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Ramsey Optimal Policy versus Multiple Equilibria with Fiscal and Monetary Interactions

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Abstract

We consider a frictionless constant endowment economy based on Leeper (1991). In this economy, it is shown that, under an ad-hoc monetary rule and an ad-hoc fiscal rule, there are two equilibria. One has active monetary policy and passive fiscal policy, while the other has passive monetary policy and active fiscal policy. We consider an extended set-up in which the policy maker minimizes a loss function under quasi-commitment, as in Schaumburg and Tambalotti (2007). Under this formulation there exists a unique Ramsey equilibrium, with an interest rate peg and a passive fiscal policy.

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Abstract

The reference model of frictionless endowment economies includes a Fisher relation for the real interest rate and government intertemporal budget constraint. For this model, Ramsey optimal policy mix is a unique equilibrium with an interest rate peg and a "passive" fiscal rule with a negative-feedback value of its parameter stabilizing public debt. This is a third equilibrium with respect to the two usual equilibria with ad hoc policy rules. The first one has passive fiscal policy and an active monetary policy rule parameter destabilizing inflation. The second one has an active fiscal policy rule parameter destabilizing public debt and a passive monetary policy which includes the case of an interest rate peg.

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1 Introduction

Monetary and fiscal interactions are often presented using Leeper's (1991) model of frictionless endowment economies. This model includes the intertemporal budget constraint of the government and a Fisher relation with a constant real interest rate equal to the representative household's discount factor. In Leeper (1991), for monetary policy, the interest rate responds in proportion to inflation. For fiscal policy, a lump-sum tax responds in proportion to the stock of real public debt.

Assuming inflation, interest rate and lump-sum tax are forward-looking variables for seeking equilibria with Blanchard and Kahn's (1980) determinacy condition, Leeper (1991) obtains two equilibria for monetary and fiscal interactions. These two equilibria are defined by ranges of values of the parameters of ad hoc policy rules. In the first equilibrium, the interest rate rule parameter destabilizes inflation ("active monetary policy" according to Leeper (1991)) and the fiscal rule parameter stabilizes public debt ("passive fiscal policy" according to Leeper (1991)). In the second equilibrium related to the fiscal theory of the price level, the fiscal rule parameter destabilizes public debt ("active policy" according to Leeper (1991)) and the interest rate rule parameter stabilizes inflation ("passive monetary policy" according to Leeper (1991)). A peg of the interest rate with a lack of response of the interest rate to inflation is a particular case of a "passive monetary policy" equilibrium.

This note demonstrates that Ramsey optimal policy is a third equilibrium with an optimal interest rate peg and "passive" fiscal policy.

Section two presents the policy transmission mechanism, the two equilibria with ad hoc policy rules and the Ramsey optimal policy equilibrium. Section three concludes.

2 Ramsey Optimal Policy in a Frictionless Endowment Economy

2.1 Policy Transmission Mechanism

Bai and Leeper (2017) and Cochrane (2019, chapter 2) omit money in the frictionless endowment economy model with respect to Leeper's (1991) seminal paper, while still obtaining Leeper's (1991) two regimes for monetary and fiscal interactions.

Consider an infinitely-lived representative consumer who receives a *constant* endowment of goods each period in the amount y and derives utility only from consumption c_t. The government purchases a *constant* quantity of goods $g > 0$ from the consumer each period. We impose equilibrium in the goods market, so that consumption: $c_t = y - g$.

The consumer makes a consumption-saving decision that produces the Fisher relation where the real rate is here equal to a *constant* discount rate:

$$E_t \frac{1}{\pi_{t+1}} = \frac{1}{\beta} \frac{1}{R_t} \tag{1}$$

where R_t is both the gross one-period nominal interest rate on nominal bonds bought at t and pay off in $t+1$ and the monetary policy instrument, and $\pi_{t+1} = P_{t+1}/P_t$ is the gross rate of inflation between t and $t+1$, with P_t the aggregate price level. In the steady state, $P_t = P_{t+1} \Rightarrow \pi^* = 1$ where π^* is the inflation target. Then, $R^* = \pi^*/\beta = 1/\beta$, is the nominal interest rate consistent with the inflation target according to the Fisher relation. The equilibrium real interest rate is constant at $r = (1/\beta) - 1$ where $0 < \beta < 1$ is the consumer's discount factor. The Fisher relation in deviation of steady state values is:

$$E_t \frac{1}{\pi_{t+1}} - \frac{1}{\pi^*} = \frac{1}{\beta}\left(\frac{1}{R_t} - \frac{1}{R^*}\right) \Rightarrow E_t \frac{1}{\pi_{t+1}} - 1 = \frac{1}{\beta}\left(\frac{1}{R_t} - \beta\right). \tag{2}$$

It is linearized around the steady state equilibrium:

$$E_t \pi_{t+1} - \pi^* = \beta\left(R_t - R^*\right) \Rightarrow E_t \pi_{t+1} - 1 = \beta\left(R_t - \frac{1}{\beta}\right).$$

Fiscal policy levies lump-sum taxes of τ_t and sets purchases to be constant, $g > 0$ with primary surplus $s_t = \tau_t - g$. Government issues one-period nominal bonds, B_t, that satisfy the flow constraint, where P_t is the aggregate price level and real debt is defined as $b_t = B_t/P_t$.

$$b_t = \frac{B_t}{P_t} = -\left(\tau_t - g\right) + R_{t-1}\frac{P_{t-1}}{P_t}\frac{B_{t-1}}{P_{t-1}} = -\left(\tau_t - g\right) + \frac{R_{t-1}}{\pi_t} b_{t-1}.$$

Using the Fisher relation, we substitute the constant real interest rate in the government intertemporal budget constraint:

$$b_t = -\left(\tau_t - g\right) + \frac{1}{\beta} b_{t-1}.$$

The dynamics of real debt does not depend on inflation or on the nominal rate R_t. The steady state level of real government debt has an exogenous value: $b_{t+1} = b_t = b^*$. To be consistent with the government intertemporal budget constraint, the steady state level of tax revenue τ^* is equal to the steady state interest expense:

$$\tau^* - g = \left(\frac{1}{\beta} - 1\right) b^*.$$

The government intertemporal budget constraint written in deviation of steady state is

$$b_t - b^* = -\left(s_t - s^*\right) + \frac{1}{\beta}(b_{t-1} - b^*).$$

The linearized dynamics are (in deviation from steady state):

$$\begin{pmatrix} E_t \pi_{t+1} - \pi^* \\ E_t b_{t+1} - b^* \end{pmatrix} = \underbrace{\begin{pmatrix} A_\pi = 0 & A_{\pi b} = 0 \\ A_{b\pi} = 0 & A_b = \frac{1}{\beta} \end{pmatrix}}_{=\mathbf{A}} \begin{pmatrix} \pi_t - \pi^* \\ b_t - b^* \end{pmatrix} + \underbrace{\begin{pmatrix} B_{\pi R} = \beta & B_{\pi s} = 0 \\ B_{bR} = 0 & B_{\pi s} = -1 \end{pmatrix}}_{=\mathbf{B}} \begin{pmatrix} R_t - R^* \\ s_{t+1} - s^* \end{pmatrix}.$$

The log-linearized dynamics of inflation is:

$$\frac{E_t \pi_{t+1} - \pi^*}{\pi^*} = \beta\left(\frac{R^*}{\pi^*}\right)\frac{R_t - R^*}{R^*} = \frac{R_t - R^*}{R^*}.$$

The log-linearized dynamics of real debt is:

$$\frac{E_t b_{t+1} - b^*}{b^*} = \frac{1}{\beta}\frac{b_t - b^*}{b^*} - \left(\frac{s^*}{b^*}\right)\frac{s_{t+1} - s^*}{s^*} \text{ with } \frac{\tau^* - g}{b^*} = \frac{1}{\beta} - 1.$$

Log-linearized dynamics are:

$$\begin{pmatrix} \frac{E_t\pi_{t+1}-\pi^*}{\pi^*} \\ \frac{E_t b_{t+1}-b^*}{b^*} \end{pmatrix} = \underbrace{\begin{pmatrix} A_\pi = 0 & A_{\pi b} = 0 \\ A_{b\pi} = 0 & A_b = \frac{1}{\beta} \end{pmatrix}}_{=\mathbf{A}} \begin{pmatrix} \frac{\pi_t-\pi^*}{\pi^*} \\ \frac{b_t-b^*}{b^*} \end{pmatrix} + \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & -\left(\frac{1}{\beta}-1\right) \end{pmatrix}}_{=\mathbf{B}_{\log}} \begin{pmatrix} \frac{R_t-R^*}{R^*} \\ \frac{s_{t+1}-s^*}{s^*} \end{pmatrix}$$

2.2 Ad Hoc Policy Rules

The fiscal authority adjusts lump-sum tax in response to the level of real government debt. Monetary policy follows an interest rate rule that responds to inflation:

$$s_t - s^* = G_b\left(b_{t-1} - b^*\right) + \varepsilon_t^s \text{ and } R_t - R^* = F_\pi\left(\pi_t - \pi^*\right) + \varepsilon_t^R$$

Shocks ε_t^R, ε_t^s are assumed to be independently and identically distributed, with mean zero and a non-zero variance-covariance matrix. The linearized dynamics are:

$$\begin{pmatrix} E_t\frac{1}{\pi_{t+1}} - \frac{1}{\pi^*} \\ E_{t-1}b_{t+1} - b^* \end{pmatrix} = \begin{pmatrix} \frac{1}{\beta}F_\pi & 0 \\ 0 & \frac{1}{\beta} - G_b \end{pmatrix}\begin{pmatrix} \pi_t - \pi^* \\ b_t - b^* \end{pmatrix} + \begin{pmatrix} \beta & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} \varepsilon_t^R \\ \varepsilon_t^s \end{pmatrix} \text{ with } b_0 \text{ given.}$$

Public debt is the only predetermined variable. Blanchard and Kahn's (1980) determinacy condition implies that one eigenvalue should be inside the unit circle and one should be outside the unit circle. Either the inflation eigenvalue is outside the unit circle $|\beta F_\pi| > 1$ and the public debt eigenvalue $\left|\frac{1}{\beta} - G_b\right| < 1$ is inside the unit circle (first equilibrium) or it is the reverse (second equilibrium): $|\beta F_\pi| < 1$ and $\left|\frac{1}{\beta} - G_b\right| > 1$ (Leeper (1991)).

2.3 Ramsey Optimal Policy under Quasi-Commitment

In a monetary policy regime indexed by j, a policy maker may re-optimize on each future period with exogenous probability $1 - q$ strictly below one. Following Schaumburg and Tambalotti (2007), we assume that the mandate to minimize the loss function is delegated to a sequence of policy makers with a commitment of random duration. The degree of credibility is modelled as if it is a change of policy-maker with a given probability of reneging commitment and re-optimizing optimal plans. The length of their tenure or "regime" depends on a sequence of exogenous independently and identically distributed Bernoulli signals $\{\eta_t\}_{t\geq 0}$ with $E_t\left[\eta_t\right]_{t\geq 0} = 1 - q$, with $0 < q \leq 1$. If $\eta_t = 1$, a new policy maker takes office at the beginning of time t. Otherwise, the incumbent stays on. A higher probability q can be interpreted as a higher credibility. A policy maker with little credibility does not give a large weight on future welfare losses. The policy maker j solves the following problem for regime j, omitting subscript j, before policy maker k starts:

$$V_0^j = -E_0 \sum_{t=0}^{t=+\infty} (\beta q)^t \left[\begin{array}{c} \frac{1}{2}\left(Q_\pi (\pi_t - \pi^*)^2 + Q_b (b_t - b^*)^2 + \mu_R (R_t - R^*)^2 + \mu_s (s_t - s^*)\right) \\ +\beta (1-q) V_t^k \end{array} \right]$$
$$R_t - R^* = \beta q (E_t \pi_{t+1} - \pi^*) + \beta (1-q) \left(E_t \pi_{t+1}^k - \pi^*\right)$$
$$b_t - b^* = \beta q (E_t b_{t+1} - b^*) + \beta (1-q) \left(E_t b_{t+1}^k - b^*\right) + \beta q (s_t - s^*) + \beta (1-q) (s_t^k - s^*).$$

Inflation and public debt expectations are an average between two terms. The first term, with weight q is the inflation and public debt, respectively, that would prevail under the current regime upon which there is commitment. The second term with weight $1-q$ is the inflation and public debt, respectively, that would be implemented under the alternative regime by policy maker k.

This optimal program is a discounted linear quadratic regulator (LQR) with a "credibility adjusted" discount factor βq. The log-linear version of the dynamic system can also be used instead of the linear version. Preferences of the policy maker are firstly given by positive weights for the two policy targets $Q_\pi \geq 0$, $Q_\pi \geq 0$ ($\mathbf{Q} = diag(Q_\pi, Q_b)$). Secondly, there are at least two non-zero policy maker's preferences for *interest rate smoothing* and *primary surplus and tax smoothing*, with *strictly* positive weights for these two policy instruments in the loss function: $\mu_R > 0$, $\mu_s > 0$ ($\mathbf{R} = diag(\mu_R, \mu_s)$). This ensures the strict concavity of the LQR program which implies the determinacy (uniqueness) of the solution of this Stackelberg dynamic game, if the system of the transmission mechanism is controllable.

All the results are valid for nearly negligible cost of changing the policy instruments: a minimal interest rate smoothing parameter ($\mu_R \geq 10^{-7}$), a minimal tax smoothing parameter ($\mu_s \geq 10^{-7}$) and a minimal credibility (a non zero probability of not reneging commitment next period: $10^{-7} \leq q \leq 1$).

If $\mu_R = 0$ or $\mu_s = 0$ or if the costs of changing the policy instruments are not strictly convex or if $q = 0$, the results are no longer valid. If $q = 0$, the policy maker knows that he is replaced next period. He does static optimization ($(\beta q)^0 = 1$) of his current period quadratic loss function subject to a static transmission mechanism where he considers the expectations terms related to the next period policy maker as exogenous intercepts (Chatelain and Ralf (2020)).

Proposition 1 *For the transmission mechanism of the Fisher relation with constant real rate and the government intertemporal budget constraint, Ramsey optimal policy has a unique equilibrium. The interest rate is pegged at its long run value $R_t = R^*$. A feedback Taylor rule is not optimal and the Taylor principle should not be satisfied: $F^* = 0 < \beta$. This is a "passive monetary policy". The optimal auto-correlation of monetary policy shocks is zero: $\rho_{\varepsilon R}^* = 0$. The optimal variance of monetary policy shocks is zero $\sigma_{\varepsilon R}^2 = 0$. Inflation jumps to its steady state value instantaneously following monetary policy shocks $\pi_0^* = 0$, as in a degenerate rational expectations model without predetermined variables. Hence, the price level is constant. Ramsey optimal fiscal rule has a negative feedback parameter ("passive fiscal policy") with ensures the local stability of public debt dynamics. There are two stable eigenvalues giving the optimal persistence of inflation ($\lambda_\pi^* = 0$) and the optimal persistence of public debt ($0 < \lambda_b^* < 1$).*

Proof. Ramsey optimal policy amounts to find the solution of a linear quadratic regulator (Chatelain and Ralf (2019)). The autocorrelation parameters of policy maker's shocks are optimally set to zero, else they increase the volatility of inflation and of public debt in the policy maker's loss function. The optimal expected value of the loss function is:

$$L^* = -\frac{1}{2}\begin{pmatrix} \widehat{\pi}_0^* & \widehat{b}_0 \end{pmatrix} \mathbf{P} \begin{pmatrix} \widehat{\pi}_0^* & \widehat{b}_0 \end{pmatrix}^T \text{ with } \mathbf{P} = \begin{pmatrix} P_\pi & P_{\pi b} \\ P_{\pi b} & P_b \end{pmatrix}, \tag{3}$$

$$\widehat{\pi}_0^* = \pi_0^* - \pi^* \text{ and } \widehat{b}_0 = b_0 - b^*. \tag{4}$$

where $\mathbf{P}$ is a positive symmetric square matrix of dimension two which is the solution of the discrete algebraic Riccati equation (DARE) of the LQR. The optimal initial anchor of inflation π_0^* on public debt b_0 is given by:

$$\left(\frac{\partial L^*}{\partial \pi_t}\right)_{t=0} = P_\pi \widehat{\pi}_0^* + P_{\pi b}\widehat{b}_0 = 0 \Rightarrow \widehat{\pi}_0^* = \frac{P_{\pi b}}{P_\pi}\widehat{b}_0 \text{ if } P_\pi \neq 0. \tag{5}$$

This initial transversality (or natural boundary) condition eliminates the indeterminacy of initial inflation π_0 put forward for the *ad hoc* policy rules solution. Because the system is decoupled ($A_{\pi b} = A_{b\pi} = 0$) and because the weight on the product $\pi_t b_t$ is zero in the loss function $Q_{\pi b} = 0$, this implies a zero weight $P_{\pi b} = 0$ in the optimal value of the loss function. Therefore, $P_\pi \geq 0$ and $P_b \geq 0$ are solutions of scalar discrete algebraic Riccati equations (DARE). Because the system of the policy transmission mechanism is decoupled, the optimal program is identical if we consolidate the central bank and the treasury as a single policy maker or if we consider that they are distinct policy makers. For inflation, this DARE equation is:

$$P_\pi = Q_\pi + \beta A_\pi' P_\pi A_\pi - \beta' A_\pi' P_\pi B_{\pi R} \left(\mu_R + \beta B_{\pi R}' P_\pi B_{\pi R}\right)^{-1} \beta B_{\pi R}' P_\pi A_\pi = Q_\pi.$$

Because $A_{\pi b} = A_{b\pi} = A_\pi = 0$, this implies that the optimal loss function parameter P_π is equal to the inflation weight in the loss function: $P_\pi = Q_\pi$. Therefore, if the policy maker has a non-zero weight on inflation volatility in his loss function $Q_\pi > 0$, optimal initial inflation is zero because $P_{\pi b} = 0$: $\pi_0^* = 0$. Because inflation dynamics are decoupled from the dynamics of predetermined public debt, inflation behaves exactly as in a degenerate rational equilibrium model where there is no predetermined variable. At any date t, if ever there is a monetary policy shock ε_t^R, inflation instantaneously jumps back to equilibrium $\pi_t^* = 0$. There are no transitory dynamics. The volatility of inflation is zero.

Because the open-loop system is already at the zero lower bound of inflation persistence ($A_\pi = 0$) and because there is a *non-zero* cost of interest rate volatility ($\mu_R > 0$) in the loss function, it is optimal to set an interest rate peg $R_t = R^*$. Therefore, the quadratic term of interest rate volatility $\mu_R (R_t - R^*)^2$ is minimized at its zero lower bound zero in the loss function. This implies a Taylor rule parameter equal to zero: $F^* = 0 < \beta$, a zero auto-correlation of monetary policy shocks $\rho_{\varepsilon^R} = 0$, and a zero variance of monetary policy shocks $\sigma_{\varepsilon^R}^2 = 0$ initiated by the policy maker.

If there is a zero cost of inflation in the loss function ($Q_\pi = 0$), the policy maker does not care about inflation. There is an indeterminacy for the optimal choice of the initial value of inflation π_0^*. However, the implied volatility originated by this indeterminacy does not matter in the policy maker's loss function.

For public debt, the solution is a scalar case of Ramsey optimal policy under quasi-commitment (we use the linear dynamics instead of the log-linear dynamics).

$$E_t b_{t+1} - b^* = \frac{1}{\sqrt{\beta q}} (b_t - b^*) - \sqrt{\beta q}(s_t - s^*) \text{ with } A_b = \frac{1}{\beta q} \text{ or } \beta q A = 1 \text{ and } B_{bs} = -1.$$

Optimal public debt persistence (or auto-correlation or closed-loop eigenvalue) is the stable root of the characteristic polynomial of the Hamiltonian system:

$$\begin{aligned} 0 &= \lambda^2 - S\lambda + \frac{1}{\beta q} \text{ with } \lambda^* \lambda_2 = \frac{1}{\beta q} \text{ with} \\ S &= A + \frac{1}{A\beta q} + \frac{B^2}{A} \frac{Q_b}{\mu_s} = 1 + \frac{1}{\beta q} + \beta q \frac{Q_b}{\mu_s} \\ 0 &< \lambda_b^* (\mu_s, \beta q) = \frac{1}{2} \left(S - \sqrt{S^2 - \frac{4}{\beta q}} \right) < 1. \end{aligned}$$

The optimal public debt persistence or autocorrelation λ_b^* increases with the cost μ_s of changing the policy instrument (lump sum taxes). Its boundaries are given by limits when the cost of changing policy instrument μ_s tends to zero and when tends μ_s to infinity:

$$\lim_{\mu_s \to 0} \lambda_b^* (\mu_s, \beta q) = \lim_{\mu_b \to 0} \frac{1}{2} \left(\frac{\beta q}{\mu_s} - \frac{\beta q}{\mu_s} \right) = 0 \text{ and } \lim_{\mu_s \to +\infty} \lambda_b^* (\mu_s, \beta q) = \frac{1}{\beta q A} = 1.$$

The fiscal rule parameter G_b remains in the range of values so that the persistence of public debt is strictly positive and strictly below one:

$$\frac{1}{\beta q} - 1 < G_b^* = \frac{\lambda_b^* - A}{B} = \frac{1}{\beta q} - \lambda_b^* < \frac{1}{\beta q}.$$

The optimal loss function parameter P_b is:

$$P_b = \frac{\frac{1}{A} \frac{Q_b}{q\beta}}{1 - \lambda_b^*} = \frac{Q_b}{1 - \lambda_b^*} .$$

It is also the solution of a scalar algebraic Riccati quadratic equation. For a given initial value of predetermined public debt b_0 and for preferences $Q_b \geq 0$ which can be equal to zero, the optimal expected value of the policy maker loss function is:

$$\text{If } Q_\pi > 0 : \; L^* = -\frac{1}{2} \begin{pmatrix} 0 & \widehat{b}_0 \end{pmatrix} \begin{pmatrix} Q_\pi & 0 \\ 0 & \frac{Q_b}{1-\lambda_b^*} \end{pmatrix} \begin{pmatrix} 0 & \widehat{b}_0 \end{pmatrix}^T = -\frac{1}{2} \frac{Q_b}{1 - \lambda_b^*} (b_0 - b^*)^2 .$$

$$\text{If } Q_\pi = 0 : \; L^* = -\frac{1}{2} \begin{pmatrix} \widehat{\pi}_0^* & \widehat{b}_0 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & \frac{Q_b}{1-\lambda_b^*} \end{pmatrix} \begin{pmatrix} \widehat{\pi}_0^* & \widehat{b}_0 \end{pmatrix}^T = -\frac{1}{2} \frac{Q_b}{1 - \lambda_b^*} (b_0 - b^*)^2 .$$

A rational policy maker with quadratic preferences including a convex cost of changing policy instruments (interest rate smoothing $\mu_R > 0$ and tax smoothing $\mu_s > 0$) taking into account private sector expectations with a minimal credibility ($q > 0$) eliminates the indeterminacy of initial inflation π_0 if the policy maker's preferences includes a non-zero weight ($Q_\pi > 0$) on inflation volatility in his loss function. Else, a rational policy maker neglects the indeterminacy of initial inflation because inflation volatility has zero weight

($Q_{\pi} = 0$) in his loss function. A SCILAB code is available from the authors to solve Ramsey optimal policy with numerical values. ■

3 Conclusion

For the model of frictionless endowment economies, Ramsey optimal policy is an interest rate peg and a "passive" fiscal policy. It is a third equilibrium with respect to Leeper's (1991) two equilibria with simple feedback rules.

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