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Dismissals scheduling and the employment of older workers

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Abstract

A model is built to study the joint effect on management of the older workforce of three crucial institutions: employment protection, the pension system, and the unemployment insurance (UI). More precisely, the interest is in the impact of corresponding settings (dismissal costs, replacement rates, UI benefit duration, statutory retirement age) on the age, at the date of job termination, of workers who experience a spell of unemployment before retiring. The opportunity cost to the worker of early exit from employment is shown to be increasing with the time distance until retirement. However, under specific assumptions particularly relevant within the continental Europe context, it is shown that corresponding marginal opportunity cost is very low provided time until retirement remains below the potential UI benefit duration (Lambda), much higher above. This makes dismissals at a time distance until retirement close to Lambda much more likely than at a shorter one. Comparative statics suggests that age (at the date of dismissal) is increasing with wage and decreasing with Lambda.

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1 Introduction

The labor market for older workers (aged 50 and older) is highly structured by three major institutions: employment protection, the pension system, and the unemployment insurance (UI). There is strong empirical evidence that corresponding settings have a joint impact on the employment of older workers - see Tuit and van Ours (2010) for the Netherlands, Grogger and Wunsch (2013) for Germany, Baguelin and Remillon (2014) for France. Understanding the nature of this joint impact is of primary interest.[1] In this note, it is shown how excessive unemployment among older workers may result from poorly coordinated institutional settings. The theoretical model presented here is designed to capture the interactions between employment protection, the pension system, and the UI. It focuses on the case of workers experiencing a spell of unemployment shortly before retiring. Although its scope is more general, one of its purposes is to provide rationales to the empirical results mentioned above; more specifically, it can be viewed as a companion paper to Baguelin and Remillon (2014).

2 The labor market institutions

The labor market for older workers is described by exogenous institutional settings. Employment protection is simply captured through the cost borne by the employer when dismissing a worker in case of legal challenge. Unemployment insurance is described with a wage replacement rate $\gamma_U \in [0;1]$, and a potential benefit duration $\lambda \geq 0$. Unemployment assistance is restricted to a benefit amount $b > 0$ independent from previous wage and paid for indefinite duration. The pension system is described with a statutory age of retirement a_R and a wage replacement rate $\gamma_R > 0$. For ease, the wage of reference for UI benefit and pension calculation is assumed to be the last received. It is further assumed that $\gamma_R \leq \gamma_U$ which is usually the case for workers with strong labor force attachment. Wage rate only depends on seniority which is assumed to be well-captured by age. Agents therefore deal with a deterministic wage profile (collectively bargained in the past, possibly) guaranteeing a worker aged α to receive $w(\alpha)$.

3 The employer

The employer of an older worker aged a_0 schedules a date for termination: he has to deal with an exogenous wage profile $w(\alpha)$ assumed to increase with the age α of the worker.[2] The statutory retirement age is $a_R > a_0$. The value for the employer of maintaining the worker in his position until age $a \in [a_0, a_R]$ is

$$\Pi(a) = \int_{a_0}^{a} (\pi - w(\alpha))\, e^{-(\alpha - a_0) r} d\alpha - C(a)$$

[1] In France, since the beginning of 2008 until 2015, the number of older workers registered as job seekers rose by +156% (against +60% for workers aged 25-49) - Dares-Analyse n°2015 − 050. In 2012, older job seekers represented 21% of those receiving UI benefits, but they absorbed 27% of UI total spending (6,2 billion € a year) - Unedic, January 2014.

[2] This assumption is supported by empirical evidence on age wage profile provided in OECD (2015).

where π is the (constant) productivity of the job held by the worker, and r is the employer's discount rate. It is assumed that $w(a_0) < \pi$. The cost $C(a)$ of terminating the job while the worker is aged a is

$$C(a) = \begin{cases} 0 & \text{with no legal challenge} \\ +\infty & \text{in case of legal challenge} \end{cases}.$$

Indeed, the employer faces the risk that the termination will be legally challenged by the worker. The employer is guaranteed to avoid a dispute if $a \geq a_R$. Otherwise, even though $a < a_R$, legal challenge can still be avoided if a is such that the dismissed worker's welfare is at least equal to what he would get assuming he stays in the same job until retirement. This is referred to as the 'no-legal-challenge (NLC) condition' .

4 The worker

It is assumed that once the job under consideration comes to an end, the older worker has no hope of finding another one. The worker is expecting to live until age $\bar{a} > a_R$ and has a constant subjective discount rate $\rho > 0$. Denoting $V(a; a_0)$ the value to the worker aged $a_0 < a_R$ of a job lasting until he reaches age $a \geq a_0$:

$$\begin{aligned} V(a; a_0) = & \int_{a_0}^{\min\{a;a_R\}} (w(\alpha) - c(\alpha)) e^{-(\alpha-a_0)\rho} d\alpha && \text{Employment} \\ + & \int_{\min\{a;a_R\}}^{\min\{a_R;a+\lambda\}} \gamma_U w(a) e^{-(\alpha-a_0)\rho} d\alpha + \int_{\min\{a_R;a+\lambda\}}^{a_R} b e^{-(\alpha-a_0)\rho} d\alpha && \text{Unemployment} \\ + & \int_{a_R}^{\bar{a}} \gamma_R w(\min\{a; a_R\}) e^{-(\alpha-a_0)\rho} d\alpha && \text{Retirement} \end{aligned}$$

where $c(\alpha)$ is the subjective instantaneous cost of holding the job under consideration at age α (money-metric disutility of work). The set of choices of the older worker is restricted to challenging or not a possible dismissal. If he does, he simply restores his 'employment until retirement' value, which will thus represent his reservation option. Note that resigning is never considered as an option.

The worker accepts a dismissal at age $a < a_R$ (*i.e.* refrains from prosecuting the employer) if and only if $V(a; a) \geq V(a_R; a)$ that is if his present out-of-employment gain between age a and $\bar{a}$ is at least equal to corresponding opportunity cost. The NLC condition can be written (in terms of flow value) as

$$\rho V(a; a) \geq \rho V(a_R; a)$$

$$\begin{bmatrix} \\ \left(1 - e^{-\min\{a_R-a;\lambda\}\rho}\right) \gamma_U w(a) \\ + \left(e^{-\min\{a_R-a;\lambda\}\rho} - e^{-(a_R-a)\rho}\right) b \\ + \left(e^{-(a_R-a)\rho} - e^{-(\bar{a}-a)\rho}\right) \gamma_R w(a) \end{bmatrix} \geq \begin{bmatrix} \rho W(a) \\ \\ \\ + \left(e^{-(a_R-a)\rho} - e^{-(\bar{a}-a)\rho}\right) \gamma_R w(a_R) \end{bmatrix},$$

where

$$\rho W(a) = w(a) - c(a) + \int_a^{a_R} (w'(\alpha) - c'(\alpha)) e^{-(\alpha-a)\rho} d\alpha - (w(a_R) - c(a_R)) e^{-(a_R-a)\rho}$$

represents the (flow-equivalent) opportunity cost from a to a_R of not being employed. The cost of early exit from employment is twofold: a shortfall below age a_R; a reduced pension above, since $w(a) < w(a_R)$.

Assume now that functions $w(.)$ and $c(.)$ are defined by $w(\alpha) = w_R + (\alpha - a_R)\phi$ and $c(\alpha) = c_R + (\alpha - a_R)\psi$ with $c_R < w_R$ but $\psi > \phi$. Instantaneous wage exceeds instantaneous disutility of work until retirement but the latter rises at a higher rate than the former. It implies that the flow value of being employed is maximal for $a = a_0$, minimal for $a = a_R$, and constantly decreasing at rate $\psi - \phi$ in between. The opportunity cost from a to a_R of not being employed, can be rewritten

$$\rho W(a) = \left(w_R - c_R + \frac{\psi - \phi}{\rho}\right)\left(1 - e^{-(a_R - a)\rho}\right) + (a_R - a)(\psi - \phi).$$

$W(a)$ is increasing with $a_R - a$ (decreasing with a): the closer from the age of retirement, the lower the direct opportunity cost of not being employed. Furthermore, all other things being equal, $W(a)$ is: increasing with $w_R - c_R$, the flow value of being employed at age a_R; increasing with $\psi - \phi$, the constant reduction rate of the flow value of being employed; decreasing with ρ, the subjective discount rate.[3] Figure 1 illustrates the role of various parameters in two cases: for $a + \lambda < a_R$, and for $a + \lambda > a_R$.

The NLC condition can be reformulated in terms of distance from retirement at the date of dismissal $a_R - a > 0$ and discounted at the date of retirement: $V(a; a) \geq V(a_R; a)$ if, and only if $f(a_R - a) \leq \gamma_U w_R - b$ where

$$\begin{aligned} f(a_R - a) = {} & (w_R - c_R - b)\frac{1 - e^{-(a_R - a)\rho}}{1 - e^{-\min\{a_R - a;\lambda\}\rho}} + \left(\frac{\psi - \phi}{\rho}\right)\frac{(a_R - a)\rho - \left(1 - e^{-(a_R - a)\rho}\right)}{1 - e^{-\min\{a_R - a;\lambda\}\rho}} \\ & + \left(1 - e^{-R\rho}\right)\gamma_R\phi\frac{(a_R - a)e^{-(a_R - a)\rho}}{1 - e^{-\min\{a_R - a;\lambda\}\rho}} + \gamma_U\phi\cdot(a_R - a), \end{aligned}$$

with $R \equiv \bar{a} - a_R$ the time spent in retirement. The term $\gamma_U w_R - b$ is the (flow) opportunity cost of being employed at the date of retirement, while $f(a_R - a)$ is the (flow-equivalent) opportunity cost of early exit from employment, discounted at the date of retirement. Now, assume $w_R - c_R > b$ so that resignation before retirement is never an option (see above).

Proposition 1 *If $w_R - c_R > b$ then the (flow-equivalent) opportunity cost of early exit from employment is generally strictly increasing with the distance $a_R - a$ from retirement. Furthermore,*

$$\lim_{a \to a_R} f(a_R - a) = \begin{pmatrix} w_R - c_R - b \\ \\ +\left(1 - e^{-R\rho}\right)\gamma_R\phi\frac{1}{\rho} \end{pmatrix} \text{ and } f(\lambda) = \begin{pmatrix} w_R - c_R - b \\ +(\psi - \phi)\left(\frac{\lambda}{1 - e^{-\lambda\rho}} - \frac{1}{\rho}\right) \\ +\left(\gamma_U - \left(1 - e^{-R\rho}\right)\gamma_R\right)\phi\lambda \\ +\left(1 - e^{-R\rho}\right)\gamma_R\phi\frac{\lambda}{1 - e^{-\lambda\rho}} \end{pmatrix},$$

with $\frac{\lambda}{1 - e^{-\lambda\rho}} > \frac{1}{\rho}$. The increase rate is very low for $a_R - a \leq \lambda$,[4] and much higher for $a_R - a > \lambda$.[5] In particular, for small ρ (a patient worker):

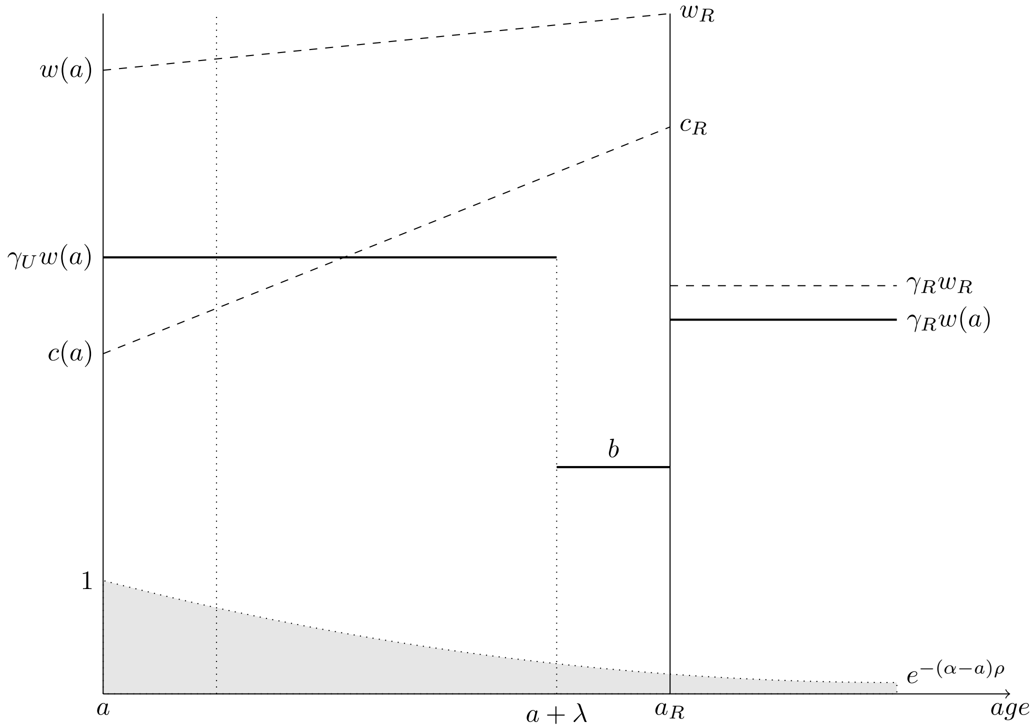
$$f'_+(\lambda) - f'_-(\lambda) \simeq \frac{w_R - c_R - b}{\lambda} + \frac{1}{2}(\psi - \phi) + \frac{R}{\lambda}\gamma_R\phi,$$

with $f'_+(\lambda) = \lim_{\substack{a_R - a \to \lambda \\ a_R - a > \lambda}} f'(a_R - a)$ and $f'_-(\lambda) = \lim_{\substack{a_R - a \to \lambda \\ a_R - a < \lambda}} f'(a_R - a)$.

[3] Note however that $\rho W(a)$ is increasing with ρ.

[4] The order of magnitude is that of $\psi - \phi$ that is 10^{-2}.

[5] The order of magnitude is that of annual money earnings.



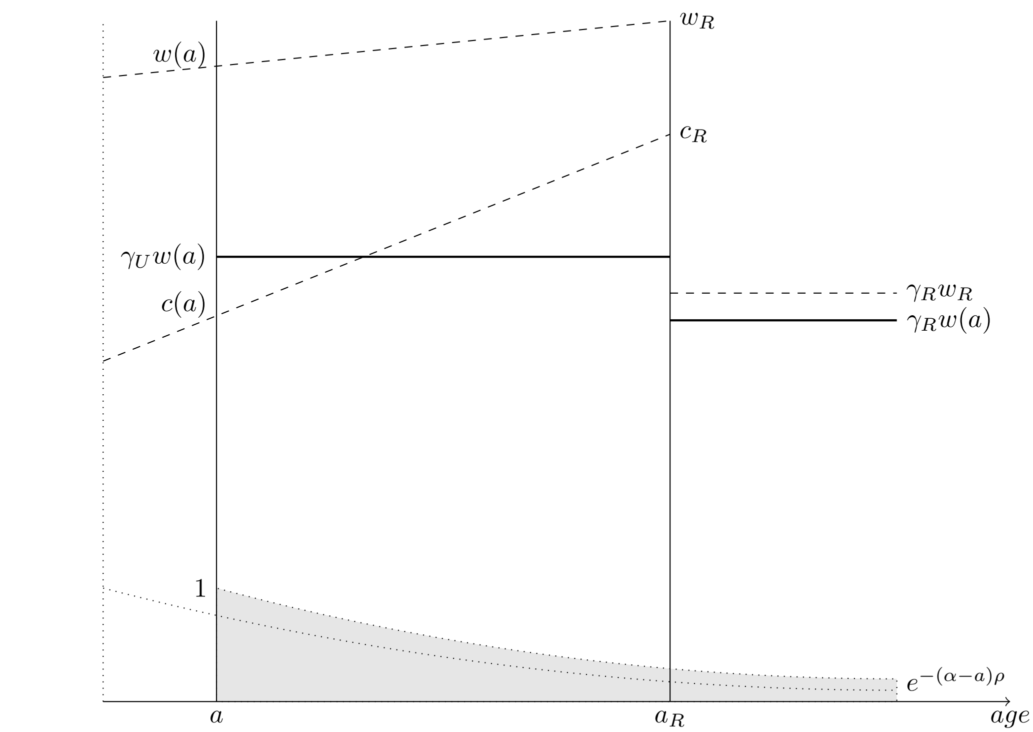


Figure 1: Main parameters, the case with $a + \lambda < a_R$ (north) versus $a + \lambda > a_R$ (south)

Proof. This is demonstrated in the analysis of the function $f(.)$ - see Appendix. ■

Suppose $a \leq a_R$, and consider a variation $da < 0$ of the date of exit from employment. The immediate lost wage $w(a)$ is offset by sparing a disutility of work $c(a)$ plus the UI compensation $\gamma_U w(a+da)$. Afterwards, the UI benefit received is therefore reduced by $\gamma_U \phi$ over the whole spell between a to $a+\lambda$, while the pension received is itself reduced by $\gamma_R \phi$ over the period between a_R to $\bar{a}$. Although the opportunity cost of early exit from employment constantly increases with time until retirement, two very different regimes can be distinguished - see Figure 1. For $a > a_R - \lambda$, the magnitude of the marginal opportunity cost of bringing forwards the exit from employment is of second order, that of $\psi - (1-\gamma_U)\phi > 0$. Now, consider the case where $a \leq a_R - \lambda$. Compared to the previous one, a variation $da < 0$ involves an additional opportunity cost of $(\gamma_U w(a+da) - b) e^{-\lambda \rho}$ *i.e.* a loss in compensation from $\gamma_U w(a+da)$ (UI) to b (unemployment assistance) occurring after a period of length λ. For ρ small enough (or a short potential benefit duration λ), the magnitude of the additional opportunity cost is of first order. This observation entails that the shape of the opportunity cost of early exit from employment can reasonably be approximated by lines bent in $a = a_R - \lambda$ such as shown in Figure 2: slopes are the average increasing rate of the early exit opportunity cost in the each of the two regimes.

5 Equilibrium

The problem of the employer is written: $\max_{a \geq a_0} \Pi(a)$ s.t. $V(a;a) \geq V(a_R;a)$. Suppose the NLC condition is not binding in equilibrium. The employer dismisses the worker at age $\tilde{a}_F$ solution of $\Pi'(\tilde{a}_F) = 0$ that is $(\pi - w(\tilde{a}_F)) e^{-(\tilde{a}_F - a_0)r} = 0 \Rightarrow w(\tilde{a}_F) = \pi$. For $w(\alpha) = w_R + (\alpha - a_R)\phi$ this leads to $\tilde{a}_F = a_R - \frac{w_R - \pi}{\phi}$. Let's consider the case where $w_R > \pi$ so that the employer would be willing to dismiss his employee before statutory retirement. Given the shape of the cost associated with breaking the NLC condition, it is clear that, if $V(\tilde{a}_F, \tilde{a}_F) < V(a_R, \tilde{a}_F)$, the employer will delay the dismissal until age $a > \tilde{a}_F$. But the cost of doing so is strictly increasing with $a - \tilde{a}_F$: if $a < a_R$ it must bind the NLC condition. This reasoning leads to the next proposition. Let $\tilde{a}_W < a_R$ denote the value of a (if it exists) such that $V(\tilde{a}_W, \tilde{a}_W) = V(a_R, \tilde{a}_W)$.

Proposition 2 *Suppose* $w_R > \max\{\pi; b + c_R\}$. *If* $\gamma_U w_R - b \leq \lim_{a \to a_R} f(a_R - a)$ *then the job terminates at the date of retirement. Otherwise, the worker is dismissed at aged* $a^* = \max\{\tilde{a}_F, \tilde{a}_W\}$ *where* $\tilde{a}_F = a_R - \frac{w_R - \pi}{\phi}$ *and* $\tilde{a}_W = a_R - f^{-1}(\gamma_U w_R - b)$.

Proof. It directly follows from proposition 1 and the reasoning above. ■

Figure 2 illustrates the case where early exit from employment is an equilibrium $a^* = \tilde{a}_W$. Since $f(\lambda)$ is very close to $\lim_{a \to a_R} f(a_R - a)$ by proposition 1, the case where $\gamma_U w_R - b$ belongs to $]\lim_{a \to a_R} f(a_R - a); f(\lambda)[$ is very unlikely. It follows that dismissal will typically occur at age a_R when γ_U and/or c_R are low, R, γ_R, and/or ϕ are high, at an age lower but close to $a_R - \lambda$ otherwise.

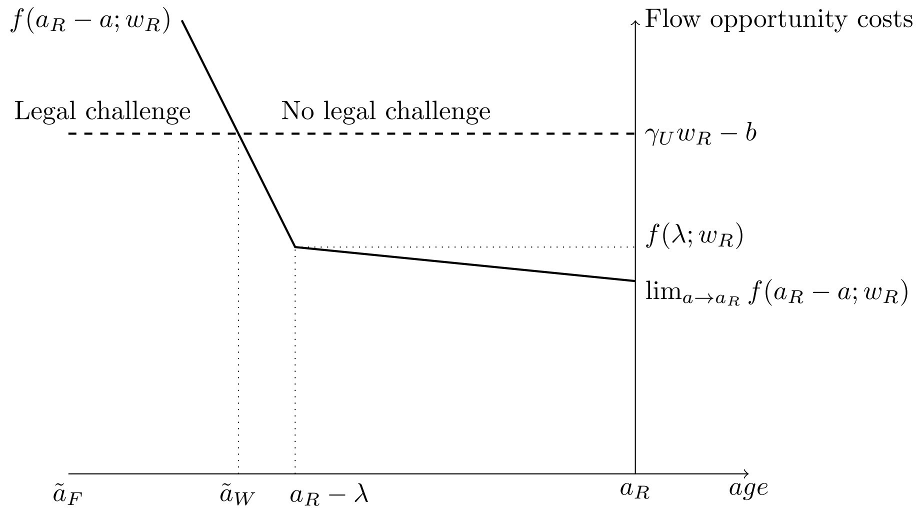


Figure 2: Early exit from employment with binding "no-legal-challenge" condition, $a^* = \tilde{a}_W$

6 Comparative statics

It is now possible to study how the model's predictions respond to changes in the value of various parameters. The main interest is in the role of λ, the potential duration of unemployment benefits.

Proposition 3 *All other things being equal, $\tilde{a}_W$ is: decreasing with λ; decreasing with c_R; increasing with w_R.*

Proof. The first two results are directly illustrated in Figures 3 and 4. Let's consider the third one (Figure 5). When w_R increases by 1 unit, the opportunity cost of being employed at the date of retirement increases by only $\gamma_U < 1$. It follows that the vertical displacement of $f(a_R - a)$ is of a larger magnitude than the vertical displacement of $\gamma_U w_R - b$. Furthermore, a higher w_R involves that the positive change of slope in $a_R - \lambda$ is itself of a bigger magnitude. The abscissa of the intersection is thus closer to $a_R - \lambda$, $\tilde{a}_W$ rises. ■

The strong influence (compared to other parameters) of λ on $\tilde{a}_W$ results from the fact that the *UI* spell immediately follows the dismissal of the worker: the discount factor is close to 1. Additionaly, since the worker, in the initial equilibrium, spends some time receiving *assistance benefits* $b < \gamma_U w(\tilde{a}_W)$, reducing λ (from a given age of exit) makes the subjective cost of the assistance spell higher. To balance these extra costs, the worker is willing to postpone his exit, taking advantage of his rising wages. Previous proposition provides rationales to Baguelin and Remillon's (2014) empirical results: all other things being equal, reducing benefit duration increases the age of older workers dismissed before statutory retirement age. It further clarifies the ambiguous role of wages: higher wages increase the opportunity cost of early exit from employment.

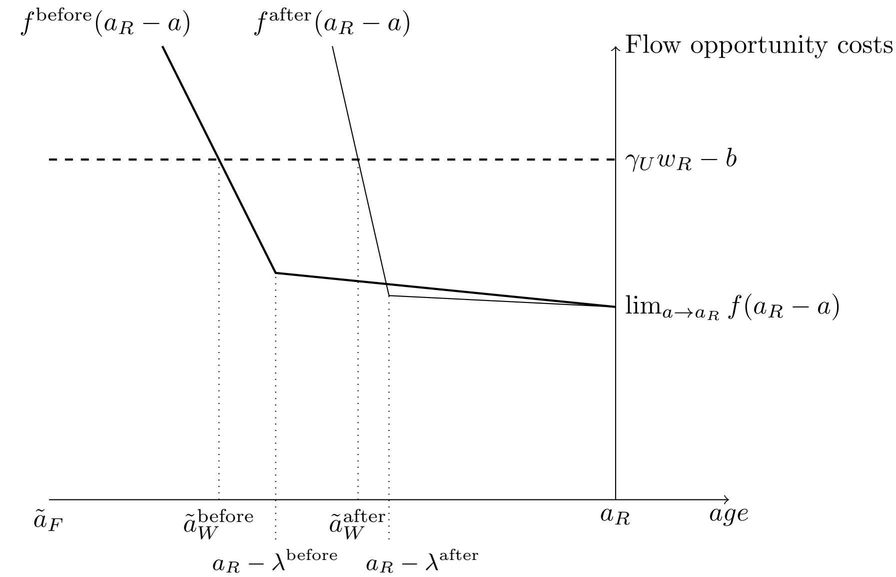


Figure 3: Comparative statics - a decrease in λ

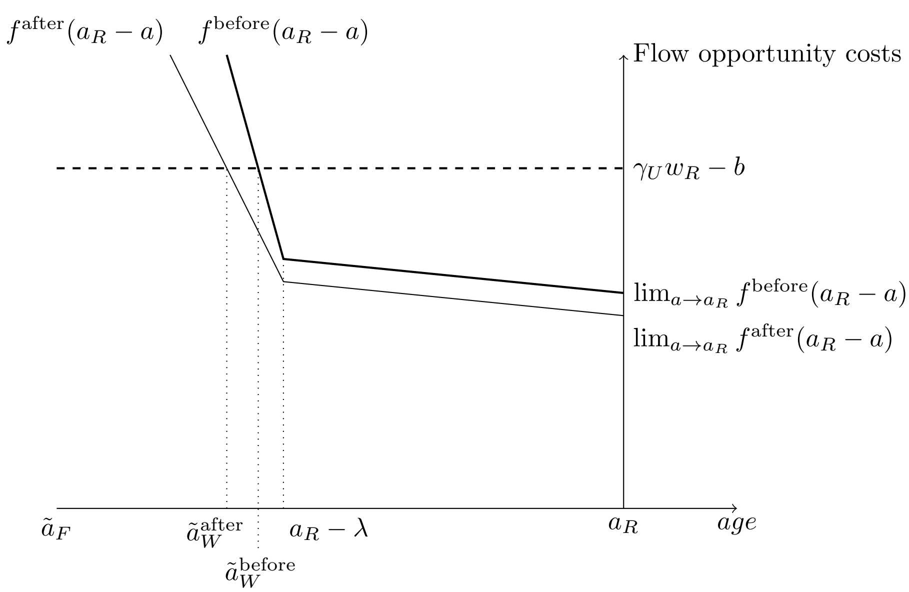


Figure 4: Comparative statics - an increase in c_R

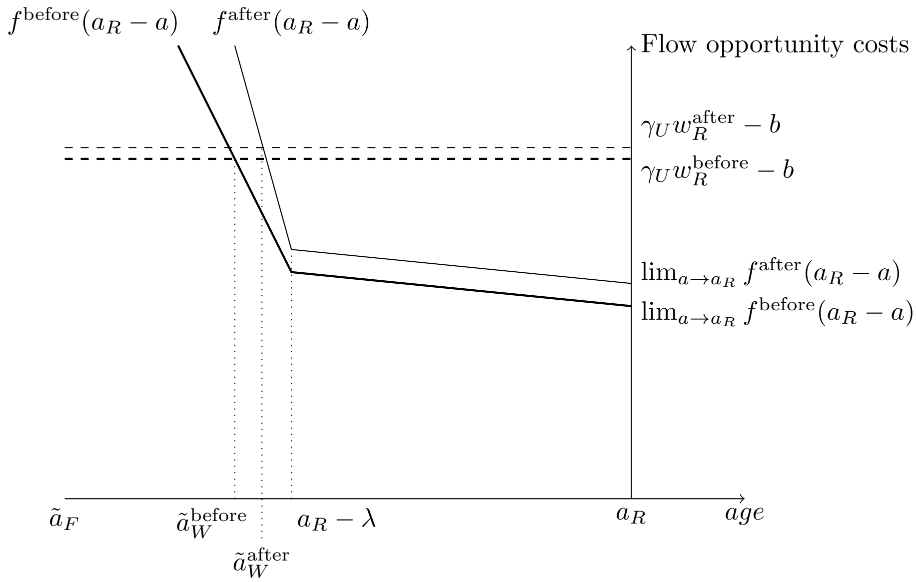


Figure 5: Comparative statics - an increase in w_R

7 Conclusion

Equilibrium and comparative statics results above focus on the case of older workers experiencing a period of unemployment before retiring from the labor force. The analysis is conducted under three main assumptions: jobs held by those workers are less and less profitable as time passes; as they get older, work disutility increases more than salaries; the employment protection is strong. These are reasonable assumptions in most OECD countries - see Boeri and van Ours (2008). Seniority generally increases wages and protection. UI is most often contributive: potential benefit duration depends on employment record, and benefit amounts are proportional to the last wages earned. Since older workers generally have longer employment record and higher wages than the average, this general arrangement is particularly favorable to them. These conditions make early exit from employment acceptable to older workers: within the context of strong employment protection, employers anxious to reduce their payroll find it relevant to concentrate dismissals on their older employees. This behavior is costly to society (through the provision of UI benefits) in particular because chances for those workers to find a new job are low.

References

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Proof 1. Let $x \equiv a_R - a$: the issue is first to study the variations of $f(x)$. Two cases are distinguished: $x \geq \lambda$ and $x < \lambda$.

$$f(x) = \begin{cases} A + \frac{\psi-\phi}{\rho}\left(\frac{x\rho}{1-e^{-x\rho}} - 1\right) + B\frac{xe^{-x\rho}}{1-e^{-x\rho}} + Cx & \text{if } x < \lambda \\ A\frac{1-e^{-x\rho}}{1-e^{-\lambda\rho}} + \frac{\psi-\phi}{\rho}\left(\frac{x\rho}{1-e^{-\lambda\rho}} - \frac{1-e^{-x\rho}}{1-e^{-\lambda\rho}}\right) + B\frac{xe^{-x\rho}}{1-e^{-\lambda\rho}} + Cx & \text{if } x \geq \lambda \end{cases}$$

with $A = w_R - c_R - b$, $B = \left(1 - e^{-R\rho}\right)\gamma_R\phi$ and $C = \gamma_U\phi$. Steps 3 and 4 are devoted to the explicit determination of specific important values.

(Step 1) If $x \geq \lambda$ then

$$\left(1 - e^{-\lambda\rho}\right) f(x) = A \cdot \left(1 - e^{-x\rho}\right) + (\psi - \phi)\left(x - \frac{1}{\rho}\left(1 - e^{-x\rho}\right)\right) + Bxe^{-x\rho} + \left(1 - e^{-\lambda\rho}\right) Cx.$$

Deriving both sides of this equality with respect to x, one gets

$$\left(1 - e^{-\lambda\rho}\right) f'(x) = A\rho e^{-x\rho} + (\psi - \phi)\left(1 - e^{-x\rho}\right) + B \cdot (1 - x\rho) e^{-x\rho} + \left(1 - e^{-\lambda\rho}\right) C.$$

Rearranging the terms, this is rewritten

$$\left(1 - e^{-\lambda\rho}\right) f'(x) e^{x\rho} = [A\rho - (\psi - \phi)] + \left[(\psi - \phi) + \left(1 - e^{-\lambda\rho}\right) C\right] e^{x\rho} + B \cdot (1 - x\rho).$$

From this expression follows that $f'(x) > 0$ if and only if

$$[A\rho - (\psi - \phi)] + \left[(\psi - \phi) + \left(1 - e^{-\lambda\rho}\right) C\right] e^{x\rho} + B \cdot (1 - x\rho) > 0,$$

that is

$$e^{x\rho} + \underbrace{\frac{A\rho - (\psi - \phi)}{(\psi - \phi) + (1 - e^{-\lambda\rho}) C}}_{>-1} > (x\rho - 1)\underbrace{\frac{B}{(\psi - \phi) + (1 - e^{-\lambda\rho}) C}}_{>0}.$$

With correct orders of magnitude for each parameter, this condition generally holds. One can nevertheless provide a sufficient condition for $f'(x) > 0$. We know that a minorant of the left term is given by $e^{x\rho} - 1$. Let θ be defined by

$$\theta \equiv \frac{B}{(\psi - \phi) + (1 - e^{-\lambda\rho}) C}.$$

The highest value of θ such that $(x\rho - 1)\theta$ always remains below $e^{x\rho} - 1$ is calculated. This value, denoted $\hat{\theta}$, solves

$$\begin{cases} (\hat{x} - 1)\hat{\theta} = e^{\hat{x}} - 1 \\ \hat{\theta} = e^{\hat{x}} \end{cases}$$

that is $\ln \hat{\theta} = 1 - \hat{\theta}^{-1}$: $\hat{\theta} \simeq 6.3$. For any set of parameters such that

$$\frac{B}{(\psi - \phi) + (1 - e^{-\lambda\rho}) C} \leq 6.3,$$

for all $x \geq \lambda : f'(x) > 0$ that is the (flow-equivalent) opportunity cost is strictly increasing.

(Step 2) If $x < \lambda$ then

$$f(x) = A - \frac{\psi - \phi}{\rho} + (\psi - \phi)\frac{x}{1 - e^{-x\rho}} + B\frac{xe^{-x\rho}}{1 - e^{-x\rho}} + Cx.$$

Since $\frac{xe^{-x\rho}}{1-e^{-x\rho}} = \frac{x}{1-e^{-x\rho}} - x$:

$$\begin{aligned} f(x) &= A - \frac{\psi - \phi}{\rho} + (\psi - \phi)\frac{x}{1 - e^{-x\rho}} + B \cdot \left(\frac{x}{1 - e^{-x\rho}} - x\right) + Cx, \\ f(x) &= A - \frac{\psi - \phi}{\rho} + \underbrace{(\psi - \phi + B)}_{>0}\frac{x}{1 - e^{-x\rho}} + \underbrace{(C - B)}_{>0}x, \end{aligned}$$

and $\frac{x}{1-e^{-x\rho}}$ is a strictly increasing function of x. It follows that $f(x)$ is itself strictly increasing with x.

(Step 3) The calculation of the limit near 0 relies on the property of exponential functions that $\lim_{x\to 0} \frac{x\exp(-x)}{1-\exp(-x)} = 1$. Indeed

$$\begin{aligned} \lim_{x\to 0} \frac{xe^{-x\rho}}{1 - e^{-x\rho}} &= \lim_{x\to 0}\left(\frac{1}{\rho}\frac{x\rho e^{-x\rho}}{1 - e^{-x\rho}}\right) = \frac{1}{\rho}\lim_{x\to 0}\frac{x\rho e^{-x\rho}}{1 - e^{-x\rho}} = \frac{1}{\rho} \\ \lim_{x\to 0} \frac{x}{1 - e^{-x\rho}} &= \lim_{x\to 0}\left(\frac{xe^{-x\rho}}{1 - e^{-x\rho}} - x\right) = \frac{1}{\rho}\lim_{x\to 0}\frac{x\rho e^{-x\rho}}{1 - e^{-x\rho}} - \lim_{x\to 0} x = \frac{1}{\rho} \end{aligned}$$

from which follows that

$$\lim_{x\to 0} f(x) = w_R - c_R - b + \frac{1 - e^{-R\rho}}{\rho}\gamma_R\phi,$$

with $\frac{1-e^{-R\rho}}{\rho} \in]0; R[$ strictly decreasing with $\rho > 0$. The expression of $f(\lambda)$ is simply

$$f(\lambda) = \left(A - \frac{\psi - \phi}{\rho}\right) + (\psi - \phi + B)\frac{\lambda}{1 - e^{-\lambda\rho}} + (C - B)\lambda.$$

(Step 4) The last step of the proof consists in comparing the slopes of $f(x)$ in the neighborhood of λ. The values to be compared are the following:

$$\begin{aligned} f'_-(\lambda) &= (\psi - \phi)\frac{1 - (1 + \lambda\rho)e^{-\lambda\rho}}{(1 - e^{-\lambda\rho})^2} + B\frac{(1 - \lambda\rho)e^{-\lambda\rho} - (e^{-\lambda\rho})^2}{(1 - e^{-\lambda\rho})^2} + C, \\ f'_+(\lambda) &= A\frac{\rho e^{-\lambda\rho}}{1 - e^{-\lambda\rho}} + (\psi - \phi) + B\frac{(1 - \lambda\rho)e^{-\lambda\rho}}{1 - e^{-\lambda\rho}} + C. \end{aligned}$$

Consequently:

$$f'_{+}(\lambda)-f'_{-}(\lambda)=\left[\begin{array}{c} A\frac{\rho e^{-\lambda\rho}}{1-e^{-\lambda\rho}} \\ +(\psi-\phi)-(\psi-\phi)\frac{1-(1+\lambda\rho)e^{-\lambda\rho}}{(1-e^{-\lambda\rho})^{2}} \\ +B\frac{(1-\lambda\rho)e^{-\lambda\rho}}{1-e^{-\lambda\rho}}-B\frac{(1-\lambda\rho)e^{-\lambda\rho}-\left(e^{-\lambda\rho}\right)^{2}}{(1-e^{-\lambda\rho})^{2}} \end{array}\right],$$

$$f'_{+}(\lambda)-f'_{-}(\lambda)=\left[\begin{array}{c} A\frac{\rho e^{-\lambda\rho}}{1-e^{-\lambda\rho}} \\ +(\psi-\phi)\left(1-\frac{1-(1+\lambda\rho)e^{-\lambda\rho}}{(1-e^{-\lambda\rho})^{2}}\right) \\ +B\cdot\left(\frac{(1-\lambda\rho)e^{-\lambda\rho}}{1-e^{-\lambda\rho}}-\frac{(1-\lambda\rho)e^{-\lambda\rho}-\left(e^{-\lambda\rho}\right)^{2}}{(1-e^{-\lambda\rho})^{2}}\right) \end{array}\right],$$

$$f'_{+}(\lambda)-f'_{-}(\lambda)=\left[\begin{array}{c} A\frac{\rho e^{-\lambda\rho}}{1-e^{-\lambda\rho}} \\ +(\psi-\phi)\left(\frac{\lambda\rho}{1-e^{-\lambda\rho}}-1\right)\frac{e^{-\lambda\rho}}{1-e^{-\lambda\rho}} \\ +B\lambda\rho\frac{e^{-\lambda\rho}}{1-e^{-\lambda\rho}}\frac{e^{-\lambda\rho}}{1-e^{-\lambda\rho}} \end{array}\right]=\left[\begin{array}{c} A\lambda^{-1} \\ +(\psi-\phi)\left(\frac{1}{1-e^{-\lambda\rho}}-\frac{1}{\lambda\rho}\right) \\ +B\frac{e^{-\lambda\rho}}{1-e^{-\lambda\rho}} \end{array}\right]\frac{\lambda\rho e^{-\lambda\rho}}{1-e^{-\lambda\rho}}>0.$$

Taking into account that $B=\left(1-e^{-R\rho}\right)\gamma_R\phi$, this can be rewritten:

$$f'_{+}(\lambda)-f'_{-}(\lambda)=\left[\begin{array}{c} A\lambda^{-1} \\ +(\psi-\phi)\left(\frac{1}{1-e^{-\lambda\rho}}-\frac{1}{\lambda\rho}\right) \\ +\left(\frac{1-e^{-R\rho}}{1-e^{-\lambda\rho}}\right)\gamma_R\phi e^{-\lambda\rho} \end{array}\right]\frac{\lambda\rho e^{-\lambda\rho}}{1-e^{-\lambda\rho}}.$$

For all $\lambda\rho\in\left]0,+\infty\right[\,:\,\frac{1}{1-e^{-\lambda\rho}}-\frac{1}{\lambda\rho}\in\left]\frac{1}{2},1\right[$ strictly increasing; $\frac{\lambda\rho e^{-\lambda\rho}}{1-e^{-\lambda\rho}}\in\left]0,1\right[$ strictly decreasing; $\frac{e^{-\lambda\rho}}{1-e^{-\lambda\rho}}\in\left]0,+\infty\right[$ strictly decreasing. This involves

$$\lim_{\rho\to0}\left\{f'_{+}(\lambda)-f'_{-}(\lambda)\right\}=\lim_{\rho\to0}\left[\begin{array}{c} A\lambda^{-1} \\ +(\psi-\phi)\left(\frac{1}{1-e^{-\lambda\rho}}-\frac{1}{\lambda\rho}\right) \\ +\left(\frac{1-e^{-R\rho}}{1-e^{-\lambda\rho}}\right)e^{-\lambda\rho}\gamma_R\phi \end{array}\right]=A\lambda^{-1}+\frac{1}{2}(\psi-\phi)+\frac{R}{\lambda}\gamma_R\phi,$$

and

$$\lim_{\rho\to+\infty}\left\{f'_{+}(\lambda)-f'_{-}(\lambda)\right\}=\left(A\lambda^{-1}+(\psi-\phi)\right)\lim_{\rho\to+\infty}\frac{\lambda\rho e^{-\lambda\rho}}{1-e^{-\lambda\rho}}=0.$$

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