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Explaining the role of commodity traders: A theoretical approach

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Abstract

Empirical models of international commodity trade ows tend to show that exchange rate volatility has either no or negative impact on export volumes. This analysis has a number of limitations. In particular, it underestimates the role of physical traders and, consequently, the importance of future markets. In this context, this article aims to provide the theoretical underpinnings to demonstrate that these traders play a very particular role and that they have an influence on the reality of export flows due to their use of future contracts. Using a very simple cobweb model, we demonstrate that exchange rate uncertainty fuel commodities' exports while futures market volatility could have a positive impact on the level of exports.

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1. Introduction

Despite the abundance of scientific works in the field of agricultural and energy economics, little seems to have been written on the role of physical traders whereas they are de facto unavoidable in commodity industries. These traders are indeed much more than mere intermediaries who would be remunerated on the basis of commissions received on the physical flows they would process. They play a key economic role: that of reconciling, in time and space, buyers and end-users of raw materials. It therefore seems hardly appropriate to be able to model the various commodity chains without taking them into account. The primary ambition of this article is, therefore, to model their presence within a given commodity chain in order to show how they can modify international trade flows. This specific approach requires to take into account the existence of futures markets which are inextricably linked to physical traders' activities. In order to hedge their trading margins, traders indeed make extensive use of this type of derivatives products that enable them to buy at a high price and possibly resell them at a lower price while maintaining a profit margin. In this respect, it is crucial, to underline that the existence of a futures market not only influences the implementation of price risk management strategies, but also has important consequences on the formation of commercial prices between the various players of the value chain. Indeed, it should be recalled that commodity futures markets are not "solely" intended to offer price risk management tools. An organized financial market offers, in fact, a public price reference, observable by all, without cost and without delay. As summarized by Black (1976), "looking at futures prices for various transaction months, participants in this market can decide on the best times to plant, harvest, buy for storage, sell from storage, or process the commodity". Considered as a fair price, this reference also serves, in most "large" markets, as a basis for negotiation for commercial transactions between producers and physical traders, traders and end-users, or even between producers and end-users. Taking into account both the trader and the availability of derivatives products (i.e futures contract) radically changes the reality of international trade flows. The second ambition of this paper is to demonstrate it. Using a simple cobweb model based on Mitra and Boussard (2012), we examine the impact both of exchange rate volatility and commodity price level and volatility on the magnitude and timing of export flows of a commodity producer who can sell his product either to domestic and foreign end users or to an international physical trader.

Our results differ from the conventional literature on international trade in several ways and highlight interesting theoretical elements which can be used to understand the reality of export flows. First, we show that an increase in commodity prices leads to an increase in demand from the trader thanks to his particular hedging strategy and to his ability to benefit from an increased discount on commodity prices. Second, while an increase in the exchange rate volatility reduces export flows from the producer to the foreign end-user when no hedging strategies are implemented, the presence of a trader changes this relationship. The effect indeed becomes undetermined. Third, an increase in the expected international/futures prices volatility may increase exports which could appear as paradoxical. The hedging strategy is similar to an arbitrage between a flat price risk and a basis risk which, although theoretically low, can cause the actual purchase or sale price of the commodity to vary (Brennan, 1958, Fama and French, 1987). To consider, from this perspective, that the existence of derivatives fully mitigates *per se* the risk linked to commodity price volatility would be inaccurate. Moreover, if one accepts the very common assumption that the use of futures contracts makes it possible to know whether the storage activity will be profitable or not, any uncertainty about the level of future prices may indeed lead producers to sell immediately and therefore, in all likelihood, to export, rather than to store. Our article is structured as follows: we specify in a first part the assumptions of our two-period cobweb model that we solve in a second part by considering successively the producer's export strategies and the corresponding trader's and buying & selling strategies. We then conclude.

2. Assumptions

Let's assume an open economy with a local producer operating in a given commodity industry, national and foreign end-users, and an international physical trader. The latter buys a given product from the producer only in period 1 and sells it to the end-user only in period 2. We consider a two-period model ($t = 1, 2$) where production is given (i.e. determined in period 0, which is not considered in this model). The whole production denoted $\bar{Q}$ is available in t=1 and can be bought in the two periods (i.e. in $t = 1$ and in $t = 2$). However, we assume that demand from end-users only appears in period 2 and that there is no storage behaviour from the end-users that could have led them to buy in period 1 for a use in period 2. In period 2, the producer can either sell his products to the national end-user or to the foreign end-user (exports). We consider that in $t = 1$, the producer can only sell his products to the physical trader. One key element can justify this assumption: the physical trader is able to buy the commodity whatever the price in $t = 1$[1]. Thus, all flows from the producer to the physical trader, or from the producer to the foreign end-user are considered as exports. Furthermore, we assume that trades to foreign end-users and physical traders (exports) are all denominated in USD.

[1] As it is demonstrated in the following model.

Since the producer can only sell to the international physical trader, we have only one possible price in period 1 which is closely related to the international benchmark and is equal to, in the producer's currency:

$$P_1^{Cash(P,T)} = e_1 P_{1,2}^F - \alpha \tag{1}$$

Where $P_1^{Cash(P,T)}$ is the price paid by the physical trader to the producer. As we consider the decision process in period 1, the futures price $P_{1,2}^F$ is observed by both the producer and the international physical trader. $\alpha > 0$ is considered as a variable capturing both the physical trader profit margin on the buy-side and transportation costs.
In period 2, the producer can sell to the national end-user or to the foreign end-user. Two prices are therefore available:

$$\begin{cases} \hat{P_2}^{Cash(P,EU)} = \theta \hat{P_2}^I \\ \hat{P_2}^{Cash(P,EU^*)} = \hat{e_2} \hat{P_2}^I \end{cases} \tag{2}$$

Where $\hat{P_2}^I$ is the expected international price in t=2 ($\hat{P_2}^I = \hat{P}_{2,3}^F$)[2]. Both the international price and the future price are denominated in USD. We assume that it is the currency used for international trade in the model. However, both agents have to form expectations on prices in period 2, that are $\hat{P_2}^{Cash(P,EU)}$, the expected price paid by the national end-user to the the producer, and $\hat{P_2}^{Cash(P,EU^*)}$, the expected price paid by the foreign end-user to the producer. θ is a parameter and $\hat{e_2}$ is the expected nominal exchange rate (direct quotation) for the next period. Although the exchange rate does not appear explicitly in the first equation, these two prices are denominated in the national currency of the domestic producer. Because it is an exchange rate, e is a strict conversion variable between a foreign price and a national price, whereas θ must be understood as a parameter which reflects the dissemination of an international price considered as the reference for setting global prices to national price. We assume that the producer is not able to rely on the futures market for hedging purposes. On the contrary, the physical trader has a full access to futures market in order to hedge his intermediation margin. He buys from the producer all the excess supply that is not stored by the producer.

[2]Hats are used to represent next-period expected values for t=2, $\hat{x} = E[x]$

3. The model

3.1- Stage one: The producer determines the optimal allocation between selling to the local end-user and selling to the foreign end-user in period 2

As in Mitra and Broussard (2012), the producer is risk adverse and myopic, he maximizes the expected utility from his sales to end-users at the end of the period 2, meaning that we need to identify the expected quantity traded to national end-users $\hat{Q}_2^{P,EU}$, and the expected quantity is traded to foreign end-users $\hat{Q}_2^{P,EU^*}$. By assuming that the producer can sell to the commodity trader only in period 1, we have $\hat{Q}_2^{P,EU} = \hat{q}_2^{P,EU}(\bar{Q} - Q_1^{P,T^S})$ and $\hat{Q}_2^{P,EU^*} = \hat{q}_2^{P,EU^*}(\bar{Q} - Q_1^{P,T^S})$, in which Q_1^{P,T^S} is the production supplied to the commodity trader in period 1, thus, $(\bar{Q} - Q_1^{P,T^S})$ is the remaining production available for sales in period 2. $\hat{q}_2^{P,EU}$ and $\hat{q}_2^{P,EU^*}$ are shares of the production sold to national end-users and foreign end-users, respectively, thus, $\hat{q}_2^{P,EU} + \hat{q}_2^{P,EU^*} = 1$. The average utility function per unit of commodity in t=2, $\hat{y}_2$, is based on the producer's revenue per unit in his local currency:

$$\hat{y}_2 = \hat{q}_2^{P,EU}\theta\hat{P}_2^{I} + \hat{q}_2^{P,EU^*}\hat{e}_2\hat{P}_2^{I} \tag{3}$$

The utility function is based on a classic mean-variance equation:

$$\hat{U}_2 = \hat{y}_2 - \frac{1}{2}A_P Var(\hat{y}_2) \tag{4}$$

Where A_P is the producer's risk aversion coefficient. Based on the proofs in Appendix 2, it gives:

$$\hat{U}_2 = \left(q_2^{P,EU}\theta\hat{P}_2^{I} + q_2^{P,EU^*}\hat{e}_2\hat{P}_2^{I}\right)$$
$$-\frac{1}{2}A_P\left(\left(\hat{q}_2^{P,EU}\right)^2\theta^2\hat{\sigma_P}^2+\left(\hat{q}_2^{P,EU^*}\right)^2\left[\hat{e}_2^{\,2}\hat{\sigma_P}^2 + \left(\hat{p}_2^{I}\right)^2\hat{\sigma}_e^{\,2} + \hat{\sigma}_e^{\,2}\hat{\sigma_P}^2\right]+2\hat{q}_2^{P,EU}\hat{q}_2^{P,EU^*}\theta\hat{e}_2\hat{\sigma_P}^2)\right)$$

Where $\hat{\sigma_P}^2$ and $\hat{\sigma}_e^{\,2}$ are the expected future price volatility and the expected exchange rate volatility respectively. The producer maximizes his utility function. The first order condition (FOC) could be represented as:

$$\frac{\partial\hat{U}_2}{\partial\hat{q}_2^{P,EU}} = 0$$

Let B be equal to:

$$B = e^2\hat{\sigma_P}^2 + \left(\hat{P}_2^{I}\right)^2\hat{\sigma}_e^{\,2} + \hat{\sigma}_e^{\,2}\hat{\sigma_P}^2 \tag{5}$$

The FOC leads us to:

$$\hat{q_2}^{P,EU} = \frac{\theta \hat{P_2}^I - \hat{e_2}\hat{P_2}^I + A_P\left(B - \theta\hat{e_2}\hat{\sigma_P}^2\right)}{A_P\left(\hat{\sigma_P}^2\theta^2 + B - 2\theta\hat{e_2}\hat{\sigma_P}^2\right)} \tag{6}$$

Our aim is to investigate both the impact of exchange rate volatility and futures price volatility on exports flows.

Proposition 1 *An increase in exchange rate volatility has a negative impact on producers export flows to foreign end-users in t=2.*

We compute the two partial derivatives of $\hat{q_2}^{P,EU}$. First, we show that:

$$\frac{\partial \hat{q_2}^{P,EU}}{\partial \hat{\sigma_e}^2} = \frac{A_P\left[\left(\hat{P_2^I}\right)^2 + \hat{\sigma_P}^2\right](\theta - \hat{e_2})\left(A_P\hat{\sigma_P}^2\theta - \hat{P_2}^I\right)}{A_P^2\left(\hat{\sigma_P}^2\theta^2 + B - 2\theta\hat{e_2}\hat{\sigma_P}^2\right)^2} \tag{7}$$

Note that the denominator of Equation (7) is always positive. Thus, in order to assess the impact of exchange rate volatility on national sales, we only have to investigate the sign of his numerator. Using the fact that $\hat{q_2}^{P,EU^*} + \hat{q_2}^{P,EU} = 1$, we know that $\hat{q_2}^{P,EU} \leq 1$, so we can demonstrate that[3]:

$$(\theta - \hat{e_2})\left(A_P\hat{\sigma_P}^2\theta - \hat{P_2}^I\right) \geq 0$$

As a consequence, we have:

$$\frac{\partial \hat{q_2}^{P,EU}}{\partial \hat{\sigma_e}^2} \geq 0$$

Thus, we demonstrate that the higher the exchange rate volatility the higher the share of the sales from the producer to the national end-user. It decreases producer's exports in period 2.

Proposition 2 *An increase in the expected futures price volatility* ***may*** *have a positive impact on commodity export flows from the producer to the international end-user in t=2.*

[3]See Appendix 2

In a second step, we investigate the relationship between international price volatility and exports. Therefore, we compute the following partial derivative[4]:

$$\frac{\partial \hat{q_2}^{P,EU}}{\partial \hat{\sigma_P}^2} = \frac{-(\theta - \hat{e_2}) P_2^I A_P \left\{ \left(\hat{e_2}^2 + \hat{\sigma_e}^2\right) + \theta(\theta - 2\hat{e_2}) + A_p \left(\hat{P_2}^I\right)^2 \theta \hat{\sigma_e}^2 \right\}}{A_P^2 [.]^2} + \frac{A_P^2 \hat{\sigma_P}^2 \theta \left((2\theta\sqrt{\hat{e_2}} - \sqrt{\hat{e_2}^3})^2 + 4\theta(\hat{e_2}^2 - \hat{\sigma_e}^2) \right)}{A_P^2 [.]^2} \quad (8)$$

As previously mentioned, the denominator is always positive. So in order to assess the impact of futures price volatility on national sales, we need to investigate the sign of the numerators. As the second term of the equation is strictly positive as long as $\hat{e_2}^2 > \hat{\sigma_e^2}$ [5], a sufficient, condition for equation (8) to be positive is that:

$$-(\theta - \hat{e_2}) \hat{P_2}^I A_p \left[(\hat{e_2} - \theta)^2 + \hat{\sigma_e}^2 + A_p \left(\hat{P_2}^I\right)^2 \theta \hat{\sigma_e}^2 \right] \leq 0 \quad (9)$$

It is straightforward to see that $A_p \left[(\hat{e_2} - \theta)^2 + \hat{e_2}\hat{\sigma_e}^2 + A_p \left(\hat{P_2}^I\right)^2 \right] \geq 0$, so the sign of the relationship is only determined by $-(\theta - \hat{e_2})$, given the fact that $\hat{P_2}^I \geq 0$ and $A_P > 0$. Based on their definition in section 2, both θ and $\hat{e_2}$ determine the sensitivity of cash prices paid by the national end-user ($\hat{P_2}^{Cash(P,EU)}$) and by the foreign end-user ($\hat{P_2}^{Cash(P,EU^*)}$), respectively, to the international price $\hat{P_2^I}$. For $\theta > \hat{e_2}$, the national price reacts more intensively to changes of the international price, hence $\hat{P_2}^{Cash(P,EU)}$ is more volatile than $\hat{P_2}^{Cash(P,EU^*)}$. As a consequence, the risk adverse producer decreases its sells to the national end-user. So, if $\theta > \hat{e_2}$, an increase in the expected international price volatility has a positive impact on exports, i.e both the share of sales to the foreign end-user in period 2 and the sales to the physical trader in period 1.

3.2- Stage two: The producer determines the optimal allocation between selling to the physical trader in period 1 and selling to end-users in period 2

In the first stage, we identified the expeced repartition of sales in period 2, but the producers also need to determine the production supplied in t=1 and

[4]See proofs in Appendix 2.

[5]Apart from extreme volatility episodes, it is reasonable to state that $\hat{e_2}^2 > \hat{\sigma_e}^2$. This condition could be understood as the squared first moment of the exchange rate being greater than the second moment of the variable e.

the production supplied in t=2. This time, the producer maximizes the expected utility function from his expected income ($\hat{Y}$) at the end of the period 2, meaning that we need to take into account both sales to commodity traders in $t = 1$ and sales to end users in $t = 2$:

$$\hat{Y} = Q_1^{P,T^S}(e_1\hat{P_1}^I - \alpha) + \hat{Q_2}^{P,EU}\theta\hat{P_2}^I + \hat{Q_2}^{P,EU^*}\hat{e_2}^{P,EU^*}\hat{P_2}^I \qquad (10)$$

Furthermore, the latter equation displays a classic mean-variance utility function, as follows:

$$\hat{U_2} = \hat{Y} - \frac{1}{2}A_P Var(\hat{Y}) \qquad (11)$$

Where A_P is the absolute risk aversion coefficient of the producer.
It can be shown that[6]:

$$\hat{U_2} = Q_1^{P,T^S}(e_1P_1^I-\alpha)+(\bar{Q}-Q_1^{P,T^S})\left(q_2^{P,EU}\theta\hat{P_2}^I+q_2^{P,EU^*}\hat{e_2}\hat{P_2}^I\right)-\frac{1}{2}A_P(\bar{Q}-Q_1^{P,T^S})^2$$
$$\left(\left(\hat{q_2}^{P,EU}\right)^2\theta^2\hat{\sigma_P^2}+\left(\hat{q_2}^{P,EU^*}\right)^2\left[\hat{e_2}^2\hat{\sigma_P^2}+\left(\hat{p_2^I}\right)^2\hat{\sigma_e^2}+\hat{\sigma_e^2}\hat{\sigma_P^2}\right]+2\hat{q_2}^{P,EU}\hat{q_2}^{P,EU^*}\theta\hat{e_2}\hat{\sigma_P}^2\right) \qquad (12)$$

To determine flows traded to the physical trader in t=1, the producer maximizes his utility function, the first order condition (FOC) is represented as:

$$\frac{\partial\hat{U_2}}{\partial Q_1^{P,T^S}} = 0$$

The FOC leads us to:

$$Q_1^{P,T^S} = \bar{Q} - \frac{\theta\hat{P_2}^I q_2^{P,EU} + \hat{e_2}\hat{P_2}^I q^{P,EU^*} - (e_1P_1^I - \alpha)}{A_P\left((\hat{q_2}^{P,EU})^2\ \theta^2\hat{\sigma_P}^2 + (\hat{q_2}^{P,EU^*})^2 B + 2\hat{q_2}^{P,EU}\hat{q_2}^{P,EU^*}\theta\hat{e_2}\hat{\sigma_p}^2\right)} \qquad (13)$$

For $Q_1^{P,T^S} \leq \bar{Q}$.

Comment: We can see from the latter equation that the production sold in t=2 to end-users by the producer is:

$$\bar{Q} - Q_1^{P,T^S} = \frac{\theta\hat{P_2}^I\hat{q_2}^{P,EU} + \hat{e}\hat{P_2}^I\hat{q_2}^{P,EU^*} - (e_1P_1^I - \alpha)}{A_P\left((\hat{q_2}^{P,EU})^2\ \theta^2\hat{\sigma_P}^2 + (\hat{q_2}^{P,EU^*})^2 B + 2\hat{q_2}^{P,EU}\hat{q_2}^{P,EU^*}\theta\hat{e_2}\hat{\sigma_p}^2\right)}$$

In the numerator we have the difference between the average revenue per unit for the sales in t=2 $\left(\theta\hat{P_2}^I\hat{q_2}^{P,EU} + \hat{e_2}\hat{P_2}^I\hat{q_2}^{P,EU^*}\right)$, and the revenue per unit for the

[6]See proofs in Appendix 3.

sales in t=1, $(P_1^I - \alpha)$. The denominator being strictly positive and $A_p > 0$, the constraint $Q_1^{P,T^S} \leq \bar{Q}$ leads to $\left(\theta \hat{P_2}^I \hat{q_2}^{P,EU} + \hat{e}_2 \hat{P_2}^I \hat{q_2}^{P,EU^*}\right) \geq (P_1^I - \alpha)$. It means than the expected average revenue for a sale in period 2 is equal or higher than the price for a sale to the physical trader in period 1. This observation makes sense because, otherwise, the producer would sell his entire production to the trader and there would not be any period 2.

As in the first stage, in order to investigate both the impact of exchange rate volatility and futures price volatility on trade flows with physical traders.

Proposition 3 *An increase in exchange rate volatility raises trade flows to physical traders.*

We compute the two partial derivatives of Q_1^{P,T^S}:

$$\frac{\partial Q_1^{P,T^S}}{\partial \hat{\sigma_e}^2} = \frac{\left(\theta \hat{P_2}^I \hat{q_2}^{P,EU} + \hat{e}_2 \hat{P_2}^I q^{P,EU^*} - (e_1 P_1^I - \alpha)\right)\left((\hat{q_2}^{P,EU^*})^2 (\hat{P_2}^I)^2 + (\hat{q_2}^{P,EU^*})^2 \hat{\sigma_p}^2\right)}{A_P\left((\hat{q_2}^{P,EU})^2 \ \theta^2 \hat{\sigma_P}^2 + (\hat{q_2}^{P,EU^*})^2 B + 2\hat{q_2}^{P,EU} \hat{q_2}^{P,EU^*} \theta \hat{e}_2 \hat{\sigma_p}^2\right)^2} \tag{14}$$

The denominator being strictly positive, the sign of $\frac{\partial Q_1^{P,T^S}}{\partial \hat{\sigma_e}^2}$ relies only on the sign of his numerator. Moreover, it is straightforward to see that the product on the right-side of equation (13) is positive. Based on the comments on equation (13): $Q_1^{P,T^S} - \bar{Q} > 0 \Leftrightarrow \theta \hat{P_2}^I q_2^{P,EU} + \hat{e}_2 \hat{P_2}^I \hat{q_2}^{P,EU^*} - (e_1 P_1^I - \alpha) > 0$. So $\frac{\partial Q_1^{P,T^S}}{\partial \hat{\sigma_e}^2} > 0$. An increase of the exchange rate volatility brings the producer to sell more to the trader in t=1 in order to reduce his risk exposure.

Proposition 4 *An increase in the international/future price volatility raises trade flows to physical traders.*

$$\frac{\partial Q_1^{P,T^S}}{\partial \hat{\sigma_p}^2} = \frac{\left(\theta \hat{P_2}^I q_2^{P,EU} + \hat{e}_2 \hat{P_2}^I q^{P,EU^*} - (e_1 P_1^I - \alpha)\right)\left((\hat{q_2}^{P,EU} \theta + \hat{q_2}^{P,EU^*} \hat{e}_2)^2 + (\hat{q_2}^{P,EU^*})^2 \hat{\sigma_e}^2\right)}{A_P\left((\hat{q_2}^{P,EU})^2 \ \theta^2 \hat{\sigma_P}^2 + (\hat{q_2}^{P,EU^*})^2 B + 2\hat{q_2}^{P,EU} \hat{q_2}^{P,EU^*} \theta \hat{e}_2 \hat{\sigma_p}^2\right)^2} \tag{15}$$

Analogously to proposition 3, we have $\frac{\partial Q_1^{P,T^S}}{\partial \hat{\sigma_p}^2} > 0$. The producer sells more to the trader in t=1 in order to reduce his price risk exposure.

Corollary *The introduction of physical traders may increase export flows.*

Without physical traders, an increase in the exchange rate volatility decreases export flows (Proposition 1). However, their introduction makes this statement uncertain because the production sold in period 1 to the physical trader is exported. We can see from Propositions 3 and 4 that when the producer's risk exposure raises because of an increase in the price and/or the exchange rate volatility, trade flows in period 2 should decrease. Consequently, an increase in the commodity price volatility raises international trade flows (Propositions 2 and 4 converge to the same conclusion), but the net effect of the impact of exchange rate volatility is uncertain: Propositions 1 and 3 diverge. Indeed, even if international trade flows coming from the producer to end-users in t=2 decrease, this concerns a smaller fraction of the production. So, paradoxically, an increase of the exchange rate volatility could even lead to more international trade if $-\Delta\hat{Q}_2^{P,EU^*} < \Delta Q_1^{P,T^S}$.

3.3- The commodity trader maximizes his profit

Until this point, we have discussed producer's sales by assuming that the trader will take the delivery of commodities in period 1 whatever the size of the volume sold, whereas the trader maximizzes his profit. The trader expresses his own demand, Q_1^{P,T^D}, in t=1, Q_1^{P,T^D}, he buys at a price $P_1^{Cash(P,T)}$, carries the commodity held until period 2 and sells it to the foreign end-user at a price $\hat{P}_2^{Cash(T,EU^*)}$. The physical trader is an international player who trades only in USD:

$$\begin{cases} P_1^{Cash(P,T)} = P_{1,2}^F - \alpha \\ \\ \hat{P}_2^{Cash(T,EU^*)} = \hat{P}_{2,3}^F + \hat{\beta} \end{cases} \tag{16}$$

α is a discount and $\hat{\beta}$ an expected premium, they represent the profit margin and the transportation costs on the buy-side and sell-side respectively. As we have assumed that the physical trader has a full access to the futures market, his long physical position in period 1 will be hedged by a short, i.e. selling, futures position according to the so-called offset hedging strategy. The physical trader buys in period 1 the quantity he can sell in period 2, thus $Q_1^{P,T^D} = \hat{Q}_2^{T,EU^*}$. Moreover, the notional amount of the futures contracts that are sold in period 1 (and therefore bought, i.e. cleared in period 2) is equal to the quantity of the commodity physically held between periods 1 and 2. The expected profit of the physical trader is therefore expressed as follow:

$$\hat{\Pi}^T = Q_1^{P,T^D}\left[\left(\hat{P}_2^{Cash(T,EU^*)} - \hat{P}_1^{Cash(P,T)}\right) + \left(P_{1,2}^F - \hat{P}_{2,3}^F\right) - c_T - i\right] \tag{17}$$

The trader's profit depends on his hedging strategy. $Q_1^{P,T^D} P_{1,2}^F$ is his long position on the futures market in t=1, and $\hat{Q}_2^{T,EU^*} \hat{P}_{2,3}^F$ is his short position on the futures market in t=2 to hedge the price risk. As exposed in section 2, $P_{t,m}^F$ is the futures price in t for a maturity m which is used as the reference price, hence $P_t^I = P_{t,m}^F$. i is the interest rate for financing the acquisition of the commodity and c_T is the carrying cost between the two periods. As a consequence, we have:

$$\hat{\Pi}^T = Q_1^{P,T^D}(\alpha + \hat{\beta} - c_T - i) \quad (18)$$

In t=1, α and i are given on the market and c_T is a constant, thus, only $\hat{\beta}$ is uncertain. It allows us to defined the expected utility function of the commodity trader:

$$\hat{U^T} = \hat{\Pi}^T - \frac{1}{2} A_T Var(\hat{\Pi}^T) \quad (19)$$

Where A_T is the trader's risk aversion. Hence:

$$Var(\hat{\Pi}^T) = Var(Q_1^{P,T^D} \hat{\beta}) = (Q_1^{P,T^D})^2 \hat{\sigma_\beta}^2$$

The mean-variance utility function is:

$$\hat{U_2^T} = Q_1^{P,T^D}(\alpha + \hat{\beta} - c_T - i) - \frac{1}{2} A_T (Q_1^{P,T^D})^2 \hat{\sigma_\beta}^2$$

The commodity trader maximizes his utility function, the FOC could be represented as:

$$\frac{\partial \hat{U_2^T}}{\partial Q_1^{P,T^D}} = 0$$

$$Q_1^{P,T^D} = \frac{\alpha + \hat{\beta} - c_T - i}{A_T \hat{\sigma_\beta}^2} \quad (20)$$

The latter equation gives the trader's demand in t=1.
The condition for the physical trader to buy the commodity whatever the price in t=1 is met for Q_1^{P,T^S} given by equation (13) equals Q_1^{P,T^D}. It gives the discount parameter α^*:

$$\alpha^* = \frac{A_P D \; A_T \hat{\sigma_\beta}^2}{A_P D + A_T \hat{\sigma_\beta}^2} \bar{Q} - \frac{A_P D(\hat{\beta} - c_T - i)}{A_P D + A_T \hat{\sigma_\beta}^2} - \frac{A_T \hat{\sigma_\beta} \left(\hat{P_2}^I (\theta \hat{q_2}^{P,EU} + \hat{e_2} \; \hat{q_2}^{P,EU^*}) - e_1 P_1^I \right)}{A_P D + A_T \hat{\sigma_\beta}^2} \quad (21)$$

Where $D = (\hat{q_2}^{P,EU})^2 \; \theta^2 \hat{\sigma_P}^2 + (\hat{q_2}^{P,EU^*})^2 B + 2 \hat{q_2}^{P,EU} \hat{q_2}^{P,EU^*} \theta \hat{e_2} \hat{\sigma_p}^2$.

3.4- The physical trader changes trade flows in the industry

Traditionally, the seller wants to trade at high prices, whereas the buyer is looking for low prices. Consequently, prices have a positive impact on supply and a negative one on demand. However, the presence of the physical trader brings to reassess these classic behaviours.

Proposition 5 *Both the producer's supply and the trader's demand react positively to international price changes.*
We can see from equation (13) that international prices do have a positive impact on the production supplied to the commodity trader in period 1:

$$\frac{\partial Q_1^{P,T^S}}{\partial P_1^I} = \frac{e_1}{A_P D} \tag{22}$$

We know that the denominator is always positive, thus $\frac{\partial Q_1^{P,T^S}}{\partial P_1^I} > 0$. It is an obvious result for a producer. Nevertheless, the physical trader's demand for commodities in period 1 appears to contrast with traditional industrial organization. Unlike the end-user, the trader does not take part in the production process. He is a midstream player whose activity is to buy and sell commodities at different specific times and locations. Thus, his profit margin does not depend on the spread between the cost of the input and the sale price of the output, but on the discount α and the expected premium $\hat{\beta}$. High prices lead to new trade opportunities for the physical trader. We can prove it by substituting α by α^* in equation (20):

$$Q_1^{P,T^D} = \frac{\alpha^* + \beta - c_T - i}{A_T \hat{\sigma_\beta}^2}$$

We have:

$$\frac{Q_1^{P,T^D}}{\partial P_1^I} = \frac{e_1}{A_P D + A_T \hat{\sigma_\beta}^2} > 0 \tag{23}$$

Moreover, we established in equation (20) that the discount α^* is neither a parameter, neither imposed by the trader but the result of $Q_1^{P,T^S} = Q_1^{P,T^D}$. We can show that:

$$\frac{\partial \alpha^*}{\partial P_1^I} = \frac{A_T \hat{\sigma_\beta}^2 e_1}{A_P D + \ A_T \hat{\sigma_\beta}^2} > 0 \tag{24}$$

This very important result of our model is logical but also illustrates the reality of commodity international trade. A high price in period 1 fosters the producer's willingness to sell now but, all things being equal, hampers the end-users' demand.

Because his price risk is fully hedged and because his profit does not depend per se on cash prices, the trader can then act as the sole counterparty to the prodcer and has, in this respect, all the more important bargainang power when prices are high.

4. Concluding remarks

This article aims to better reflect the role of physical traders and derivatives markets in the reality of commodity chains. Contrary to the theoretical corpus which tends to minimize the importance both of this type of economic agents and of the prices which are formed on commodity exchanges, we show that the presence of traders can significantly change the pattern of trade flows. We indeed reach two significant results. First, an increase in prices raises the producer's exports thanks to the trader's ability to absorb these volumes. Second, an increase in the volatility of international/futures prices could introduce uncertainty into the opportunity cost of holding inventories and could lead producer to sell their stocks and, therefore, to export commodities.
This model is of course very simple when compared to the reality of international trade and is subject to some limitations. In particular, we have not considered, for the face of simplicity, market structure volatility (i.e. variation in the contango or backwardation levels, but also the probability to move from contango to backwardation, or vice versa) whereas his impact is decisive on the effective commodity cash prices. This article should therefore be seen as a first step towards better describing and understanding, from a theoretical modelling perspective, the behaviour of traders and their impacts on the reality of trade flows.

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