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Cooperation and Effort in Group Contests

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Abstract

We consider a two group contest over a group specific public good comparing two situations: (i) where all players act independently; and (ii) where the players of each group cooperate. This comparison leads us to the conclusion that it is possible for one group to contribute more (and have a higher expected payoff) in the non-cooperative regime than in the cooperative regime. .

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1. Introduction

We consider a two-group contest over a group specific public good in which a member of each group can invest efforts so that the group wins the contest. Our purpose is to examine the equilibrium efforts invested by individual players in each group. We consider two groups and compare two situations: (i) where all players act independently; and (ii) where each of the group players cooperates. This comparison leads us to the conclusion that, in certain circumstances, players may contribute more in scenario (i) than in scenario (ii).

Economic policy involves a struggle between interest groups: one group which defends the status-quo and another group which challenges it by fighting for an alternative policy. There may be different examples such as taxation, pollution standards, a monopoly facing opposition, capital owners and a workers' union which can be engaged in a contest over minimum wages and so on. In Israel, there was a public committee headed by Professor Eytan Sheshinski to determine the taxation level on natural resources which had just been found (gas). Different sides tried to affect the outcome. On the one side there was the public and on the other side were the firms leading the extraction of the natural resource. Both sides tried to affect the final outcome of the committee. Before the committee began its sessions the members of the interest groups tried to influence the politicians in determining whether to establish the committee or not. At this stage the members of the interest groups did not cooperate with each other.[1] In this paper we consider this stage in the contest and the struggle regarding the formation of the committee. We wish to compare the situation under which the members of the interest group do not cooperate with each other in the group (which occurred in this situation) to the case where the members do cooperate with each other. The outcome of the contest depends on the stakes of each contestant, and, in turn, on their exerted efforts. These contests may involve group specific public-goods.

There exists a vast literature dealing with contests with group-specific public-good prizes. [2] In the literature, free-riding is a well known problem and it may overshadow a specific public-good. For example, Nitzan (1991) presents a sharing

[1] After the committee started working, the public continued not to cooperate while the firms cooperated.

[2] See for example: Katz, Nitzan and Rosenberg (1990), Ursprung (1990), Riaz, Shogren and Johnson (1995), Konrad (2009, and references within), Baik (2008) and Cheikbossian (2008a).

rule to decrease the free-ridding problem, while Baik (2008) studies a case of free-riding where only one player invests the effort to win the contest while the other contestants free-ride.[3] Cheikbossian (2008a) presents a model of endogenous public-good provision and group rent-seeking influence. Specifically, two groups with different preferences over public policy and different sizes, engage in rent-seeking or lobbying activities to influence policy making in their preferred direction. When there is within-group cooperation in lobbying, both groups neutralize each other in the political process. Without within-group cooperation, the free-rider problem in lobbying gives the smaller group more political influence. In both cases, the total level of rent-seeking activities is shown to be increasing in taste heterogeneity while decreasing in group size asymmetry.[4] In a similar type of model Epstein and Mealem (2009) consider a situation in which two groups contest a group-specific public good. They show that the level of free-riding depends on the return on investments and consider the situation in which one group initiates a contest, adding different players and/or groups. The question they pose is: what would be the optimal structure of the added groups?

The early literature on coordination of games suggests that coordination failure is common in the laboratory (for example, Cooper et al., 1992). This important finding has been interpreted as relevant for environments ranging from individual organizations to macro-economies, and has led to an active research agenda to investigate possible mechanisms to resolve this coordination failure.[5]

In our paper, we consider the generalized logit contest success function. The idea behind this assumption is that one tries to affect the policy outcome at low cost such as writing an e-mail, signing a petition on the internet or sending a text message by phone. This was very common during the sessions of the Sheshinski committee. Many petitions were signed via the internet and many e-mails were sent by different members of each side of the contest. Emails and signing positions are costless. Sending the first e-mail has a stronger effect than sending the second one; signing the

[3] Baik (2008) considers a model with n groups competing to win a group-specific public-good prize. The main difference between Baik's paper and ours is that while one can aggregate the total effort invested in the contest, our model can only aggregate effort after using non-linear transformation.

[4] In a similar paper Cheikbossian (2008b) presents a model that deals with spillovers in decentralized provision of local public-goods. It is shown that the spillovers may lead to a higher surplus than centralized provisions even though the players have identical references.

[5] There is a growing literature on experimental economics of group contests with and without cooperation for example Riechmann and Weimann (2008), Reuben and Tyran (2010), Cason, Sheremeta and Zhang (2010) and Leibbradt and Saaksvouri (2010) deal with similar issues.

first petition has a stronger effect than the second etc. Thus these investments have decreasing returns in the contest. Epstein and Mealem (2009) describe this situation in detail and present these types of effort, showing them to have a low marginal cost with a decrease returns to scale.

Our main results show that the sufficient condition for one of the groups to "over invest" (invest more than the situation in which the group cooperates) is that the number of players in this group has to be sufficiently smaller than the other group. Moreover, in the case where one of the groups invests more effort than the amount invested under cooperation, we would obtain that the expected payoff of this group would be higher than when there is cooperation.

2. The Model

2.1. No Cooperation

Consider a contest with two groups competing for a prize, as in Epstein and Nitzan (2004) and Epstein and Mealem (2009). Suppose that a status-quo policy is challenged by one interest group and defended by the other. For example, in the contest over monopoly regulations, one firm defends the status-quo, lobbying for the profit-maximizing monopoly price (and against any price regulation) while the consumers challenge the status-quo lobbying preferring a competitive price (a tight price cap).[6]

Assume that in group 1 there are N players, while in group 2 there are M players. In group 1, each player has a payoff of n from winning the contest, while in group 2 each player has a payoff m from winning the contest. Each player from group 1 invests x_i $\left(i=1,...,N\right)$ units to change the status-quo to the new policy and each player from group 2 invests y_j $\left(j=1,...,M\right)$ units so that the policy will not be changed.

The probability that the new policy will be accepted and the status-quo changed, p_x, is a function of the resources both groups invest in the contest. It is assumed that the probability is given by the generalized logit contest success function:

[6] See for example Epstein and Nitzan (2003, 2007).

$$p_x = \frac{\sum_{i=1}^{N} x_i^{\alpha}}{\sum_{i=1}^{N} x_i^{\alpha} + \sum_{j=1}^{M} y_j^{\alpha}} \text{ with } 0 < \alpha < 1 \tag{1}$$

We restrict our analysis to the case in which $0 < \alpha < 1$.[7] The expected payoff of each player in group 1 will equal:

$$E(U_i) = \frac{\sum_{i=1}^{N} x_i^{\alpha}}{\sum_{i=1}^{N} x_i^{\alpha} + \sum_{j=1}^{M} y_j^{\alpha}} n - x_i \quad \forall\ i = 1,\dots,N \tag{2}$$

and for each player in group 2:

$$E(U_j) = \frac{\sum_{j=1}^{M} y_j^{\alpha}}{\sum_{i=1}^{N} x_i^{\alpha} + \sum_{j=1}^{M} y_j^{\alpha}} m - y_j \quad \forall\ j = 1,\dots,M\ . \tag{3}$$

Solving the first order conditions (it can be verified that the second order conditions hold); we obtain that the Nash equilibrium investment of the players of each of the groups equals:

$$x_i^* = \frac{\alpha n k^{\alpha} M^{1-\alpha}}{N^{\alpha}\left(N^{1-\alpha}k^{\alpha} + M^{1-\alpha}\right)^2} \quad (i = 1,\dots,N)$$
$$\text{and} \tag{4}$$
$$y_j^* = \frac{\alpha m k^{\alpha} N^{1-\alpha}}{M^{\alpha}\left(N^{1-\alpha}k^{\alpha} + M^{1-\alpha}\right)^2} \quad (j = 1,\dots,M),$$

where $k = \frac{n}{m}$. The expected payoff becomes:

$$E(U_i^*) = \frac{nk^{\alpha}\left[N^{2-\alpha}k^{\alpha} + M^{1-\alpha}(N-\alpha)\right]}{N^{\alpha}\left(N^{1-\alpha}k^{\alpha} + M^{1-\alpha}\right)^2}$$
$$\text{and} \tag{5}$$
$$E(U_j^*) = \frac{m\left[M^{2-\alpha} + N^{1-\alpha}k^{\alpha}(M-\alpha)\right]}{M^{\alpha}\left(N^{1-\alpha}k^{\alpha} + M^{1-\alpha}\right)^2}.$$

[7] For the other cases where $\alpha = 1$ see Baik (2008). For $\alpha > 1$ second order conditions may not hold.

2.2. Cooperation

Consider the case of cooperation. Under the scenario in which one of the players (the leading player or a central planner) in each group will determine the optimal investments of each player in his group. The objective function for group 1 would be to maximize:

$$\sum_{i=1}^{N} E(U_{ic}) = \frac{\sum_{i=1}^{N} x_{ic}^{\alpha}}{\sum_{i=1}^{N} x_{ic}^{\alpha} + \sum_{i=1}^{M} y_{jc}^{\alpha}} Nn - \sum_{i=1}^{N} x_{ic} \tag{6}$$

and in the case of group 2:

$$\sum_{j=1}^{M} E(U_{jc}) = \frac{\sum_{j=1}^{M} y_{jc}^{\alpha}}{\sum_{i=1}^{N} x_{ic}^{\alpha} + \sum_{j=1}^{M} y_{jc}^{\alpha}} Mm - \sum_{j=1}^{M} y_{jc} \ . \tag{7}$$

The Nash equilibrium investments of each player in both groups under cooperation will equal:

$$x_{ic}^{*} = \frac{\alpha n k^{\alpha} NM}{\left(Nk^{\alpha} + M\right)^2} \ \left(i = 1,...,N\right) \text{ and } \ y_{jc}^{*} = \frac{\alpha m k^{\alpha} NM}{\left(Nk^{\alpha} + M\right)^2} \ \left(j = 1,...,M\right) \tag{8}$$

and the equilibrium expected payoffs becomes:

$$E(U_{ic}^{*}) = \frac{Nnk^{\alpha}\left[Nk^{\alpha} + M(1-\alpha)\right]}{\left(Nk^{\alpha} + M\right)^2}$$

and (9)

$$E(U_{jc}^{*}) = \frac{Mm\left[M + Nk^{\alpha}(1-\alpha)\right]}{\left(Nk^{\alpha} + M\right)^2} .$$

2.3. Comparison

Let us now compare the investments in both of the cases and see if it is possible that, under cooperation, the players will invest less effort than they would without cooperation. The investment under cooperation is lower than with no cooperation, $x_i^{*} > x_{ic}^{*}$, if:

$$\frac{\alpha n k^{\alpha} M^{1-\alpha}}{N^{\alpha}\left(N^{1-\alpha}k^{\alpha} + M^{1-\alpha}\right)^2} > \frac{\alpha n k^{\alpha} NM}{\left(Nk^{\alpha} + M\right)^2} . \tag{10}$$

Writing (10) differently we obtain:

$$\left(\frac{Nk^{\alpha}+M}{N^{1-\alpha}k^{\alpha}+M^{1-\alpha}}\right)^2 > N^{1+\alpha}M^{\alpha} \tag{11}$$

and after some manipulation (see appendix) we obtain:

$$M\left(1-N^{0.5\alpha+0.5}M^{-0.5\alpha}\right) > Nk^{\alpha}\left(N^{0.5-0.5\alpha}M^{0.5\alpha}-1\right) \tag{12}$$

Inequality (11) may well hold. For example, if $\alpha = 0.5$, $N = 2$, $M = 32$ and $k = 4$, we obtain that inequality (11) becomes $18 > 16$. On the other hand, $y_j^* < y_{jc}^*$. This means that for group 1 we obtain higher levels of investment than without cooperation, and for group 2 we obtain Easy-riding.[8]

Proposition 1:

(a) a necessary condition for $x_i^* > x_{ic}^*$ *is* $M > N^{1+\frac{1}{\alpha}}$.

(b) a sufficient condition for $x_i^* > x_{ic}^*$ *is that M is sufficiently large*

For proof see appendix.

The question that comes up is why, for a sufficiently large number of players in group 2, *M*, the investments of each player in group 1 without cooperation is greater than with cooperation? It is important to note that the investments of the different players, in the same group, are substitutes but not perfect substitutes. Since they are not perfect substitutes they are to some extent complementary to each other in affecting the outcome. To answer the question above let us consider the following two situations:

1. Under cooperation, if group 2 is sufficiently large (*M* is sufficiently large), increasing the size of this group will decrease each of the players' investment, since each investments of each player is a substitute for the investment of a different player in the same group.[9] However, since the investments are not perfect substitutes, the increase in the size of the group overcompensates for the decrease in the investment of each player, therefore, the total investment will increase. This means that the central planner of group 2 takes advantage of the complementarity which exists between the investments of the players. By increasing the size of the group each player decreases his investment while the

[8] Since investments are not zero, we consider this to be easy-riding, see Cornes and Sandler (1984).

[9] $\frac{\partial\left(\sum_{1}^{M} y_{jc}^*\right)}{\partial M} > 0$. $\frac{\partial y_{jc}^*}{\partial M} < 0$ if and only if $k^{\alpha}N < M$.

total investment increases. As a result of the increase in the total investment by group 2, and the increase in its size, the central planner of group 1 "substantially" decreases the total investment of his group;[10] therefore, the investment of each player in his group decreases.[11]

2. In the case of no cooperation, and when the number of players in group 2 increases (*M* increases), each player in the group decreases their efforts (easy riding), thus the intensity of this reduction depends on the size of the group. If *M* is sufficiently large, then increases in the size of the group will also raise the level of free-riding and thus decrease the total investment made by the group.[12] This means that the effect of the decrease in the investments of each player dominates the increase in the size of the group. In this case the investments of the players are substitutes, in addition to the free-riding which already exists and which enhances the substitution between the players' investments. The increase in *M* results in a decrease in the investments of each player in group 1 and therefore in the total investments of group 1.[13] However, since there is no coordination in group 1, and each player easy-rides, the decrease in the investments of this group (and therefore by each player) will be "moderate" in comparison to the first case because group 2 has decreased its investments. This is also reflected in a "moderate" decrease in the winning probability.

Mathematically we can show these arguments by comparing the effect of a change in the size of group 2 (M) on the investments of each player in group 1 under the two

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[10] $\frac{\partial\left(\sum_{1}^{N} x_{ic}^{*}\right)}{\partial M} < 0$ and $\frac{\partial x_{ic}^{*}}{\partial M} < 0$ if and only if $k^{\alpha} N < M$.

[11] The “larger” group can take advantage of its position by increasing its investment, and, as a result, the “smaller” group decreases its investment. This result has the same type of flavor as the result presented in Epstein and Nitzan (2006) where increasing both players' stakes may increase the effort invested by the players.

[12] $\frac{\partial y_{j}^{*}}{\partial M} < 0$. $\frac{\partial\left(\sum_{j=1}^{M} y_{j}^{*}\right)}{\partial M} < 0$ if and only if $k^{\frac{\alpha}{1-\alpha}} N < M$.

[13] $\frac{\partial x_{i}^{*}}{\partial M} < 0$ and $\frac{\partial\left(\sum_{i=1}^{N} x_{i}^{*}\right)}{\partial M} < 0$ if and only if $k^{\frac{\alpha}{1-\alpha}} N < M$.

cases, cooperation (x_i^*) and no cooperation (x_{ic}^*). As we can see from the calculations presented in the appendix, we obtain that for $M \to \infty$, $\frac{\partial x_i^*}{\partial M} > \frac{\partial x_{ic}^*}{\partial M}$.

A lower boundary to the expression $\left(1+\frac{1}{\alpha}\right)$ is 2; thus, from Proposition 1 we may conclude the following Corollaries:

Corollary 1: If $x_i^* > x_{ic}^*$ then $M > N^2$.

Corollary 2: If $M \le N^{1+\frac{1}{\alpha}}$ then $x_i^* \le x_{ic}^*$ independent of the values of *m* and *n*.

Corollary 3: If $x_i^* > x_{ic}^*$ then $y_j^* < y_{jc}^*$.

Corollary 3 is a direct outcome of Corollary 1. Let us consider the following proof using a contradictory argument: Assume that when $x_i^* > x_{ic}^*$ it holds that $y_j^* \ge y_{jc}^*$. According to Proposition 1 part (a), since $x_i^* > x_{ic}^*$ then $M > N^{1+\frac{1}{\alpha}}$ or $M^{\frac{\alpha}{1+\alpha}} > N$. Also, if $y_j^* \ge y_{jc}^*$, in a similar way to the proof of proposition 1, we would obtain $N > M^{1+\frac{1}{\alpha}}$. Thus it must hold that $M^{\frac{\alpha}{1+\alpha}} > N > M^{1+\frac{1}{\alpha}}$ (or $M^{\frac{\alpha}{1+\alpha}} > M^{1+\frac{1}{\alpha}}$). This inequality is impossible. Thus, if the investment of a group under non-cooperative is higher than under cooperative, the opposite would hold for the other group.

Proposition 2: *If* $x_i^* > x_{ic}^*$ *then* $E(U_i^*) > E(U_{ic}^*)$.

For proof see appendix**.**

Proposition 2 states that in the case where group 1 invests more effort than that which would have been invested under cooperation, we would obtain that the expected payoff of each player in group 1 would be higher than that of cooperation ($E(U_i^*) > E(U_{ic}^*)$). Let us explain this result. From proposition 1 and corollary 3 we obtain that when we have cooperation, and as group 2 is sufficiently large, the central planner of group 2 uses its advantage, with regard to the group's size, increasing the effort of each player in the group relatively to the effort they would have invested under no cooperation. Therefore, moving from cooperation to no cooperation, the winning probability of group 2 decreases, and the winning probability of group 1 increases.[14] Indeed, the effort of each player in group 1 has increased (proposition 1); however, the increase in the probability dominates the increase in efforts and thus the expected payoff is also increased.[15]

To conclude: Our main results show that for one of the groups, the sufficient condition to invest more under no cooperation rather than under cooperation is that the number of players in the other group has to be sufficiently large. Moreover, in the case where each player in one of the groups invests more effort than would have been invested under cooperation, we would obtain that the expected payoff of each player in this group would be higher than in the case of cooperation.

[14] The winning probability of group 1 increases from moving from cooperation to no cooperation if and only if $M > N$.

[15] A question that comes to mind is whether the rent dissipation under cooperation will be higher than without cooperation: $Nx_i^* + My_j^* < Nx_{ic}^* + My_{jc}^*$. It can be shown that a sufficient condition for this would be that $N^2 \geq M \geq N^{0.5}$. However it is not clear that the rent dissipation under cooperation will be higher than without cooperation. For example, for $k = 1000$, $\alpha = 0.5$, $N = 2$ and $M = 2000$ this inequality will not hold.

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Appendix

Proof of Proposition 1:

Part (a): From (11) we take the square root from both sides and obtain

$$\frac{Nk^{\alpha}+M}{N^{1-\alpha}k^{\alpha}+M^{1-\alpha}} > N^{0.5+0.5\alpha}M^{0.5\alpha}$$

Multiplying both sides by $\left(N^{1-\alpha}k^{\alpha}+M^{1-\alpha}\right)$ we obtain

$$Nk^{\alpha}+M > N^{0.5+0.5\alpha}M^{0.5\alpha}\left(N^{1-\alpha}k^{\alpha}+M^{1-\alpha}\right)$$

thus

$$Nk^{\alpha}+M > N^{1.5-0.5\alpha}M^{0.5\alpha}k^{\alpha}+N^{0.5\alpha+0.5}M^{1-0.5\alpha}.$$

Rewriting the inequality

$$M-N^{0.5\alpha+0.5}M^{1-0.5\alpha} > N^{1.5-0.5\alpha}M^{0.5\alpha}k^{\alpha}-Nk^{\alpha}$$

thus

$$M\left(1-N^{0.5\alpha+0.5}M^{-0.5\alpha}\right) > Nk^{\alpha}\left(N^{0.5-0.5\alpha}M^{0.5\alpha}-1\right).$$

Since $0<\alpha<1$ then $N^{0.5-0.5\alpha}M^{0.5\alpha}>1$, therefore the right hand side of the above inequality is positive. A necessary condition for (11) is that the left hand side is also positive; thus, $1>N^{0.5\alpha+0.5}M^{-0.5\alpha}$ which is identical to $M>N^{1+\frac{1}{\alpha}}$.

Part (b) - Dividing (11) by $M^{2\alpha}$ we obtain

$$\frac{1}{M^{2\alpha}}\left(\frac{Nk^{\alpha}+M}{N^{1-\alpha}k^{\alpha}+M^{1-\alpha}}\right)^2 > \frac{N^{1+\alpha}}{M^{\alpha}}.$$

Rewriting the inequality we obatin

$$\left[\frac{Nk^{\alpha}+M}{M^{\alpha}\left(N^{1-\alpha}k^{\alpha}+M^{1-\alpha}\right)}\right]^2 > \frac{N^{1+\alpha}}{M^{\alpha}}$$

thus

$$\left(\frac{Nk^{\alpha}+M}{M^{\alpha}N^{1-\alpha}k^{\alpha}+M}\right)^2 > \frac{N^{1+\alpha}}{M^{\alpha}}$$

Dividing the nominator and denominator in the brackets of the LHS by *M* we obtain

$$\left(\frac{\dfrac{Nk^{\alpha}}{M}+1}{\dfrac{N^{1-\alpha}k^{\alpha}}{M^{1-\alpha}}+1}\right)^2 > \frac{N^{1+\alpha}}{M^{\alpha}}.$$

As we can see for $M\to\infty$ the LHS converges to 1 and the RHS to 0. ∎

Comparing $\frac{\partial x_i^*}{\partial M}$ to $\frac{\partial x_{ic}^*}{\partial M}$:

Let us establish conditions so that $\frac{\partial x_i^*}{\partial M} > \frac{\partial x_{ic}^*}{\partial M}$. Since $\frac{\partial x_{ic}^*}{\partial M} = \frac{\alpha n k^\alpha N\left(Nk^\alpha - M\right)}{\left(Nk^\alpha + M\right)^3}$ and $\frac{\partial x_i^*}{\partial M} = \frac{\alpha n k^\alpha (1-\alpha)\left(N^{1-\alpha}k^\alpha - M^{1-\alpha}\right)}{(NM)^\alpha\left(N^{1-\alpha}k^\alpha + M^{1-\alpha}\right)^3}$ thus $\frac{\partial x_i^*}{\partial M} > \frac{\partial x_{ic}^*}{\partial M}$ hold if $\frac{\alpha n k^\alpha (1-\alpha)\left(N^{1-\alpha}k^\alpha - M^{1-\alpha}\right)}{(NM)^\alpha\left(N^{1-\alpha}k^\alpha + M^{1-\alpha}\right)^3} > \frac{\alpha n k^\alpha N\left(Nk^\alpha - M\right)}{\left(Nk^\alpha + M\right)^3}$. Rewriting this expression we obtain $\left(\frac{Nk^\alpha + M}{N^{1-\alpha}k^\alpha + M^{1-\alpha}}\right)^3 > \frac{N(NM)^\alpha\left(M - Nk^\alpha\right)}{(1-\alpha)\left(M^{1-\alpha} - N^{1-\alpha}k^\alpha\right)}$. Multiplying both sides by $M^{-2\alpha}$ we obtain that $M^{-2\alpha}\left(\frac{Nk^\alpha + M}{N^{1-\alpha}k^\alpha + M^{1-\alpha}}\right)^3 > M^{-2\alpha}\frac{N(NM)^\alpha\left(M - Nk^\alpha\right)}{(1-\alpha)\left(M^{1-\alpha} - N^{1-\alpha}k^\alpha\right)}$, and rewriting this expression we obtain $\left(\frac{\frac{Nk^\alpha}{M^{1-\frac{1}{3}\alpha}} + M^{\frac{1}{3}\alpha}}{\frac{N^{1-\alpha}k^\alpha}{M^{1-\alpha}} + 1}\right)^3 > \frac{N^{1+\alpha}\left(1 - \frac{Nk^\alpha}{M}\right)}{(1-\alpha)\left(1 - \frac{N^{1-\alpha}k^\alpha}{M^{1-\alpha}}\right)}$. If $M \to \infty$ the LHS converges to infinity $\left(M^\alpha\right)$ while the RHS converges to $\frac{N^{1+\alpha}}{1-\alpha}$. Thus, if M is sufficiently large it holds that $\frac{\partial x_i^*}{\partial M} > \frac{\partial x_{ic}^*}{\partial M}$.

Proof of Proposition 2:

$E(U_i^*) > E(U_{ic}^*)$ is identical to $\left(\frac{Nk^\alpha + M}{N^{1-\alpha}k^\alpha + M^{1-\alpha}}\right)^2 > \frac{N^{1+\alpha}\left[Nk^\alpha + M(1-\alpha)\right]}{\left[N^{2-\alpha}k^\alpha + M^{1-\alpha}(N-\alpha)\right]}$. Also $x_i^* > x_{ic}^*$ is identical to inequality (11). We will show that the right hand side of inequality (11) is larger than the right hand side of $\left(\frac{Nk^\alpha + M}{N^{1-\alpha}k^\alpha + M^{1-\alpha}}\right)^2 > \frac{N^{1+\alpha}\left[Nk^\alpha + M(1-\alpha)\right]}{\left[N^{2-\alpha}k^\alpha + M^{1-\alpha}(N-\alpha)\right]}$, and by that we have proven our proposition. $N^{\alpha+1}M^\alpha > \frac{N^{1+\alpha}\left[Nk^\alpha + M(1-\alpha)\right]}{\left[N^{2-\alpha}k^\alpha + M^{1-\alpha}(N-\alpha)\right]}$ is identical to

$Nk^{\alpha}\left(N^{1-\alpha}M^{\alpha}-1\right)>M\left(1-N\right)$. The left hand side of this inequality is positive while the right hand side is negative. ■