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Monotonicity of additive indices of revealed comparative advantage

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Abstract

This note extends Hillman's analysis of the appropriateness of the Balassa Index to the recently proposed additive measures of revealed comparative advantage (RCA). While previous studies suggest that Hillman's monotonicity condition is not restrictive, monotonicity conditions for the additive indices are overly restrictive, as they will fail to hold unless the indices reveal a comparative advantage.

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1. Introduction

Although Liesner's (1958) assessment of the impact of the liberalizing of intra-European trade on British industries could be regarded as the first attempt to which post-trade data was employed to quantify comparative advantage, the term 'revealed' was introduced by Balassa (1965), who appraised the prospective lasting effects of trade liberalization resulting from the Kennedy-Round within the comparative advantage framework, whereupon revealing comparative advantage became an empirical practice.

Furthermore, the independence of the measurement of comparative advantage from the underlying theory provided a certain degree of freedom for subsequent researchers in altering the Balassa Index (Benedictis & Tamberi, 2002). Justification of such practices, on the other hand, can be argued to be overlooked. Particularly Hillman's (1980) monotonicity condition, which can easily be verified, has been partially addressed in the substantial empirical literature that has employed either Balassa's index or its altered versions[1].

Even if used, Hillman's monotonicity condition can only be employed for multiplicative revealed comparative advantage (RCA) indices, while such a condition may not be applicable for additive RCA indices. In this note, similar monotonicity conditions for the recently proposed additive RCA indices are derived following Hillman. It is concluded that such monotonicity conditions are quite restrictive for the additive indices, as they will fail to hold unless the indices reveal a comparative advantage.

2. Revealing Comparative Advantage

As Balassa emphasized, comparative advantage may arise from different factors, of which some may not be quantified or observed. Thus he challenged whether observed patterns of trade could provide evidence for the underlying determinants of those patterns of trade. He employed an export performance index in evaluating RCA such that

$$BRCA_a^i = \frac{X_a^i}{X_t^i} \Big/ \frac{X_a^w}{X_t^w}, \tag{1}$$

where subscripts a and t, respectively, refer to the reference and the aggregate of all traded commodities; while superscripts i and w, respectively, represent the examined country and the world; and X stands for the export flows.

Although revealing comparative advantage using the Balassa Index became a prevailing practice, related empirical inconsistencies prompted subsequent researchers in altering the Balassa Index. Asymmetry and the variable mean were the characteristics of the Balassa Index first challenged as being the source of those empirical inconsistencies. Vollrath (1991)

[1]This condition is empirically investigated by only Marchese & De Simone (1989), and Hinloopen & van Marrewijk (2008) and addressed by only Hinloopen & van Marrewijk (2001) to our knowledge.

and Dalum et al. (1998) employed logarithmic and quasi-logarithmic transformations to re-normalize the original index to have a symmetric distribution, whereas Proudman & Redding (2000) normalized the Balassa Index by its cross-sectional mean to fix its varying mean so that the average commodity would be at the comparative-neutral state.

On the other hand, the multiplicative nature of the Balassa Index could also be argued to cause the associated inconsistencies considering Liesner's and Balassa's concerns about the conditionality of the change in the value of the RCA index on the size of the country and the latter points made by Benedictis & Tamberi (2002) on the compositional effects of the components of the Index. With that regard, Hoen & Oosterhaven (2006) and Yu et al. (2009) proposed additive RCA indices, the former being

$$ARCA_a^i = \frac{X_a^i}{X_t^i} - \frac{X_a^w}{X_t^w}, \tag{2}$$

and the latter being

$$NRCA_a^i = \frac{X_a^i}{X_t^w} - \frac{X_t^i}{X_t^w} \cdot \frac{X_a^w}{X_t^w}, \tag{3}$$

which scales $ARCA_a^i$ by country i's relative exports and hence can be rewritten as

$$NRCA_a^i = ARCA_a^i \left(\frac{X_t^i}{X_t^w} \right). \tag{4}$$

3. Rationalization of such 'Reveal'ations

It was first Kunimoto (1977), who attempted a theoretical rationale using a probabilistic framework for interpreting the positive (negative) deviations from unity in such indices as an indicator of comparative advantage (disadvantage). Although Kunimoto's approach provides an interpretative justification for the Balassa Index and its variants, Hillman (1980) provided a sound theoretical relation. By assuming identical homothetic preferences among the reference countries, Hillman derived a necessary and sufficient condition for the consistency between RCA identified by the Balassa Index and relative autarky prices in cross-country comparisons.

In his two-country two-good setting, the second good being a Hicksian composite good excluding the first good (denoted by a), countries are assumed to have identical technologies and identical capital-labor endowment ratios but different absolute endowments. It is shown that under such a setting the countries' trade exchanges result in identical RCA scores as the countries' exports in good a and total exports would differ by a factor of proportionality (ψ), which depends on their absolute endowments. More explicitly,

$$BRCA_a^1 = BRCA_a^2 = \frac{X_a^1}{X_a^w} \Big/ \frac{X_t^1}{X_t^w} \quad \text{as} \quad \begin{cases} X_a^2 = \psi X_a^1 \\ X_t^2 = \psi X_t^1 \end{cases} \quad \text{where} \;\; 0 < \psi < 1. \tag{5}$$

Hillman imposes an increase in the first country's labor endowment with an equal reduction in its capital endowments such that the autarkic value of national output remains constant. This orchestrated change in factor endowments results in a comparative advantage in good a for the first country and the correspondence of the RCA scores would entail $BRCA_a^1 > BRCA_a^2$ as $P_a^1 < P_a^2$, where the latter are autarky prices. To confirm that such an orchestrated change in endowments affect the relative RCA scores as denoted, Hillman rearranges (1) as

$$BRCA_a^i = \left(\frac{X_a^i}{X_a^i + X_a^{w-i}} \right) \Big/ \left(\frac{X_a^i + X_{t-a}^i}{X_a^i + X_{t-a}^{w-i}} \right). \tag{6}$$

Having exports X_a^i increased, (6) is then differentiated with respect to X_a^i and the outcome is

$$\frac{X_a^i}{BRCA_a^i} \cdot \frac{dBRCA_a^i}{dX_a^i} = \frac{X_{t-a}^i}{X_{t-a}^i + X_a^i} + \frac{X_a^{w-i}}{X_a^{w-i} + X_a^i} + \frac{X_{t-a}^{w-i}}{X_{t-a}^{w-i} + X_a^i}, \tag{7}$$

which can be rewritten as

$$\frac{X_a^i}{BRCA_a^i} \cdot \frac{dBRCA_a^i}{dX_a^i} = \left(1 - \frac{X_a^i}{X_a^w} \right) - \frac{X_a^i}{X_t^i} \left(1 - \frac{X_t^i}{X_t^w} \right). \tag{8}$$

Although (8) has an indeterminate sign, Hillman provides a necessary and sufficient condition; for $BRCA_a^1 > BRCA_a^2$ as $P_a^1 < P_a^2$ to hold, $\frac{X_a^i}{BRCA_a^i} \cdot \frac{dBRCA_a^i}{dX_a^i} > 0$ must hold. Hence the Hillman condition can be stated as;

$$\Big[1 - (X_a^i / X_a^w) \Big] > (X_a^i / X_t^i) \cdot \Big[1 - (X_t^i / X_t^w) \Big], \tag{9}$$

where (X_a^i / X_a^w) measures the share of country's exports of a particular commodity in that of world exports; (X_a^i / X_t^i) measures the share of a particular commodity in the country's total exports, hence the degree of export specialization; and (X_t^i / X_t^w) measures the share of a country's exports in world exports. In other words, Hillman's condition requires "scaling of a country's exports by a measure of its size and by a measure of commodity size to be a monotonic transformation" and that "the changes in the RCA index are consistent with changes in countries' relative factor endowments" (Marchese & Simone, 1989, p.159).

Yet the proposition of relative autarky prices as the determinant of the pattern of trade would fail if the simple two-country two-good model were extended in plausible ways. Although the employed two-by-two model can easily be extended to having more goods or

more countries, in which the above proposition would hold in terms of a chain of comparative advantage (Deardorff, 2005), increasing the number of both would cease the capability of relative autarky prices in determining the pattern of trade. Alternatively, introduction of barriers to trade would alter the existing patterns of trade based on relative autarky prices (Travis, 1972). Nevertheless what can be assured is that "a negative correlation exists between any country's relative autarky prices and its pattern of net exports" (Deardorff, 1980, p.942), thus the above proposition would hold if it is restated in terms of averages across all goods.

Considering Hillman's analysis, associating the increase in a country's exports in a particular good that results in an increase in its RCA score with the country's lower autarky prices, is in line with Deardorff's (1980) model, where lower autarky prices are associated with exports. Thus, this exercise may provide to be useful in making partial statements for two goods and two countries, whereas under realistic assumptions, it only gives a necessary and sufficient condition for an exogenous increase in exports of a particular good to result in an increase in the country's revealed comparative advantage[2].

4. Monotonicity of Additive RCA Indices

Although Hillman's condition can be employed for other subsequent normalized multiplicative indices, it may not be employed for the proposed additive RCA indices. Thus, following Hillman (1980), similar monotonicity conditions will be derived for the additive indices proposed by Hoen & Oosterhaven (2006) and Yu et al. (2009).

Given the same two-country two-good setting, countries' trade exchanges will result again in identical RCA scores that are now defined by $ARCA_a^i$. More explicitly, given $X_a^2 = \psi X_a^1$ and $X_t^2 = \psi X_t^1$ where the factor of proportionality is $(0 < \psi < 1)$,

$$ARCA_a^1 = ARCA_a^2 = \frac{X_a^1}{X_t^1} - \frac{X_a^w}{X_t^w}. \tag{10}$$

Again, imposing similar changes in the first country's endowments that would leave the autarkic value of national output constant, would result in a comparative advantage in good a for the first country and the correspondence of the RCA scores defined by $ARCA_a^1$ would entail $ARCA_a^1 > ARCA_a^2$ as relative autarkic prices are $P_a^1 < P_a^2$. On the other hand if RCA scores are identified by $NRCA_a^1$, then given the same settings,

$$NRCA_a^1 = \frac{X_a^1}{X_t^w} - \frac{X_t^2 \cdot X_a^w}{X_t^w \cdot X_t^w} > NRCA_a^2 = \frac{X_a^2}{X_t^w} - \frac{X_t^2 \cdot X_a^w}{X_t^w \cdot X_t^w}. \tag{11}$$

Thus, even though the countries have equal capital-labor ratios, first country would be revealed to have a comparative advantage in good a due to its richer endowments, if RCA

[2]This was indicated by the anonymous referee.

is defined by $NRCA_a^i$. That is, the proportionality fails to hold for $NRCA_a^i$, as it scales the previous index by the country's relative exports. Yet one may still ascertain whether country 1's increased exports in good a due to this orchestrated endowment change would affect comparative advantage in a similar fashion, which will be a further increase in RCA as defined by $NRCA_a^i$.

Hence both $\frac{X_a^i}{ARCA_a^i} \cdot \frac{dARCA_a^i}{dX_a^i} > 0$ and $\frac{X_a^i}{NRCA_a^i} \cdot \frac{dNRCA_a^i}{dX_a^i} > 0$ should hold for the consistency between the RCA's indicated by the relevant additive index and the relative autarkic prices. (2) and (3) are rearranged as

$$ARCA_a^i = \left(\frac{X_a^i}{X_a^i + X_{t-a}^i}\right) - \left(\frac{X_a^i + X_a^{w-i}}{X_i^a + X_{w-i}^{t-a}}\right) \text{ and} \tag{12}$$

$$NRCA_a^i = \left(\frac{X_a^i}{X_a^i + X_{t-a}^{w-i}}\right) - \left(\frac{(X_a^i + X_{t-a}^i) \cdot (X_a^i + X_a^{w-i})}{(X_a^i + X_{t-a}^{w-i}) \cdot (X_a^i + X_{t-a}^{w-i})}\right). \tag{13}$$

Having exports X_a^i increased, (12) and (13) are differentiated with respect to X_a^i and the outcomes are

$$\frac{X_a^i}{ARCA_a^i} \cdot \frac{dARCA_a^i}{dX_a^i} = \frac{X_a^i \cdot \left[(X_t^i)^2 \cdot (X_t^w - X_a^w) - (X_t^w)^2 \cdot (X_t^i - X_a^i)\right]}{X_t^i \cdot X_t^w \cdot (X_t^i \cdot X_a^w - X_a^i \cdot X_t^w)} \text{ and} \tag{14}$$

$$\frac{X_a^i}{NRCA_a^i} \cdot \frac{dNRCA_a^i}{dX_a^i} = \frac{X_a^i \cdot \left[X_t^i \cdot X_t^w - 2 \cdot X_t^i \cdot X_a^w + X_a^w \cdot X_t^w + X_a^i \cdot X_t^w - (X_t^w)^2\right]}{X_t^w \cdot (X_t^i \cdot X_a^w - X_a^i \cdot X_t^w)}. \tag{15}$$

For $\left\{ \begin{array}{c} ARCA_a^i > ARCA_a^j \\ NRCA_a^i > NRCA_a^j \end{array} \right\}$ as $P_a^i < P_a^j$ to hold, $\frac{X_a^i}{ARCA_a^i} \cdot \frac{dARCA_a^i}{dX_a^i} > 0$ and $\frac{X_a^i}{NRCA_a^i} \cdot \frac{dNRCA_a^i}{dX_a^i} > 0$ should hold. In both calculated derivatives, the numerators will obtain negative values unless country i's exports in commodity a are zero. Hence for both inequalities to hold $(X_t^i \cdot X_a^w - X_a^i \cdot X_t^w) > 0$ must hold. This last condition can be rewritten as

$$\frac{X_a^i}{X_t^i} - \frac{X_a^w}{X_t^w} > 0 \text{ or } \frac{X_a^i}{X_t^w} - \frac{X_t^i}{X_t^w} \cdot \frac{X_a^w}{X_t^w} > 0 \tag{16}$$

and these correspond to $ARCA_a^i$ and $NRCA_a^i$, respectively. More explicitly,

> *Monotonicity conditions for the additive indices will hold as long as the index scores are positive hence reveal a comparative advantage, whereas indices revealing a comparative disadvantage will always violate such monotonicity conditions.*

5. Conclusion

This note extends Hillman's analysis of the appropriateness of the Balassa Index to the recently proposed additive measures of revealed comparative advantage. In a two-country two-good setting, Hillman asks whether higher Balassa scores are associated with lower autarky prices. He shows that the Balassa Index identifies countries that differ only in scale as having same index score, and provides a necessary and sufficient condition for an exogenous increase in exports of a particular good to result in an increase in the country's RCA, which under two-by-two setting corresponds to lower autarky prices.

Although the verification of this condition is a matter of empirical investigation, its applicability is limited to the Balassa Index and its normalized multiplicative variants. Thus following Hillman, similar monotonicity conditions are derived for the proposed additive RCA indices. It is shown that the proportionality holds for only $ARCA_a^i$ but not $NRCA_a^i$. Furthermore, while previous studies by Marchese & De Simone (1989) and Hinloopen & van Marrewijk (2001, 2008) suggest that the Hillman condition is not restrictive, it is shown here that similar monotonicity conditions for the additive indices are overly restrictive, as they will fail to hold if the indices reveal a comparative disadvantage.

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