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Intergroup Conflict with Intragroup Altruism

Wei Hu
ETH-Zürich, Switzerland

Nicolas Treich
Toulouse School of Economics, INRA

Abstract

In this paper, we consider an intergroup contest game with intragroup altruism. We show that more altruism within a group increases conflict intensity by increasing total groups' efforts. Moreover, we show that, unlike the celebrated Olson's group size paradox, group size increases the probability of winning the contest provided that intragroup altruism is high enough.

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Contact: Wei Hu - weihu@ethz.ch, Nicolas Treich - ntreich@toulouse.inra.fr.

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1. Introduction

Scholars in different disciplines have argued that intragroup cooperation plays a central role in situations of intergroup conflict.[1] Military discourse for instance repeatedly appeals to group loyalty and group identity, suggesting that intragroup cooperation is instrumental in wars. Moreover, it has often been advanced that altruism is a key behavioral factor explaining intragroup cooperation (Dawes *et al*. 1977). Extensive empirical as well as numerical research provides an evolutionary basis for intragroup altruism in the context of intergroup conflict (Bowles 2006, Rusch 2014).

In this paper, we examine theoretically the role of intragroup altruism in an intergroup contest game. As a first step, we consider a static game with exogenous intragroup altruism. We show that an increase in altruism within a group confers an advantage to that group but also increases conflict intensity. We also examine the impact of group size, and in turn revisit the celebrated Olson (1965)'s "group size paradox" that smaller groups have an advantage in intergroup conflicts (Esteban and Ray 2001, Cheikbossian 2012).

2. The Model

We consider a simple intergroup contest game in which two groups compete for a rent (Katz *et al*. 1990, Nitzan 1991), except that we allow for intragroup altruism.[2] For analytical simplicity, we suppose that all members of the same group share the same degree of altruism. Denoting it by $\alpha_A \in [0,1]$ for group A and $\alpha_B \in [0,1]$ for group B, the utility functions of agent i in groups A and B are written respectively:

$$\begin{aligned} u_i &= p_A w_A - a_i + \alpha_A \sum_{j \in \mathrm{A}, j \neq i} \left[p_A w_A - a_j \right], i \in \mathrm{A} \\ u_i &= (1-p_A) w_B - b_i + \alpha_B \sum_{j \in \mathrm{B}, j \neq i} \left[(1-p_A) w_B - b_j \right], i \in \mathrm{B}, \end{aligned} \tag{1}$$

where a_i and b_i indicate an individual agent's effort in the respective groups. p_A is the probability that group A wins the rent, determined according to the standard Tullock (1980)'s contest success function, i.e. $p_A = \frac{A}{A+B}$, with $A \equiv \sum_{i \in \mathrm{A}} a_i$ and $B \equiv \sum_{i \in \mathrm{B}} b_i$. w_A and w_B represent the value of rent to an agent when her group wins the rent. Following Esteban and Ray (2001), we assume that the rent is composed of both a pure public goods component P and an impure component R:

$$w_A = \lambda P + (1-\lambda)\frac{R}{m}, \quad w_B = \lambda P + (1-\lambda)\frac{R}{n},$$

where m and n are the respective size (i.e. the number of members) of group A and B and $\lambda \in [0,1]$ is the degree of publicness of the rent.

[1] See, e.g., in biology (Wilson and Wilson 2008), political science (Hardin 1995), psychology (Bornstein and Ben-Yossef 1994) and economics (Abbink *et al*. 2010, Bowles and Gintis 2011).

[2] This game is more thoroughly studied in our previous working paper (Hu and Treich 2014).

We define $W_A \equiv [1+\alpha_A(m-1)]w_A$ and $W_B \equiv [1+\alpha_B(n-1)]w_B$ so that (1) can be written:

$$u_i = p_A W_A - a_i - \alpha_A \sum_{j\in \mathrm{A}, j\neq i} a_j, \quad i \in \mathrm{A} \tag{2}$$
$$u_i = (1-p_A) W_B - b_i - \alpha_B \sum_{j\in \mathrm{B}, j\neq i} b_j, \quad i \in \mathrm{B}.$$

Hence, W_A and W_B correspond to the "perceived rent" to an altruistic agent in either group.[3] We then obtain the best response function of either group, $A(B)$ and $B(A)$, in terms of aggregate efforts:

$$A(B) = \max\{\sqrt{W_A B} - B, 0\}; \quad B(A) = \max\{\sqrt{W_B A} - A, 0\}. \tag{3}$$

We can then easily find the equilibrium efforts and winning probability as follows:

$$A = \frac{W_A^2 W_B}{(W_A+W_B)^2}; \quad B = \frac{W_A W_B^2}{(W_A+W_B)^2}; \quad p_A = \frac{A}{A+B} = \frac{W_A}{W_A+W_B}, \tag{4}$$

leading to:

$$\begin{cases} \dfrac{\partial A}{\partial W_A} > 0, \quad \dfrac{\partial A}{\partial W_B} \gtreqless 0 \iff W_A \gtreqless W_B; \\ \dfrac{\partial (A+B)}{\partial W_A} > 0, \quad \dfrac{\partial (A+B)}{\partial W_B} > 0; \\ \dfrac{\partial p_A}{\partial W_A} > 0, \quad \dfrac{\partial p_A}{\partial W_B} < 0. \end{cases} \tag{5}$$

We recognize here some common equilibrium features of contest games. A group with a larger perceived rent collects more effort than its rival. As a consequence, this group also has a greater probability to win the contest (Nitzan 1991). Moreover, the equilibrium effort of the group with the higher perceived rent would increase if the other group exerts more effort, whereas the reverse is true for the group with the lower perceived rent. The contest game is therefore neither a game of strategic complements or a game of strategic substitutes (Dixit 1987): A group may "give up" or "keep up" when the rival group increases its effort depending on whether the rival group exerts relatively more or less effort.

3. The Effects of Intragroup Altruism and of Group Sizes

In this section, we first examine the effects of intragroup altruism and of group sizes across equilibria by comparing two equilibria that only differ by the degree of altruism or by the group size. Then, we examine these effects within an equilibrium. That is, we compare the two groups' respective efforts at a specific equilibrium, and study how this comparison is affected by the groups' differences in intragroup altruism and group size.

[3] Alger (2010) uses a similar terminology in a public goods game with altruistic agents.

3.1 Comparing equilibria

By observing the composition of W_A and W_B, it is immediate that both α_A and α_B have a positive effect on the respective group's perceived rent. Following (5), this implies that an increase of altruism in a group has a positive effect on the group's effort and its winning probability. Moreover, although altruism may have either a positive or a negative effect on the rival group's effort, its impact on the total efforts of both groups is positive. In other words, intragroup altruism always increases conflict intensity, namely the positive effect on own group's effort dominates the negative effect of the rival group's effort, no matter the size difference between the two groups.[4]

We now study the impact of group size. Take group A for instance. Note that W_A varies with m in the following way:

$$\frac{\partial W_A}{\partial m} = -(1-\lambda)\frac{R}{m^2} + \alpha_A \left[\lambda P + (1-\lambda)\frac{R}{m^2}\right]. \tag{6}$$

This identifies two opposing effects of group size. First, there is a well known negative effect that occurs in the standard intergroup contest game without altruism (i.e. $\alpha_A = 0$). That is, without altruism, the free riding problem within a group is so severe that the size of group is detrimental rather than an advantage. Only when $\lambda = 1$ is this effect nil, and the group size does not affect equilibrium efforts. This corresponds to the neutrality result of group sizes first identified in Katz *et al.* (1990). Second, there is a positive effect of size due to intragroup altruism represented by the second term on the right hand side of (6). This is because intragroup altruism reduces free riding within the group. As a result of these two effects, when the degree of intragroup altruism is high enough, i.e., $\alpha_A \geq \frac{(1-\lambda)R}{(1-\lambda)R+\lambda P m^2}$, the overall effect of an increase in group size increases this group's effort, and in turn the group's winning probability as well as the conflict intensity. Interestingly, note that the threshold value such that group size affects the group's effort positively is decreasing in group size. Hence, the larger a group the lower the level of altruism required to overturn the Olson paradox. We summarize the above observations in the following proposition.

Proposition 1. *An increase in altruism in a group increases the efforts in that group and also increases conflict intensity. An increase in the size of a group increases efforts in that group and increases conflict intensity if and only if the degree of altruism in this group is high enough.*

3.2 Within equilibrium comparison

We now focus on the comparison of efforts between the two groups at an equilibrium. Given above results, it is obvious that if the groups have the same size, the one with a higher degree of altruism collects more effort and wins the contest with a higher probability. Assuming symmetry *within* each group, this implies that any agent in such a group also exerts more effort than any agent in the other group. Furthermore, an agent in the more altruistic group also has a higher expected material payoff (i.e., her expected utility excluding the altruistic component) than any agent in the other group.[5]

[4] This result is not obvious. In asymmetric (group) contests, conflict intensity usually declines relative to the symmetric case (Nti 1999).

[5] By assuming $m = n$, the comparison of agents' material payoffs reduces to: $p_A w_A - A/m > (1-p_A)w_A - B/m \iff \frac{1+\alpha_A(m-1)}{1+\alpha_B(m-1)} > 1 \iff \alpha_A > \alpha_B$.

Regarding the effect of group sizes, we first consider two simple polar cases. For $\alpha_A = \alpha_B = 0$, we have $W_A < W_B$ when $m > n$. Without intragroup altruism, a larger group is disadvantaged. This case is consistent with the Olson's group size paradox. In contrast, for $\alpha_A = \alpha_B = 1$, $W_A > W_B$ when $m > n$ and the group size paradox fails. Indeed, under full intragroup altruism, agents behave as "one for all", so that size becomes an advantage.

We now assume $\alpha_A = \alpha_B = \alpha \in (0, 1)$. In this case, it is easy to see that $W_A > W_B$ when $m > n$ iff $\alpha > \frac{(1-\lambda)R}{(1-\lambda)R+\lambda Pmn}$. That is, the larger group has an advantage provided that the degree of altruism is high enough. Note that, this time, the threshold value for a positive size effect expressed in the degree of intragroup altruism is decreasing in the size of either group. One may also want to compare agents' material payoffs between the two groups. This comparison critically depends on the publicness of the rent. It can be shown that there exists a threshold value $\lambda^* \in (0, 1)$, such that an agent in a larger group has a higher expected material payoff iff $\lambda > \lambda^*$.[6] We have just assumed that groups of different sizes have the same degree of intragroup altruism. The following proposition provides a general result that essentially confirms previous observations.

Proposition 2. *The larger group collects more effort than the smaller group as long as intragroup altruism or the number of agents in the larger group is high enough.*

Proof: Suppose $m > n$, $\alpha_A > 0$, $\alpha_B > 0$. Then, we have

$$\begin{aligned} W_A > W_B \quad &\Leftrightarrow \quad [1 + \alpha_A(m-1)](\lambda P + (1-\lambda)\frac{R}{m}) > [1 + \alpha_B(n-1)](\lambda P + (1-\lambda)\frac{R}{n}) \\ &\Leftrightarrow \quad \alpha_A > \alpha_A^*(\alpha_B) \equiv \frac{(1-\lambda)R(\frac{1}{n} - \frac{1}{m})}{\lambda P(m-1) + (1-\lambda)R\frac{m-1}{m}} + \alpha_B \frac{\lambda P(n-1) + (1-\lambda)R\frac{n-1}{n}}{\lambda P(m-1) + (1-\lambda)R\frac{m-1}{m}} \end{aligned}$$

in which $\alpha_A^*(\alpha_B) \in (0, 1)$, because $\alpha_A^*(\alpha_B)$ increases in α_B and $0 < \alpha_A^*(0) < \alpha_A^*(1) < 1$.

Alternatively, the comparison can be presented in terms of group sizes:

$$\begin{aligned} W_A > W_B \Leftrightarrow \qquad & (1-\lambda)R(1-\alpha_A)\frac{1}{m} + \lambda P\alpha_A m + \alpha_A[(1-\lambda)R - \lambda P] \\ > \quad & (1-\lambda)R(1-\alpha_B)\frac{1}{n} + \lambda P\alpha_B n + \alpha_B[(1-\lambda)R - \lambda P]. \end{aligned}$$

Hence the left hand side of the last inequality increases in m as soon as $m > \sqrt{\frac{(1-\lambda)R(1-\alpha_A)}{\lambda P\alpha_A}}$ with $\alpha_A > 0$. Q.E.D.

4. Conclusion

In this paper, we have shown that more altruism within a group increases conflict intensity in an intergroup contest game. Moreover, we have shown that in contrast to the group size paradox (Olson 1965) group size increases the probability of winning the contest provided that intragroup altruism is high enough. Some extensions of our simple model may consider that the size of the rent depends on altruism (Nitzan and Ueda 2009), that altruism may depend on the size of the group or that other-regarding preferences take different forms (e.g., inequity aversion, spite). More

[6]The proof is available upon request.

fundamentally, it may be interesting to study the evolutionarily stable degree of altruism (Alger 2010).

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