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Portfolio Choice and Precautionary Savings

Riccardo Calcagno
EMLYON Business School & CeRP Collegio Carlo Alberto

Mariacristina Rossi
University of Turin & CeRP Collegio Carlo Alberto

Abstract

We study the effect on savings of an increase in the capital risk of the investment opportunities when the representative consumer is allowed to optimally choose her portfolio. Sandmo (1970) and Levhari and Srinivasan (1969) prove that individuals with high risk-aversion and time-separable, power utility increase their optimal savings when capital risk increases holding constant the expected return of the risky asset. We obtain the opposite effect when the consumer chooses her portfolio allocation optimally.

Riccardo Calcagno. EMLYON Business School & CeRP Collegio Carlo Alberto. Mariacristina Rossi, Dipartimento di Scienze Economiche e Finanziarie University of Turin, Italy, and CeRP, Collegio Carlo Alberto.
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1 Introduction

A key insight of the modern consumption theory is that agents try to keep the marginal utility of their expenditure constant over time, also known as the"consumption smoothing" behavior. The former behaviour is central in the life-cycle and permanent income theory (LCPI), which dates at least back, in its simplest form, to Brumberg and Modigliani (1954) and Friedman (1957).

Several modifications have been added to the basic LCPI model to allow for a more realistic environment in which the individuals make their choices on savings. Among these extensions, uncertainty plays a big role. More specifically, Kimball (1990) shows that agents exhibiting prudence in their utility function (that is, when the marginal utility is convex) behave very differently from what is predicted by the basic LCPI in a certain environment. When their preferences exhibit prudence, Kimball (1990) proves that the agent's total savings increase with future earnings uncertainty, generating the so-called precautionary saving. Some theoretical studies could attribute half of individual savings to precautionary motives (e.g. Caballero (1991)), while empirical studies, such as Dynan (1993) and Lusardi (1996) do not find strong evidence of a precautionary wealth accumulation process on U.S data.

The aim of this paper is to shed light on whether the precautionary motive for saving is still at work when uncertainty on the risky asset return (i.e. capital risk), instead of income risk, is considered and when the consumer optimally chooses her portfolio composition. Sandmo (1970) argues that when capital investment becomes riskier, the agent will respond to the higher potential future losses by generating additional savings, similarly to the case of income uncertainty. While the net effect of higher capital risk on saving cannot be determined in general, Levhari and Srinivasan (1969) prove that individuals with high risk-aversion and time-separable, isoelastic utility increase their optimal savings when capital risk increases.

We extend the two-period analysis of Sandmo (1970) by endogenizing the choice of the optimal investment portfolio between a risky and a risk-free asset. The representative agent thus maximizes her lifetime utility over consumption in both periods and over the portfolio composition. We assume that her utility function exhibits constant relative risk aversion and that she faces uncertainty in the rate of return of the risky asset, which is not known when the resource allocation decisions are made. Following Sandmo (1970), we increase the volatility of the risky asset return with a mean-preserving shock and we obtain the opposite result as Levhari and Srinivasan (1969), where the consumer invests all her savings in the risky asset. In our model, opposite to theirs, consumers exhibiting higher risk aversion *reduce* their savings in the first period when capital risk increases. The intuition for the result is the following. Risk-averse consumers reduce their exposure towards the risky asset when its return becomes more volatile. Two opposite forces are at work in presence of a more volatile asset return. First, a more uncertain and riskier environment would generate additional savings, the "income effect" in Sandmo (1970)). At the same time, given that the frontier of investment opportunities becomes less attractive, the consumer will modify her portfolio choice by investing less in the risky asset (the "substitution effect"). We show that, at optimum, this second effect always prevails if the individual is sufficiently risk-averse, thus she decreases her optimal amount of savings. Notice that our increase in capital risk is obtained holding constant the expected return of the risky asset: in this respect, our result differs from Gollier and Kimball (1996), who study the effect on total saving of introducing a risky asset with expected return higher than the risk-free one in the investment possibilities of the consumer. This would generate a positive wealth effect that we do not consider in our model.

Finally, it is important to remind that fluctuations on asset returns have a strong impact on household welfare, particularly at older age, when the stock of assets reaches its peak value, while income represents a small fraction of the individual wealth.

2 The model

A representative consumer lives two periods $t = 1, 2$ and is endowed with a power utility function with constant degree of relative risk aversion $\gamma > 0$. At the beginning of the first period she chooses the level of savings s and the portfolio allocation α in order to maximize her lifetime welfare:

$$\max_{s,\, \alpha} \frac{(y-s)^{1-\gamma}}{1-\gamma} + \frac{1}{1-\gamma} E\left[s\left(1+\alpha(1+\widetilde{R}-1)\right)\right]^{1-\gamma} \tag{1}$$

under the budget constraints

$$\begin{aligned} c_1 &= y - s \\ \tilde{c}_2 &= \alpha s + s\,(1-\alpha)\left(1+\widetilde{R}\right) \end{aligned}$$

where c_t denotes the consumption in period t, α the fraction of total saving invested in the risky asset and $\widetilde{R}$ the excess rate of return of the risky asset over the riskless rate. The consumer can invest a quota $(1-\alpha)$ of her saving in a riskless asset whose return is normalized to zero. Initial endowments and bequests are also set to zero. The realization of $\widetilde{R}$ is unknown to the consumer when she decides the optimal portfolio allocation. Following a standard practice in the literature[1] we assume that the excess return of the risky asset is log-normal, i.e. $1+\widetilde{R} = e^{\widetilde{X}}$ with $\widetilde{X} \sim N(m, \sigma^2)$. Denoting by μ the mean of the log-normal distribution, we posit $m = \mu - \frac{\sigma^2}{2}$.[2]

[1] See for example Campbell and Viceira (2002).

[2] Since the mean of the log-normal distribution is equal to $m + \frac{\sigma^2}{2}$ in this way we obtain that the mean of the log-normal μ is independent of σ^2.

2.1 Optimal Portfolio Choice

As the choice of the optimal portfolio allocation does not involve the first period utility, problem (1) can be seen as a dynamic problem in which s is selected in period one and α in the final period[3]. We introduce then a value function $v(s)$ that can be written as

$$v(s) = \max_{\alpha} E\left[\frac{\left(s(1+\alpha\widetilde{R})\right)^{1-\gamma}}{1-\gamma}\right] \tag{2}$$

so that (1) becomes

$$\max_{s} \frac{(y-s)^{1-\gamma}}{1-\gamma} + v(s) \tag{3}$$

Proceeding backward, we first solve for the optimal α^* given the level of total savings.

Proposition 1: *The optimal portfolio allocation is given by*

$$\alpha^* = \frac{m+\Sigma/2}{\gamma\Sigma} \tag{4}$$

where $\Sigma = m^2 + \sigma^2$.

Proof: The solution of (2) is characterized by its first-order condition

$$s^{1-\gamma} E\left[(1+\alpha\widetilde{R})^{-\gamma}\widetilde{R}\right] = 0 \tag{5}$$

that can be expressed as:

$$E\left[(1+\alpha(e^{\widetilde{X}}-1))^{-\gamma}(e^{\widetilde{X}}-1)\right] = 0 \tag{6}$$

using the fact that $1+\widetilde{R} = e^{\widetilde{X}}$. We then approximate $f(\widetilde{X}) = (1+\alpha(e^{\widetilde{X}} - 1))^{-\gamma}(e^{\widetilde{X}}-1)$ with its Taylor expansion around $\widetilde{X} = 0$ writing $f(\widetilde{X}) = f(0) + f'(0)\widetilde{X} + \frac{1}{2}f''(0)\widetilde{X}^2$ where

$$\begin{aligned} f(0) &= 0 \\ f'(0) &= 1 \\ f''(0) &= 1 - 2\gamma\alpha \end{aligned}$$

Taking the expected value of $f(\widetilde{X})$ we obtain $E[f(\widetilde{X})] = f(0) + f'(0)E[\widetilde{X}] + \frac{1}{2}f''(0)E[\widetilde{X}^2]$ so that we can approximate $E[f(\widetilde{X})]$ locally as

$$E[f(\widetilde{X})] \simeq f(0) + f'(0)m + \frac{1}{2}f''(0)\Sigma$$

[3] For a similar procedure, see Gollier (2003).

where $\Sigma = m^2 + \sigma^2 = \left(\mu - \frac{\sigma^2}{2}\right)^2 + \sigma^2$.[4] Finally, substituting for $f(0)$, $f'(0)$, $f''(0)$ into (6):

$$\begin{aligned} m + \frac{1}{2}(1 - 2\gamma\alpha)\Sigma &= 0 \\ \alpha^* &= \frac{m}{\gamma\Sigma} + \frac{1}{2\gamma} \end{aligned}$$

that can be rewritten as (4). ■

Proposition 1 yields the usual optimal portfolio rule in a log-normal model with power utility (see for example Campbell and Viceira (2002)): the optimal share invested in the risky asset α^* is increasing in its expected excess log-return[5] and decreasing in its variance. Finally notice that the optimal portfolio allocation (4) is independent of the level of savings.

2.2 Optimal savings decision

We now proceed solving (3) where the optimal portfolio allocation α^* characterized in Proposition 1 is taken as given.

Proposition 2: *Given the optimal portfolio allocation α^* in (4), the optimal level of savings is given by*

$$s(\alpha^*) = \frac{y}{1 + \exp\left(\frac{\gamma-1}{\gamma}\left(\alpha^* m + \frac{1}{2}\alpha^*(1 - \gamma\alpha^*)\Sigma\right)\right)} \tag{7}$$

Proof: For simplicity, in this proof we write α instead of α^*. The first order condition of (3) is

$$\begin{aligned} (y - s)^{-\gamma} &= E\left[s^{-\gamma}\left(1 + \alpha(1 + \widetilde{R} - 1)\right)^{1-\gamma}\right] \\ \left(\frac{y - s}{s}\right)^{-\gamma} &= E\left[\left(1 + \alpha(1 + \widetilde{R} - 1)\right)^{1-\gamma}\right] \end{aligned} \tag{8}$$

Use a Taylor expansion around $\widetilde{X} = 0$ of $f(\widetilde{X}) = \left(1 + \alpha(e^{\widetilde{X}} - 1)\right)^{1-\gamma}$. Given that

$$\begin{aligned} f'(x) &= (1-\gamma)(1 + \alpha(e^x - 1))^{-\gamma}\alpha e^x, \\ f''(x) &= -\gamma(1-\gamma)(1 + \alpha(e^x - 1))^{-\gamma-1}(\alpha e^x)^2 + (1-\gamma)(1 + \alpha(e^x - 1))^{-\gamma}\alpha(e^x)^2 \\ &\Rightarrow f(0) = 1, \\ &\Rightarrow f'(0) = (1-\gamma)\alpha, \\ &\Rightarrow f''(0) = (1-\gamma)\alpha(1 - \gamma\alpha). \end{aligned}$$

[4] Notice that Σ is increasing in σ^2 if and only if $\mu < \frac{\sigma^2}{2} + 1$. In this case the comparative statics w.r.to σ have the same sign as the ones w.r.to Σ.

[5] Remember that the mean of the excess log-return $\mu = m + \frac{\sigma^2}{2}$, which explains the addition of the term with one-half of the variance.

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we can approximate $\left(1+\alpha(e^{\widetilde{X}}-1)\right)^{1-\gamma}$ as:

$$\left(1+\alpha(1+\widetilde{R}-1)\right)^{1-\gamma} \simeq 1+(1-\gamma)\alpha m+\frac{1}{2}(1-\gamma)\,\alpha\,(1-\gamma\alpha)\,\Sigma \tag{9}$$

Substituting (9) into (8) and taking the log of both sides we obtain the following equality:

$$-\gamma\log\left(\frac{y}{s}-1\right)=\log\left(1+(1-\gamma)\alpha m+\frac{1}{2}(1-\gamma)\,\alpha\,(1-\gamma\alpha)\,\Sigma\right)$$

Finally, using the approximation $\log(1+z)=z$, we have

$$\begin{aligned}
-\gamma\log\left(\frac{y}{s}-1\right) &= (1-\gamma)\alpha m+\frac{1}{2}(1-\gamma)\,\alpha\,(1-\gamma\alpha)\,\Sigma && (10)\\
\log\left(\frac{y}{s}-1\right) &= \tfrac{(1-\gamma)\alpha m+\frac{1}{2}(1-\gamma)\alpha(1-\gamma\alpha)\Sigma}{-\gamma} && (11)\\
&\Rightarrow \frac{y}{s}=1+\exp\left(\tfrac{1-\gamma}{-\gamma}\left(\alpha m+\frac{1}{2}\alpha\,(1-\gamma\alpha)\,\Sigma\right)\right)
\end{aligned}$$

which rearranged provides (7). ■

The interpretation of Proposition 2 becomes clear analyzing (11): with $\alpha\in]0,1]$, the RHS of (11) is negative for $\gamma\leq 1$. Hence $\frac{y}{s}-1<1$, that is $\frac{y}{2}<s$ where $s=y/2$ is the optimal level of saving when there is no uncertainty (or, equivalently, the consumer can invest only in the riskless asset). Thus Proposition 2 obtains a result similar to Gollier and Kimball (1996): the possibility to invest in a risky asset increases (resp. reduces) saving compared to the case with only a risk-free asset if and only if absolute prudence is larger (resp. smaller) than twice the absolute risk aversion.[6] However, notice that this conclusion is not obvious *a priori*. In fact, Gollier and Kimball (1996) consider the effect of introducing a risky asset with an expected return higher than the risk-free rate in the market. Thus they analyze a situation in which not only the riskiness, but also the returns possibilities change. This in turn generate a positive wealth effect since it increases the expected future income of the consumer. We abstract from this latter wealth effect, but still reach similar conclusions on the precautionary saving for a pure change in the riskiness of the investment possibilities.

3 The effect on savings of capital risk

In this section we show the main result of our paper: increasing the uncertainty on the return of capital provides effects on the level of savings opposite to the ones already known in the literature when we allow the representative consumer to choose for the optimal portfolio allocation.

[6] With a CRRA utility function this last condition is satisfied for $\gamma\leq 1$.

Reconsider the f.o.c. (10) and substitute for the optimal value for α^*:

$$\begin{aligned}
\log\left(\frac{y}{s}-1\right) &= \frac{(1-\gamma)\alpha^* m + \frac{1}{2}(1-\gamma)\alpha^*(1-\gamma\alpha^*)\Sigma}{-\gamma} \qquad (12)\\
&= \frac{(\gamma-1)}{\gamma}\alpha^*\left(m+\frac{1}{2}(1-\gamma\alpha^*)\Sigma\right)\\
&= \frac{(\gamma-1)}{\gamma}\left(\frac{m}{\gamma\Sigma}+\frac{1}{2\gamma}\right)\left(m+\frac{1}{2}\left(1-\gamma\left(\frac{m}{\gamma\Sigma}+\frac{1}{2\gamma}\right)\right)\Sigma\right)\\
&= \frac{(\gamma-1)}{\gamma^2}\left(\frac{m}{\Sigma}+\frac{1}{2}\right)\left(\frac{m}{2}+\frac{\Sigma}{4}\right)
\end{aligned}$$

Our goal is to obtain the comparative statics of saving when we increase the variance of the risky asset returns, σ^2, keeping its mean μ constant[7].

Proposition 3: *When the return of the risky asset is distributed log-normal with an expected excess return sufficiently close to zero, a mean-preserving increase in its variance leads to lower savings (resp. higher savings) when $\gamma > 1$ (resp. when $\gamma < 1$).*

Proof: By substituting the expressions corresponding to m and Σ into (*12*) we obtain:

$$\log\left(\frac{y}{s}-1\right) = \frac{(\gamma-1)}{\gamma^2}\left(\frac{1}{2}\left(\mu-\sigma^2\right)+\frac{1}{8}\left(2\sigma^2+\left(\mu-\sigma^2\right)^2\right)+\frac{1}{2}\frac{\left(\mu-\sigma^2\right)^2}{\left(2\sigma^2+\left(\mu-\sigma^2\right)^2\right)}\right)$$

For $\gamma > 1$ saving decreases with σ^2 if the RHS of the above equality increases with σ^2. We compute the sign of the partial derivative:

$$\begin{aligned}
&\frac{\partial\left(\frac{1}{2}\left(\mu-\sigma^2\right)+\frac{1}{8}\left(2\sigma^2+\left(\mu-\sigma^2\right)^2\right)+\frac{1}{2}\frac{\left(\mu-\sigma^2\right)^2}{\left(2\sigma^2+\left(\mu-\sigma^2\right)^2\right)}\right)}{\partial\sigma^2}\\
&= -\frac{1}{4}\frac{\left(2\sigma^2+\left(\mu-\sigma^2\right)^2\right)^2+\left(\mu-\sigma^2\right)\left(2\sigma^2+\left(\mu-\sigma^2\right)^2\right)^2+4\left(\mu-\sigma^2\right)\left(\mu+\sigma^2\right)}{\left(\sigma^4-2\sigma^2+2\sigma^2\mu+\mu^2\right)^2}
\end{aligned}$$

Taking the limit of the above expression for $\mu \to 0$:

$$\begin{aligned}
&\lim_{\mu\to 0} -\frac{1}{4}\frac{\left(2\sigma^2+\left(\mu-\sigma^2\right)^2\right)^2+\left(\mu-\sigma^2\right)\left(2\sigma^2+\left(\mu-\sigma^2\right)^2\right)^2+4\left(\mu-\sigma^2\right)\left(\mu+\sigma^2\right)}{\left(\sigma^4-2\sigma^2\mu+2\sigma^2+\mu^2\right)^2}\\
&= \frac{\sigma^6}{4}\frac{1}{\left(\sigma^4+2\sigma^2\right)^2}\left(\sigma^2+3\right) > 0
\end{aligned}$$

In this case, with $\gamma > 1$ the RHS is increasing in σ^2. Hence an increase in the variance of portfolio, holding constant the average return μ, causes a reduction in the optimal level of saving. ■

[7] With a log-normal distribution, an increase in the average excess return would also generate an increase in the portfolio variance. However, we want to isolate the effect of an increase in the variance that does not affect the average expected return. For this reason, we concentrate on the increase in σ^2.

Notice that in Proposition 3 we obtain the opposite result as with $\alpha = 1$ fixed exogenously (Sandmo (1970) and Levhari and Srinivasan (1969)): in these papers, more risk-averse consumers (those with $\gamma > 1$) *increase* savings when the riskiness of their portfolio increases if they are forced to invest in the risky asset all theirs savings. Indeed, putting $\alpha^* = 1$ in (8) we have:

$$\begin{aligned}\left(\frac{y-s}{s}\right)^{-\gamma} &= E\left[\left(1+\widetilde{R}\right)^{1-\gamma}\right] = Ee^{(1-\gamma)\widetilde{X}} \\ &= e^{(1-\gamma)\left(\mu-\frac{1}{2}\sigma^2\right)+(1-\gamma)^2\frac{1}{2}\sigma^2} \\ &= e^{(1-\gamma)\left(\mu-\gamma\frac{1}{2}\sigma^2\right)}\end{aligned}$$

so that

$$\frac{y}{s} = 1 + e^{(1-\gamma)\left(\mu-\gamma\frac{1}{2}\sigma^2\right)}$$

from which it is easy to derive the result of Sandmo (1970) and Levhari and Srinivasan (1969).

The intuition for our result is the following. In our model, the representative consumer is allowed to choose optimally his portfolio allocation between a riskless and a risky asset. When the volatility of the risky asset return increases, the consumers reduce their exposure to this asset. The market overall provides a less attractive investment opportunity set to the consumer/investor who in turn then saves less (a "substitution effect"). For high levels of risk-aversion this substitution effect is stronger. In the absence of a riskless investment, higher riskiness on the market forces the consumer to save more in order to protect herself against future low realizations of returns (an "income effect"): however, with the possibility of investing in a riskless asset, this income effect is much lower than when the riskless asset is not available. This explains the difference between our result and the one in Sandmo (1970) and Levhari and Srinivasan (1969).

4 Conclusions

In this paper we study the effect on savings of an increase in the capital risk when the representative consumer optimally chooses her portfolio allocation between risky and riskless assets. We show that if the agent is allowed to invest only in aggregate risk and if the volatility of this unique asset increases holding constant its expected return, then savings increases (resp. decreases) for more (resp. less) risk-averse consumers. Our results are opposite to the ones obtained when the portfolio composition of the agent is fixed exogenously.

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