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Product proliferation to prevent entry: a pedagogical note

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Abstract

In Cabral's (2000) industrial organization textbook, a simple model of strategic product proliferation is sketched out, in which an established firm – which would offer a homogeneous product, absent the threat of entry – launches two product varieties in order to deter entry, provided the fixed component of the cost function falls within a certain range. Among other simplifying assumptions, Cabral's model restricts the potential entrant to offering just one product variety. In this pedagogical note we allow for multiple varieties also on the part of the potential entrant and solve the two-stage location game for any level of the fixed cost. This analysis will permit a deeper understanding of the relationship between the degree of scale economies and the intensity of product proliferation to deter entry.

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1 Introduction

Entry deterrence refers to any conduct whereby an established firm prevents potential competitors from entering the market. Industrial organization textbooks provide stylized models showing how this conduct may pertain to dimensions such as capacity building, advertising, R&D, and the array of product varieties to offer. For example, a simple model of strategic product proliferation is sketched out in Cabral (2000, pp. 265-267). The model considers an industry whose output need not be homogeneous in that different product varieties may differ in terms of a single characteristics. Production of each variety exhibits economies of scale, due to a fixed component in the cost function. In the face of the varieties chosen by the established firm (incumbent), a potential competitor chooses whether to enter the industry, in which case he is assumed to offer just one variety. As in Schmalensee's seminal work (1978), the price is taken to be fixed. A well defined range is found for the fixed cost which makes it most profitable for the incumbent to offer two varieties in order to deter entry whereas, absent the threat of entry, he would offer just one variety.

One question I have often been asked when lecturing on this topic is how the location choice made by the incumbent would change if the fixed cost went below the aforementioned range. Another issue is whether it would make a difference for the results if the entrant too were allowed to offer multiple varieties.[1] Addressing these two points is the aim of this pedagogical note.

The paper is organized as follows. Section 2 reviews Cabral's model. In Section 3 we allow the entrant to offer multiple varieties and consider how the equilibrium of the two-stage location game depends on the size of the fixed cost. Quite interestingly, the extent of strategic product proliferation turns out to be the same as when the potential entrant may produce just one variety. And, most importantly, the analysis will deliver the main message for students of an industrial organization course: the density of the incumbent's varieties in the output space will tend to be the higher as the degree of scale economies is the lower.

[1] According to Cabral, "the results are qualitatively similar if we consider multiple varieties" (2000, p. 266) on the part of the entrant.

2 Cabral's model

In Cabral's model, the industry output, say, ready-to-eat (RTE) breakfast cereals,[2] need not be homogeneous in that different product varieties may differ in terms of a single characteristics, say, sweetness. The space of this characteristics is the interval $[0, 1]$. A large number of consumers are uniformly distributed over this interval in terms of their preference: therefore, the consumers whose most preferred grade of sweetness is less than x are a fraction x of the total.

A two-stage location game of perfect information is analyzed. There are two players, the "incumbent" and the "potential entrant" (hereinafter, firms I and E, respectively). In the first stage firm I chooses which varieties to offer; in the knowledge of firm I's choice, in the second stage firm E chooses whether to enter and with which varieties. The price is assumed to be fixed at $\overline{p}$, independently of how many producers are active and the varieties on offer. Demand by each consumer is positive, no matter how far his location from the closest variety on offer; total industry demand is normalized to one. The long-run production cost of any variety is assumed to be the constant F, no matter the output level q of that variety. Thus there are economies of scale, average cost being $AC(q) = F/q$ for each variety.

It will be convenient to assume that, whenever there are two or more most profitable alternatives, the one entailing the largest output is chosen. As an implication, if the maximum profit from being active is zero, the firm chooses to be active rather than not.

If a pure monopolist, firm I would offer just one variety, thereby obtaining profits of $\overline{p} - F$, since additional varieties would only raise cost. We let

$$F \leq \overline{p} \tag{1}$$

in order for the industry to be viable for a a pure monopolist.

Further notation will be helpful. Profits are denoted by Π_I and Π_E for firms I and E, respectively. Any variety i offered by firm I is denoted by its location $l_{I,i} \in [0, 1]$. Firm I's choice is a location vector, $\mathbf{n_I} = (l_{I,1}, ..., l_{I,n_I})$, where n_I is the number of varieties offered. Without loss of generality, we

[2] According to Schmalensee (1978), strategic brand proliferation might explain the absence of substantial entry which characterized the industry of RTE breakfast cereals over almost three decades since 1940, in spite of the rapid growth of demand, the high profitability and the small degree of scale economies.

let $l_{I,i+1} > l_{I,i}$ (each $i \geq 1$). We similarly denote E's location choice by

$\mathbf{n_E} = (l_{E,1}, ..., l_{E,n_E})$, where $l_{E,i} \in [0,1]$ (each $i = 1, ..., n_E$) and n_E is the number of varieties offered. In the following, $l_{E,j} = l_{I,i}^-$ ($l_{E,j} = l_{I,i}^+$) means that variety $l_{E,j}$ is located to the left (to the right) and arbitrarily close to variety $l_{I,i}$.

In Cabral's model, firm E is allowed to produce a single variety. As for firm I, the following two options are compared: to produce a single variety or the pair of varieties $(\frac{1}{4}, \frac{3}{4})$. This pair effectively prevents entry if $F > \frac{\overline{p}}{4}$. In fact, firm E's maximum output is then $\frac{1}{4}$, obtained by locating in the interval $[\frac{1}{4}, \frac{3}{4}]$. Thus, E's maximum profit if entering is $\frac{\overline{p}}{4} - F$, which is negative if $F > \frac{\overline{p}}{4}$. Thus, holding this condition, E does not enter and $\Pi_I = \overline{p} - 2F$, which is non-negative if $F \leq \frac{\overline{p}}{2}$. Alternatively, firm I might consider offering

a single variety: then it would locate at $l_{I,1} = \frac{1}{2}$, which minimizes firm E's maximum output if entering, equal to $\frac{1}{2}$ and obtained by locating at $l_{E,1} = \frac{1}{2}$ on the product line. This leads to $\Pi_I = \Pi_E = \frac{\overline{p}}{2} - F$. If $F \leq \frac{\overline{p}}{2}$, this is nonnegative and hence E enters the market; at the same time, I ends up with profits lower than (or at least not higher) than $\overline{p} - 2F$, which is its profits (holding $F > \frac{\overline{p}}{4}$) with the vector of locations $(\frac{1}{4}, \frac{3}{4})$. To conclude, the incumbent offers a pair of varieties to prevent entry if

$$F \in \left(\frac{\overline{p}}{4}, \frac{\overline{p}}{2}\right]. \tag{2}$$

More formally, holding (2), it is part of a subgame perfect equilibrium for firm I to offer $\mathbf{n_I} = (\frac{1}{4}, \frac{3}{4})$, which prevents firm E from entering the market. This illustrates strategic product proliferation: I would offer just one variety (for example, variety $\frac{1}{2}$) if there were no threat of entry or $F \in \left(\frac{\overline{p}}{2}, \overline{p}\right]$.

3 Allowing multiple varieties for the entrant

Here we extend Cabral's model by considering any value of $F \in (0, \overline{p})$ and, most importantly, by allowing the production of multiple varieties by firm E. This will further clarify the relationship between product proliferation and the degree of scale economies.[3]

[3]Unlike our model, Bonanno (1987) and Neven *et al.* (1989) analyze entry deterrence in a location model in which the price is endogenously determined; at the same time,

Further notation is needed: $q_E(n_E \mid \mathbf{n_I})$ and $\Pi_E(n_E \mid \mathbf{n_I}) = \overline{p}q_E(n_E \mid \mathbf{n_I}) - n_E F$ denote, respectively, the maximum output and profit obtained by E with n_E varieties, conditional on $\mathbf{n_I}$; $\Pi_I(\mathbf{n_I})$ denotes I's profit with $\mathbf{n_I}$.

As a preliminary step, it must be understood how the density of I's varieties on the product line will affect E's profit opportunities. To start with, we define $\mathbf{n_I^*}$-location vectors.[4]

Definition. *For any number of varieties n_I, let $\mathbf{n_I^*} := (l^*_{I,1}, ..., l^*_{I,n_I})$ be the location vector such that:*

$$l^*_{I,i+1} - l^*_{I,i} = 2l^*_{I,1}, \qquad (i = 1, ..., n_I - 1) \tag{3}$$

$$l^*_{I,n_I} = 1 - l^*_{I,1}. \tag{4}$$

In words: the interval between any two subsequent locations is twice as large as the interval $[0, l^*_{I,1}]$ and the interval $[l^*_{I,n_I}, 1]$. Side summation over the equations (3) yields $l^*_{I,n_I} - l^*_{I,1} = 2(n_I - 1)l^*_{I,1}$; along with (4), this leads to $l^*_{I,1} = 1 - l^*_{I,n_I} = \frac{1}{2n_I}$ and $l^*_{I,i+1} - l^*_{I,i} = \frac{1}{n_I}$ (each $i = 1, ..., n_I - 1$). Vectors of locations $\mathbf{n_I^*}$ are $(\frac{1}{4}, \frac{3}{4})$, $(\frac{1}{6}, \frac{1}{2}, \frac{5}{6})$, $(\frac{1}{8}, \frac{3}{8}, \frac{5}{8}, \frac{7}{8})$, and so on and so forth.

It will be shown that

$$q_E(n_E \mid \mathbf{n_I^*}) = \frac{n_E}{2n_I} \qquad \textit{for } n_E = 1, ..., 2n_I. \tag{5}$$

For example, $q_E(n_E = 1 \mid \mathbf{n_I^*}) = \frac{1}{2n_I}$, obtained with $l_{E,1} \in [l^*_{I,i}, l^*_{I,i+1}]$ $(1 \leq i \leq n_I - 1)$.[5] With $n_E = n_I$, E's output is maximal with $\mathbf{n_E} = \mathbf{n_I^*}$: if all of firm I's varieties are matched, then one may take E's output to be $\frac{1}{2}$, which coheres with (5). As n_E increases above n_I, E's output keeps on increasing at the rate $\frac{1}{2n_I}$, so long as $n_E < 2n_I$. In fact, with $n_E = n_I + h$ (any $h \leq n_I$), firm E would closely "surround" h of I's varieties to maximize its sales. For example, with $n_E = n_I + 1$, E might choose $\mathbf{n_E} = (l_{E,1}, l_{E,2}, ..., l_{E,n_E}) = (l^{*-}_{I,1}, l^{*+}_{I,1}, l^*_{I,2}, ..., l^*_{I,n_I})$: then $l_{E,1}$ and $l_{E,2}$ capture a market share of $\frac{1}{2n_I}$ each,[6] and the

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however, the potential entrant is allowed to produce just one variety.

[4]These location vectors are discussed by Neven (1985), Bonanno (1987) and Neven *et al.* (1989) in a context where the price is endogenously determined.

[5]Or with $l_{E,1} = l^{*-}_{I,1}$ or $l_{E,1} = l^{*+}_{I,n_I}$.

[6]Variety $l_{I,1}$ is completely displaced: consumers in the interval $[0, \frac{1}{2n_I})$ will purchase variety $l_{E,1}$ and consumers in the interval $(\frac{1}{2n_I}, \frac{2}{2n_I})$ will purchase variety $l_{E,2}$.

same holds for the remaining $n_E - 2 = n_I - 1$ varieties; therefore, $q_E(n_E = n_I + 1 \mid \mathbf{n_I^*}) = \frac{n_I+1}{2n_I} = \frac{n_E}{2n_I}$. This reasoning extends to any $n_E = n_I + 1, ..., 2n_I$. For instance, with $n_E = 2n_I$, E will choose $\mathbf{n_E} = (l^{*-}_{I,1}, l^{*+}_{I,1}, ..., l^{*-}_{I,n_I}, l^{*+}_{I,n_I})$, thereby serving the whole market, again coherently with (5).

Consequently,

$$\Pi_E(n_E \mid \mathbf{n_I^*}) = \frac{n_E}{2n_I}\overline{p} - n_E F = n_E\left(\frac{\overline{p}}{2n_I} - F\right) \quad \textit{for } n_E = 1, ..., 2n_I. \tag{6}$$

Thus $\Pi_E(n_E \mid \mathbf{n_I^*})$ is proportional to n_E: as a consequence, if it does not pay to enter with one variety, which leads to profits of $\frac{\overline{p}}{2n_I} - F$, *a fortiori* it does not pay to offer several varieties; if, instead, entering with one variety is profitable, then E will offer twice as many varieties as the incumbent.

Among the $\mathbf{n_I}$-location vectors corresponding to any number n_I of varieties, $\mathbf{n_I^*}$ minimizes the output E may sell with any number of varieties n_E: more specifically,

$$q_E(n_E \mid \mathbf{n_I} \neq \mathbf{n_I^*}) > \frac{n_E}{2n_I} \qquad \textit{for } n_E = 1, ..., 2n_I - 1 \tag{7}$$

$$q_E(n_E \mid \mathbf{n_I} \neq \mathbf{n_I^*}) = \frac{n_E}{2n_I} = 1 \quad \textit{for } n_E = 2n_I. \tag{8}$$

In fact, for any vector $\mathbf{n_I} \neq \mathbf{n_I^*}$, $l_{I,1} > \frac{1}{2n_I}$ or $l_{I,n_I} < 1 - \frac{1}{2n_I}$ or $l_{I,i+1} - l_{I,i} > \frac{1}{n_I}$ (some $i = 1, ..., n_I - 1$): as a consequence, $q_E(n_E = 1 \mid \mathbf{n_I} \neq \mathbf{n_I^*}) > \frac{1}{2n_I}$. It is similarly $q_E(n_E \mid \mathbf{n_I} \neq \mathbf{n_I^*}) > \frac{n_E}{2n_I}$ for any $n_E \leq 2n_I - 1$. By way of contradiction, denote by n_E° the minimum $n_E \in \{2, ..., 2n_I - 1\}$ such that $q_E(n_E \mid \mathbf{n_I} \neq \mathbf{n_I^*}) \leq \frac{n_E}{2n_I}$: *i.e.*, $q_E(n_E^\circ \mid \mathbf{n_I} \neq \mathbf{n_I^*}) \leq \frac{n_E^\circ}{2n_I}$ while $q_E(n_E \mid \mathbf{n_I} \neq \mathbf{n_I^*}) > \frac{n_E}{2n_I}$ (any $n_E < n_E^\circ$). Hence $q_E(n_E^\circ \mid \mathbf{n_I} \neq \mathbf{n_I^*}) - q_E(n_E^\circ - 1 \mid \mathbf{n_I} \neq \mathbf{n_I^*}) < \frac{1}{2n_I}$. At the same time, since $q_E(n_E \mid \mathbf{n_I} \neq \mathbf{n_I^*}) = \frac{n_E}{2n_I}$ for $n_E = 2n_I$, there exists $n_E^{\circ\circ} \in \{n_E^\circ + 1, ..., 2n_I - 1\}$ such that $q_E(n_E^{\circ\circ} + 1 \mid \mathbf{n_I} \neq \mathbf{n_I^*}) \geq \frac{n_E^{\circ\circ}+1}{2n_I}$ and $q_E(n_E^{\circ\circ} \mid \mathbf{n_I} \neq \mathbf{n_I^*}) \leq \frac{n_E^{\circ\circ}}{2n_I}$. Hence, the variety introduced when moving from $n_E^{\circ\circ}$ to $n_E^{\circ\circ} + 1$ varieties will raise E's output by no less that $\frac{1}{2n_I}$, revealing that the variety introduced when moving from $n_E^\circ - 1$ to n_E° varieties was inappropriately located.

It follows that a specified number of varieties n_I can prevent entry only if $\Pi_E(n_E \mid \mathbf{n_I^*}) < 0$, *i.e.*, only if $\frac{\overline{p}}{2n_I} - F < 0$. In fact, if $\frac{\overline{p}}{2n_I} - F \geq 0$ so that $\Pi_E(n_E \mid \mathbf{n_I^*}) \geq 0$, then $\Pi_E(n_E \mid \mathbf{n_I} \neq \mathbf{n_I^*}) > 0$ because of (7).

3.1 Solving the game for any $\mathbf{F<\overline{p}}$

We denote by $\underline{\mathbf{n}}_\mathbf{I}^*$ the $\mathbf{n}_\mathbf{I}^*$-location vector with a number of varieties, $\underline{n}_I$, equal to the integer such that

$$\frac{\overline{p}}{2\underline{n}_I} < F \leq \frac{\overline{p}}{2(\underline{n}_I - 1)}. \tag{9}$$

Note, first of all, that $\Pi_E(n_E \mid \underline{\mathbf{n}}_\mathbf{I}^*) = n_E\left(\frac{\overline{p}}{2\underline{n}_I} - F\right) < 0$ for $n_E = 1, ..., 2\underline{n}_I$: thus E does not enter and hence $\Pi_I(\underline{\mathbf{n}}_\mathbf{I}^*) = \overline{p} - \underline{n}_I F$. To check that $\Pi_I(\underline{\mathbf{n}}_\mathbf{I}^*) \geq 0$, note that, for any $\underline{n}_I \geq 2$, inequality $F \leq \frac{\overline{p}}{2(\underline{n}_I - 1)}$ leads to $\overline{p} - \underline{n}_I F \geq (\underline{n}_I - 2)F \geq 0$.[7] Secondly, $\Pi_I(\mathbf{n}_\mathbf{I}) \leq \Pi_I(\underline{\mathbf{n}}_\mathbf{I}^*)$ for any $\mathbf{n}_\mathbf{I} \neq \underline{\mathbf{n}}_\mathbf{I}^*$. Any $\mathbf{n}_\mathbf{I}$ with $n_I > \underline{n}_I$ would reduce I's profits by increasing costs. Any $\mathbf{n}_\mathbf{I} \neq \underline{\mathbf{n}}_\mathbf{I}^*$ with $n_I = \underline{n}_I$ is either effective in preventing entry - hence leading to $\Pi_I(\mathbf{n}_\mathbf{I}) = \overline{p} - \underline{n}_I F$, just as $\underline{\mathbf{n}}_\mathbf{I}^*$ - or else invites entry, in this case obviously leading to lower profits $(1-\alpha)\overline{p} - \underline{n}_I F$ (α denoting the market share captured by firm E). Any $\mathbf{n}_\mathbf{I}$ with $n_I < \underline{n}_I$ would definitely invite entry since then $\Pi_E(n_E \mid \mathbf{n}_\mathbf{I}) \geq n_E\left(\frac{\overline{p}}{2(\underline{n}_I - 1)} - F\right) \geq 0$, due to (6), (7), and (9). This in turn results in lower (or at least not higher) profits for I. The last point is immediately proven for $\mathbf{n}_\mathbf{I}^*$-location vectors: taking into account (9) and (6), firm E will introduce $n_E = 2n_I$ varieties, thereby completely displacing firm I. Turn next to vectors of locations $\mathbf{n}_\mathbf{I} \neq \mathbf{n}_\mathbf{I}^*$ with $n_I < \underline{n}_I$ and such that E finds it most profitable not to completely displace I. Again, $\Pi_I(\mathbf{n}_\mathbf{I}) \leq \overline{p} - \underline{n}_I F$. To see this, suppose first that E's best response is some $\mathbf{n}_\mathbf{E}$ with $n_E < n_I$. Denoting E's resulting output by α, this means that $\alpha\overline{p} - n_E F \geq \frac{\overline{p}}{2} - n_I F$ (the right-hand side being E's profits with $\mathbf{n}_\mathbf{E} = \mathbf{n}_\mathbf{I}$). Then $\Pi_I(\mathbf{n}_\mathbf{I}) = (1-\alpha)\overline{p} - n_I F \leq \overline{p} - \underline{n}_I F$: this follows from $\alpha\overline{p} \geq n_E F + \frac{\overline{p}}{2} - n_I F$ and the fact that $F \leq \frac{\overline{p}}{2(\underline{n}_I - n_E)}$ due to (9). Suppose next that $n_I = 1$ or that $\Pi_E(n_E = n_I \mid \mathbf{n}_\mathbf{I}) > \Pi_E(n_E < n_I \mid \mathbf{n}_\mathbf{I})$. Then, $\Pi_I(\mathbf{n}_\mathbf{I}) \leq \frac{\overline{p}}{2} - n_I F$,[8] where $\frac{\overline{p}}{2} - n_I F \leq \overline{p} - \underline{n}_I F$ since $F \leq \frac{\overline{p}}{2(\underline{n}_I - n_I)}$, again as a consequence of (9).

By now, we have reached the following result.

Proposition 1 *At any subgame perfect equilibrium of the game, firm I offers* $\underline{n}_I$ *varieties and* $\Pi_I = \overline{p} - \underline{n}_I F$*; firm E does not enter on the equilibrium path. In one such equilibrium firm I chooses precisely the* $\underline{\mathbf{n}}_\mathbf{I}^*$*-location vector.*

[7] If $\underline{n}_I = 1$, firm I's profit is $\overline{p} - F \geq 0$ because of (1).

[8] If E's best response is some $\mathbf{n}_\mathbf{E}$ such that $n_E > n_I$, then $\Pi_I(\mathbf{n}_\mathbf{I}) < \frac{\overline{p}}{2} - n_I F$ since E's market share will be higher than 1/2.

It is checked from (9) that $\underline{n}_I$ tends to increase as F decreases. Firm I will offer one variety if $F \in (\frac{\overline{p}}{2}, \overline{p}]$: this is the case of "blockaded entry", in which firm I behaves like a pure monopolist. Strategic product proliferation obtains when $F \leq \frac{\overline{p}}{2}$: firm I offers two varieties if $F \in (\frac{\overline{p}}{4}, \frac{\overline{p}}{2}]$, three varieties if $F \in (\frac{\overline{p}}{6}, \frac{\overline{p}}{4}]$, four varieties if $F \in (\frac{\overline{p}}{8}, \frac{\overline{p}}{6}]$; and so on and so forth.

Finally, by reviewing the argument in this section, one can easily check that Proposition 1 would hold even if firm E were allowed to produce just one variety.

4 Concluding remarks

Our extension of Cabral's model sheds further light on the relationship between product proliferation and the degree of scale economies. In this connection it is worth to reformulate the results in a slightly different way. One way of measuring the degree of scale economies is in terms of the firm minimum efficient size: the degree of scale economies is the greater as the minimum efficient size is the greater. As in Cabral (2000, p. 244), let us define the minimum efficient size, call it $\widehat{q}$, as the level of output to be crossed in order for average cost $AC(q) = \frac{F}{q}$ to fall below some critical level: clearly, the degree of scale economies is increasing in F. Next, take this critical level to be equal to the exogenously given price: then $\widehat{q} = \frac{F}{\overline{p}}$. We have seen that, faced with an $\mathbf{n}_\mathbf{I}^*$-location vector, firm E's output with one variety is $\frac{1}{2n_I}$. The strategy of entry deterrence amounts to populating the output space with sufficiently many (and properly located) varieties so as to reduce below the minimum efficient output what the potential entrant may sell with a single variety; namely, $\underline{n}_I$ is the minimum n_I such that $\frac{1}{2n_I} < \widehat{q} = \frac{F}{\overline{p}}$. Therefore, product proliferation will be the higher as the degree of scale economies is the lower.

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