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Duration-dependent Markov-switching model: an empirical study for the Brazilian business cycle.

Fernando H.P.S Mendes
Federal University of Rio Grande do Sul

João Frois Caldeira
Federal University of Rio Grande do Sul

Guilherme Valle Moura
Federal University of Santa Catarina

Abstract

This paper uses a duration-dependent Markov-switching model to identify business cycles in the Brazilian economy and to test for the presence of duration dependence in periods of expansion and contraction. The model is estimated using the growth rate of quarterly GDP from 1980:II to 2016:II. In the empirical application we found evidence of significant asymmetry in growth rates and duration dependence in the business cycle transition probabilities. The parameter estimates indicated that as the recession ages, the probability of a transition into an expansion increases (positive duration dependence in recessions). On the other hand, as the expansions ages, the probability of a transition into a recession decreases (negative duration dependence in expansions). The smoothed probabilities of the model captured several periods of contraction during the last three decades, matching the recession dates of the Business Cycle Dating Committee (CODACE) from the Getúlio Vargas Foundation.

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Contact: Fernando H.P.S Mendes - fernandohpsm@hotmail.com, João Frois Caldeira - joao.caldeira@ufrgs.br, Guilherme Valle Moura - guilherme.moura@ufsc.br.
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1 Introduction

Measuring the state of the economy and understanding the transition between periods of recession and expansion has been an important topic in research regarding business cycles and has its foundations in the works of Fisher (1925) and Burns & Mitchell (1946). Based on these studies, various authors have sought to develop methodologies in order to capture regularities in economic activity that can define the phases of the business cycle and also the transition probabilities from an expansion to a recession and vice-versa.

The knowledge of the timing and duration of the business cycle is important for economic decision making, such as adopting anti-cyclical fiscal and monetary policies (see, for example, Castro 2010). However, the business cycle is characterized by nonlinearities (see, for example, Terasvirta & Anderson 1992, Beaudry & Koop 1993). More specifically, Keynes (1936) has argued that recession, although more aggressive, tend to be more short-lived than expansions; therefore, the research on duration dependence in business cycles attempts to answer the following question: "Are periods of expansion or contraction in economic activity more likely to end as they become older? More technically, do business cycles exhibit positive duration dependence?" Sichel (1991, p. 254).

Earlies studies about duration dependence analyzes the NBER chronology using non-parametric methods or hazard models (see, for example, Diebold & Rudebusch 1990, Sichel 1991, Diebold et al. 1993, among others). Based on the length of each phase, these studies found significant evidence of positive duration dependence for pre-WWII expansions and post-WWII contractions in U.S. economy. Another strand of the literature based on the Markov-Switching models, which defines the switches between expansions and recessions through a first-order Markov chain. Different to the existing studies, Durland & McCurdy (1994) extended Hamilton (1989) Markov-switching model to allow for duration dependence in recessions and in expansions. This methodology defines business cycle through an unobservable stochastic process, so that the business cycle chronology is not necessary.

Durland & McCurdy (1994) showed that as a contraction ages the probability of moving into an expansion increases, i.e., coming out of the recession is more plausible should the crisis be prolonged. In the opposite scenario, the model did not find significant results for duration dependence associated with the probability of a transition out of expansions, but could nicely match NBER business cycle dates. Lam (2004) generalized the model of Durland & McCurdy (1994) incorporating the duration dependence in the mean growth rate. The main conclusion of the author is that the probability of the expansion ending gradually decreases as the expansions ages, while the probability of the contraction ending increases as the contraction ages.

The duration dependence business cycle studies have generally focused on the developed countries, especially, U.S. economy, since their chorology are well documented by NBER. Empirical research have remained limited for the developing countries (notable exceptions include: Ozun & Turk 2009, Castro 2015). Brazil is one of the emerging market economies that can constitute an important case of study. Markov-switching models and its variants have also been applied in the study of Brazilian business cycles, as shown in Chauvet (2002), Correa & Hillbrecht (2004), Céspedes et al. (2006), and Valls Pereira & Vieira (2014). However, none of the previous studies investigate the duration dependent feature of the Brazilian business fluctuations. Thus, our paper contribution try to fill the gap that can be observed in the empirical literature devoted to the Brazilian economy.

The purpose of this paper is to identify the timing, behavior and duration of business cycles in the Brazilian economy. In addition to identifying periods of recession and expansion, we test for the presence of duration dependence. More specifically, we implement a procedure to identify periods of recessions and expansions starting from the 1980s until 2016 and simultaneously test for the presence of duration dependence. We employ a duration-dependent Markov-switching model, developed by Maheu & McCurdy (2000*a*) to study the U.S. bull and bear market and employed by Lam (2004) to study the U.S. business cycle.

The remainder of this paper is organized as follows: Section 2 presents the methodology; Section 3 describes the data and our empirical results; Section 4 concludes.

2 Methodology

2.1 Regime switching model with duration dependence

The Markov-switching model proposed by Hamilton (1989) is defined by:

$$y_t = \mu(S_t) + \sum_{i=1}^{p} \phi_i \{y_{t-i} - \mu(S_{t-i})\} + \varepsilon_t, \tag{1}$$

where y_t is the GDP growth rate, $\mu(S_t)$ is presented with the state variable S_t where $\mu(S_t) = \mu_0(1-S_t) + \mu_1 S_t$ with μ_0 and μ_1 being parameters. If $S_t = 0$, then $\mu(S_t) = \mu_0$. If $S_t = 1$, then $\mu(S_t) = \mu_1$. The evolution of the unobserved state variable S_t follows a first-order Markov chain with transition probabilities and takes value 0 when the economy is in recession and 1 when the economy is in expansion, $\phi_1, \ldots, \phi_p$ are parameters, p the number of lags and ε_t is an error term at time t following an identically and independently normal distribution.

In an attempt to investigate the duration dependence in the business cycle, Durland & McCurdy (1994) extended the traditional Markov-switching model exploring high order Markov chains. In the duration-dependent Markov-switching specification, the probability of a regime change is a function of the previous state, as well as the duration of the previous state. Following Maheu & McCurdy (2000*b*), the length of occurrences of the state, S_t, can be characterized by:

$$D_t = min(D_{t-1} I(S_t, S_{t-1}) + 1, \tau) \tag{2}$$

where $I(S_t, S_{t-1}) = 1$ if $S_t = S_{t-1}$ and 0 otherwise. To make the estimation tractable is necessary to define a limiting parameter τ. The transition probabilities are parametrized using a logistic function. This ensures that the probabilities are between 0 and 1. Using i to index the state and d the duration (in quarters), where $\gamma_1(i)$ and $\gamma_2(i)$ are the parameters, the transition probability of staying in state i, given that we have been in state i for d_{t-1} periods is given by:

$$P(S_t = i | S_{t-1} = i, D_{t-1} = d_{t-1}) = \begin{cases} \dfrac{\exp(\gamma_1(i) + \gamma_2(i) d_{t-1})}{1 + \exp(\gamma_1(i) + \gamma_2(i) d_{t-1})}, & \text{if } d \le \tau \\ \dfrac{\exp(\gamma_1(i) + \gamma_2(i)\tau)}{1 + \exp(\gamma_1(i) + \gamma_2(i)\tau)}, & \text{if } d > \tau \end{cases} \quad i = 0, 1. \tag{3}$$

The conditional probability of a state change, given that the state has achieved a duration d is described by the hazard function. Using the transition probabilities, it is given by:

$$1 - P(S_t = i | S_{t-1} = i, D_{t-1} = d_{t-1}) = \frac{1}{1 + \exp(\gamma_1(i) + \gamma_2(i) d_{t-1})}, \quad i = 0, 1. \tag{4}$$

A decreasing hazard function is referred to as negative duration dependence whereas an increasing hazard function characterizes the positive duration dependence. The parameter $\gamma_2(i)$ summarizes the duration effect on the hazard function. For example, $\gamma_2(i) < 0$ means that a long period in regime i implies a higher probability of state switching (positive duration dependence); $\gamma_2(i) = 0$ means that the transition probability is independent of the regime duration; and $\gamma_2(i) > 0$ implies that the longer the duration of regime i, the higher the chance of the process to remain at i (negative duration dependence).

The parameters of the duration-dependent Markov-switching model can be estimated using two different approaches: the maximum-likelihood method following Maheu & McCurdy (2000*a*), or using MCMC methods, such as in Pelagatti (2001). In this study, we employed the first method. For the model with two states, $S_t = i$ where $i = 0, 1$ and p is the number of lags of y_t, Maheu & McCurdy (2000*a*) defined a new latent variable $\mathcal{S}_t$, which covers all possible paths from $S_t = i$ to $S_{t-p} = i$ as well as the respective duration in the sequence of states up to τ. Using this approach, the duration-dependent model collapses into a first-order Markov model, where the transition matrix for $\mathcal{S}_t$, is given by:

$$P = \begin{bmatrix} p_{11} & p_{21} & \cdots & p_{N1} \\ p_{12} & p_{22} & \cdots & p_{N2} \\ \vdots & \vdots & & \vdots \\ p_{1N} & p_{2N} & \cdots & p_{NN} \end{bmatrix}, \tag{5}$$

where $p_{ii} = P(\mathcal{S}_t = i|\mathcal{S}_{t-1} = i)$, $i, i = 1, \ldots, N$ is constructed using equations (3) and (4) and N represents the number of all states, $N = 2^{p+1} + 2(\tau - p - 1)$. Based on that, the authors show that estimation and smoothing can be performed with the usual techniques as seen in Hamilton (1989).

3 Data and Empirical Results

3.1 Data

We consider quarterly data from the Brazilian Gross Domestic Product (GDP) growth for the empirical analysis spanning the period from 1980:II to 2016:II, a total of 145 observations. Due to the fact that the current GDP series begins in 1996:1, we used two different bases to depict the data series for the entire period. The first data series refers to quarterly GDP with seasonal adjustment from 1996:I to 2016:II (base year 2010). The second data series refers to quarterly GDP with seasonal adjustment from 1980:I to 2014:III (base year 2000). In order to obtain the GDP from 1980 to 2016, the first data series was retropolated using the growth rate of the second data series. After the data treatment, the first difference of its logarithm was taken[1].

Before going to the estimates results, Table 1 presents an overview of the economic cycles in Brazil during the period considered in our data sample. The analysis is based on the Brazilian Business Cycle Dating Committee (CODACE) from the Getúlio Vargas Foundation.[2] During the last 36 years, the Brazilian economy went through 9 recessions.

[1]The GDP data is obtained from Ipeadata (http://www.ipeadata.gov.br).

[2]The CODACE is a committee created in 2008 by the Getúlio Vargas Foundation to determine a chronology of reference for the Brazilian business cycles. Its form of organization and method of work follows the model adopted in many countries, notably the North American Data Committee, created in 1978 by the National Bureau of Economic Research (NBER).

In this period, the economic growth was weak and volatile, excepted for some periods, for example, the most prosperous growth phase started at the end of 2003 lasting until the world crises of 2008-2009. The recessions were short and less severe in the two decades that followed the Real Plan in 1994, but it changed in 2014, the beginning of the last recession in our data sample. For further details summarizing the key events in recessionary (expansionary) phases, see, Weller (2019).

Table 1: Quarterly chronology of business cycles in Brazil - CODACE

Recessions			Expansions		
Periods	Number of Quarters	Accumul. Growth %	Periods	Number of Quarters	Accumul. Growth %
1981:1 - 1983:1	9	-8.9	1983:2 - 1987:2	17	26.2
1987:3 - 1988:4	6	-4.2	1989:1 - 1989:2	2	8.1
1989:3 - 1992:1	11	-8.0	1992:2 - 1995:1	12	17.6
1995:2 - 1995:3	2	-2.8	1995:4 - 1997:4	9	8.5
1998:1 - 1999:1	5	-1.7	1999:2 - 2001:1	8	7.3
2001:2 - 2001:4	3	-0.9	2002:1 - 2002:4	4	5.1
2003:1 - 2003:2	2	-1.5	2003:3 - 2008:3	21	26.7
2008:4 - 2009:1	2	-6.0	2009:2 -2014:1	20	20.8
2014:2 - 2016:2	9	-7.6			

Note: The third (sixth) column of this table refers to the peak-trough (trough-peak) accumulated GDP growth rate in the period. Peak refers to the end of an expansion and is followed by the start of a recession in the next quarter. Trough refers to the final quarter of a recession, which is followed by the beginning of economic expansion in the next quarter.

3.2 Markov Model with Duration-Dependent Transition Probabilities

Table 2 reports the estimates for the duration-dependent Markov-switching model. For the quarterly frequency ranging from 1980:II to 2016:II, this specification capture a dichotomous pattern in the series associated with high and low economic growth phases. The recession is characterized by a negative mean value, $\mu_0 = -2.142\%$, while expansion is denoted by a positive mean value, $\mu_1 = 0.993\%$. These estimates are statistically significant and evidenced the asymmetry in the growth rates.[3] Regarding the duration dependence coefficients, our preliminary results indicated an asymmetry on the state temporal dependence in cyclical data. These estimates were also statistically significant implying a positive duration dependence in the recession, $\gamma_2(0) = -1.048$ and negative duration dependence in the expansion, $\gamma_2(1)$= 0.289.

Figure 1 shows the transition probability of Eq. (3). The dotted line represents the probability of remaining in the recession and the continuous line denotes the probability of remaining in the expansion. Note that the chance of remaining in the recession falls gradually over the quarters, that is, the probability of moving from recession to expansion increases as a function of time (increasing hazard function). On the contrary, the chance of remaining in the expansion increases gradually over the quarters, that is, the probability

[3]We follow Chauvet (2002) setting $p = 0$ (see Eq. 1). In this case, we have $N = 2+2(\tau-1)$.

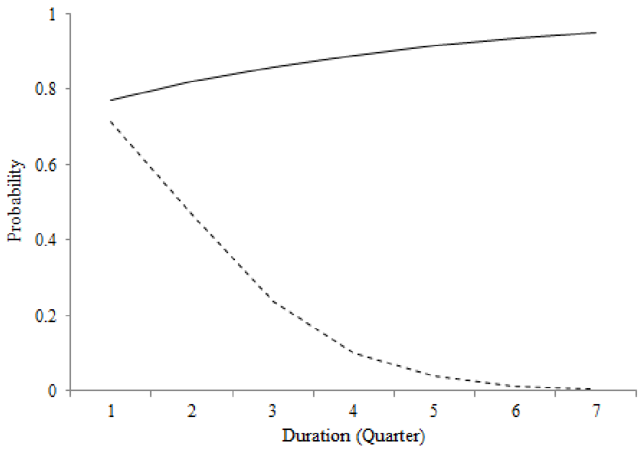
Table 2: Markov Model with Duration-Dependent Transition Probabilities

Parameter	Estimate	Standard Error
μ_0	-2.142^{***}	0.536
μ_1	0.993^{***}	0.142
σ^2	1.993^{***}	0.452
$\gamma_1(0)$	1.968^{*}	1.582
$\gamma_2(0)$	-1.048^{*}	0.827
$\gamma_1(1)$	0.937	0.825
$\gamma_2(1)$	0.289^{**}	0.163
τ	7	
$\ln L$	-282.96	

Note: This table reports the estimates for the Markov model with duration-dependent transition probabilities for the quarterly Brazilian GDP growth rate from 1980:2 to 2016:2. $\ln L$ is the value of the log-likelihood. $^{***},^{**},^{*}$ denote significance at the 1%, 5% and 10% levels, respectively.

of moving from expansion to recession does not increase as a function of time (decreasing hazard function). After 7 quarters, the probability of switching the regime is duration independent and its value is represented by parameter τ, which was calibrated using grid search from $[5,25]$ with the log-likelihood values as the criterion as advocated by Maheu & McCurdy (2000*a*).

Figure 1: Transition Probability: Recession (dotted line), Expansions (continuous line)



3.3 General Markov Model with Duration-Dependent Transition Probabilities

Despite the statistical evidence of negative duration dependence in expansions, much of the conventional wisdom state that a very long expansion is unstable and contraction is increasingly imminent (see, for example, Burns 1969, Neftici 1982). To further investigate our preliminary results, we apply a general type of duration-dependent Markov-switching model in the same spirit of Lam (2004). In this specification, the duration variable enters both in the transition probabilities and in the mean process. The model is defined as:

$$y_t = \mu(S_t) + \psi(S_t)D_t + \sum_{i=1}^{p} \phi_i\{y_{t-i} - \mu(S_{t-i}) - \psi(S_{t-i})D_{t-i}\} + \varepsilon_t, \tag{6}$$

where $\psi(S_t) = \psi_0(1-S_t) + \psi_1 S_t$ is the new component which ψ_0 and ψ_1 are parameters. This model allows duration to be a conditioning variable in the mean growth rates characterizing the dynamic behavior within each regime. The persistence in particular state implies that D_t increases and its effects are measured by the coefficient $\psi(S_t)$, which captures the relationship between the mean growth rate and the age of economic condition.[4]

The estimates of the general Markov model with duration-dependent transition probabilities are presented in Table 3. This specification also captured the asymmetry in the duration dependence dynamics. For the parameter ψ_1, the estimate value is negative (-0.244) and statistically significant, which implies that GDP growth rate declines as the expansion ages. For example, taking the value of τ, after 7 quarters, the GDP growth rate would be $\mu_1 = 0.52\%$. This result is related to the characterization of the business cycle, which suggest that expansion tends to be rapid in its early stages than its endings (Sichel 1994). On the other hand, Burns (1969) suggests that during recessions the rate of decline is usually fasted in the middle states than the early stages. This statement would be related to the estimates value of ψ_0, which is also negative (-0.27). However our result lacks statistical significance.

Table 3: General Markov Model with Duration-Dependent Transition Probabilities

Parameter	Estimate	Standard Error
μ_0	-1.454	1.416
μ_1	2.445^{***}	0.585
σ^2	1.927^{***}	0.253
$\gamma_1(0)$	2.549^{*}	1.758
$\gamma_2(0)$	-1.067^{*}	0.734
$\gamma_1(1)$	0.458	0.910
$\gamma_2(1)$	0.422^{**}	0.181
ψ_0	-0.278	0.536
ψ_1	-0.244^{**}	0.093
τ	7	
$\ln L$	-279.60	

Note: This table reports the estimates for the general Markov model with duration-dependent transition probabilities for the quarterly Brazilian GDP growth rate from 1980:2 to 2016:2. $\ln L$ is the value of the log-likelihood. ***,**,* denote significance at the 1%, 5% and 10% levels, respectively.

Overall results identified evidence of duration dependence in Brazilian business cycle. Our estimates are consistent with the findings of Lam (2004). However, the U.S. and Brazilian economy are very different, and there is not a unique justification for these kinds of features. Theoretically, the negative duration could be referred to the increasing

[4] We adopted the linear parametrization following Maheu & McCurdy (2000*a*). Other types of specifications can also be applied, for example, Lam (2004) assumes that the relationship between the mean growth rate and the age of current phase is quadratic.

probability of remaining in the expansion in the absence of external disturbances, while, the positive duration, could be related to the use of anti-cycle policies to mitigate the effect of the recessions. Nevertheless, there are several economic reasons why duration dependence might occur.

More recently, Rudebusch (2016) discuss several postwar changes in the U.S. economy that contributed to more robust and longer-lived expansions. For example, the increased share of services instead of tangible goods in the GDP would tend to diminish the importance of inventory fluctuations and moderate the business cycle. The author also highlights the postwar influence of the federal government actively focused on stabilizing the economy, which also included attempts to curtail recessions. Particularly, the Employment Act of 1946, applied broadly to the federal government, including to the Federal Reserve and in the conduct of monetary policy is a example of countercyclical policy helped prolong business expansions and alter the pattern of business cycle age dependence (Diebold & Rudebusch 1999).

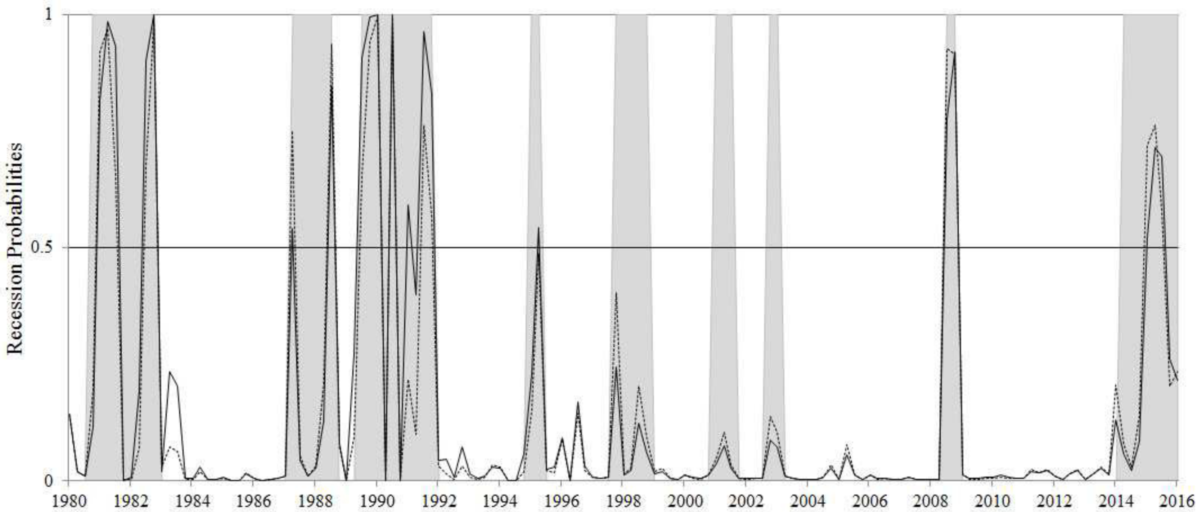
Notably, the Brazilian business cycle is characterized by volatility and stagnations. From 1980 to 2016, the overall growth results were mediocre compared to other developing economies. These stylized facts could be related to our general duration-dependent estimates since negative duration dependence could be associated with a growth rate that declines over the expansion. This results support what Brazilian economists refer to the stop-and-go process, whereas the country went through 9 recessions in the recent last thirty years. For the positive duration dependence, one possible narrative of our results could be referred to the fact that recessions were shorter after the Real Plan until 2014, as previously analyzed in Table 1. However, the economic forces behind the duration dependence effects could be investigated in future works.

3.4 Business Cycle Identification

Concerning the business cycle identification, Figure 2 plot the filtered probability of being in a recession and expansion phases. It is worth mentioning that the objective of this analysis is not to describe historical facts behind these results. Instead of this type of investigation, we compared our estimates to Table 1. In general, the models captured the main business cycle phases vis-à-vis to the CODACE. The general duration-dependent model seems to be slightly superior to the baseline duration specification. Using the Hamilton's 0.5-rule (recession probabilities higher or, equal to 0.5) the general model identified 6 out of 9 recessions from 1980 to 2016. For the expansion phases, both specifications match to the CODACE chronology.

It is important to point out that CODACE decisions are made on the basis of analyzing the most comprehensive set of variables, statistics and taking the point of view of its members. Moreover, the chronology is carried out many months after the facts have occurred, and therefore the duration-dependent model is not expected to be entirely accurate to this chronology. However, the filtered probabilities are in line with previous studies in Brazil (see, for example, Chauvet 2002, Céspedes et al. 2006, Valls Pereira & Vieira 2014). Besides updating the data set, which takes account the latest recession in the Brazilian economy, the attractiveness of our empirical application stems for including the parameter that captures the duration-dependent, as the same time, we obtain the state probabilities. In both models, the hypothesis of duration dependence was confirmed suggesting this is a characteristic of the data.

Figure 2: Duration-dependent Markov-switching model (dotted line), General duration-dependent Markov-switching model (continuous line) and Recession according to CODACE-FGV (shaded areas)



3.5 Model Comparisons

Finally, we test the overall significance of duration dependence comparing our models to the classic Markov-switching model, as seen in Durland & McCurdy (1994) and Lam (2004). For our first duration model, we were not able to reject the classic model in favor of the duration-dependent specification. The LR test statistic is 3.84, which has a p-value > 0.05 according to the χ^2 distribution with 2 degrees of freedom. On the order hand, we rejected the classic Markov-switching model in favor of the general duration-dependent model. The LR test statistic is 10.56, which has a p-value < 0.05 according to the χ^2 distribution with 4 degrees of freedom. Having these distinctive results, we compare the duration-dependent models. The LR test statistic is 6.72, which also has a p-value < 0.05, according to the χ^2 distribution with 2 degrees of freedom. Overall results reject the null hypothesis of absence of duration dependence in mean growth rates and transition probabilities since the general model was statistically different from the classic Markov-switching and the baseline duration model.

Table 4: Likelihood-Ratio Test

Model	ln$L$	Test	d.f.	LR	p-value
$\mathcal{M}1$	- 284.88	$\mathcal{M}2$ x $\mathcal{M}1$	2	3.84	0.147
$\mathcal{M}2$	-282.96	$\mathcal{M}3$ x $\mathcal{M}1$	4	10.56	0.032
$\mathcal{M}3$	-279.60	$\mathcal{M}3$ x $\mathcal{M}2$	2	6.72	0.034

Note: $\mathcal{M}1$ refers to the Markov-switching model, $\mathcal{M}2$ is the duration-dependent Markov-switching model, and $\mathcal{M}3$ covers the General duration-dependent Markov-switching model. LR is the test statistics and d.f is the degrees of freedom.

4 Conclusion

In this article, we have identified business cycles in the Brazilian economy as well as evidence of duration dependence in the respective phases of expansion and recession. Using the duration-dependent Markov-switching model in growth rate of quarterly GDP from 1980:II to 2016:II, the estimates indicated that as the recession ages, the probability of a transition into an expansion increases (positive duration dependence in recessions). On the other hand, as the expansions ages, the probability of a transition into a recession decreases (negative duration dependence in expansions).

Regarding to the business cycles identification, the model probabilities proved to have a reasonable capacity of discerning periods of contraction and expansion. Instead of describing the historical facts behind these identifications, we compare the recession probabilities to the Business Cycle Dating Committee (CODACE) from the Getúlio Vargas Foundation, which is a reliable ex post reference for the Brazilian business cycle turning points. Using the Hamilton's 0.5-rule, the probabilities of the general duration-dependent Markov-switching model captured 6 out of the 9 recessions from 1980 to 2016.

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