Volume 39, Issue 2

Formal and Informal Interactions in a Duopoly Model

Kahina Cherfi-Feroukhi
CREAD (Applied Eonomy Research Center for Development)

Said Souam
University Paris Nanterre, EconomiX, CREST and CREAD

Abstract

This article theoretically analyzes the interactions between formal and informal firms in a vertically differentiated duopoly in which a regulatory authority intervenes to control the compliance of the firms to the law. In our simple framework, it turns out that it is never optimal to deter completely the informal firm from entering the market. This is mainly due to a production expansion effect and to an increased price competition, which increase the consumers' surplus. Moreover, it appears that a complete 'laissez-faire' policy is never optimal.

Introduction

One of the main characteristics of developing countries is that the majority of small enterprises are informal. Moreover, the cost of compliance with all legal obligations may be relatively high. Being formal or informal is thus determined by the public audit policy to control non-compliance with law. The presence of informal firms raises many questions dealing with unfair competition, tax burden and quality of the produced goods. Specifically, McGahan (2012) argues that formal and informal firms should be studied together, since they compete for the same customers and resources. However, the existence of an informal sector can have a positive effect on social welfare, since it always implies an expansion of the production in the economy. At the same time, the existence of large differences in terms of quality between formal and informal products can lead to a rearrangement of sales, which eventually may have a negative impact on social welfare. Indeed, the presence of informal products on the market could induce a change in the behavior of some consumers, who will now be inclined to buy inferior products. In order to understand this mutual interaction between formal and informal firms, we propose to analyze it theoretically within a vertically differentiated duopoly model. The analysis is based on the pioneer articles in the domain, Mussa and Rosen (1978), Gabszewicz and Thisse (1979) and Shaked and Sutton (1982, 1983). According to the standard result developed by this literature, the firms choose, at equilibrium, to differentiate their qualities. However, these results are valid only in a hedged market. Choi and Shin (1992) and Motta (1993) have extended them to include the case in which the market is not fully covered: some consumer groups may not consume in equilibrium.

This article is divided into three sections. In the first section, we propose a theoretical modeling based on a formal/informal duopoly in which a regulatory authority could intervene in order to control the compliance to law by the firms. In the second section, we solve by backward induction, the different choices made by the firms (quality and price) in both configurations in which the informal firm is deterred or not from entering into the market. The third section discusses the optimality of the public intervention in terms of controlling the compliance to the law of the firms. Our main results are the following ones. It is never optimal, from the regulatory authority's point of view, to deter completely the informal firm from entering into the market. Moreover, a complete *laissez-faire* policy is also never optimal.

## 1. Modeling the interaction between formal and informal firms

In the light of the above discussion, we propose to analyze the aforementioned questions within a theoretical model of a formal/informal duopoly. More precisely, we do consider three types of agents: the consumers, the firms (formal and informal) and the regulatory authority. The latter is responsible for the market control. It carries out audits and, if it finds infringement of the law, punishes the informal firm that does not comply with the law. The firms (formal and informal) first choose the quality of the good they produce and thus compete in price to satisfy consumer demand. We do consider that they sell vertically differentiated goods. By assumption, we do assume that the quality of the formal firm is higher than that of the informal firm. Finally, the consumers are heterogeneous according to their willingness to pay for quality.

1.1 The structure of the duopoly game

The situation is modeled according to a game with the following steps (see Figure 1 below).
1. The regulatory authority announces that it carries out an audit with a probability τ on all firms (formal and informal). If an informal firm is detected, the authority stops the informal activity and a level of fine F is imposed.
2. The informal firm decides to be active or not on the market.
3. If the informal firm is active, it finds itself in a situation of a duopoly with the formal enterprise. They decide, first of all, on the quality of the goods they produce. Secondly, they compete in price. On the other hand, if the informal firm is discouraged from entering the market, the formal firm finds itself in a monopoly. It produces a certain quality and sells at a monopoly price.

Figure 1: The structure of the duopoly game
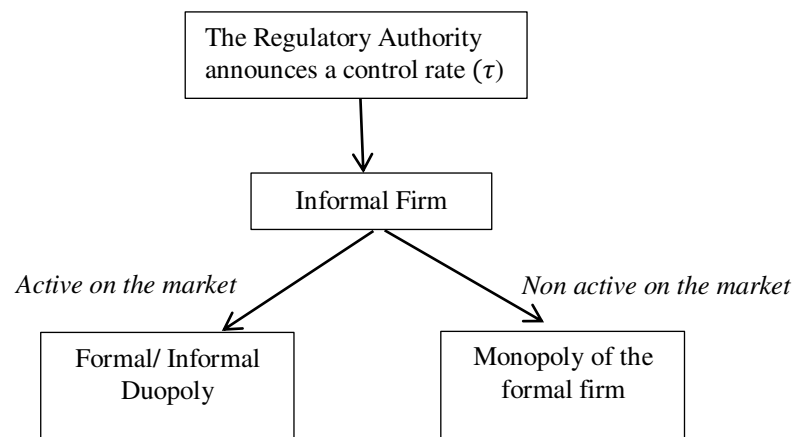
1.2 The different actors of the game

In the sequel, we describe the behavior of all the actors (the regulatory authority, the firms and the consumers).

The regulatory authority

The regulatory authority is considered in our model as an anti-fraud authority that receives no dividends if the firm complies with the law. When the authority conducts an audit with a probability τ and discovers the fraud, it will impose a penalty F to the audited informal firm. However the audit is costly and depends on the probability of detection, $c(\tau)$ with $c(0) = 0, c'(\tau) > 0$ and $c''(\tau) > 0$.

The firms

The firms form a duopoly (formal/informal). They first choose the quality of the sold goods and thus compete in price. The formal firm produces legally, the informal firm produces illegally, and the regulatory authority is responsible for market control and punishes the informal firm when it is controlled and discovered. We assume that an informal firm decides to enter the market if and only if its expected profit is positive. If the activity of the informal firm is detected, which happens with a probability τ, it must pay the penalty F, and loses in addition its income.

The consumers

We do consider that there is a continuum of consumers indexed by the parameter $\theta \epsilon [0,1]$ that follows a uniform law. This parameter represents the heterogeneity of the consumers and their willingness to pay for the quality of the good. We do assume that each consumer maximizes a utility function *à la* Mussa and Rosen (1978). More precisely, we do assume that each consumer buys a unit of good or nothing. The utility is represented by $U(\theta, q, p) = \theta q - p$, where q is the quality of the good and p is the price.

2. The profits at market equilibrium

In this section, we solve the model by backward induction. We first determine the price equilibrium in the final price competition stage. In a second step, we do determine the qualities chosen by the firms while anticipating the equilibrium prices. In a third step, we do consider the entry decision by the informal firm. We finally determine the optimal audit policy from the regulatory authority's point of view.

We do assume that the detection of the informal firm is possible only after the sale. Two configurations are thus possible. In the first one, the informal firm is deterred from entering the market (monopoly of the formal firm). In the second one, the informal firm enters the market and there is a duopoly. Of course, there is a risk of being audited and thus paying a fine. Hereafter, we analyze these two configurations.

In order to simplify the analysis and focus on some important issues, we assume that the production costs of both firms are independent of the quality choices and are standardized to 0 as in Wauthy (1996) and Banerjee (2003).

2.1 The monopoly configuration: deterrence of the entry of the informal firm

An aggressive policy from the regulatory authority deters the entry of the informal firm. In this case, there is a monopoly of the formal firm in the market. It thus prices at a level that maximizes its profits. The indifferent consumer between buying a unit from the formal firm and not buying is given by $\theta_m = \frac{p_f}{q_f}$. The consumer whom parameter exceeds this threshold buys a unit of the formal good. The others do not buy any unit. The profit of the formal monopoly is thus given by:

$$\pi_f^m = (1-t)p_f D_f = (1-t)p_f(1-\theta_m) = (1-t)p_f\left(1 - \frac{p_f}{q_f}\right).$$

The parameter t represents the unit tax rate of the formal firm. It is thus easy to show that the optimal monopoly price is $p_f^m = \frac{q_f}{2}$.

The profit of the formal firm is thus $\pi_f^m = (1-t)\frac{q_f}{2}\left(1 - \frac{q_f}{2}\frac{1}{q_f}\right) = \frac{(1-t)q_f}{4}$.

It turns out that the formal firm has an interest, when in a monopoly configuration, to choose the highest possible quality ($q_f = \bar{q}$, see hereafter).

2.2 The duopoly configuration: entry not deterred

Suppose now that the informal firm is not deterred from entering the market. Let us first determine the demand addressed to each firm.

The utility of the consumer is given by:

$$U(\theta) = \begin{cases} \theta q_f - p_f & \textit{if the consumer buys a formal good} \\ \theta q_i - p_i & \textit{if the consumer buys an informal good} \\ 0 & \textit{if the consumer does not buy the good} \end{cases}$$

(p_f, p_i) and (q_f, q_i) represent respectively the prices and the qualities of the formal and informal goods.

The higher the parameter θ, the greater the satisfaction that the consumer derives from the q quality. Therefore θq represents its willingness to pay for the quality.

Firms first simultaneously choose their qualities. We suppose that q_f and q_i must belong to the interval $[0, \overline{q}]$ where $\overline{q}$ represents the maximum quality of the produced good. They secondly compete in prices given the qualities.

2.2.1 Price competition

As well known in this kind of models, the difference between the utilities from buying the highest and the lowest qualities increases with the parameter θ. Consumers with higher parameter θ are thus more attracted by the firm proposing the highest quality. This will basically give the structure of the demand functions addressed to each type of firm. For that purpose, we determine the indifferent consumer between buying a formal or an informal good, indexed θ_0, given by the equalization of the net utilities $U(p_f) = U(p_i)$.

$\theta q_f - p_f = \theta q_i - p_i \Leftrightarrow \theta_0 = \frac{p_f - p_i}{q_f - q_i}$ where $q_f > q_i > 0$ and $p_f \geq p_i$.

Let us index θ_p the consumer that is indifferent between buying the informal product or not buying the good. It is given by $\theta_p = \frac{p_i}{q_i}$.

The market breaks down as follows. Consumers who do not sufficiently value the quality ($\theta \in [0, \theta_p]$) do not buy it. Those with a moderate valuation ($\theta \in [\theta_p, \theta_0]$) choose the lowest quality. Finally, those with the highest valuations of the quality ($\theta \in [\theta_0, 1]$) buy the highest quality (see Figure 2).

Figure 2: The consumers' positioning

| $\emptyset$ | Buy the informal good | Buy the formal good |

0   θ_p θ_0 1
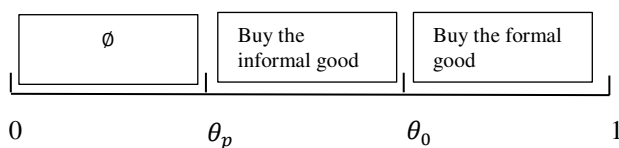
Demand functions addressed to the formal and informal firms are given by:

$$D_f = 1 - \theta_0 = 1 - \frac{p_f - p_i}{q_f - q_i}, \quad \text{and} \quad D_i = \theta_0 - \theta_p = \frac{p_f - p_i}{q_f - q_i} - \frac{p_i}{q_i}.$$

As a result, the profit of the formal firm (with payment of a unit tax t on profits) is given by:

$$\pi_f(p_f, p_i) = (1 - t)\, p_f D_f = (1 - t)\, p_f(1 - \theta_0) = (1 - t) p_f(1 - \frac{p_f - p_i}{q_f - q_i}).$$

The first-order condition for maximizing the profit of the formal firm is:

$$\frac{\partial \pi_f(p_f, p_i)}{\partial p_f} = \frac{\partial}{\partial p_f}\left((1 - t)(p_f - \frac{p_f(p_f - p_i)}{q_f - q_i})\right) = 0.$$

This gives the following condition: $1 - \frac{2p_f - p_i}{q_f - q_i} = 0 \Leftrightarrow p_f = \frac{p_i}{2} + \frac{q_f - q_i}{2}$.

The best response function of the formal firm is thus:

$$p_f = BR_f(p_i) = \frac{p_i}{2} + \frac{q_f - q_i}{2}.$$

The expected profit of the informal firm is given by:

$$\pi_i(p_f, p_i) = (1 - \tau)p_i D_i - \tau F = (1 - \tau)p_i(\theta_0 - \theta_p) - \tau F = (1 - \tau)p_i\left(\frac{p_f - p_i}{q_f - q_i} - \frac{p_i}{q_i}\right) - \tau F.$$

The condition of the first order maximization of the profit of the informal firm is:

$$\frac{\partial \pi_i(p_f, p_i)}{\partial p_i} = (1 - \tau)\left(\frac{p_f - p_i}{q_f - q_i} - \frac{p_i}{q_i}\right) + (1 - \tau)p_i(\frac{-1}{q_f - q_i} - \frac{1}{q_i}) = 0.$$

$$q_i(p_f - 2p_i) = 2\,p_i(q_f - q_i) \Leftrightarrow p_i = \frac{q_i p_f}{2q_f}.$$

Hence the best response function for the informal firm is: $p_i = BR_i(p_f) = \frac{q_i p_f}{2q_f}$.

The Nash equilibrium prices are thus given by: $p_f = \frac{(q_f - q_i)2q_f}{4q_f - q_i}$, and $p_i = \frac{(q_f - q_i)q_i}{4q_f - q_i}$.

We finally obtain the reduced form of the price competition game.

$$\pi_f(p_f, p_i) = (1 - t)p_f D_f = 4(1 - t)\frac{q_f^2(q_f - q_i)}{(4q_f - q_i)^2},$$

$$\pi_i(p_f, p_i) = (1 - \tau)p_i D_i - \tau F = (1 - \tau)\frac{q_i q_f(q_f - q_i)}{(4q_f - q_i)^2} - \tau F.$$

2.2.2 Quality choice

We solve now the Nash equilibrium choices of the qualities by the firms. Let us begin by analyzing the choice of the quality from the formal firm. For a given quality chosen by the informal firm, let us determine the impact of an increase in the quality offered by the formal firm. We have:

$$\frac{\partial \pi_f}{\partial q_f} = 4(1 - t)\frac{[2q_f(q_f - q_i) + q_f^2][4q_f - q_i]^2 - 8(4q_f - q_i)q_f^2(q_f - q_i)}{(4q_f - q_i)^4}.$$

This is easily simplified to:

$$\frac{\partial \pi_f}{\partial q_f} = 4(1 - t)q_f\frac{\{[2q_f - q_i]^2 + q_i^2 + q_f q_i\}}{(4q_f - q_i)^3} > 0.$$

Thus, the optimal quality for the formal firm is the highest one, i.e. $q_f = \bar{q}$.

We determine now the optimal choice of the informal firm when it enters the market. It is provided by:

$$\frac{\partial \pi_i}{\partial q_i} = 0 \Leftrightarrow \frac{[q_f(q_f - q_i) + q_i q_f(-1)](4q_f - q_i)^2 - 2(-1)(4q_f - q_i)\,q_i q_f(q_f - q_i)}{(4q_f - q_i)^4}(1 - \tau) = 0.$$

$$\frac{\partial \pi_i}{\partial q_i} = 0 \Leftrightarrow q_i = \frac{4}{7}q_f = \frac{4}{7}\bar{q}.$$

The equilibrium prices are given by:

$$p_f = \frac{\left(\bar{q} - \frac{4}{7}\bar{q}\right)2\bar{q}}{4\bar{q} - \frac{4}{7}\bar{q}} = \frac{1}{4}\bar{q}, \quad \text{and} \quad p_i = \frac{\left(\bar{q} - \frac{4}{7}\bar{q}\right)\frac{4}{7}\bar{q}}{4\bar{q} - \frac{4}{7}\bar{q}} = \frac{1}{14}\bar{q}.$$

The threshold parameters are thus $\theta_0 = \frac{5}{12}$ and $\theta_p = \frac{1}{8}$.

Given the qualities chosen by the two firms in the case of duopoly, it is now possible to write their profits according to the different parameters of the model.

$$\pi_f = \frac{7}{48}(1 - t)\bar{q}, \text{ and } \quad \pi_i = \frac{1}{48}(1 - \tau)\bar{q} - \tau F.$$

2.2.3 Entry decision

Traditionally, as in the literature on crime deterrence, the firm will be discouraged from entering the market, if the probability of auditing, and therefore the threat of payment of fine, is sufficiently high. In our setting, the informal firm will be deterred from entering the market if its expected profit is negative, $\frac{1}{48}(1-\tau)\bar{q} - \tau F \leq 0$. This is equivalent to having a sufficiently large audit probability: $\tau \geq \tau^* = \frac{\bar{q}}{48F+\bar{q}}$. Since auditing is costly, the authority has no interest in going beyond this threshold probability which is just sufficient to deter entry of the informal firm. Otherwise, there is no deterrence and the market structure is a duopoly one.

3. The optimal audit policy

The regulatory authority chooses its optimal policy according to (F, τ) which maximizes social welfare. We consider here, as is very often done in the literature on the economic analysis of crime, that authority does not take into account the profit of informal firms, but only the one from formal firms. As a result, it is interested in social welfare which includes the profit of the formal firm, the consumer surplus and its expected net income.

In order to determine the optimal policy from this point of view, it is necessary to determine the total welfare in the two configurations (monopoly and duopoly).

3.1 The total welfare

The total welfare in the monopoly configuration is given by: $W^M = CS^M + \pi_f^m - c(\tau^*)$.
The consumer surplus is determined as follows:

$$CS^M = \int_{\theta_m}^{1} (\theta\bar{q} - p_f^m)\, d\theta = \int_{1/2}^{1} \left(\theta\bar{q} - \frac{\bar{q}}{2}\right) d\theta = \bar{q}\left[\left(\theta - \frac{1}{2}\right)^2\right]_{\frac{1}{2}}^{1} = \frac{1}{8}\bar{q}.$$

This gives a total surplus of:

$$W^M = \frac{3}{8}\bar{q} - c(\tau^*).$$

In the case of a duopoly, the consumer surplus is equal to the sum of the surplus for the purchase of the formal and informal goods.

$$CS^D = CS_f^D + CS_i^D = \int_{\frac{5}{12}}^{1}\left(\theta\bar{q} - \frac{\bar{q}}{4}\right) d\theta + \int_{\frac{1}{8}}^{\frac{5}{12}}\left(\theta\frac{4\bar{q}}{7} - \frac{\bar{q}}{14}\right) d\theta.$$

The calculations give the following results:

$$CS_i^D = \frac{\bar{q}}{14}\left[\frac{1}{16}(8\theta - 1)^2\right]_{1/8}^{5/12} = \frac{7}{288}\bar{q}, \text{ and } CS_f^D = \bar{q}\left[\frac{1}{2}\left(\theta - \frac{1}{4}\right)^2\right]_{5/12}^{1} = \frac{77}{288}\bar{q}.$$

The total surplus of consumers is thus given by: $CS^D = \frac{7}{24}\bar{q}$.

We can immediately notice that $CS^D > CS^M$. The presence of the informal firm allows more consumers to buy the good. This is due to the fact that on the one hand, it reduces the price of the formal firm and, on the other hand, it offers two varieties of the good to the consumers.

The total welfare is given by:

$$W^D = CS^D + gross\ profit\ of\ the\ formal\ firm + \tau F - c(\tau)$$

$$W^D = \frac{7}{24}\bar{q} + \frac{7}{48}\bar{q} + \tau F - c(\tau) = \frac{7}{16}\bar{q} + \tau F - c(\tau).$$

3.2 The optimal policy

In our case, it is clear that it is never in the interest of the regulatory authority to completely deter the entry of the informal firm. Indeed, even a *laissez-faire* policy in which no audit effort is supported by the regulatory authority (*i.e.* $\tau = 0$) is better than a complete deterrence of the entry of the informal firm: $W^D = \frac{7}{16}\bar{q} > W^M = \frac{3}{8}\bar{q} - c(\tau^*)$.

Thus, in the context of a model of price competition with vertical differentiation of qualities, it is never optimal to completely deter an informal firm that would compete with a formal firm even if the regulatory authority exercises no control. However, in a very general context, it appears that it is in the interest of the authority to exercise effective control which actually enables it to increase its resources. Indeed, it is sufficient to assume, for example, that the cost of the audit is such that $c'(0) < F$ to be sure that it is optimal, for the regulatory authority, to carry out audits ($\tau > 0$). Two situations are then possible. In the first, the unconstrained optimum, defined by $F = c'(\hat{\tau})$, is less than $\tau^* = \frac{\bar{q}}{48F+\bar{q}}$. In this case, it is optimal to choose an audit probability equal to $\hat{\tau}$. In the second situation, $\hat{\tau} > \tau^* = \frac{\bar{q}}{48F+\bar{q}}$. In this case, and in order to maximize the net income from its control, the regulatory authority would like to increase its probability of audit so much that it would deter the informal firm from doing so, in which case it would recover no income. The best it can do is to choose the highest probability that does not discourage the informal firm from entering to the market ($\tau^* - \varepsilon$). To better understand this discussion, it is appropriate to consider the case of a quadratic audit cost function $c(\tau) = \frac{c}{2}\tau^2$. It is then sufficient to compare $\hat{\tau} = \frac{F}{c}$ with 1 and $\tau^* = \frac{\bar{q}}{48F+\bar{q}}$. It turns out that when the fine F is sufficiently small ($F < \frac{2c\bar{q}}{\sqrt{\bar{q}^2+192c\bar{q}}+\bar{q}}$), the optimal audit policy is $\hat{\tau} = \frac{F}{c}$ (unconstrained duopoly). In the complementary case, the optimal audit policy consists in fixing the probability at the highest level that does not deter the informal firm from entering to the market.

Conclusion

In this paper, we propose a theoretical analysis of a market in which could be present formal and informal firms that produce vertically differentiated goods and that compete in prices. We also consider a public action to control the compliance of the firms with the law. The regulatory authority in charge of this control tries to maximize collective welfare. It clearly appears, in our simple framework, that a complete deterrence of the entry of informal firms is never optimal. This is mainly due to the fact that the presence of an informal firm induces a surplus of production and also forces the formal firm to decrease its price. The consumers buy more units of the good whatever its quality. In practice, this informal production makes it possible to satisfy the needs of an economically fragile social category that would not have been satisfied with only the formal good. Moreover, we show that a *laissez-faire* policy is not optimal. Indeed, even when the level of the fine is relatively small and deterrence not granted, it is optimal to audit the market since this would enhance the revenues of the regulatory authority coming from the fines paid by the informal firm.

However, and at least in the long term, the presence of informal firms can be highly detrimental to the formal firms, which could not *ad vitam aeternam*, withstand to this unfair competition. Indeed, inequality of opportunity, facing the tax authority, allows the informal firm to avoid taxes, whereas the formal firm is statutorily obliged to comply. This could even lead, in the extreme situations, to the elimination of the formal firm. In this case, the informal firm would be in a monopoly position. However, this would have both positive and negative effects. The advantage would be that the less affluent population groups could benefit from a range of products that they would not have been able to consume in the absence of the informal firm. On the other hand, the negative effect would inevitably materialize by the decline in the quality of all the products available. As a result, the impact of such situation of informal monopoly would have an ambiguous effect on social welfare. Godfrey and Mendi (2015) propose a theoretical analysis in which the formal firms produce two levels of quality and possibly a competitive informal fringe is present and offers a third level of quality. The presence of the competitive informal fringe has two opposite effects in terms of welfare: a positive one dealing with the expansion of the total output and a negative one dealing with the fact that the fierce price competition will decrease the quality of the offered intermediate good, this could make the entry into the market less profitable for those firms.

To conclude, our partial equilibrium analysis shows that, in the short term, a complete deterrence of informal firms is not always an optimal policy. However, in the long term the existence of informality seems to be detrimental to the optimal growth path (Cherfi-Feroukhi and Souam, 2019).

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