*	*[-1,00 ; -0,30]*
Rational spending (R)	*438*	*438*	*438*
Gain on rational spending (G)	[168 ; 274]	[168 ; 274]	[27 ; 90]
Excess spending (E)	*3 124*	*3 124*	*3 124*
Loss on excess spending (L)	[2839 ; 2949]	[8570 ; 13926]	[1370 ; 4566]
Net consumer surplus for addicted gamblers (CS)	[-2675 ; -2671]	[-13652 ; -8401]	[-4477 ; -1343]

Source: Data in italics are taken from the APC (1999). Other data are calculations by the authors. The computed results for the APCM exhibit a slight difference with respect to the original results. This negligible difference is certainly due to rounding issues since original calculations were performed at a disaggregated level, distinguishing the type of gambling and the degree of addiction (moderate or severe).

It appears that both APCM and PBM largely underestimate the welfare loss on excess spending and hence substantially overestimate the net consumer surplus for addicted gamblers. We can also observe that the APCM for the net consumer surplus loss for addicted gamblers represents less than one-third of the RM (2671/8401=0.318 and 2675/13652=0.196).

On another note, it is interesting to look at the effect of the different approaches on the aggregate results. The APC estimates the aggregate consumer surplus by adding the consumer surplus for rational (or recreational) gamblers and taxes on gambling. The comparisons are shown in Table 2.

Table II. Aggregate consumer surplus for gambling in Australia (1997–98, $ million)

	APCM	RM	PBM
Consumer surplus for rational gamblers	*[2745 ; 4460]*	*[2745 ; 4460]*	*[2745 ; 4460]*
Net consumer surplus for addicted gamblers	[-2675 ; -2671]	[-13652 ; -8401]	[-4477 ; -1343]
Tax, licence fees and community contributions	*4 312*	*4 312*	*4 312*
Aggregate consumer surplus	[4386 ; 6097]	[-4880 ; -1344]	[4295 ; 5714]

Source: Data in italics are taken from the APC (1999). Other data are calculations by the authors.

The difference between the three approaches becomes dramatic at the aggregate level. Both APCM and PBM indicate that gambling is a source of large net benefit for the country while the RM shows that it imposes an important net loss. This implies that the correction we propose is unfavourable to the industry and may lead to recommend increased regulation of addictive goods.

7. Concluding remarks

The present note proposes an adjustment of the measure for consumer surplus in the case of addiction in the rational benchmark framework. As pointed out by Cutler et al. (2016), the main advantages of the rational benchmark approach are to be grounded in the standard consumer surplus analysis framework, to account for internalities without requiring specific assumptions about the type of cognitive bias at play, and to be quite parsimonious in terms of the data needed.

There are also limitations to this approach. One of them is related to the need to distinguish between two groups of consumers: the rational consumers and the addicted consumers. Even if there exist reliable measurement tools developed by clinical research enabling to classify consumers rigorously, the use of a specific instrument (e.g. the SOGS instead of the Canadian Problem Gambling Index to identify problem gamblers) can be a source of disagreement and can affect the results. Considering that the actual preferences of addicts are identical to the preferences of rational consumers is also a strong assumption. For this reason, estimates obtained with the rational benchmark approach should probably be considered as an upper bound for internalities and as a lower bound for the net surplus. Sensitivity analyses with regard to the choices of key parameters (e.g. price elasticity of rational demand) should also be encouraged.

Taking these considerations into account, the adjustment we propose makes the measure algebraically more accurate. The magnitude of the bias in the APCM and the PBM is closely related to the difference between the level of rational consumption and the level of addicted consumption. A significant difference between these two levels of consumption is likely to be characteristic of many types of addictions. It follows that the biases highlighted with gambling data from the APC (1999) should also be substantial for other addictive goods and more recent data. Besides, this contribution raises potential issues with models based on the parallelism assumption. It would be valuable to assess the impact of relaxing this assumption on existing results.

Addictions are widespread in our societies. Welfare assessment tools are needed to better regulate them. We hope our measure of addicted consumers' surplus can be a useful contribution to this aim.

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Appendix A: Welfare gain on rational spending

The rational demand is described by a linear inverse demand function:

$$P = a - bQ.$$

We can rewrite this function as:

$$P = a + \frac{dP}{dQ}Q = a + \frac{dP}{dQ}\frac{Q}{P}P = a + \frac{1}{\eta_{R(P,Q)}}P,$$

where $\eta_{R(P,Q)}$ is the price elasticity of rational demand at any point (P, Q).

It follows that

$$a = \left(1 - \frac{1}{\eta_R^*}\right) P^*,$$

with η_R^*, the price elasticity at the point of equilibrium (P^*, Q_R^*).

Welfare gain on rational spending is then given by

$$G_C = \frac{(a - P^*)Q_R^*}{2} = \frac{1}{2}\left[\left(1 - \frac{1}{\eta_R^*} - 1\right) P^*\right] Q_R^* = -\frac{P^* Q_R^*}{2\eta_R^*} = \frac{R}{2|\eta_R^*|}.$$

Appendix B: Revised measure of welfare loss on excess spending

The rational demand is described by a linear inverse demand function:

$$P = a - bQ.$$

Thus, the welfare loss on excess spending is given by[8]

$$L = P^*(Q_A^* - Q_R^*) - \int_{Q_R^*}^{Q_A^*} a - bQ\, dQ, \tag{B.1}$$

where

$$\int_{Q_R^*}^{Q_A^*} a - bQ\, dQ = a(Q_A^* - Q_R^*) - \frac{b}{2}\left({Q_A^*}^2 - {Q_R^*}^2\right). \tag{B.2}$$

Both parameters a and b can be expressed as functions of the price elasticity η_R^* at the point of equilibrium (P^*, Q_R^*):

$$b = -\frac{1}{\eta_R^*}\frac{P^*}{Q_R^*}$$

and

$$a = P^* + bQ_R^* = \left(1 - \frac{1}{\eta_R^*}\right) P^*.$$

Putting these expressions in equation (B.2), we get

$$\int_{Q_R^*}^{Q_A^*} a - bQ\, dQ = \left(1 - \frac{1}{\eta_R^*}\right) P^*(Q_A^* - Q_R^*) + \frac{1}{2\eta_R^*}\frac{P^*}{Q_R^*}\left({Q_A^*}^2 - {Q_R^*}^2\right).$$

Substituting this last result in equation (B.1), we obtain

$$\begin{aligned} L &= P^*(Q_A^* - Q_R^*) - \left(1 - \frac{1}{\eta_R^*}\right) P^*(Q_A^* - Q_R^*) - \frac{1}{2\eta_R^*}\frac{P^*}{Q_R^*}\left({Q_A^*}^2 - {Q_R^*}^2\right) \\ &= \frac{P^*}{\eta_R^*}(Q_A^* - Q_R^*) - \frac{P^*}{\eta_R^*}\left(\frac{{Q_A^*}^2 - {Q_R^*}^2}{2Q_R^*}\right) \end{aligned}$$

[8] The first term represents the rectangle of excess spending, namely $t + L_C$ in Figure 2.a and $t_1 + L_C + t_2$ in Figure 2.b. The second term represents $t - \varepsilon$ in Figure 2.a and t_1 in Figure 2.b. It follows that $L = L_C + \varepsilon$ in Figure 2.a and $L = L_C + t_2$ in figure 2.b.

$$= \frac{P^*}{\eta_R^*}\left(\frac{2Q_A^*Q_R^* - {Q_R^*}^2 - {Q_A^*}^2}{2Q_R^*}\right) = -\frac{P^{*2}(Q_A^* - Q_R^*)^2}{2P^*Q_R^*\eta_R^*} = \frac{E^2}{2R|\eta_R^*|}.$$

Appendix C: Welfare gain and loss under the parallelism assumption

Both rational and addicted demands are described by linear inverse demand functions:

$$P = a_R - b_R Q_R$$

and

$$P = a_A - b_A Q_A.$$

The parallelism assumption implies that $b_R = b_A$. To capture this assumption, we replace b_R by b_A in the rational demand function on which the welfare measures are based:

$$P = a_R - b_A Q_R,$$

with

$$b_A = -\frac{P^*}{\eta_A^* Q_A^*}$$

and

$$a_R = P + b_A Q_R^* = \left(1 - \frac{Q_R^*}{\eta_A^* Q_A^*}\right) P^*.$$

By recalculating both measures of welfare gain and loss with these parameter values, we obtain

$$G_P = \frac{(a_R - P^*)Q_R^*}{2} = \frac{1}{2}\left[\left(1 - \frac{Q_R^*}{\eta_A^* Q_A^*} - 1\right) P^*\right] Q_R^* = -\frac{{P^*}^2 {Q_R^*}^2}{2P^* Q_A^* \eta_A^*} = \frac{R^2}{2A|\eta_A^*|}$$

and

$$L_P = P^*(Q_A^* - Q_R^*) - \int_{Q_R^*}^{Q_A^*} a_R - b_A Q_R \, dQ_R$$

$$= P^*(Q_A^* - Q_R^*) - \left(1 - \frac{Q_R^*}{\eta_A^* Q_A^*}\right) P^*(Q_A^* - Q_R^*) - \frac{P^*}{2\eta_A^* Q_A^*}\left({Q_A^*}^2 - {Q_R^*}^2\right)$$

$$= \frac{P^*}{\eta_A^* Q_A^*}\left(\frac{2Q_A^*Q_R^* - {Q_R^*}^2 - {Q_A^*}^2}{2}\right) = -\frac{P^{*2}(Q_A^* - Q_R^*)^2}{2P^*Q_A^*\eta_A^*} = \frac{E^2}{2A|\eta_A^*|}.$$