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On the performance of the representative agent during out-of-equilibrium dynamics

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Abstract

This note contributes to the discussion about the representative agent's ability to characterize the collection of agents. We perform two exercises in context of a fairly general environment and show that even if the representative agent is powerful enough to describe the society in equilibrium, she might fail in describing the out-of-equilibrium dynamics of the system. It is established that this feature is not specific to the systems with or without interacting agents.

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1 Introduction

Whether the behavior of one individual can accurately describe the behavior of the collection of agents is the topic of decades-long debate in economics. Some people believe the approximation is useful, even if not very accurate, while others think it is fundamentally misleading. This central player, on which the bulk of the discipline is built, is called the "representative agent."

This paper aims to contribute to the debate by demonstrating that the representative agent can well describe the economic system only in special cases. Most of the previous effort has gone into the analysis of the representative agent's powers in equilibrium (Kirman, 1989; 1992). Here we are concerned with her ability to describe the out-of-equilibrium behavior of dynamic economy.

To analyse the problem we set up a very simple model and study its behavior out of equilibrium. The important feature of the model is that agents are learning while the system is out of equilibrium. Learning is the force that drives the economy to a time-invariant state, which we use as the definition of equilibrium. The equilibrium of the model is not particularly interesting: it is characterized by agent homogeneity. Thus, we can predict without any mathematical analysis that the representative agent will be powerful in equilibrium. This is done intensionally so that we can analyse the out-of-equilibrium performance of the representative that is powerful in equilibrium.

In our simple model agents are video gamers, and they do only one thing: upgrade their gaming skills. We consider two variants. In the first, gamers are learning from their own gaming history. This is the case where agents are heterogenous but do not interact with each other. In the second variant, gamers interact and share skills with each other. We demonstrate that in both cases one needs additional requirements for the representative agent to be able to describe the dynamics of the society out-of-equilibrium.

2 The Representative Agent and Its Problems

The predecessor of the modern representative agent is, without a doubt, Marshall's (1890) "representative firm," that was defined as a "fairly" successful firm that was managed with "normal" ability. However, today the representative agent is a statistical construct. It is an "average" agent of the economy. Although we were not able to pin down an exact definition of the modern representative agent anywhere in the literature, after the analysis of numerous uses of the concept we feel confident to state that the representative agent is the construct that describes the "average" values of agent-specific variable distributions.

The decades-long literature on the validity of the approach has brought up two major topics of discussion. The first is, whether the representative agent can be constructed at all. In this respect the fundamental contribution is due Rubinstein (1974) who provided fairly general sufficient conditions under which a representative agent can be constructed. However, as Kirman (1992) argues there can be cases where this kind of "representatives" will not accurately represent the society. In particular, he shows that one can construct a setup in which this representative agent prefers one option over the other, while *every* member of the society has the reverse preference ordering. Then we have an issue of the usefulness of the constructed representative agent.

To demonstrate the second discussion point, consider we construct the agent that closely describes the behavior of the average of some variable. It might well be that behavioral rules of this agent are different from the the behavioral rules of the society that it represents. Caballero (1992) points out that the representative agent framework has “blured the distinction between statements that are valid at the individual level and those that apply to the aggregate.” Schlee (2001) calls this the problem of “normative representativeness.” By his definition, representative agent is a “normative representative” of the society if, and only if, his behavioral rules are not different from those of agents comprising the society.

Therefore, we can distinguish two essential features of the representative agent. One is the “normativeness” in Schlee’s sense, the other is the “functionality,” which means that the construct is able to describe the averages of important variables. Then, we can have two types of representative agents: “normative” and “functional.” These two constructs need not coincide. There are serious problems associated with approaches where “normative” and “functional” representatives diverge (Babutsidze, 2010). Therefore, we believe that the only useful agent has to be “functional” as well as “normative” representative of the society.

We think that, by now, merits and problems of the representative agent are well understood in economic literature. But, this only concerns the performance of the agent in describing the equilibrium properties. This is natural, as economics has largely been concerned with equilibria. However, in recent years there has been a substantial increase in research into out-of-equilibrium behavior. It is tempting to use the representative agent for these types of analyses. Therefore, it is important to evaluate the performance of the representative agent during out-of equilibrium dynamics. This is the aim of this note.

Closest to the exercise presented herein comes the research by Geweke (1985) who analyses the performance of the representative agent in evaluating the effect of the policy change. He studies the adjustment process to the new policy, which is similar to the analysis of the out-of-equilibrium dynamics, as the original arrangement is not equilibrium after the policy change. Geweke (1985) constructs a model where firms are playing three roles: they produce, demand production factors and supply products. He demonstrates that in this economy there will be three different representative agents: the average producer, the average supplier and the firm placing average demand on production factors. In all three cases representatives are different from each other and none of the three predict the average effect of the policy change correctly.

What we do in this note is somewhat similar. We start our system from the point out-of-equilibrium and study its transition to equilibrium. But we analyse a simpler and more general setup. Our agents play a single role and they are homogenous at all times with respect to all characteristics except the one under discussion. Therefore, there is no need for several representative agents (which was the case in Geweke, 1985). Agents in our setup do one simple task: they learn.

3 The model

We use a simple setup of the economy. There is continuum of agents indexed by i conveniently placed on interval $[0; 1]$. Think of them as gamers that have to decide which video game to play. They have to make this decision every time period. Time is discrete. Number of products/games is finite. Each agent has a certain skill level s for each game, such that $s_t(i)$ represents the skill level of the gamer i at time t for the game under discussion. Besides their skills, agents are identical (e.g. laws of motion of skill levels are identical across agents).

Consider a setup where quality is constant across options and is known to consumers. There is no additional network value (direct or indirect) attached to games. Therefore, consumer decisions are solely based on respective skills. Skills cannot be negative. We assume they do not depreciate, therefore $s_t(i)$ is non-decreasing in t. We also assume they are bounded from above by unity. This assumption is relevant to any product that requires utilization skills (e.g. consumer electronics). Once the user has become familiar with all the features of the product her skills cannot increase further. The bound of unity can be viewed as having 100% of skills for a particular option.

In this note we consider two versions of the model. In the first variant gamers can probabilistically choose from multiple options. In this environment we can define $P(x)$ as the probability that the consumer will choose the game if her skill level for it is x. $P(\cdot)$ is continuous and $\partial P/\partial x > 0$: the more skills an agent has for the game, the higher the chance that she will play it (*ceteris paribus*). We assume that skills are augmented by playing this particular game, therefore this mechanism is similar to increasing returns. However, in our model this is only a temporary phenomenon: due to the bounded skill level choice probability for any particular option will increase to a certain point and then it will decrease as agent continues enhancing skills for alternative options.

In the second variant of the model we discuss an environment where learning takes place only through interaction with other gamers. Therefore, consumer's choices do not affect the skill dynamics in society. This gives us the opportunity to simplify the framework and discuss a setup with only one option on the market.

Our concern is with the gaming skill levels in the society. Therefore, we can define the equilibrium of the economy as follows.

Definition 1 (Equilibrium)**.** *The system is in equilibrium when* $s_{t+1}(i) = s_t(i)$, $\forall i$.

Both of the examples that we discuss ensure the homogeneity of learning prospects of all gamers (due to equal option qualities and homogenous bounds on skills). Therefore, the equilibrium as defined here will be characterized by agent homogeneity: everybody will have the same skill level for all the games. Of course, in this case the representative agent will have no problem describing the equilibrium. However, our concern is with the out-of-equilibrium dynamics.

In this environment we can define the representative agent.

Definition 2 (Representative Agent)**.** *The representative agent is an agent that behaves identically to every other agent and describes the evolution of the average skill level from the initial state to the equilibrium.*

From the definition above it is obvious that we are looking for the representative agent that is "functional" and "normative" at the same time. If the representative agent, as described in definition 2, exists it can be further used for the analysis. If it does not exist, in order to construct another representative agent (that would clearly be a non-normative representative) we have to first solve for the evolution path of the average skill level and only after that create the representative agent that would mimic it. Even if one ignores the possibility of many behavioral protocols being able to replicate the same route of average skill evolution, the use of such representative agent will be clearly limited. The analysis of a slightly different problem might call for a different representative of the economy.

4 Results

4.1 Learning by doing

Consider the situation where the only way to augment the skills for a game is to play it. This is essentially learning by doing (Arrow, 1962). Assume that every time the gamer plays the game her skills for this particular game increase by $\gamma[1 - s_t]$, where s_t is the skill level before the play and $\gamma \in [0; 1]$ controls the speed of learning. With time gamers learn and ultimately (as $t \to \infty$) everybody's skill level for every game converges to the maximum. To simplify the presentation assume that there are several games on the market for already long enough so that every agent already has maximum skill levels for all of them. Then the system is in equilibrium and choices are time invariant.

Now consider a new game entering the market with an arbitrary initial skill distribution, which is described by $s_0(i)$. Clearly, the skill level dynamics for each of the agents depends on her product choices. To analyze it we can use the expected law of motion for the skill of every agent. The agent i with skill $s_t(i)$ plays the game at time t with the probability $P\left(s_t(i)\right)$, to simplify notations we refer to this value as $P_t(i)$. Therefore, with the same probability her skills increase by $\gamma[1 - s_t(i)]$. However, with the probability $1 - P_t(i)$ skills remain at the same level. Using this logic, we can write down the expected law of motion:

$$s_{t+1}(i) = s_t(i) + \gamma\left[1 - s_t(i)\right] P_t(i). \tag{1}$$

Notice that the equation (1) is agent-specific and if one assumes that the initial distribution is not given by Dirac's delta, we will have heterogeneity in skill development paths across population. In this context the question arises: do we need to track the skill level of every agent or we can construct a representative agent which can describe the dynamics of the society?

Proposition 1. *The representative agent will accurately describe the development of the society if the following requirement is satisfied*

$$\left[1 - \bar{s}_t\right] \bar{P}_t = \int_0^1 [1 - s_t(i)] P_t(i) di,$$

$\forall t$*, where* $\bar{s}_t$ *describes the average skill level in population at time* t *and* $\bar{P}_t = P\left(\bar{s}_t\right)$*.*

The proof of proposition 1 is given in appendix A

From proposition 1 it is clear that the representative agent, which is clearly powerful in equilibrium, cannot be directly used for the out-of-equilibrium analysis. For it to be powerful out of equilibrium we have to impose additional restrictions on our society. Hence, it is only certain special cases when the representative agent can be used for out-of-equilibrium analysis. Few of the special cases are presented here.

Examples: There are three straightforward examples that one can construct when the representative gamer will describe the economy precisely. First of them is when γ is zero. This is easy to infer from equation (3) in appendix A. However, this case means that there is no learning in the economy, thus no dynamics of skills. Consequently the economy is already in equilibrium. And by construction, the representative agent is perfectly able to describe the economy. Another example is when $s_0(i) = s$, $\forall i$. This case implies that there is no heterogeneity across population (initial distribution is Dirac's delta). In this case our best guess of the skill level development is the same for every agent, of course including the representative. Therefore, representative agent is powerful. This case is not interesting as is does not permit any heterogeneity, and consequently, it does not require the representative agent. An example involving heterogeneity and learning with powerful representative is the case when the function $P(\cdot)$ is constant. However, this case completely undermines the model's central assumption that gamers' choices are determined by the skill levels for games. Here, although there is skill level dynamics, there is no dynamics in choices. Thus, the skill levels themselves become irrelevant for describing agent behavior.

Any other instance where the representative agent combining features of "functionality" and "normativeness" will exist in the economy will involve restrictions on functions $P(\cdot)$ and/or $s_0(\cdot)$ in order to satisfy the requirement in proposition 1. Unfortunately, these restrictions are not straightforward to derive.

4.2 Skill Sharing

Now consider the case when there is an interaction among agents. We do not assume anymore that gamers learn by playing. Rather, we assume that they learn by interaction with each other. In this case the multiplicity of options does not play a role, thus we discuss the environment with only one available game. The probability density of the population skill for this game at time zero is described by $f_0(s)$. The interaction structure is as follows. Every period, every agent (i) randomly picks one other agent (j) from the population. If j has higher skill then i, i learns from j. As a result, her skills increase by $\mu\,[s(j) - s(i)]$, where $\mu \in [0; 1]$ controls the speed of learning. If j's skill level is lower than that of i's, i cannot learn anything.

One thing to note about this scheme is that the agent who has the highest skill level in initial distribution (denote it with $\hat{s}$), cannot learn anything from anybody in the population. Everyone else's skill level approaches her skill level asymptotically. Here again, we are interested in out-of-equilibrium dynamics, and whether the representative agent can predict how the average skill level approaches its time-invariant value.

Given the description of the model we can specify the expected law of motion for skill level of an agent i as

$$s_{t+1}(i) = s_t(i) + \mu \int\limits_{s_t(i)}^{\hat{s}} f_t(s)[s - s_t(i)]ds, \tag{2}$$

Due to the fact that $f_t(s)$ is the density function of skill distribution at time t, the second summand in the right hand side gives the expected increase in the skill level.

In this environment we can formulate following proposition.

Proposition 2. *The representative agent will accurately describe the economy if the following requirement is satisfied*

$$\int\limits_{\bar{s}_t}^{\hat{s}} f_t(s)[s - \bar{s}_t]ds = \int\limits_0^1 \int\limits_{s_t(i)}^{\hat{s}} f_t(s)[s - s_t(i)]dsdi$$

$\forall t$.

The proof of proposition 2 is given in appendix B.

As one can see, again, the representative agent that is powerful in equilibrium can describe the out-of-equilibrium dynamics of the system only in special cases.

Examples: Examples of these cases are when $\mu = 0$ or when $s_0(i) = s$, $\forall i$. Both of these cases imply that the economy starts off at equilibrium, thus, we cannot evaluate the performance of the representative agent out-of-equilibrium.

Any non-trivial case in which the representative agent of the economy can be constructed would require restrictions on the skill probability density functions $f_t(\cdot)$ $\forall t$ in order the requirement in proposition 2 to be satisfied. Similar to the case discussed in section 4.1, derivation of these restrictions is not straight forward.

5 Conclusion

In this note we have discussed two types of behavior of collection of heterogeneous agents. We have demonstrated that even if the representative agent is powerful in equilibrium she might not be able to describe the out-of-equilibrium dynamics of the society. In particular, we have shown that the representative agent at time t will not be representative at time $t+1$ unless certain functional requirements are satisfied. This warns against using representative agent setups for the analysis of out-of-equilibrium dynamics even if one can demonstrate that the setup is insulated from the standard criticism (which usually concerns the equilibrium properties of the setup). By analysing two setups, one with interaction, another – without, we have also established that this phenomenon is not specific to models with (or without) agent interaction.

Appendix

A Proof of Proposition 1.

Proof. The representative agent has the average skill level in the initial skill distribution: $s_0^r = \bar{s}_0 = \int_0^1 s_0(i)di$. Then, her skill level next period should be

$$s_1^r = \bar{s}_0 + \gamma \left[1 - \bar{s}_0\right] \bar{P}_0. \tag{3}$$

The average skill level in the economy at time one is

$$\bar{s}_1 = \int_0^1 s_1(i)di = \int_0^1 \left[s_0(i) + \gamma \left[1 - s_0(i)\right] P_0(i)\right] di = \bar{s}_0 + \gamma \int_0^1 [1 - s_0(i)] P_0(i)di. \tag{4}$$

From equations (3) and (4) it is obvious that for $s^r(1) = \bar{s}(1)$ to hold we need

$$\left[1 - \bar{s}_0\right] \bar{P}_0 = \int_0^1 [1 - s_0(i)] P_0(i)di. \tag{5}$$

Equality (5) is the requirement for the representative agent at time zero to be representative at time one. But the representative agent has to be able to describe the average skill level in the society at every time period. Thus, we need the following general equality to hold

$$[1-\bar{s}_t]\,\bar{P}_t = \int_0^1 [1-s_t(i)]P_t(i)di,\ \forall t.$$

$\square$

B Proof of Proposition 2.

Proof. Our representative agent at time zero has to have a skill level equal to the average of the society $s_0^r = \bar{s}_0 = \int_0^1 s_0(i)di$. Then, her skill level at time one will be

$$s_1^r = \bar{s}_0 + \mu \int_{\bar{s}(0)}^{\hat{s}} f_0(s)[s-\bar{s}_0]ds. \tag{6}$$

We can also calculate the average skill level in the economy at time one

$$\bar{s}_1 = \int_0^1 s_1(i)di = \int_0^1 \left[s_0(i) + \mu \int_{s_0(i)}^{\hat{s}} f_0(s)\left[s - s_0(i)\right]ds \right] di = \bar{s}_0 + \mu \int_0^1 \int_{s_0(i)}^{\hat{s}} f_0(s)\left[s - s_0(i)\right]dsdi. \tag{7}$$

By looking at equations (6) and (7) for the equality $s^r(1) = \bar{s}(1)$ to hold we need

$$\int_{\bar{s}_0}^{\hat{s}} f_0(s)\left[s-\bar{s}_0\right]ds = \int_0^1 \int_{s_0(i)}^{\hat{s}} f_0(s)\left[s - s_0(i)\right]dsdi. \tag{8}$$

Or, in general

$$\int_{\bar{s}_t}^{\hat{s}} f_t(s)[s-\bar{s}_t]ds = \int_0^1 \int_{s_t(i)}^{\hat{s}} f_t(s)\left[s - s_t(i)\right]dsdi,\ \forall t.$$

$\square$

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