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A legislative bargaining approach to earmarked public expenditures

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Abstract

This paper develops a model of legislative spending in which revenues can be spent through earmarks or a general fund. Legislative choice is modeled as a Baron and Ferejohn style legislative bargaining game. The novel approach is to model the bargaining process as a two-stage game reflecting the reality that earmarked expenditures precede general fund appropriations. This drives the result that all revenue is spent by way of earmarking leaving no revenue in the general fund.

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1. Introduction

Earmarking has recently become a popular buzz word in the political press usually referring to some form of pork barrel spending. Throughout this paper I will refer to the traditional public finance definition of earmarking: the dedication of a specific revenue (tax) to a specific purpose. This definition highlights a fundamental feature of earmarking: earmarked revenues are diverted into dedicated accounts that circumvent the usual legislative appropriations process whereby revenues go into a general fund and are then distributed. Thus, earmarking expenditure decisions must precede general fund expenditure decisions. This note develops a model of earmarking in a legislative bargaining game in the style of Baron and Ferejohn (1989) whereby earmarking decisions precede general fund spending decisions.

Earmarking has long been discussed in the public finance literature yet its treatment has been far from exhaustive. The classic paper in this literature is Buchanan (1963). Buchanan uses a median voter approach to public decision making to analyze how the different institutions of earmarking or general fund financing influence public spending. This approach is continued with the papers of Browning (1975), Goetz (1968), and Goetz and McKnew (1972) and more recently by Athanassakos (1990).

The most recent earmarking research takes a game theoretic approach. Dhillon and Perroni (2001) and Bös (2000) take an information economics approach to earmarking. Leblanc, Snyder and Tripathi (2000) briefly address earmarking inside a bargaining model of public investment. Brett and Keen (2000) present a model of earmarking in the context of a signalling game. Jackson (2011) presents a model of earmarking based on partisan politics and separation of powers reminiscent of the policy insulation literature(Moe (1989), Moe (1990), Moe (1991), Figueiredo (2002), and Figueiredo (2003)). Anesi (2006) develops a two period model of earmarking with endogenous elections whereby incumbents can use earmarking as a means to commit themselves to a less preferred policy in order to gain electoral advantage. See Jackson (2011) for a detailed review of the earmarking literature.

This paper is organized as follows. Section 2 presents the model with section 3 giving the main results. The paper concludes in section 4. All proofs are relegated to the appendix.

2. The Model

A legislature with 3 members must decide how to spend a given amount of tax revenue. Tax revenue is given exogenously and is denoted by R. I refer to a generic legislator as legislator i and it is often convenient to refer to legislators other than i as j or k. Revenue can be spent on distribution to each legislator's district and on a public good. Let x_i denote the amount of distribution made to legislator i's district. Spending on the public good is denoted by x_y. Production of the public good is represented by a constant returns to scale cost function with constant K so that when spending on the public good is x_y the level of public good provision is $y = \frac{x_y}{K}$. Spending levels are determined in a two stage bargaining game with stage one being the earmarking stage and the second stage being the general fund stage.

Each legislator has preferences given by a utility function which depends on the quantity of funds distributed to a legislator's district and level of public good provision. To keep the model simple all legislators have identical preferences regarding public good provision. Legislators disagree, however, when it comes to spending directed at legislator's individual districts. Utility takes a quasilinear form as given in (1) below.

$$U_i(x_i, y) = x_i + Bln(y) \tag{1}$$

In the earmarking stage, one legislator is randomly selected (with equal probability on each) to make an earmarking proposal. This proposal contains a proposed amount of distribution to each legislator's district and a spending proposal for the public good. Denote the earmarking proposal made by legislator i when she is recognized by $x^{ie} = \left(x_i^{ie}, x_j^{ie}, x_k^{ie}, x_y^{ie}\right)$. This proposal is feasible if all proposed expenditures are positive and sum to no more than available revenue. Denote the feasible earmarking proposal set by X given below

$$X \equiv \left\{ x^{ie} \in \mathbb{R}^4 \;\middle|\; x_l^{ie} \geq 0 \; \forall \; l \; and \; \sum_{l=1}^{3} x_l^{ie} + x_y^{ie} \leq R \right\}. \tag{2}$$

The proposal, x^{ie}, is voted on using majority rule against a status quo, denoted q, which must be feasible itself. Denote the realization, after voting, in the earmarking stage by x^e; if i's proposal is passed then $x^e = x^{ie}$ and if the proposal is not passed then $x^e = q$. At the earmarking stage, proposal and vote decisions are made by evaluating expected utility.

In the general fund stage one legislator is randomly recognized to make a general fund proposal. This is a proposal on how to spend any revenue left over after the earmarking stage; it is denoted $x^i = \left(x_i^i, x_j^i, x_k^i, x_y^i\right)$. This proposal is feasible if all proposed expenditures are positive and expenditure doesn't exceed revenue available. The general fund feasible set is denoted $X^{gf}(x^e) \subset \mathbb{R}^4$ as given formally in (3).

$$X^{gf}(x^e) \equiv \left\{ x^i \in \mathbb{R}^4 \;\middle|\; x_l^i \geq 0 \; \forall \; l \; and \; \sum_{l=1}^{3} x_l^i + x_y^i \leq R - \left(\sum_{l=1}^{3} x_l^e + x_y^e \right) \right\} \tag{3}$$

This proposal is voted on against the status quo that all general fund revenue is spent on the public good. Denote the realization, after voting, in the general fund stage by x^{gf}. After voting on the general fund proposal each legislator now gets utility with $x_i = x_i^e + x_i^{gf}$ for each $i = 1, 2, 3$ and $y = \frac{x_y^e + x_y^{gf}}{K}$.

A strategy consists of: an earmarking proposal plan, $x^{ie}(q) \subset \mathbb{R}^4$; A general fund proposal plan, $x^i(x^e) \subset \mathbb{R}^4$; probability distributions that determine which player is bought off to be a part of a majority coalition; voting rules over earmarking and general fund proposals. The set $M^{gf}(x^i(x^e))$ specifies the set of all legislators who vote to pass general fund proposal $x^i(x^e)$ and the set $M(x^{ie}(q))$ denotes all legislators who vote to pass earmarking proposal $x^{ie}(q)$. The equilibrium concept employed is the standard subgame perfect Nash equilibrium.

It is easy to verify that many unrealistic equilibria will exist under the assumption of simple Nash voting behavior. For example, it is an equilibrium for each legislator to take a voting strategy in which all proposals are voted against. I wish to rule out this type of unrealistic Nash voting behavior and do so by imposing the assumption of weak dominance in voting strategies throughout. Voting strategies satisfy weak dominance when a legislator votes to pass any proposal that gives her at least as much (expected) utility as not passing the proposal. It is obvious that weakly dominant voting strategies are Nash voting strategies.[1]

3. Results

I begin by identifying optimal general fund proposal strategies which must be a solution to the following constrained optimization problem.

$$x^{i*}(x^e) \equiv \underset{x^i(x^e)}{\arg\max}\; x_i^e + x_i^i(x^e) + B\ln\left(\frac{x_y^e + x_y^i(x^e)}{K}\right) \tag{4}$$

$$\begin{array}{rl} & s.t. \\ a). & x^i(x^e) \in X^{gf}(x^e) \\ b). & i \in M^{gf}(x^i(x^e)) \\ c). & \left|M^{gf}(x^i(x^e))\right| \geq 2. \end{array}$$

It is clear from the objective function that any solution will have to spend all available revenue; constraint a) will be binding. Likewise, it is optimal to induce a minimally sized coalition to vote for the proposal and it is optimal to do so in a minimal way so that the chosen coalition memeber $j \in M(x^i(x^e))$ will be indifferent between passing the proposal or not. Legislator i is entirely indifferent to which legislator she will buy off; they are equally costly to buy off in the general fund stage. Thus, I let legislator i assign a probability of one half to each legislator to be chosen to maintain symmetry. The following equations describe the optimal general fund proposal strategies referred to in proposition 1 below. The sets X^1, X^2, X^3, and X^4 cover X and are defined in the appendix.

[1] The sets $M^{gf}(x^i(x^e))$ and $M(x^{ie}(q))$ are written formally in the appendix under the assumption weakly dominant voting.

$$x_i^{i*}(x^e) = \begin{cases} R - \sum_{l=1}^{3} x_l^e - 2B & if\ x^e \in X^1 \\ \quad -B\left[\ln\left(\frac{R-\sum_{l=1}^{3} x_l^e}{K}\right) - \ln\left(\frac{2B}{K}\right)\right] & \\ 0 & if\ x^e \in X^2 \bigcup X^4 \\ R - \sum_{l=1}^{3} x_l^e - x_y^e & if\ x^e \in X^3 \\ \quad -B\left[\ln\left(\frac{R-\sum_{l=1}^{3} x_l^e}{K}\right) - \ln\left(\frac{x_y^e}{K}\right)\right] & \end{cases} \tag{5}$$

$$x_j^{i*}(x^e) = \begin{cases} B\left[\ln\left(\frac{R-\sum_{l=1}^{3} x_l^e}{K}\right) - \ln\left(\frac{2B}{K}\right)\right] & if\ x^e \in X^1 \\ 0 & if\ x^e \in X^2 \bigcup X^4 \\ B\left[\ln\left(\frac{R-\sum_{l=1}^{3} x_l^e}{K}\right) - \ln\left(\frac{x_y^e}{K}\right)\right] & if\ x^e \in X^3 \end{cases} \tag{6}$$

$$x_k^{i*}(x^e) = 0 \tag{7}$$

$$x_y^{i*}(x^e) = \begin{cases} 2B - x_y^e & if\ x^e \in X^1 \\ R - \sum_{l=1}^{3} x_l^e - x_y^e & if\ x^e \in X^2 \bigcup X^4 \\ 0 & if\ x^e \in X^3 \end{cases} \tag{8}$$

Proposition 1. $x^{i*}(x^e) = \left(x_i^{i*}(x^e), x_j^{i*}(x^e), x_k^{i*}(x^e), x_y^{i*}(e)\right)$ *defined by equations 5, 6, 7, and 8 form optimal general fund proposals.*

In a subgame perfect equilibrium the legislator chosen to make a proposal will take general fund proposals from proposition 1 as given and choose an earmarking proposal to maximize expected utility. This is formally stated in the earmarking constrained optimization problem defined below.

$$x^{ie*}(q) \equiv \arg\max_{x^{ie}(q)} \sum_{j=1}^{3} \frac{1}{3}\left(x_i^{ie}(q) + x_i^{j*}(x^{ie}(q)) + B\ln\left(\frac{x_y^{ie}(q) + x_y^{j*}(x^{ie}(q))}{K}\right)\right) \tag{9}$$

$$\begin{aligned} &s.t. \\ &a).\ \ x^{ie}(q) \in X \\ &b).\ \ i \in M(x^{ie}(q)) \\ &c).\ \ |M(x^{ie}(q))| \geq 2. \end{aligned}$$

It is not immediately apparent that a solution to the earmarking constrained optimization problem will spend all revenue. Theorem 1 demonstrates that if a solution exists that does not earmark all revenue then there must be another solution in which all revenues are earmarked leaving no money on the table to be spent in the general fund stage of the game.

Theorem 1. *If a solution to the earmarking constrained optimization problem exists, then there is always a solution in which all revenue is earmarked.*

4. Conclusion

I develop a legislative bargaining model in which public spending is accomplished via earmarking to the neglect of a general fund. The fact that earmarking decisions must precede general fund decisions is what makes earmarks appealing. To not earmark is to give up the opportunity to secure ones place in a winning coalition. The recognized legislator must compensate another for the forgone opportunity to make a proposal in the general fund stage thus securing majority support for the earmarked proposal. However, there is no requirement in the model that an earmarking proposal spend all revenue. It is possible to buy votes to pass an earmarking proposal by either earmarking revenue to coalition members or by leaving unspent revenue to be allocated in the general fund stage of the game. This feature of the model constitutes a significant deviation from the traditional bargaining literature (Baron and Ferejohn (1989), Rubinstein (1982)) which requires proposals to allocate the entire economic pie and bargaining coming to an end with an accepted proposal.

This analysis suggests that when the institutions of legislative decision making allow earmarking to occur, it will be done. This is not consistent with the available data on earmarking. Though some US states earmark large portions of their revenue, none earmark it all. This is likely due to frictions that prevent such behavior from occurring as may arise from partisan politics or the separation of powers among branches of government as argued by Jackson (2011) but are absent in the model of this paper. Future work on earmarking should incorporate a richer political structure and address the dynamic implications of earmarking.

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A. Appendix

A.1 Voting Strategies

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The sets $M^{gf}(x^i(x^e))$ and $M(x^{ie}(q))$ can be written formally as below for voting strategies that satisfy weak dominance.

$$M^{gf}(x^i(x^e)) \equiv \left\{ j \in (1,2,3) \middle| x^i_j(x^e) \geq B\left(\ln\left(\frac{R - \sum_{k=1}^3 x^i_k(x^e)}{K}\right) - \ln\left(\frac{x^e_y + x^i_y(x^e)}{K}\right)\right)\right\}$$

$$M(x^{ie}(q)) \equiv \left\{ j \in (1,2,3) \middle| x^{ie}_j(q) + \sum_{k=1}^3 \frac{x^k_j(x^{ie}(q))}{3} + \sum_{k=1}^3 \frac{1}{3}\left(B\ln\left(\frac{x^{ie}_y(q) + x^k_y(x^{ie}(q))}{K}\right)\right) \geq \right.$$

$$\left. q_j + \sum_{k=1}^3 \frac{x^k_j\left(R - \sum_{l=1}^3 q_l - q_y\right)}{3} + \sum_{k=1}^3 \frac{1}{3}\left(B\ln\left(\frac{q_y + x^k_y\left(R - \sum_{l=1}^3 q_l - q_y\right)}{K}\right)\right)\right\}$$

A.2 Optimal GF Proposal Strategy

Divide the feasible set X into 4 regions: X^1, X^2, X^3, and X^4 as follows

$$X^1 \equiv \left\{ x \in X \middle| 2B \geq x_y \textit{ and } R - \sum_{l=1}^3 x_l - 2B > 2B\left[\ln\left(\frac{R - \sum_{l=1}^3 x_l}{K}\right) - \ln\left(\frac{2B}{K}\right)\right]\right\}$$

$$X^2 \equiv \left\{ x \in X \middle| 2B \geq x_y \textit{ and } R - \sum_{l=1}^3 x_l - 2B \leq 2B\left[\ln\left(\frac{R - \sum_{l=1}^3 x_l}{K}\right) - \ln\left(\frac{2B}{K}\right)\right]\right\}$$

$$X^3 \equiv \left\{ x \in X \middle| 2B \leq x_y \textit{ and } R - \sum_{l=1}^3 x_l - x_y > 2B\left[\ln\left(\frac{R - \sum_{l=1}^3 x_l}{K}\right) - \ln\left(\frac{x_y}{K}\right)\right]\right\}$$

$$X^4 \equiv \left\{ x \in X \middle| 2B \leq x_y \textit{ and } R - \sum_{l=1}^3 x_l - x_y \leq 2B\left[\ln\left(\frac{R - \sum_{l=1}^3 x_l}{K}\right) - \ln\left(\frac{x_y}{K}\right)\right]\right\}.$$

The sets X^1, X^2, X^3, and X^4 have been constructed to cover the space X; $\bigcup_{i=1}^4 X^i = X$.

A.3 Proofs

Proof of Proposition 1. The proof is broken into four parts; each reflecting a different location of the earmarking outcome $x^e \in X$.

1. Let $x^e \in X^1$.

   The utility that j receives from passage of the proposal is

$$x^e_j + x^i_j(x^e) + B\ln\left(\frac{x^e_y + x^i_y(x^e)}{K}\right).$$

The utility to j from rejecting the proposal is

$$x_j^e + B \ln\left(\frac{R - \sum_{l=1}^3 x_l^e}{K}\right).$$

Equating the two we derive the distribution made to legislator j

$$x_j^i(x^e) = B\left(\ln\left(\frac{R - \sum_{l=1}^3 x_l^e}{K}\right) - \ln\left(\frac{x_y^e + x_y^i(x^e)}{K}\right)\right).$$

Legislator i's optimal spending on the public good can be characterized as the solution to an unconstrained concave optimization problem maximizing i's utility choosing spending on the public good.

$$x_y^{i*}(x^e) = \arg\max_{x_y^i(x^e)} R - \sum_{l=1}^3 x_l^e - x_y^e - x_y^i(x^e) - B\ln\left(\frac{R - \sum_{l=1}^3 x_l^e}{K}\right) + 2B\ln\left(\frac{x_y^e + x_y^i(x^e)}{K}\right)$$

The solution to this problem is $x_y^{i*}(x^e) = 2B - x_y^e$. Substitution directly implies that i's distribution to j will be

$$x_j^{i*}(x^e) = B\left(\ln\left(\frac{R - \sum_{l=1}^3 x_l^e}{K}\right) - \ln\left(\frac{2B}{K}\right)\right)$$

with the remaining revenue going to i

$$x_i^{i*}(x^e) = R - \sum_{l=1}^3 x_l^e - 2B - B\left(\ln\left(\frac{R - \sum_{l=1}^3 x_l^e}{K}\right) - \ln\left(\frac{2B}{K}\right)\right).$$

These are proposals are consistent with equations 1-4. By construction we know that j will vote to pass the proposal. To show that this is the optimal proposal two more conditions must be demonstrated; the proposal must be feasible and individually rational for i. It is trivial to show that the proposal is feasible.

Individual rationality for i (i votes for her own proposal) requires

$$R - \sum_{l=1}^3 x_l^e - 2B \geq 2B\left(\ln\left(\frac{R - \sum_{l=1}^3 x_l^e}{K}\right) - \ln\left(\frac{2B}{K}\right)\right).$$

This is true by the assumption that $x^e \in X^1$.

2. Let $x^e \in X^2$.

Suppose that

$$R - \sum_{l=1}^{3} x_l^e - 2B = 2B\left(\ln\left(\frac{R - \sum_{l=1}^{3} x_l^e}{K}\right) - \ln\left(\frac{2B}{K}\right)\right).$$

In this case, the proposal derived in part 1 above is still an optimal proposal. However, this proposal yields the proposer the same utility as the status quo. Therefore, the status quo (all revenue is spent on the public good) is also an optimal proposal.

Now suppose that

$$R - \sum_{l=1}^{3} x_l^e - 2B < 2B\left(\ln\left(\frac{R - \sum_{l=1}^{3} x_l^e}{K}\right) - \ln\left(\frac{2B}{K}\right)\right).$$

Further suppose that a feasible proposal, $\hat{x}^i(x^e)$, exists which gives the proposer, i, more utility than the status quo and j at least as much utility as the status quo. This implies the following inequalities

$$\hat{x}_i^i(x^e) > B\ln\left(\frac{R - \sum_{l=1}^{3} x_l^e}{K}\right) - B\ln\left(\frac{x_y^e + \hat{x}_y^i(x^e)}{K}\right)$$

and

$$\hat{x}_j^i(x^e) \geq B\ln\left(\frac{R - \sum_{l=1}^{3} x_l^e}{K}\right) - B\ln\left(\frac{x_y^e + \hat{x}_y^i(x^e)}{K}\right).$$

Feasibility of the proposal can be written as follows

$$R - \sum_{l=1}^{3} x_l^e - x_y^e - 2B\left(\ln\left(\frac{R - \sum_{l=1}^{3} x_l^e}{K}\right) - \ln\left(\frac{x_y^e + \hat{x}_y^i(x^e)}{K}\right)\right) > \hat{x}_y^i(x^e). \qquad (10)$$

Notice that when $\hat{x}_y^i(x^e) = 2B - x_y^e$ combined with the assumption that $x^e \in X^2$, equation (10) cannot hold. The proof proceeds by showing that there is no feasible $\hat{x}_y^i(x^e)$ that satisfies equation (10). This contradiction then implies that no feasible proposal can be passed which offers the proposer more utility than the status quo.

Differentiate both sides of equation (10) with respect to $\hat{x}_y^i(x^e)$. The derivative of the right hand side is one and the derivative of the left hand side is $\frac{2B}{x_y^e + \hat{x}_y^i(x^e)}$. Notice that when $\hat{x}_y^i(x^e) = 2B - x_y^e$ the derivative of the left hand side is equal to one and when $\hat{x}_y^i(x^e) > 2B - x_y^e$ the derivative of the l.h.s. is positive but less than one. Increasing $\hat{x}_y^i(x^e)$ from $\hat{x}_y^i(x^e) = 2B - x_y^e$ will increase the lhs of equation (10) at a slower rate than the rhs increases. Thus, there is no $\hat{x}_y^i(x^e) > 2B - x_y^e$ that can satisfy equation

(10).

Now consider $0 \leq \hat{x}_y^i(x^e) < 2B - x_y^e$. Note that the derivative of the l.h.s. of equation (10) is greater than one. Decreasing $\hat{x}_y^i(x^e)$ from $\hat{x}_y^i(x^e) = 2B - x_y^e$ will decrease the lhs of equation (10) at a faster rate than the rhs will decrease. Thus, there is no $\hat{x}_y^i(x^e)$ with $0 \leq \hat{x}_y^i(x^e) < 2B - x_y^e$ that can satisfy equation (10).

With no $\hat{x}_y^i(x^e)$ feasible that can satisfy equation (10), then there is no feasible proposal that can be passed by a majority that gives more utility to the proposer than the status quo. The the optimal proposal is the status quo of spending all revenue on the public good.

3. Let $x^e \in X^3$. The utility of the proposer as a function of $x_y^i(x^e)$ can be written as follows

$$x_i^e + R - \textstyle\sum_{l=1}^3 x_l^e - x_y^e - x_y^i(x^e) - B\left(\ln\left(\frac{R-\sum_{l=1}^3 x_l^e}{K}\right) - \ln\left(\frac{x_y^e + x_y^i(x^e)}{K}\right)\right)$$
$$+\ln\left(\frac{x_y^e + x_y^i(x^e)}{K}\right).$$

The marginal utility to the proposer with respect to $x_y^i(x^e)$ is $\frac{2B}{x_y^e + x_y^i(x^e)} - 1$.

The sign of marginal utility at $x_y^i(x^e) = 0$ is less than or equal to zero because $x_y^e \geq 2B$. Therefore, the optimal feasible level of public good spending for i to propose is zero, $x_y^{i*}(x^e) = 0$.

This directly gives the distribution paid to j to buy her vote,

$$x_j^{i*}(x^e) = B\left(\ln\left(\frac{R - \sum_{l=1}^3 x_l^e}{K}\right) - \ln\left(\frac{x_y^e}{K}\right)\right),$$

and the remainder of available revenue goes to the proposer

$$x_i^{i*}(x^e) = R - \sum_{l=1}^{3} x_l^e - x_y^e - B\left(\ln\left(\frac{R - \sum_{l=1}^3 x_l^e}{K}\right) - \ln\left(\frac{x_y^e}{K}\right)\right).$$

It is trivial to show that this proposal is feasible. The proposal was constructed in a way that legislator j will vote to pass the proposal. It remains to verify that this proposal satisfies individual rationality for i which requires the following equation

$$R - \textstyle\sum_{l=1}^3 x_l^e - x_y^e - B\left(\ln\left(\frac{R-\sum_{l=1}^3 x_l^e}{K}\right) - \ln\left(\frac{x_y^e}{K}\right)\right)$$
$$\geq B\left(\ln\left(\frac{R-\sum_{l=1}^3 x_l^e}{K}\right) - \ln\left(\frac{x_y^e}{K}\right)\right).$$

This holds because $x^e \in X^3$.

4. Let $x^e \in X^4$. Suppose that there exists a feasible proposal, $\hat{x}^i(x^e)$, such that the

proposer gets strictly more utility than with the status quo and legislator j gets at least as much utility as under the status quo. Such a proposal must satisfy

$$\hat{x}_i^i(x^e) > B\ln\left(\frac{R-\sum_{l=1}^{3}x_l^e}{K}\right) - B\ln\left(\frac{x_y^e+\hat{x}_y^i(x^e)}{K}\right)$$

$$\hat{x}_j^i(x^e) \geq B\ln\left(\frac{R-\sum_{l=1}^{3}x_l^e}{K}\right) - B\ln\left(\frac{x_y^e+\hat{x}_y^i(x^e)}{K}\right)$$

and

$$R-\sum_{l=1}^{3}x_l^e - x_y^e \geq \hat{x}_i^i(x^e)+\hat{x}_j^i(x^e)+\hat{x}_y^i(x^e).$$

Combining these three equations yields the following

$$R-\sum_{l=1}^{3}x_l^e - x_y^e > 2B\left(\ln\left(\frac{R-\sum_{l=1}^{3}x_l^e}{K}\right) - \ln\left(\frac{x_y^e+\hat{x}_y^i(x^e)}{K}\right)\right)+\hat{x}_y^i(x^e).$$

Since $x^e \in X^4$ the following holds

$$2B\left(\ln\left(\frac{x_y^e+\hat{x}_y^i(x^e)}{K}\right) - \ln\left(\frac{x_y^e}{K}\right)\right) > \hat{x}_y^i(x^e). \tag{11}$$

I now show that there is no feasible $\hat{x}_y^i(x^e)$ which satisfies equation (11).

When $\hat{x}_y^i(x^e)=0$ both sides of equation (11) are zero; the equation cannot be satisfied. All possible solutions require that $\hat{x}_y^i(x^e)>0$. Differentiate both sides of equation (11) with respect to $\hat{x}_y^i(x^e)$. The derivative of the r.h.s is one and the derivative of the l.h.s. is $\frac{2B}{x_y^e+\hat{x}_y^i(x^e)}$. Because $x_y^e \geq 2B$ it follows that the derivative of the l.h.s is less than one. Therefore, when increasing $\hat{x}_y^i(x^e)$ from zero, the left hand side of equation (11) increases more slowly than does the right hand side. Hence, there is no feasible $\hat{x}_y^i(x^e)$ which satisfies equation (11). Thus, when $x^e \in X^4$ it is optimal for the proposer to propose the status quo of all revenue being spent on the public good.

□

Proof of Theorem 1. Expected utility for the earmarking proposer is

$$EU_i(x^{ie})) = \sum_{j=1}^{3}\frac{1}{3}\left(x_i^{ie}+x_i^{j*}(x^{ie})+B\ln\left(\frac{x_i^{ie}+x_y^{j*}(x^{ie})}{K}\right)\right).$$

Let $\hat{x}^{ie}(q)$ be a solution to the earmarking constrained optimization problem. Then

$$EU_i(\hat{x}^{ie}(q)) = \hat{x}_i^{ie}(q) + \sum_{j=1}^{3} \frac{1}{3}\left(x_i^{j*}(\hat{x}^{ie}(q)) + B\ln\left(\frac{\hat{x}_y^{ie}(q) + x_y^{j*}(\hat{x}^{ie}(q))}{K}\right)\right)$$

Further, suppose that $\hat{x}^{ie}(q) \in X^2 \bigcup X^4$ and $\sum_{l=1}^{3} \hat{x}_l^{ie}(q) + \hat{x}_y^{ie}(q) < R$.

Let $\epsilon = R - \left(\sum_{l=1}^{3} \hat{x}_l^{ie}(q) + \hat{x}_y^{ie}(q)\right)$ and define $x^{ie*}(q)$ as follows

$$x^{ie*}(q) \equiv \begin{cases} \hat{x}_i^{ie}(q) \\ \hat{x}_j^{ie}(q) \\ \hat{x}_k^{ie}(q) \\ \hat{x}_y^{ie}(q) + \epsilon \end{cases}$$

The expected utility from the modified earmarking proposal $x^{ie*}(q)$ to the proposer can be written as

$$EU_i\left(x^{ie*}(q)\right) = \hat{x}_i^{ie}(q) + B\ln\left(\frac{\hat{x}_y^{ie}(q) + \epsilon}{K}\right).$$

Because $\hat{x}^{ie}(q) \in X^2 \bigcup X^4$ all general fund proposals (following a passage of $\hat{x}^{ie}(q)$) will spend all left over revenue (ϵ) on the public good. Therefore $EU_i\left(x^{ie*}(q)\right) = EU_i\left(\hat{x}^{ie}(q)\right)$. It is simple to confirm that $x^{ie*}(q)$ will pass under majority rule; this is omitted for brevity. Therefore $x^{ie*}(q)$ is also a solution to the constrained optimization problem when $\hat{x}^{ie}(q) \in X^2 \bigcup X^4$. It is trivial to show that $x^{ie*}(q)$ spends all revenue in the earmarking stage.

Now suppose $\hat{x}^{ie}(q)$ is a solution to the earmarking constrained optimization problem and $\hat{x}^{ie}(q) \in X^1$ with $\sum_{l=1}^{3} \hat{x}_l^{ie}(q) + \hat{x}_y^{ie}(q) < R$.

Pick $p \in \left(\frac{1}{3}, \frac{2}{3}\right)$ and define $x^{ie*}(q)$ as follows

$$x^{ie*}(q) \equiv \begin{cases} \hat{x}_i^{ie}(q) + \epsilon p \\ \hat{x}_j^{ie}(q) + \epsilon(1-p) \\ \hat{x}_k^{ie}(q) \\ \hat{x}_y^{ie}(q) \end{cases}$$

Write equation (12) as below.

$$R - \sum_{l=1}^{3} x_l^{ie*}(q) - 2B > 2B\left[\ln\left(\frac{R - \sum_{l=1}^{3} x_l^{ie*}(q)}{K}\right) - \ln\left(\frac{2B}{K}\right)\right] \tag{12}$$

Equation (12) is continuous in ϵ and holds at $\epsilon = 0$ because $\hat{x}^{ie}(q) \in X^1$. Therefore, $\exists\, \bar{\epsilon}$ such that $R - \left(\sum_{l=1}^{3} \hat{x}_l^{ie}(q) + \hat{x}_y^{ie}(q)\right) > \bar{\epsilon} > 0$ and letting $\epsilon = \bar{\epsilon}$, equation (12) still holds; that is $x^{ie*}(q) \in X^1$.

Let $\epsilon = \bar{\epsilon}$. Then the expected utility to the proposer from proposal $x^{ie*}(q)$ can be written as

$$EU_i(x^{ie*}(q)) = \hat{x}_i^{ie}(q) + \epsilon p + \frac{1}{3}\left(R - \sum_{l=1}^{3} \hat{x}_l^{ie}(q) - \epsilon - 2B\right) + B\ln\left(\frac{2B}{K}\right)$$

and the expected utility to the proposer from proposal $\hat{x}^{ie}(q)$ can be written as

$$EU_i(\hat{x}^{ie}(q)) = \hat{x}_i^{ie}(q) + \frac{1}{3}\left(R - \sum_{l=1}^{3} \hat{x}_l^{ie}(q) - 2B\right) + B\ln\left(\frac{2B}{K}\right).$$

Because $\epsilon > 0$ and $p > \frac{1}{3}$ it follows that $EU_i(x^{ie*}(q)) > EU_i(\hat{x}^{ie}(q))$. It remains to be shown that j prefers such a proposal as well. Do a similar calculation for j

$$EU_j(x^{ie*}(q)) = \hat{x}_j^{ie}(q) + \epsilon(1-p) + \frac{1}{3}\left(R - \sum_{l=1}^{3} \hat{x}_l^{ie}(q) - \epsilon - 2B\right) + B\ln\left(\frac{2B}{K}\right)$$

and

$$EU_j(\hat{x}^{ie}(q)) = \hat{x}_j^{ie}(q) + \frac{1}{3}\left(R - \sum_{l=1}^{3} \hat{x}_l^{ie}(q) - 2B\right) + B\ln\left(\frac{2B}{K}\right)$$

so that $EU_j(x^{ie*}(q)) > EU_j(\hat{x}^{ie}(q))$ because $p < \frac{2}{3}$.

Hence, we have found a feasible proposal that both i and j strictly prefer to $\hat{x}^{ie}(q)$. This provides a contradiction to $\hat{x}^{ie}(q)$ being a solution to the constrained optimization problem. Thus any solution to the earmarking constrained optimization problem with $x^{ie*}(q) \in X^1$ must earmark all revenue.

Lastly, suppose $\hat{x}^{ie}(q)$ is a solution to the earmarking constrained optimization problem and $\hat{x}^{ie}(q) \in X^3$ with$\sum_{l=1}^{3} \hat{x}_l^{ie}(q) + \hat{x}_y^{ie}(q) < R$. Construct $x^{ie*}(q)$ as was previously done when $x^{ie*}(q) \in X^1$ but with $\epsilon = R - \left(\sum_{l=1}^{3} \hat{x}_l^{ie}(q) + \hat{x}_y^{ie}(q)\right)$ and $p \in \left(\frac{1}{3}, \frac{2}{3}\right)$.

Then

$$EU_i(x^{ie*}(q)) = \hat{x}_i^{ie}(q) + \epsilon p + B\ln\left(\frac{\hat{x}_y^{ie}(q)}{K}\right)$$

and

$$EU_i(\hat{x}^{ie}(q)) = \hat{x}_i^{ie}(q) + \frac{\epsilon}{3} + B\ln\left(\frac{\hat{x}_y^{ie}(q)}{K}\right).$$

So that $EU_i(x^{ie*}(q)) > EU_i(\hat{x}^{ie}(q))$ because $p > \frac{1}{3}$. It remains to show that j prefers such a proposal as well.

$$EU_j(x^{ie*}(q)) = \hat{x}_j^{ie}(q) + \epsilon(1-p) + B\ln\left(\frac{\hat{x}_y^{ie}(q)}{K}\right)$$

and

$$EU_j(\hat{x}^{ie}(q)) = \hat{x}_j^{ie}(q) + \frac{\epsilon}{3} + B\ln\left(\frac{\hat{x}_y^{ie}(q)}{K}\right)$$

so that $EU_j(x^{ie*}(q)) > EU_j(\hat{x}^{ie}(q))$ because $p < \frac{2}{3}$.

Hence, we have found a feasible proposal that both i and j strictly prefer to $\hat{x}^{ie}(q)$. This provides a contradiction to $\hat{x}^{ie}(q)$ being a solution to the earmarking constrained optimization problem. Any solution with $x^{ie*}(q) \in X^3$ must earmark all revenue. $\square$