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Stability and Taxation in Monopolistic Competition

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Abstract

This paper shows that the standard monopolistically competitive general equilibrium is stable if and only if a particular tax-subsidy policy induces higher utility in equilibrium. This policy is taxing profits, and subsidizing labor income at a rate less than the price markup. Therefore, the government can increase the utility in any stable equilibrium using this tax/subsidy scheme without knowing the parameters of technology and preferences. Finally, even if the laissez-faire equilibrium is unstable, a subsidy rate sufficiently close to the price markup always ensures that the equilibrium is stable. That is, the government intervention can stabilize the free market equilibrium when the equilibrium is unstable.

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1. Introduction

Monopolistic competition in general equilibrium is the workhorse model in many fields of economics. This paper analyzes the standard general equilibrium model with monopolistic competition in product markets assuming identical consumers and firms. As is well-known, the equilibrium without government intervention, i.e. *laissez-faire equilibrium,* is inefficient due to markup pricing. I show that a particular tax-subsidy policy increases the represenative consumer's utility in equilibrium if and only if the equlibrium is stable. This particular tax-subsidy policy is taxing profits, and subsidizing labor income at a rate less then the price markup. The government budget is balanced.

Therefore, the stability of equilibrium is sufficient for higher utility under this tax-subsidy policy regardless of the preference and technology parameters. This means that no information about the preference/technology parameters is germane to increasing the equilibrium utility with this policy when the equilibrium is stable.

As for the actions, nominal wages are assumed to be perfectly observed to avoid a complicated a tax/subsidy scheme. Observing no other action such as output, prices, etc., is necessary.

It is noteworthy that assuming identical agents does not rule out the problem of unknown parameters of technology and preference. To demonstrate this claim as clearly as possible, an example where the representative consumer has a Cobb-Douglas utility function and the representative firm has a simple production function is also analyzed. Applying the general results of the present paper to this simple example, it is shown that the tax/subsidy scheme discussed here increases the equilibrium utility despite unknown parameters of the Cobb-Douglas utility and the production function. This example can, of course, be extended to cases with arbitrary number of unknown parameters.

The regulation of a single-representative monopolist with unknown parameters is extensively studied in the partial equilibrium framework (see Laffont and Martimort (2009)). To the best of my knowledge, however, mitigating the inefficiency caused by monopolistic competition in general equilibrium (with or without representative agents) by an appropriate tax-subsidy scheme is not studied in the literature.

Pareto-improving taxation is, however, a fairly standard result in some other general equilibrium concepts and market structures. For example see Villanacci and Zenginobuz (2012) for public goods , Citanna et. al. (1998, 2006) for incomplete markets, Geanakoplos and Polemarchakis (2008) for externalities, Greenwald and Stiglitz (1986) for imperfect information. A particularly interesting example is Bisin et. al. (2011) which considers how to induce a Pareto-improvement under the problem of asymmetric information among consumers. Nonetheless, due to the impressive degree of generality of these studies, their results are typically confined to the existence of Pareto-improving taxation.

In contrast, by virtue of the identical agents assumption, this paper makes a clear statement on the direction of all welfare enhancing tax policies. To be specific, the main result says that the labor subsidy rate should always be increased if it is originally below the price markup, given a stable equilibrium. This is certainly more informative than only proving the existence of Pareto-improving taxation.

Moreover, this study also shows the surprising fact that the stability of the equilibrium is inherently related to whether subsidizing labor and taxing profits is welfare-improving. Note that general equilibrium modelling is vital to this insight, which is hardly possible in the partial equilibrium analysis. That is because, the general equilibrium approach is built on the the idea that employees (who supply labor) are also the consumers (who own firms and demand commodities) in any given economy.

The possibility of an unstable equilibrium is also discussed. Even if the equilibrium without any government intervention is unstable, a sufficiently high subsidy rate on labor income ensures that the equilibrium with government intervention is stable. Thus, the tax/subsidy scheme discussed in this essay has a stabilizing potential too. Note that this is another curious instance of the inherent relation between stability of equilibrium and subsidizing labor income/taxing profits.

The next section introduces the economy. The equilibrium with an active government intervention is defined in Section 3. The notion of stability is discussed in Section 4. The main result of the paper is presented in Section 5.

2. The economy

Let us consider the standard general equilibrium model of monopolistic competition which subsumes well-known models such as Blanchard and Kiyotaki (1987) or Cooper (2004). There are n number of produced commodities whose prices are $(p_1, \ldots, p_n)$. There are also m number of individuals. Given $(c_{i1}, \ldots c_{in})$, the consumption of each good by individual i, define

$$c_i = \left(c_{ij}^{1/\mu}\right)^{\mu}$$

which is the composite consumption by individual i. The preference of individual i is given by the utility function $u(c_i, l_i)$ where $l_i \in [0,1]$ is leisure enjoyed by individual i. Hence, $1 - l_i$ is the labor supply of individual i. Assume $u(\cdot,\cdot)$ is strictly quasi-concave, smooth, and monotonically increasing in c_i and l_i. To avoid boundary behavior in equilibrium, assume $u_c(0,\cdot) = u_l(\cdot\,,0) = \infty$. No closed functional form is imposed on u.

Each individual i solves

$$\max u(c_i, l_i) \quad (1)$$
$$\text{s.t.}$$
$$p_j c_{ij} \leq \rho w(1 - l_i) + \pi^i$$

by choosing $(c_{i1}, \ldots, c_{in})$ and l_i given the wage rate w, and the profit income that individual i earns denoted by π^i. Here ρ is the policy parameter chosen by the government.

Remark 1 *Laissez-faire corresponds to* $\rho = 1$*: no government intervention. If* $\rho > 1$*, then labor income is subsidized. Of course,* $\rho < 1$ *says that labor income is taxed.*

Define $\varepsilon = \mu/(\mu - 1)$ which is the elasticity of substitution, and assume $\mu > 1$. Then aggregating the solution of Eq (1) in c_{ij} over i yields the aggregate demand for good j:

$$a(p_j) = \frac{1}{n}\left(\frac{p_j}{p}\right)^{-\varepsilon}\frac{y}{p}$$

where $y = \sum_i (\pi_i + \rho(1 - l_i)w)$ is interpreted as total income of all individuals, and $p = \left(1/n \sum_j p_j^{1-\varepsilon}\right)^{1/(1-\varepsilon)}$ is interpreted as the general price level.

Each commodity j is produced by firm j. The technology is represented by a smooth, monotonically increasing, and concave function $f(e_j)$ where e_j is the level of employment by firm j. The profit of the firm is $\pi_j = p_j f(e_j) - we_j - t$ where t is the tax paid to the government. By choosing (e_j, p_j) tuple, firm j solves

$$\max \pi_j$$
$$\text{s.t.}$$
$$f(e_j) \leq a(p_j).$$

Individuals have equal profit shares implying $\pi^i = \sum_j \pi_j/n$ which is the profit income that individual i earns. The budget balancedness condition of the government is

$$t = (\rho - 1)\frac{m}{n}\sum_i (1 - l_i)w. \tag{2}$$

Let us briefly comment on this tax/subsidy scheme. First note that observing the average labor cost per firm, $m\sum_i (1 - l_i)w/n$, is sufficient for the government to compute t in Eq (2) to be imposed on firms. So assume that the government perfectly observes the nominal labor income of workers. Yet no other information on actions such as production, employment, etc. is necessary to make the tax/subsidy scheme operational. According to Kleven (2014), observing labor income is a substantially weak information constraint due to third party reporting. Finally, also note that t is a lump-sum tax from the perspective of the firms despite the fact that t certainly involves choices variables of the workers.

3. Monopolistically competitive equilibrium

Symmetric equilibrium is a solid standard in the monopolistically competitive equilibrium literature. Hence let us also focus on the symmetric equilibrium in the present model too.

Definition 2 *Given the policy tuple* (ρ, t), *the symmetric monopolistically competitive equilibrium with fiscal policy is a vector* $(c^*, l^*, e^*, p^*, w^*)$ *which satisfies the following conditions*

1) (c^*, l^*) solves the utility maximization problem for all individuals.
2) (e^*, p^*) solves the profit maximization problem for all firms.
3) Goods and labor markets clear: $mc^* = nf(e^*)$ and $ml^* = ne^*$.
4) Government budget is balanced (i.e. Eq (2) holds).

After setting $w = 1$ to normalize prices, the symmetric equilibrium here is a vector $\xi = (c, l, e, p)$ that solves $\Gamma(\xi) = 0$ where

$$\Gamma(\xi) = \begin{pmatrix} u_c/u_l - p/\rho \\ pf'(e) - \mu \\ mc - nf(e) \\ ml + ne - m \end{pmatrix}$$

and u_l/u_c is the marginal rate of substitution between leisure and consumption evaluated at (c, l).

The first line of the equilibrium condition, $\Gamma = 0$, says that price taking individuals maximize utility, the second line says that price making firms maximize profits, and the last two lines say that product and labor markets clear. Observe that market clearing condition for the product market is equivalent to the representative individual's budget constraint. The equilibrium conditions given by $\Gamma(\xi) = 0$ do not involve the policy parameter t. That is because, the lump-sum tax t is expressed as a function ρ using the balanced budget condition Eq (2). Therefore, only ρ appears in the equilibrium conditions as a policy variable. The equilibrium, given the policy parameter ρ, is denoted by $\xi^*(\rho) = (c^*, l^*, e^*, p^*)$.

Proving the existence and generic local uniquness of the equilibrium is a routine exercise, and available from the author upon request. Define the equilibrium utility as $u^*(\rho)$ which is $u(c, l)$ evaluated at $\xi^*(\rho)$. That is, the equilibrium utility depends on the subsidy rate. An important special case is $\rho = 1$ implying no government intervention, i.e. *laissez-faire*. Therefore, $\xi^*(1)$ is said to be the laissez-faire equilibrium. In a similar vein, $u^*(1)$ is the laissez-faire utility. The laissez-faire equilibrium - $\xi^*(1)$ - is Pareto-inefficient since the marginal rate of substitution is not equal to marginal rate of transformation.

The objective of this essay is to show that this inefficiency decreases by arbitrarily choosing the subsidy rate ρ such that $\rho \leq \mu$ if and only if the equilibrium $\xi^*(\rho)$ is stable. Therefore, discussing the stability of the equilibrium from a formal standpoint is the next subject to be discussed.

4. Stability of equilibrium

This paper adopts the approach of Dixit (1986) to define stability. That is, the firms increase their prices if higher prices yield higher profits, and decrease their prices otherwise at any feasible point. That is to say, the rate of change in price, $\dot{p}$, is proportional to $\mu - pf'(e)$. In a similar vein, individuals consume more if utility is increasing in consumption under the feasibility constraints. So the rate of change in consumption, $\dot{c}$ is proportional to $u_c/u_l - p/\rho$ evaluated at (c, l) such that the allocations are feasible:

$$mc = nf(e) \text{ and } m = ml + ne. \tag{3}$$

This means that the adjustment process takes place over the manifold of feasible allocations. The feasibility conditions in Eq (3) can be used to solve for (e, l) as a smooth function of consumption, c. In words, the feasible amounts of employment and leisure are functions of consumption decision.

Lemma 3 *Given the price making general equilibrium $\xi^*(\rho) = (c^*, l^*, e^*, p^*)$, there is a smooth function $\left(\bar{e}(\cdot), \bar{l}(\cdot)\right)$ such that $\left(\bar{e}(c), \bar{l}(c), c\right)$ solves Eq (3) for any c in a sufficiently small neighborhood of c^*.*

Proof. All proofs are at the end of the paper as an appendix.

Hence, the equilibrium adjustment process is summarized by three pieces of information: i) $\dot{p}$ is proportional to $\mu - pf'(e)$, ii) $\dot{c}$ is proportional to $u_c/u_l - p/\rho$, and iii) Eq (3) holds which means all allocations over the course adjustment are feasible. This can be represented in matrix notation as a differential equation system. Given a vector of positive constants $\sigma = (\alpha, \beta)$, the rates of change in p and c are

$$\begin{bmatrix} \dot{p} \\ \dot{c} \end{bmatrix} = \begin{bmatrix} \alpha \times \left(\mu - pf'\left(\bar{e}(c)\right)\right) \\ \beta \times (u_c/u_l - p/\rho) \end{bmatrix} \tag{4}$$

such that u_c/u_l is evaluated at $\left(c, \bar{l}(c)\right)$. Recall that $\left(\bar{e}(c), \bar{l}(c)\right)$ gives market clearing levels of the tuple (e, l) as a function of c. The initial points of p and c are arbitrary.

By design, the process evolves only over feasible allocations. In this adjustment process, $\sigma = (\alpha, \beta) \in \mathbb{R}^2_{++}$ is the speed of adjustment. The stationary point of the adjustment process is the price-making equilibrium, $\xi^*(\rho)$. The crucial property that we are interested is whether the adjustment process in Eq (4) is locally asymptotically stable around its stationary point $\xi^*(\rho)$ for some speed of adjustment σ. Thus, we make the following definition.

Definition 4 (Stability) *The price-making equilibrium $\xi^*(\rho)$ is said to be stable when the adjustment process in Eq (4) is locally asymptotically stable around its stationary point, i.e. $\xi^*(\rho)$, for some speed of adjustment σ.*

The following technical lemma is crucial for the analysis that will follow.

Lemma 5 (Stability) *Let $D_\xi \Gamma$ denote the derivative of $\Gamma(\xi)$ with respect to ξ evaluated at $\xi^*(\rho)$. Then $\xi^*(\rho)$ is stable if and only if $det\left|D_\xi \Gamma\right| < 0$.*

Now we are ready to discuss the main results of the present essay.

5. Increasing equilibrium utility

This section shows that the government can increase the utility in equilibrium by subsidizing labor income at a rate no higher than the price markup if and only if the equilibrium is stable. In formal terms, when $\rho < \mu$, the stability of equilibrium is equivalent to that $u^*(\rho)$ increases in ρ. To see this equivalence relation, write $D_\rho u^*(\rho)$ for the derivative of $u^*(\rho)$ with respect to ρ.

Theorem 6 *Given a subsidy rate $\rho < \mu$, the price-making equilibrium $\xi^*(\rho)$ is stable if and only if $D_\rho u^*(\rho) > 0$.*

Therefore, if the equilibrium is stable then welfare increases with a subsidy rate less than the price markup. Moreover this sufficiency relation is also necessary. Now the obvious questions is under which condtions we can be sure that the equilibrium is stable. A simple technical condition for the stability of equilibrium is strict concavity of $u(c, l)$ and $u_{cl} = 0$ which is known as additively separable preferences. This claim is formally stated in the next proposition:

Proposition 7 *If $u(c, l)$ is additively separable, and strictly concave then $\xi^*(\rho)$ is stable.*

These results also imply that the government's information constraints are surprisingly weak. To see this, consider the following example: the prefences are Cobb-Douglas

$$u(c, l) = \alpha \ln c + (1 - \alpha) \ln l$$

and the production technology is $f(e) = e^\beta$ where $\alpha \in (0,1)$ and $\beta \in (0,1)$. Suppose that the government does not know the true values of α and β. Finally, suppose that $\mu \geq 1.1$ is public information but the precise value of the price-markup μ is unknown to the government.

Applying the general results above to this simple example, we can easily prove that the government can increase the equilibrium utility despite unknown parameters of technology and preference which are α and β. First of all, Proposition 7 guarantees that the equilibrium is always stable due to additively seperable preferences in this example. Because the equilibrium is stable, Theorem 6 implies

$$u^*(1.1) > u^*(1) \text{ for all } \alpha \in (0,1) \text{ and } \beta \in (0,1) \text{ and } \mu \geq 1.1.$$

In words, $\rho = 1.1$ induces higher utility in equilibrium regardless of the values of technology and preferences: α and β. Thus, 10% subsidy on labor income unambigously increases utility in equilibrium when price markup is at least %10 regardless of the parameters of utility and technology.

Our final result pertains to the case in which the equilibrium is unstable. It is easy to concoct examples with unstable equilibria when the utility function is not additively seperable. Can the government prevent this instability problem? The next result shows that the answer is positive if the government intervention is strong enough.

Theorem 8 *If ρ is sufficiently close to μ, then the equilibrium $\xi^*(\rho)$ is stable.*

As a consequence, even if the laissez-faire equilibrium $\xi^*(1)$ fails to be stable, a sufficiently high subsidy rate ρ ensures that the equilibrium $\xi^*(\rho)$ is stable. The interpretation is that subsidizing labor income and taxing profits has a stabilizing property when the laissez-faire equilibrium is unstable.

6. Conclusion

The market power of firms are observed to be significant in free market economies (see Martins and Scarpetta (1996), Cooper (2004)). In this essay, the equivalence between the stability of equilibrium when firms enjoy market power and the existence of a particular welfare increasing economic policy is proved. To be specific, the standard general equilibrium with monopolistic competition is stable if and only if taxing profits and subsidizing labor income increases equilibrium utility when the subsidy rate is less than the price markup. Moreover, this tax/subsidy scheme discussed here has an obvious virtue: parsimony regarding the amount of information that the government should have. No technology or preference parameter is necessary or relevant to finding a tax/subsidy scheme which increases utility in a stable equilibrium. Finally, this policy also has a stabilizing role. Even if the laissez-faire equilibrium is unstable, a sufficiently high subsidy rate ensures stability of the equilibrium.

7. Appendix

Proof of Lemma 3: Define $\Theta: \mathbb{R}^2_{++} \times \mathbb{R}_{++} \to \mathbb{R}^2$ such that

$$\Theta(e,l,c) = \{mc - nf(e), m - ml - ne\}.$$

Differentiating Θ with respect to (e,l) gives

$$D_{(e,l)}\Theta = \begin{bmatrix} -nf'(e), 0 \\ -n, -m \end{bmatrix}.$$

Note that $\det\left(D_{(e,l)}\Theta\right) = mnf'(e) \neq 0$. Therefore, by the implicit function theorem, there is a function $\left(\bar{e}(c), \bar{l}(c)\right)$ that solves

$$\Theta\left(\bar{e}(c), \bar{l}(c), c\right) = 0$$

for all c in a sufficiently small neighborhood of c^* where c^* satisfies $\xi^*(\rho) = (c^*, l^*, e^*, p^*)$. Also deduce that

$$\frac{d\bar{e}}{dc} = \frac{m}{nf'(e)}$$
$$\frac{d\bar{l}}{dc} = -\frac{1}{f'(e)}$$

again due to the implicit function theorem.

Proof of Lemma 5 (Stability): Note that

$$D_\xi\Gamma = \begin{bmatrix} \frac{d(u_c/u_l)}{dc} & \frac{d(u_c/u_l)}{dl} & 0 & -1/\rho \\ 0 & 0 & pf'' & f' \\ m & 0 & -nf' & 0 \\ 0 & m & n & 0 \end{bmatrix} \quad (5)$$

evaluated at $\xi^*(\rho)$ implying

$$\det\left|D_\xi\Gamma\right| = \frac{d(u_c/u_l)}{dc}mn(f')^2 - \frac{d(u_c/u_l)}{dl}mnf' + \frac{p}{\rho}f''m^2.$$

Now we shall see that $\det\left|D_\xi\Gamma\right| < 0$ if and only if $\xi^*(\rho)$ is stable.

The linear approximation of the differential equations in Eq (4) around $\xi^*(\rho)$ is

$$\begin{bmatrix} \dot{p} \\ \dot{c} \end{bmatrix} = \begin{bmatrix} -\alpha f' & -\alpha f'' \frac{p}{\rho}\frac{d\bar{e}}{dc} \\ -\beta & \beta\left(\frac{d(u_c/u_l)}{dc} + \frac{d(u_c/u_l)}{dl}\frac{d\bar{l}}{dc}\right) \end{bmatrix} \begin{bmatrix} p - p^* \\ c - c^* \end{bmatrix}$$

where the coefficient matrix is evaluated at $\xi^*(\rho)$. The asymptotic stability of this linearized system is equivalent to that the eigenvalues of the coefficient matrix have negative real parts. This occurs if and only if its trace is negative and its determinant is positive.

Let A denote the coefficient matrix of the linearized system. Then the trace of A is

$$tr|A| = -\alpha f' + \beta\left(\frac{d(u_c/u_l)}{dc} + \frac{d(u_c/u_l)}{dl}\frac{d\bar{l}}{dc}\right)$$

while its determinant is

$$\begin{aligned}\det|A| &= -\alpha\beta f'\left(\frac{d(u_c/u_l)}{dc} + \frac{d(u_c/u_l)}{dl}\frac{d\bar{l}}{dc} - \frac{pmf''}{\rho n(f')^2}\right)\\ &= -\alpha\beta\left(f'\frac{d(u_c/u_l)}{dc} - \frac{d(u_c/u_l)}{dl} - \frac{pmf''}{\rho nf'}\right)\\ &= -\alpha\beta\frac{\det|D_\xi\Gamma|}{mnf'}\end{aligned}$$

It is easy to see that there always exists σ such that $tr|A| < 0$. Therefore, the existence of σ such that the linearized system is asymptotically stable is equivalent to $\det|D_\xi\Gamma| < 0$. However, the asymptotically stability of the linearized system is equivalent to the local asymptotically stability of the original system given by Eq (4).

Thus far, I have shown that local asymptotically stability of the adjustment system for some σ is equivalent to $\det|D_\xi\Gamma| < 0$. As a consequence, due to Definition (Stability), $\xi^*(\rho)$ is stable if and only if $\det|D_\xi\Gamma| < 0$. This proves the desired result.

Proof of Theorem 6: Write $\xi^*(\rho) = (c^*, l^*, e^*, p^*)$ for the equilibrium when the rate of subsidy is ρ. By the chain rule,

$$D_\rho u^*(\rho) = D_\xi u \cdot D_\rho \xi^*.$$

where $D_\rho\xi^*$ is the derivative of $\xi^*(\rho)$ with respect to ρ. By the implicit function theorem,

$$\begin{aligned}D_\rho\xi^* &= \left(\frac{dc^*}{d\rho}, \frac{dl^*}{d\rho}, \frac{de^*}{d\rho}, \frac{dp^*}{d\rho}\right)\\ &= -\left(D_\xi\Gamma\right)^{-1} \cdot D_\rho\Gamma\end{aligned}$$

where $D_\xi\Gamma$ is given in Eq (5), and

$$D_\rho\Gamma = [p/\rho^2 \quad 0 \quad 0 \quad 0]$$

is the derivative of Γ with respect to ρ. Note that $D_\rho\Gamma$ is denoted as a row vector due to space considerations although it is a column vector.

Now let us prove that $\det|D_\xi\Gamma| < 0$ if and only if $D_\rho u^*(\rho) > 0$ given $\rho < \mu$. By the Cramer's rule, deduce that

$$\frac{dc^*}{d\rho} = -\frac{\det|D_\xi\Gamma_1|}{\det|D_\xi\Gamma|} \text{ and } \frac{dl^*}{d\rho} = -\frac{\det|D_\xi\Gamma_2|}{\det|D_\xi\Gamma|}$$

where $D_\xi\Gamma_i$ is obtained by replacing the i^{th} of column of $D_\xi\Gamma$ with $D_\rho\Gamma$ while keeping the other columns of $D_\xi\Gamma$ fixed. Routine computations show that

$$\begin{aligned}\det|D_\xi\Gamma_1| &= pmn(f'/\rho)^2\\ \det|D_\xi\Gamma_2| &= -pmnf'/\rho^2.\end{aligned}$$

By the chain rule,

$$D_\rho u^*(\rho) = \frac{\partial u}{\partial c}\frac{dc^*}{d\rho} + \frac{\partial u}{\partial l}\frac{dl^*}{d\rho} \qquad (6)$$

$$= \lambda\left(-p\frac{\det\left|D_\xi \Gamma_1\right|}{\det\left|D_\xi \Gamma\right|} - \rho\frac{\det\left|D_\xi \Gamma_1\right|}{\det\left|D_\xi \Gamma\right|}\right)$$
$$= \frac{\lambda}{\det\left|D_\xi \Gamma\right|}\left(-p\det\left|D_\xi \Gamma_1\right| - \rho\det\left|D_\xi \Gamma_2\right|\right)$$
$$= \frac{\lambda}{\det\left|D_\xi \Gamma\right|}\left(-ppmn(f'/\rho)^2 + \rho pmnf'/\rho^2\right)$$
$$= \frac{pmn\lambda f'/\rho^2}{\det\left|D_\xi \Gamma\right|}(-pf' + \rho)$$
$$= \frac{pmn\lambda f'/\rho^2}{\det\left|D_\xi \Gamma\right|}(-\mu + \rho)$$

Conclude that $\det\left|D_\xi \Gamma\right| < 0$ if and only if $D_\rho u^*(\rho) > 0$ assuming $\rho < \mu$. This proves the desired result due to Lemma 5 (Stability).

Proof of Proposition 7: Assume $d(du/dc)/dl = 0$. Moreover, concavity of $u(c, l)$ implies $u_{cc} < 0$ and $u_{ll} < 0$. Deduce

$$\det\left|D_\xi \Gamma\right| = \frac{d(u_c/u_l)}{dc} mn(f')^2 - \frac{d(u_c/u_l)}{dl} mnf' + \frac{p}{\rho} f'' m^2$$
$$= \frac{u_{cc}}{u_l} mn(f')^2 + u_{ll}\frac{u_c}{(u_l)^2} mnf' + \frac{p}{\rho} f'' m^2 < 0.$$

The first line ensues by definition. The second line is a consequence of $d(du/dc)/dl = 0$. The inequality is due to $u_{cc} < 0$ and $u_{ll} < 0$ and $f'' \leq 0$. However, $\det\left|D_\xi \Gamma\right| < 0$ implies that $\xi^*(\rho)$ is stable due to Lemma 5 (Stability), proving the desired result.

Proof of Theorem 8: Define

$$Q = \begin{bmatrix} u_{cc} & u_{cl} & u_c \\ u_{cl} & u_{ll} & u_l \\ u_c & u_l & 0 \end{bmatrix}.$$

Strict quasi-concavity of $u(c, l)$, already postulated in Section 2, implies

$$\det|Q| = -u_{cc}(u_l)^2 + u_{cl}u_c u_l + u_c(u_{cl}u_l - u_{ll}u_c) > 0.$$

Expanding $\det\left|D_\xi \Gamma\right|$ gives

$$\det\left|D_\xi \Gamma\right| = \frac{d(u_c/u_l)}{dc} mn(f')^2 - \frac{d(u_c/u_l)}{dl} mnf' + \frac{p}{\rho} f'' m^2$$
$$= \left(\frac{u_{cc}}{u_l} - \frac{u_c}{(u_l)^2}u_{cl}\right) mn(f')^2 - \left(\frac{u_{cl}}{u_l} - \frac{u_c}{(u_l)^2}u_{ll}\right) mnf' + \frac{p}{\rho} f'' m^2$$
$$= \left(\frac{u_{cc}}{u_l} - \frac{u_c}{(u_l)^2}u_{cl}\right) mn\left(\frac{\mu}{\rho}\frac{u_l}{u_c}\right)^2 - \left(\frac{u_{cl}}{u_l} - \frac{u_c}{(u_l)^2}u_{ll}\right) mn\frac{\mu}{\rho}\frac{u_l}{u_c} + \frac{p}{\rho} f'' m^2$$
$$= \frac{mn}{u_l}\left(\frac{1}{u_c}\right)^2 \frac{\mu}{\rho}\left((u_{cc}(u_l)^2 - u_c u_l u_{cl})\frac{\mu}{\rho} - (u_{cl}u_l - u_{ll}u_c)\right) + \frac{p}{\rho} f'' m^2.$$

This implies

$$\lim_{\rho \to \mu} \det\left|D_\xi \Gamma\right| = -\det|Q| + \frac{p}{\rho} f'' m^2 < 0$$

proving the desired result since $\det\left|D_\xi \Gamma\right| < 0$ is equivalent to the stability of $\xi^*(\rho)$ (Lemma 5 (Stability)).

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