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Majority Rule and Coalitional Stability

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Abstract

In this note we study the centralization vs. decentralization issue for the management of a given collective activity. The aim is to characterize a class of decision rules that guarantees the stability of global cooperation (i.e centralization) against the incentive of coalitions of citizens to opt-out, towards forms of decentralized organizations. We show that a simple majority rule required to break global cooperation guarantees the existence of core-stable allocations, independently of the expected behaviour of individuals in the minority. We also show that if majorities can extract resources from minorities, stability may require a supermajority rule, whose threshold is increasing in its extraction power.

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1. Introduction

In a number of political and economic situations, individuals have to decide whether certain activities will be managed centrally, by the entire collectivity, or rather locally, by groups or communities of citizens. Such problems typically combine a *political* and a *strategic* dimension. The rules by which coalitions can propose and object to collective agreements are set by constitution or by some other laws and regulations. At the same time, the way in which groups and communities interact, should global cooperation fail, largely depends on the widespread externalities stemming from the decisions taken at community level on matters as, for instance, health, education, taxation or pollution control.

In this note we study cooperative decision problems in which these two dimensions, political and strategic, coexist. The aim is to characterize a class of decision rules that guarantees the stability of global cooperation (or *centralization*) in the management of the activities against the incentive of coalitions of citizens to opt-out, towards *decentralized* organizations of such activities.[1] We model the interaction of individuals under decentralization as a game in strategic form, in which players can be organized in coalition structures that implicitly define the extent of local cooperation. More precisely, players cooperate within coalitions, and compete across coalitions. This is the framework of coalitional games with externalities, studied, for instance, in Bloch (1996), Yi (1997), Ray and Vohra (1997), Maskin (2003), Hafalir (2007), and surveyed in Bloch (2003), Yi (2003), Ray (2007) and Marini (2009). In objecting to a centralized cooperative allocation, a coalition anticipates the equilibrium in which it will autonomously decide on the actions of its members, exposed to the externalities coming from the actions of outside coalitions. For this reason, as well acknowledged by the above literature, the profitability of any objection crucially depends on what a coalition expects on the reaction of nonmembers. In this context, the rules of political decision-making matter in the following sense: decisions that attain to the decentralization of certain goods and services are taken through the rules and procedures of the political game, and violating such rules would imply prohibitively large costs, such as the secession from the central State or the exclusion from nation-wide public goods.

We find that when the underlying strategic form game satisfies strong symmetry properties, if the decision about the centralization or decentralization of a given activity is taken by the majority of players, this always guarantees the existence of core-stable allocations (and, hence, of global cooperation) independently of the assumptions on the behaviour of individuals in the minority. We also show that if majorities can extract resources from the minorities, stability requires a supermajority rule, whose threshold is increasing in its extraction power. At the end of the paper we provide examples of widely studied instance of cooperative problems in which our symmetry and regularity conditions hold, and in which the required supermajority for stability is explicitly computed.

2. Setup and Notation

2.1. **The Strategic Form Game.** Let $G = (N, (X, u_i)_{i\in N})$ be a game in strategic form, with finite set players $N = \{1, 2, ..., n\}$, strategy set X_i and payoff function $u_i : X_1 \times .. \times X_n \to \mathbb{R}_+$ for each $i \in N$. We assume that G is *symmetric and monotone* in the following sense:

[1] In this respect our contribution differs from the recent literature trying to characterize endogenously self-stable (Barbera and Jackson, 2004) or dynamically stable (Acemoglu, *et al*, 2012) constitutional rules.

A.1 *(Symmetry)*. $X_i = X$ for all $i \in N$. Moreover, for all $x \in X^n$ and all permutations $p : N \to N$: $u_{p(i)}\left(x_{p(1)}, ..., x_{p(n)}\right) = u_i\left(x_1, ..., x_n\right)$.

A.2 *(Monotone Externalities)*. One of the following two cases must hold:

(1) *Positive Externalities* (PE): $u_i(x_i, x_{-i})$ increasing in x_{-i} for all $x_i \in X$ and all $i \in N$;
(2) *Negative Externalities* (NE): $u_i(x_i, x_{-i})$ decreasing in x_{-i} for all $x_i \in X$ and all $i \in N$.

2.2. **Coalitions and Coalitional Worth.** A partition $\Pi = (S_1, S_2, .., S_j, .., S_m)$ of N describes the cooperation patterns in the game G. Let s_j denote the cardinality of S_j for all $j = 1, 2, .., m$. Players belonging to the same coalition in Π are assumed to cooperate to achieve their maximal joint payoff. Across coalitions agents set strategies noncooperatively (see, for instance, Ichiishi, 1981). Formally, for each partition $\Pi = (S_1, S_2, ..., S_j, ..., S_m)$ we define the game $G(\Pi)$ with player set $\{1, 2, ...j, .., m\}$, each with strategy set $X_j = X_{S_j}$ and payoff function $U_j = \sum_{i \in S_j} u_i$. Note that the way in which we define the game $G(\Pi)$ implies that our symmetry assumption holds both within and across coalitions in $G(\Pi)$ (compare this with the weaker assumption adopted, for instance, in Yi 1997).

Under suitable assumptions on payoff functions, best-replies in $G(\Pi)$ are such that all members within a coalition play the same strategy (see Currarini and Marini, 2006).[2] When $G(\Pi)$ possesses a unique Nash equilibrium payoffs unambiguously define the worth of all coalitions in Π. Note that in the present setting, the partition formed by the grand coalition N always generates the maximal aggregate payoff, and is, in this sense, *efficient*.

2.3. **Core Stability.** We will be interested in efficient outcomes of the game G that are stable against objection by subcoalitions of the set N. Because of the presence of externalities, what a coalition obtains by objecting to a proposed allocation depends on what partition emerges in response to the objection. Specific cases include the *gamma* assumption, where players in $N \backslash S$ split up into singletons, the *delta* assumption, where remaining players merge into the coalition $N \backslash S$, and the *rational* assumption, where remaining players re-organize in the partition of the set $N \backslash S$ that guarantees them the highest aggregate payoff. Let then $\Pi(S)$ denote the partition that is associated with the formation of coalition S. For instance, under the δ assumption, $\Pi(S) = \{S, N \backslash S\}$, while under the γ assumption $\Pi(S) = \left\{S, \{j\}_{j \in N \backslash S}\right\}$. Given $\Pi(S)$, the worth of S, denoted by $v(S)$ is identified by the aggregate payoff of S in the game $G(\Pi(S))$. The function v, together with the players' set N, defines the characteristic function game (N, v) associated with the underlying game G in strategic form.

Definition 1. *The core of the characteristic function game* (N, v) *consists of all efficient allocations* $u \in R_+^n$ *such that* $\sum_{i \in S} u_i \leq v(S)$ *for all* $S \subset N$.

Variants of the core solution concept can be derived by restricting the blocking power to only certain types of subsets of the players' set N. Greenber and Weber (1981), Demange (2004) and Currarini (2007) have considered exogenous restrictions based on partial orderings of players in N, possibly representing the organizational structure through which efficient allocations are achieved. Here, being concerned with collective decision making, we specifically consider situations in which only coalitions encompassing at least a given percentage $\beta(n)$ of the set of players are allowed to propose or object to given allocations.

[2] In Currarini and Marini (2006) it is shown that either under increasing differences or strict quasiconcavity of players' payoff the members of a coalition plays the same equilibrium strategy in $G(\Pi)$.

We will denote by $\mathcal{M}^{\beta}(N)$ the set of such coalitions. When $\beta(n) = (n/2)/n$ for n even and $\beta(n) = [(n+1)/2]/n$ for n odd, this reduces to the simple (weak) majority rule and, for brevity, we will use the notation $\mathcal{M}(N)$; when the value required for $\beta(n)$ is higher than the above, we will generally use the term *supermajority*.

Definition 2. *The $\mathcal{M}^{\beta}$-core of the characteristic function game (N, v) consists of all efficient allocations $u \in R^n_+$ such that $\sum_{i \in S} u_i \leq v(S)$ for all $S \subset N$ and $s > \beta(n) \cdot n$.*

3. Majorities and Core Stability

We start by recording an important property of the class of games considered here, that was proved in Currarini and Marini (2006). We denote by u_S the per capita payoff for members of coalition S, that is, $u_S = U_S/s$. For any partition Π such that $S_j \in \Pi$ and $S_k \in \Pi$, we also denote by $x_j(x_k)$ the efficient choice of members of S_j as a function of the choice of members of S_k, keeping all strategies played by other coalitions in Π fixed (these are omitted for ease of notation).

Definition 3. *The game $G(\Pi)$ satisfies the "contraction property" if for each two coalitions $S_j, S_k \in \Pi$ such that $s_j = s_k$, the function $x_j(x_k)$ is a Banach contraction mapping.*

We also introduce the well-known properties of increasing and decreasing differences of payoff functions.

Definition 4. *The function u_i has increasing differences if for any $x_i, x_i' \in X$ such that $x_i < x_i'$ and for any $x_{-i}, x_{-i}' \in X_{-i}$ such that $x_{-i} < x_{-i}'$:*

$$u_i(x_i', x_{-i}') - u_i(x_i, x_{-i}') > u_i(x_i, x_{-i}) - u_i(x_i, x_{-i})$$

Definition 5. *The function u_i has decreasing differences if for any $x_i, x_i' \in X$ such that $x_i < x_i'$ and for any $x_{-i}, x_{-i}' \in X_{-i}$ such that $x_{-i} < x_{-i}'$:*

$$u_i(x_i', x_{-i}') - u_i(x_i, x_{-i}') < u_i(x_i, x_{-i}) - u_i(x_i, x_{-i}).$$

Lemma 1. *Let G be a symmetric monotonic game. Let also payoffs display either increasing or decreasing differences and the contraction property. For all partition Π and coalitions S, T in Π such that $t < s$, the Nash equilibrium $x(\Pi)$ of the game $G(\Pi)$ satisfies $u_S(x(\Pi)) < u_T(x(\Pi))$.*

Under the *gamma* assumption, Lemma 1 immediately implies that the efficient equal-split allocation in G belongs to the core of (N, v). In fact, any objecting coalition S would face smaller coalitions in the induced partition $\Pi(S)$, whose members are better of than the members of S by Lemma 1, contradicting efficiency of the equal-split allocation in the first place. In contrast, under the *delta* assumption this argument does not apply to minority coalitions, who face a larger (complement) coalition in partition Π induced by the objection of S. The core under the *delta* assumption may in fact be empty even under the assumptions of Lemma 1. The next Proposition shows that core-stability is guaranteed by the simple majority rule independently of the assumption that is used to originate the partition $\Pi(S)$ for a generic objecting coalition S.

Proposition 1. *Let G be a symmetric monotonic game. Let also payoffs display either increasing differences or decreasing differences and the contraction property. Then, the $\mathcal{M}$-core of the game (N, v) is nonempty and global cooperation is, therefore, stable.*

Proof. By definition, if S is a majority, every coalition $T \neq S$ in $T \in \Pi(S)$ is such that $t < s$. Suppose then that S objects to the equal split efficient allocation assigning to each $i \in N$ the payoff $u^e \equiv v(N)/n$. If this is the case, then $v(S) > su^e$. By Lemma 1, $v(T)/t > v(S)/s > u^e$ for all $T \in \Pi(S)$. This implies that at the Nash equilibrium of the game $G(\Pi(S))$, all players are strictly better off than at the efficient allocation u^e, a contradiction. $\square$

3.1. **Exploitation of Minorities and Supermajority Rules.** The result of Proposition 1 essentially exploits the strategic disadvantage of large coalitions in the game $G(\Pi)$ to obtain stability of the equal split efficient allocation under the majority rule. Since within a partition Π a majority is larger than any other coalition - and then worse off in per capita terms under the conditions of Lemma 1 - efficient allocations must meet the claim of the majority. In the above analysis, we have defined the worth of an objecting majority S by only referring to the payoffs that result from the equilibrium actions associated with the partition $\Pi(S)$ emerging after the objection. Majorities have, in this sense, no coercive power over minorities. As explained in the introduction, they can only decide whether a given collective issue will be managed in a centralized or a decentralized fashion.

In the real political arena though, majorities often possess ways of extracting resources from minorities, either through taxation or through other redistributive decisions to which minorities are subjected. Even under the conditions of Lemma 1 (and the resulting strategic advantage of minorities in the underlying game), the exploitation of minority members may provide additional power to objecting majorities and, eventually, undermine the stability of global cooperation. We can say that the existence of core allocations ultimately depends on the composition of two opposite forces: on the one hand, the strategic advantage of minority members, free-riding on the externalities that come from larger coalitions, and on the other hand the political advantage of majorities, extracting resources from minorities.

In this section we provide a simple framework for the analysis of these two forces, and a sufficient condition that resolves this trade-off in favour of the stability of centralized decision-making.

We first define an exploitation rule that bounds the amount of resources that a majority can extract from minority members. We first refer to the case in which a majority coalition S expects, on objecting, that the minority members would form the complementary coalition $\Pi(S) = \{S, N \backslash S\}$ (the δ assumption). We also prove that under assumptions A1-A3, if a core-allocation exists under the δ assumption, it is also a core-allocation under the γ assumption, so that our Proposition 2 below also applies to the characteristic function derived under the γ.

We assume that a majority coalition S can appropriate a fixed per capita worth of z from each member of the complement coalition $N \backslash S$. In the context of transferable utility, such extraction would not modify the way in which the underlying game is played by the coalitions S and by the players in $N \backslash S$. The worth of a coalition S is now expressed in terms of a new characteristic function v_δ^*, accounting for the payoff that originates in the underlying game $G(\Pi(S))$ and of the per capita extraction z. For S such that $s > n/2$ this worth is given by:

$$v_\delta^*(S) = u_S(x(\Pi(S))) + \frac{n-s}{s} z$$

for S such that $s < n/2$ it is given by:

$$v_\delta^*(S) = u_S(x(\Pi(S))) - sz.$$

When $s = n/2$ we set $v^*_\delta(S) = u_S(x(\Pi))$.

Majorities are here allowed to appropriate a fixed per capita amount from minority members at the decentralized equilibrium. This type of rule is meant to capture a simple feature of political exploitation: as the size of the exploiting group increases, the number of agents to exploit decreases - and with it the amount of exploitable resources - and the number of agents that have a claim on such resources increases. This implies that the per capita benefits for majorities tend to die out as their relative size grows very large.

We then state a condition that essentially strengthens the property of Lemma 1, requiring that not only majorities suffer from a strategic disadvantage, but also that such disadvantage is larger the larger the size of the majority. In section 4 we show that this property is satisfied in several well-known games in which coalition formation and cooperation are a relevant issue. We will show in Proposition 2 that this condition implies that core-stable allocations exist for appropriate supermajority rules.

Formally, let us define

$$\Delta(S) \equiv u_{N\backslash S}(x(\Pi(S))) - u_S(x(\Pi(S))) \tag{3.1}$$

the difference in per capita payoffs between the coalitions $N\backslash S$ and S in the partition $\Pi(S) = \{S, N\backslash S\}$ at the Nash equilibrium associated with Π. Under the conditions of Lemma 1, $\Delta(S) > 0$ for $S > N\backslash S$.

A3. *The difference $\Delta(S)$ is increasing in s for all $s \geq n/2$.*

Once we account for the extraction power of majorities, the difference between the per-capita payoffs of a majority coalition S and the per capita payoff of its complement becomes:

$$\Delta^*(S) \equiv u_{N\backslash S}(x(\Pi(S)) - z - u_S(x(\Pi(S)) - \frac{n-s}{s}z$$

Proposition 2. *Let Assumptions A1-A3 hold. Let z denote the per-capita worth that majorities can extract from minorities. Then, for each $z < \Delta(N\backslash\{i\})\frac{n-1}{n}$, there exists $1 > \beta(n) > (n/2)/n$ for n even and $1 > \beta(n) > [(n+1)/2]/n$ for n odd such that the $\mathcal{M}^\beta$-core of the game in characteristic function (N, v^*_δ) is nonempty. Moreover, $\beta(n)$ is increasing in z.*

Proof. Rewrite expression $\Delta^*(S)$ as follows:

$$\Delta^*(S) = \Delta(S) - z(\frac{n}{s})$$

which under A3 is increasing in s for all $s \geq \frac{n}{2}$. Note than that the assumption that $z < \Delta(N\backslash\{i\})\frac{n-1}{n}$ (a bound on the extraction power) guarantees that $\Delta^*(N\backslash\{i\}) > 0$. Thus, in such range of z there exists some size $s^* < n$ such that $\Delta^*(S) > 0$ for all $s \geq s^*$. We can therefore apply the same argument used in Proposition 1 to all coalitions of size larger than s, and show that the equal split efficient payoff u^e is not objected by any such coalition and belongs therefore to the $\mathcal{M}^\beta$-core of the game (N, v^*_δ). Since $\Delta^*(S)$ is decreasing in z, we also conclude that this size s^* is increasing in z, which concludes the proof. □

To fully appreciate the key insight behind Proposition 2, let us look closer at the two main ingredients of the present model: the extraction rule and the patterns of strategic interaction. As we said, the extraction rule used here is such that the per capita benefits for majorities tend to die out as their relative size grows very large. At the same time, our assumption

A3 captures a technological property that, as we show in the next section, underlies many instances of economic interaction where cooperation is a relevant issue: larger groups take on most of the effort of production, from which smaller groups benefit. These two features of the model immediately suggest that large groups will suffer from a substantial strategic disadvantage due to the latter force, which they are not able to outweigh by means of resource exploitation. Thus, the weak objecting power of large coalitions implies the stability property of supermajority rules.

We end this section by showing that the result of Proposition 2 extends to the characteristic function derived under the γ assumption. This is proved by showing that in the present context (assumptions A1-A2), the δ assumption assigns to a coalition S a higher worth than the γ assumption, and that the cores of the games that originate from these assumptions are therefore ordered by inclusion.

Proposition 3. *Let assumptions A1-A2 hold. Then, for all $\beta \in [0,1]$ the $\mathcal{M}^\beta$-core of the game (N, v_δ^*) is weakly included in the $\mathcal{M}^\beta$-core of the game (N, v_γ^*).*

Proof. We prove the argument by showing that for any $S \backslash N$ under assumptions A1-A2 we have $u_s(x(\{S, N\backslash S\})) \geq u_s(x(\{S, \{j\}_{j\in N\backslash S}\}))$. For simplicity, let us denote $T \equiv N\backslash S$, and by x_s the per capita strategy of each member of S in the efficient profile $x(\Pi)$. We start by showing that for any fixed x_s, the efficient strategy x_t is larger than the strategy y_t played by any player outside S in the partition $\{S, \{j\}_{j\in N\backslash S}\}$. By efficiency of x_t we write:

$$u_T(x_t, .., x_t, x_S)) \geq u_T(y_t, ..., y_t, x_S),$$

Also, by Nash equilibrium property of y_t we have that for all $i \in T$:

$$u_i(y_t, ., y_{t,}., y_t, x_S)) \geq u_i(y_t, ., x_t, ., y_t, x_S)),$$

implying, together with symmetry, that:

$$tu_i(y_t, ., y_{t,}., y_t, x_S)) \geq tu_i(y_t, ., x_t, ., y_t, x_S)).$$

Using again symmetry and the above inequalities we obtain:

$$tu_i(x_t, ., x_{t,}., x_t, x_S)) \geq tu_i(y_t, ., y_{t,}., y_t, x_S)) \geq tu_i(y_t, ., x_t, ., y_t, x_S)).$$

This implies that under positive externalities $x_{t,.} \geq y_t$, while under negative externalities $x_{t,.} \leq y_t$. Consider now the case of positive externalities - the opposite case can be proven along similar lines. If there are increasing differences in u, then the efficient strategy $x_s(x_t)$ is such that $x_s(x_t) \geq x_s(y_t)$. Let now $\overline{x}_s \equiv x_s(x_t)$. Increasing differences again imply that $x_s(\overline{x}_s) > x_t$. This process of increasing reactions of coalition S and T converges to the equilibrium $x_s(\{S,T\})$ and $x_t(\{S,T\})$ where $x_t(\{S,T\}) \geq y_t$. Since we are assuming positive externalities, we have $u_s(x(\{S, N\backslash S\})) \geq u_s(x(\{S, \{j\}_{j\in N\backslash S}\}))$. When there are decreasing differences, the same argument applies, noting that the adjustment process of the efficient strategies x_s and x_t is such that x_s decreases at each step, while x_t increases at each step. We obtain again the conclusion that $x_t(\{S,T\}) \geq y_t$, implying the result. □

In this final section we present three well-know examples of games falling in the present framework and for which assumption **A.3**. holds. For these games the size of supermajority required for stability as a function of extraction power can be easily computed.

4. Examples

4.1. **Oligopoly Games.** Although oligopoly models are far from being "political", many political issues can be modelled as an oligopoly game. When firms are identical and there are no synergies, a merger (or cartel) is usually assumed to behave as a macro-player (i.e., as a single firm). Therefore, in the partition $\Pi(S) = \{S, T\}$ equilibrium profits are such that $U_{N\setminus S}(x(\Pi(S)) = U_S(x(\Pi(S))$. This implies that $\Delta(S)$ is strictly increasing in s for $s \geq n/2$ and, therefore, Proposition 2 applies. The level of the supermajority required for stability of global collusion increases monotonically with the intensity of minority exploitation. For instance, under linear Cournot oligopoly and normalized demand and cost such that $(a-c)^2 = 1$, for $n = 10$ if the extraction power is $z = 0.0055$, the required supermajority is $s^* = 6$, while, for instance, if $z = 0.088$, $s^* = 9$. Similar results can be obtained in all games in which, as in Cournot, the coalitional worths in $G(\Pi)$ are indipendent of coalitional sizes.

4.2. **Public Good Games.** Ray and Vohra (1997) consider a game of public good provision in which each agent $i \in N$ contributes x_i to the public good and receives a payoff $U_i(x) = \sum_{j\in N} x_j - cx_i^2$. The worth of coalition S is $U_S(x) = s\sum_{j\in N} x_j - \sum_{i\in S} cx_i^2$. By computing the equilibrium profile $x(\Pi(S))$ we obtain:

$$\Delta(S) = \frac{2s^2 + (n-s)^2}{4c} - \frac{s^2 + 2(n-s)^2}{4c} = \frac{(2s-n)\,n}{4c},$$

which is positive and monotonically increasing in s for $s > n/2$. Therefore, **A.3** holds and Proposition 2 applies. Note that similar results are obtainable in all games in which the function $U_S(x(\Pi(S))$ increases in s more than proportionally for $s \in \left[0, \frac{n}{2}\right]$ and less than proportionally for $s \in \left(\frac{n}{2}, n\right]$.

4.3. **Alliances in Contests.** Following a number of recent contributions on alliance formation in contests (see, for instance, Bloch, 2011 for a survey), let n players exert effort $e_i \in E_i$ and obtain a payoff $u_i : E^n \to \mathbf{R}_+$ given by

$$u_i(e) = p(e)R - c(e_i)$$

where R is a fixed prize, $c(e_i)$ each player cost of effort, and

$$\begin{cases} p(e) = e_i\left(\sum_{i\in N} e_i\right)^{-1} \text{ if } \sum_{i\in N} e_i > 0 \\ \quad\quad \text{and } \frac{1}{|N|} \text{ otherwise} \end{cases}$$

is a contest success function typical of rent-seeking games (Tullock, 1987). The effort of each player affects the probability to access the prize (here fixed for simplicity).
When only two coalitions S and T compete for prize, it can be easily shown that Lemma 1 applies. Moreover, numerical simulations show that, when costs are given by $c(e_i) = e_i^2/2$,

$$\Delta(s) = \frac{\sum\limits_{i\in N\setminus S}\left(e_i\left(\sum_{i\in N} e_i\right)^{-1} - \frac{e_i^2}{2}\right)}{n-s} - \frac{\sum\limits_{i\in S}\left(e_i\left(\sum_{i\in N} e_i\right)^{-1} - \frac{e_i^2}{2}\right)}{s}$$

is increasing in s and the supermajority required for the stability increases monotonically in the extraction power of the majority. For example, for $n = 10$,

$$s^* = 6 \text{ for } z = 0.00618, \quad s^* = 7 \text{ for } z = 0.01589$$
$$s^* = 8 \text{ for } z = 0.03328, \quad s^* = 9 \text{ for } z = 0.07497.$$

5. Concluding Remarks

We have shown that in symmetric games with no synergies if the decisions about centralization or decentralization of a given activity are taken by the majority of players, this always guarantees the existence of core-stable allocations (and, hence, of global cooperation) irrespective of the assumptions on the behaviour of individuals excluded from blocking coalitions. We have also argued that supermajorities may be required to ensure core-stability when majorities possess some forms of exploitation rights over minorities.

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