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A Note on the Weak Condition of “Globally” Ricardian

Kazuki Hiraga
Faculty of Economics, Keio University

Abstract

This paper shows that there is the weak “Globally” Ricardian rules that sustains the sustainability of the government debt in the sense that it binds intertemporal government budget constraint. And we compare our result with the former paper about testing the government debt sustainability. As a result, we declare that all of these papers are sufficient condition of our condition.

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Contact: Kazuki Hiraga - khiraga581470@gmail.com.
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1. Introduction

Following "the Lehman Shock" of September 11, 2008, many countries addressed the economic recession through a fiscal stimulus. Among other measures, for example, the U.S. Congress passed the $787 billion American Recovery and Reinvestment Act of 2009. This legislation provoked concerns about the sustainability of the U.S. federal deficit, which President Obama pledged to cut in half by 2013. Around that time, events in Greece raised worldwide awareness of impending sovereign debt crises in many countries. Maastricht criteria for acceptance into the European Union (E.U.) imposed a ceiling on member states' deficits and debt, but Greece's government disguised its true deficit to satisfy the criteria, and a crisis arose in 2010 when Prime Minister Papandreou revealed the previous regime's window-dressing. Several countries still face serious sovereign crises, especially Portugal, Ireland, Italy, Greece, Spain, and Japan. Meanwhile, the U.S. deficit remains worrisome. In 2011, the U.S. Congress increased the debt ceiling, and Standard & Poor downgraded the long-term credit rating of the U.S. from AAA to AA+ for the first time. International concern about government debt has increased.

Drawing upon Woodford (1998), this note shows that a weak "Globally" Ricardian condition exists. This condition supports the sustainability of government debt where it binds the government's intertemporal budget constraint. I relax the assumptions of linear stabilization and invariance and show that the "Globally" Ricardian condition can sustain the binding intertemporal budget constraint when these assumptions are violated. This note is organized as follows. Section 2 reviews related literature concerning fiscal sustainability. Section 3 analyzes the weak "Globally" Ricardian condition. Section 4 concludes.

2. Related literature concerning fiscal sustainability

Hamilton and Flavin (1986) suggest a test to determine whether an intertemporal government budget constraint is in place, that is, whether current government debt equals the present value of the primary surplus. They find that the constraint pertains under the following condition:

$$\lim_{i\to\infty}\frac{B_{t+i}}{(1+r)^{i}}=0, \tag{1}$$

where B_t is the level of the government debt and r is the interest rate.

Hamilton and Flavin (1986) use two methods to investigate whether an existing state of affairs satisfies Eq. (1). First, they check for a unit root in a succession of government debt issues using an Augmented Dickey-Fuller test Second, they check it to apply the test for bubble.

However, Bohn (1998, 2008) criticizes Hamilton and Flavin's (1986) test as "ad-hoc" sustainability. He argues that their specified conditions do not capture the rule under which the government stabilizes its government debt and that government does not determine the fiscal activities that fulfill Eq. (1). Bohn (1998, 2008) proposes the following rule:

$$\frac{PS_t}{Y_t} = \phi \frac{B_t}{Y_t} + \mu_t, \tag{2}$$

where Y_t is output (GDP) and μ_t is a constant term. Other stationary components, such as YVAR and GVAR, are defined by Barro (1986).[1]

Bohn (1998) shows that conditions are sufficient for the sustainability of government debt if $\varphi > 0$ is statistically significant. Several researches satisfy the Bohn (1998) condition.

Woodford (1995, 1998) describes “Locally” and “Globally” Ricardian policies as two fiscal stabilization rules.[2] To understand these policies, we set the following policy:

$$PS_t = \lambda_t B_t + \nu_t, \tag{3}$$

where λ_t is a time-varying parameter satisfying $0 < \lambda_t \leq 1$, which represents the government’s responsiveness to changes in level of government debt and resembles Bohn’s (1998) rule. By substituting (3) into the government budget constraint,

$$B_{t+1} = (1 + r - \lambda_t) B_t - \nu_t. \tag{4}$$

If $\lambda_t > r$, the locally Ricardian condition is satisfied. That is, government debt converges to zero, its present value represented by Eq. (1) is also zero, and the debt is sustainable. The “Globally” Ricardian condition is satisfied if $0 < \lambda_t \leq r$, meaning the growth rate of government debt is below the interest rate, and the present value of debt converges to zero. Therefore, the “Globally” Ricardian condition for sustainability of the government debt is weaker than the “Locally” Ricardian condition, but both rules satisfy Eq. (1). Figures 1 and 2 depict both Ricardian conditions. Later, we examine the weaker “Globally” Ricardian condition for sustainability of government debt.

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[1] $GVAR_t = \frac{G_t - G_t^*}{Y_t}$, $YVAR_t = \left(1 - \frac{Y_t}{Y_t^*}\right)\frac{G_t^*}{Y_t}$, where G_t is government consumption, Y_t^* and G_t^* are the trend components of output and real government consumption, respectively.

[2] For simplicity, we omit money and price level from Woodford’s (1995, 1998) original discussions to examine the debt level and primary surplus.

Figure 1: Depiction of the "Locally" Ricardian Condition

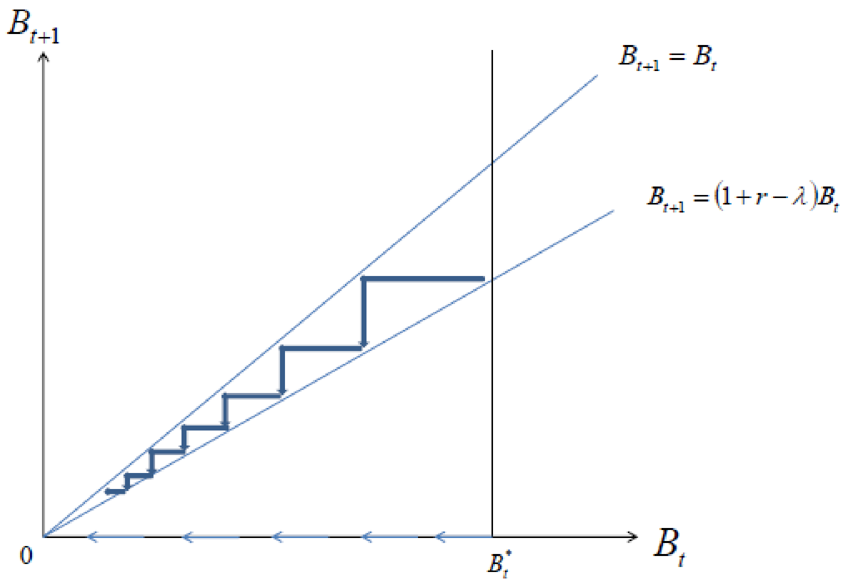
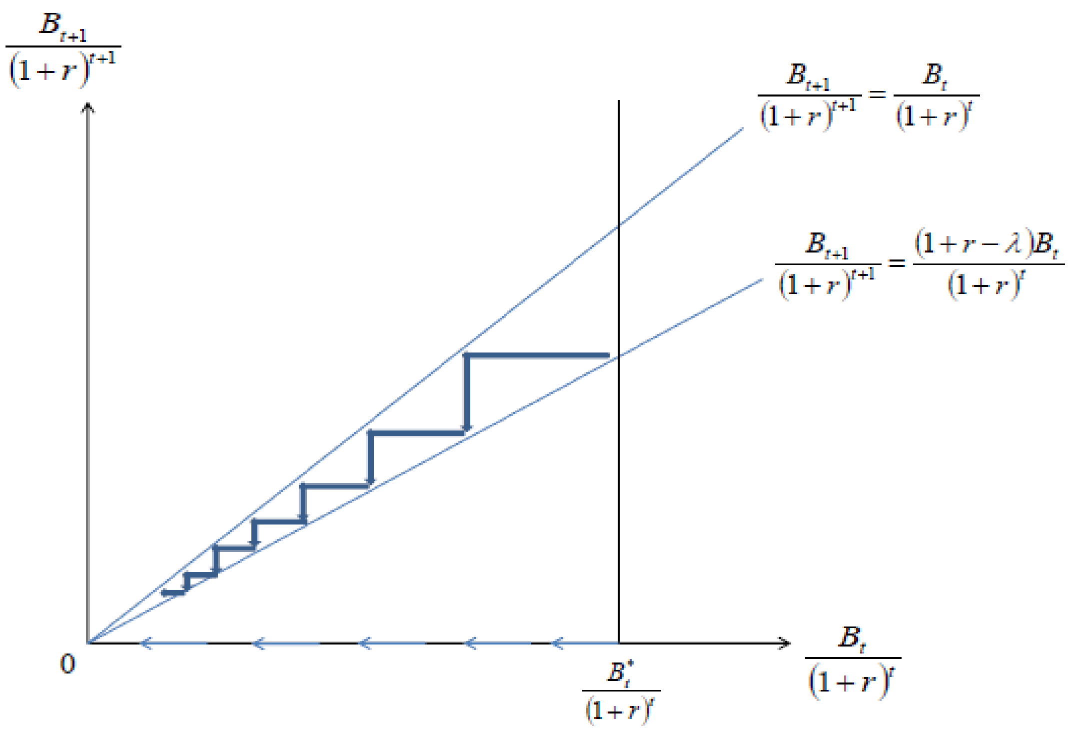


Figure 2: Depiction of the "Globally" Ricardian Condition



3. The Model and Basic Analyses

This section explains the weak "Globally" Ricardian condition by invoking a government budget constraint and explains its applicability for a non-stochastic and a stochastic economy.

3.1. Non-Stochastic Economy

For simplicity, assume a constant interest rate $(r_t = r)$. Government's budget constrain at period t can be written as

$$B_{t+1} = (1+r)B_t - PS_t, \tag{5}$$

where B_t is government debt stock at the start of period t and PS_t is the primary surplus (tax revenues minus government expenditures excluding interest payments). By rewriting Eq. (5) and expanding into the intertemporal budget constraint, we get

$$B_t = \sum_{i=0}^{\infty}\left(\frac{1}{1+r}\right)^i PS_{t+i} + \lim_{N\to\infty}\left(\frac{1}{1+r}\right)^N B_{t+N}. \tag{6}$$

Some studies, notably Hamilton and Flavin (1986), show that government debt is sustainable if the second term on the right-hand side of Eq. (1) is zero. That is, $\lim_{N\to\infty}\left(\frac{1}{1+r}\right)^N B_{t+N} = 0$. We additionally assume that the primary surplus is a positive value—PS_t for all t > 0—and define the ratio of the primary surplus to government debt as $x_t \equiv \frac{PS_t}{B_t} > 0$[3]. Therefore, we can rewrite government's budget constraints at each period:

$$\begin{aligned} B_{t+1} &= (1+r)B_t + PS_t = (1+r-x_t)B_t, \\ B_{t+2} &= (1+r)B_{t+1} + PS_{t+1} = (1+r-x_{t+1})B_{t+1} = (1+r-x_{t+1})(1+r-x_t)B_t, \\ &\vdots \\ B_{t+N} &= \prod_{i=1}^{N}(1+r-x_{t+i-1})^i B_t. \end{aligned} \tag{7}$$

Using Eqs. (2) and (3), we illustrate the following proposition.

Proposition 1:

If Bt is a finite value and PSt generally remains positive following any finite period t, the right-hand side of Eq. (2) is zero.

Proof:

Substituting Eq. (3) for the second term in the right-hand side of Eq. (2), we obtain $\lim_{N\to\infty}\prod_{i=1}^{N}\left(\frac{1+r-x_{t+i-1}}{1+r}\right)^i B_t$. If B_t is a finite value and x_t is positive, the second term in the right-hand side of Eq. (4) is zero ($\lim_{N\to\infty}\prod_{i=1}^{N}\left(\frac{1+r-x_{t+i-1}}{1+r}\right)^i B_t = 0$), and the second term in the

[3] This rule resembles that in Bohn (1998), who argues that the government improves its fiscal stance by increasing its debt.

right-hand side of Eq. (2) is also zero. Then, the government budget constraint is binding under the circumstances above, as illustrated in Figure 1. Q.E.D.

3.2.Stochastic Economy

Next, we consider a stochastic economy. We illustrate the following proposition.

Proposition 2:

If we assume that all sequences of the primary surplus per level of government debt $\{x_t, x_{t+1}, \cdots, x_{t+N}, \cdots\}$ are at any period t + n ≧ t, the following transversality condition is satisfied:

$$\lim_{N\to\infty} E_t \prod_{i=1}^{N} \left(\frac{1}{1+r_{r+i-1}} \right)^i B_{t+N} = 0. \quad (8)$$

Proof:

Adding expectations to Eq. (5):

$$B_{t+N} = E_t \prod_{i=1}^{N} (1+r_{t+i-1} - x_{t+i-1})^i B_t . \quad (9)$$

Substituting Eq. (9) for the second term on the right-hand side of Eq. (6), we obtain

$$\lim_{N\to\infty} E_t \prod_{i=1}^{N} \left(\frac{1+r_{t+i-1} - x_{t+i-1}}{1+r_{t+i-1}} \right)^i B_t . \quad (10)$$

If B_t is a finite value, Eq. (10) equals zero, $\lim_{N\to\infty} E_t \prod_{i=1}^{N} \left(\frac{1+r_{t+i-1} - x_{t+i-1}}{1+r_{t+i-1}} \right)^i B_t = 0,$ because the sequences $\frac{1+r_{t+i} - x_{t+i}}{1+r_{t+i}} < 1$ for any i ≧ 0. The government budget constraint is binding under those circumstances, even in a stochastic economy. Q.E.D.

3.3. Sustainability of Debt per GDP

Domar (1944), Blanchard et al. (1990), and Uctum and Wickens (2000) discuss sustainability using the government budget constraint divided by GDP (Y_t), following Eq. (11). They suggest that we can determine the sustainability of government deficits to test the condition that Eq. (6) satisfies:

$$\frac{B_t}{Y_t} = \sum_{i=0}^{\infty} \left(\frac{1+g}{1+r} \right)^i \frac{PS_{t+i}}{Y_{t+i}} + \lim_{N\to\infty} \left(\frac{1+g}{1+r} \right)^N \frac{B_{t+N}}{Y_{t+N}} . \quad (11)$$

$$\text{where} \quad \lim_{N\to\infty}\left(\frac{1+g}{1+r}\right)^N \frac{B_{t+N}}{Y_{t+N}} = 0. \tag{12}$$

Therefore, we investigate the weak sustainability of the deficit per GDP.

For comparison with earlier literature, especially Domar (1944), we assume that interest rate r and the economic growth rate g (GDP growth rate) are constant and that the primary surplus is zero. Under these assumptions, we rewrite government's budget constraint per GDP for each period in the manner presented above:

$$\begin{aligned} \frac{B_{t+1}}{Y_{t+1}} &= \frac{1+r}{1+g}\frac{B_t}{Y_t} \\ \frac{B_{t+2}}{Y_{t+2}} &= \frac{1+r}{1+g}\frac{B_{t+1}}{Y_{t+1}} = \left(\frac{1+r}{1+g}\right)^2 \frac{B_t}{Y_t} \\ &\vdots \\ \frac{B_{t+N}}{Y_{t+N}} &= \left(\frac{1+r}{1+g}\right)^N \frac{B_t}{Y_t}. \end{aligned} \tag{13}$$

When we set N→∞, we obtain the following form:

$$\lim_{N\to\infty}\frac{B_{t+N}}{Y_{t+N}} = \lim_{N\to\infty}\left(\frac{1+r}{1+g}\right)^N \frac{B_t}{Y_t}. \tag{14}$$

Next, we relax the assumption that the primary surplus is zero and assume that the primary surplus per government debt is constant ($\frac{PS_t}{B_t} = x$). Then, we can rewrite the government's budget constraint per GDP:

$$\begin{aligned} \frac{B_{t+1}}{Y_{t+1}} &= \frac{1+r-x}{1+g}\frac{B_t}{Y_t} \\ \frac{B_{t+2}}{Y_{t+2}} &= \frac{1+r}{1+g}\frac{B_{t+1}}{Y_{t+1}} = \left(\frac{1+r-x}{1+g}\right)^2 \frac{B_t}{Y_t} \\ &\vdots \\ \frac{B_{t+N}}{Y_{t+N}} &= \left(\frac{1+r-x}{1+g}\right)^N \frac{B_t}{Y_t}. \end{aligned} \tag{15}$$

Substituting Eq. (15) into Eq. (14), we obtain

$$\lim_{N\to\infty}\frac{B_{t+N}}{Y_{t+N}} = \lim_{N\to\infty}\left(\frac{1+r-x}{1+r}\right)^N \frac{B_t}{Y_t} = 0. \tag{16}$$

Therefore, we obtain the following proposition.

Proposition 3:

If primary surplus x is usually positive, the weak Ricardian condition is satisfied.

3.4.Model Featuring Money and Prices

Following Woodford (1995, 1998), we introduce base money, M_t, into the model (i.e., we assume a central bank). The consolidated budget constraint for the government and central bank takes this form:

$$M_{t+1} + B_{t+1} = (1 + r_t)B_t + M_t - S_t. \tag{17}$$

Dividing both sides of this equation by nominal GDP Y_t, we obtain

$$m_{t+1} + b_{t+1} = \frac{1 + r_t}{1 + g_{t+1}} b_t + \frac{1}{1 + g_{t+1}} m_t - \frac{1}{1 + g_{t+1}} s_t, \tag{18}$$

where $m_t \equiv \frac{M_t}{Y_t}, b_t \equiv \frac{B_t}{Y_t}, s_t \equiv \frac{S_t}{Y_t}$.

Denoting total consolidated liabilities as ω_t ($\equiv b_t + m_t$), we rewrite Eq. (18) as follows:

$$\omega_{t+1} - \omega_t = \frac{r_t}{1 + g_{t+1}} \omega_t - \frac{g_{t+1}}{1 + g_{t+1}} \omega_t - \left(\frac{r_t}{1 + g_{t+1}} m_t + \frac{1}{1 + g_{t+1}} s_t \right). \tag{19}$$

Note that $\frac{r_t}{1 + g_{t+1}} m_t$ represents seigniorage. The second term on the right-hand side, $-\frac{g_{t+1}}{1 + g_{t+1}} \omega_t$, represents the "growth dividend" (Bohn, 2008).

We rewrite Eq. (19) as follows:

$$\omega_t = \frac{1 + g_{t+1}}{1 + r_t} \omega_{t+1} + \frac{1}{1 + r_t} s_t + \frac{r_t}{1 + r_t} m_t . \tag{20}$$

Integrating Eq. (20) forward from the current period and taking expectations into account, we obtain this present value expression of the budget constraint, conditional on information available in period t:

$$\omega_t = E_t \sum_{j=0}^{T-1} \left(\prod_{k=0}^{j} \left(\frac{1 + g_{t+k+1}}{1 + r_{t+k}} \right)^k \right) \frac{1}{1 + r_t} s_{t+k} + E_t \sum_{j=0}^{T-1} \left(\prod_{k=0}^{j} \left(\frac{1 + g_{t+k+1}}{1 + r_{t+k}} \right)^k \right) \frac{r_t}{1 + r_t} m_{t+k}$$
$$+ E_t \prod_{k=0}^{T-1} \left(\frac{1 + g_{t+k+1}}{1 + r_{t+k}} \right)^k \omega_{t+T} . \tag{21}$$

Eq. (21) implies that the transversality condition is satisfied if

$$\lim_{T \to \infty} E_t \prod_{k=0}^{T-1} \left(\frac{1 + g_{t+k+1}}{1 + r_{t+k}} \right)^k \omega_{t+T} = 0. \tag{22}$$

Eq. (18) is also called "Globally" Ricardian in Woodford (1995, 1998).

We find that the weak Ricardian condition exists in this model and suggest the following clarifying proposition.

Proposition 4:

The weak Ricardian condition exists if the total government surplus (seigniorage + primary surplus) is usually positive.

Proof:

First, we rewrite Eq. (19) as

$$\omega_{t+1} = \frac{1 + r_t - x_t^{\omega}}{1 + g_{t+1}} \omega_t, \tag{23}$$

where $\chi_t^{\omega} \equiv \dfrac{r_t m_t + s_t}{\omega_t}$.

As in the preceding discussion, we obtain government's total consolidated liabilities at period t + T:

$$\omega_{t+T} = \prod_{k=0}^{T-1} \left(\frac{1 + r_{t+k} - x_{t+k}^{\omega}}{1 + g_{t+k+1}} \right)^{k+1} \omega_t. \tag{24}$$

When we set T→∞ and add the symbol of expectation E_t, we obtain

$$\lim_{T\to\infty} E_t \omega_{t+T} = \lim_{T\to\infty} E_t \prod_{k=0}^{T-1} \left(\frac{1 + r_{t+k} - x_{t+k}^{\omega}}{1 + g_{t+k+1}} \right)^{k+1} \omega_t. \tag{25}$$

Multiplying $\prod_{k=0}^{T-1} \left(\frac{1 + g_{t+k+1}}{1 + r_{t+k}} \right)$ by Eq. (25), we obtain

$$\lim_{T\to\infty} E_t \prod_{k=0}^{T-1} \left(\frac{1 + g_{t+k+1}}{1 + r_{t+k}} \right)^{k} \omega_{t+T} = \lim_{T\to\infty} E_t \prod_{k=0}^{T-1} \left(\frac{1 + r_{t+k} - x_{t+k}^{\omega}}{1 + r_{t+k}} \right)^{k+1} \omega_t = 0. \tag{26}$$

Therefore, Eq. (22) converges to zero when x_t^{ω} is usually positive. Q.E.D.

3.5.Introducing Regime Switch

This section applies the weak "Globally" Ricardian condition to the regime-switch model and examines sustainability.

3.5.1. Threshold Model

First, we assume that the government has the following policy, which it adjusts when debt exceeds the threshold $\overline{B}$:

$$\begin{cases} x_t = x_1 < 0, & if \quad B_t < \overline{B} \\ x_t = x_2 > 0, & if \quad B_t > \overline{B} \end{cases}, \tag{27}$$

where $\overline{B}$ is the threshold value of the debt and its finite value.

Under this rule, Proposition 1 is satisfied. The debt is sustainable because government adjusts its policy rule from regime 1 to regime 2. Figure 2 depicts this situation.

3.5.2. Markov-Switching Model

Next, we consider the situation in which adjustments in the fiscal rule follow the stochastic regime change. That is, the fiscal rule follows a Markov-switching process. We assume that the fiscal policy has the following regimes:

$$\begin{cases} x_t = x(s_t = 1) < 0, & if \quad \mathrm{Re}\,gime 1 \\ x_t = x(s_t = 2) > 0, & if \quad \mathrm{Re}\,gime 2 \end{cases}, \tag{28}$$

The transition matrix is

$$P = \begin{bmatrix} p_1 & 1 - p_1 \\ 1 - p_2 & p_2 \end{bmatrix}. \tag{29}$$

We derive the state system as follows:

$$\begin{bmatrix} p_1 & 1 - p_1 \\ 1 - p_2 & p_2 \end{bmatrix} \begin{bmatrix} B_{1,t+1} \\ B_{2,t+1} \end{bmatrix} = \begin{bmatrix} (1 + r - x_1) B_{1,t+1} \\ (1 + r - x_2) B_{2,t+1} \end{bmatrix},$$

$$\Leftrightarrow \begin{bmatrix} B_{1,t+1} - B_{1,t} \\ B_{2,t+1} - B_{2,t} \end{bmatrix} = \begin{bmatrix} \dfrac{p_1 p_2 (1 + r - x_1)}{p_1 + p_2 - 1} - 1 & -\dfrac{(1 - p_1) p_2 (1 + r - x_2)}{p_1 + p_2 - 1} \\ -\dfrac{p_1 (1 - p_2)(1 + r - x_1)}{p_1 + p_2 - 1} & \dfrac{p_1 p_2 (1 + r - x_2)}{p_1 + p_2 - 1} - 1 \end{bmatrix} \begin{bmatrix} B_{1,t+1} \\ B_{2,t+1} \end{bmatrix} \equiv S \begin{bmatrix} B_{1,t+1} \\ B_{2,t+1} \end{bmatrix}. \tag{30}$$

Therefore, we can investigate the sustainability of the debt to ascertain the following proposition:

Proposition 5:

Government debt is sustainable in this economy when det(S) > 0 and tr(S) < 0.

Proof:

If this proposition is satisfied, the stable dynamics of Eq. (26) is obtained. Q.E.D.

4. Conclusion

This note has discussed several fiscal policy circumstances under which a weak "Globally" Ricardian condition exists. It has demonstrated that governments can secure a primary surplus within budget constraints when the "Globally" Ricardian rule is satisfied.

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