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Equity premium under multiple background risks

Yoichiro Fujii
Graduate School of Systems and Information Engineering, University of Tsukuba

Yutaka Nakamura
Graduate School of Systems and Information Engineering, University of Tsukuba

Abstract

In a static Lucas's tree economy, we explore the effect of two types of background risk, uninsurable risk for labor income and miscalibrated risk for payoff distribution of risky asset, on the equilibrium price of the risky asset. Then we analyze the data of U.S. stock market and GDP growth rates during 1871-2004 to verify that our simple static model could provide appropriate magnitudes of equity premium.

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1. Introduction

It was pointed out by Mehra and Prescott (1985) that the representative agent model in a Lucas (1978) tree economy dramatically underestimates the equity premium. In a two-period version of Lucas economy, Weil (1992) examined whether or not the existence of uninsurable background risk, such as a risk on labor income, could explain the equity premium puzzle. He showed that if preferences exhibit standardness (i.e., decreasing absolute risk aversion and decreasing absolute prudence), then the magnitude of the equity premium is increased. The question is whether or not it yields a sizable effect.

Recently, in a static Lucas economy, Gollier (2001) examined a simplified version of Weil model by adding a background risk to initial wealth, and concluded that the existence of idiosyncratic background risk, when considered in isolation, cannot explain the equity risk premium puzzle, that is, the model cannot yield a sizable effect. On the other hand, Gollier and Schlesinger (2002) attached the background risk to the initial asset-payoff distribution, rather than to initial wealth. They showed how such a miscalibration of risk, together with an assumption that preferences are standard, offers a new potential explanation for empirically high equity premium.

This article further elaborates Gollier-Schesinger's static Lucas model by adding background risks to both of initial wealth and asset-payoff distribution. Thus we explore the effect of two types of background risk, uninsurable risk for labor income and miscalibrated risk for payoff distribution of risky asset, on the equilibrium price of the risky asset. We analyze the data of U.S. stock market and GDP growth rates during 1871-2004 and estimate the magnitude of the equity premium. Our calibrations show that the impact of unisurable risk on the magnitude of the equity premium is much smaller than miscalibrated risk, and verify that our simple static model could provide appropriate magnitudes of equity premium with the observed standard deviation 9% of labor income risk when the standard deviation of miscalibrated risk is between 3% and 8%, depending on the relative risk aversion coefficients.

2. Equilibrium Prices under Multiple Background Risks

Consider a static Lucas tree economy consisting of risk-averse individuals, all of whom may be portrayed by a representative agent. Let u denote the representative agent's von Neumann-Morgenstern utility function over the final wealths. In addition to the one unit of physical capital, each agent in the economy is endowed with some initial human capital. Let $\tilde{x}$ denote the revenue generated by each unit of physical capital, which is perfectly correlated across firms. The revenue generated by the human capital is denoted by $\tilde{w}$. We assume that it is independently distributed across agents, and that it is also independent of $\tilde{x}$. Following

Gollier and Schlesinger (2002), the equilibrium price of $\tilde{x}$ will be equal to

$$P_{u,\tilde{w}}(\tilde{x}) = \frac{E\tilde{x}u'(\tilde{w}+\tilde{x})}{Eu'(\tilde{w}+\tilde{x})};$$

the equity premium in this economy will be equal to

$$\phi_{u,\tilde{w}}(\tilde{x}) = \frac{E\tilde{x}}{P_{u,\tilde{w}}(\tilde{x})} - 1.$$

Now we shall consider two types of background risks, $\tilde{\epsilon}_x$ and $\tilde{\epsilon}_w$, and replace $\tilde{x}$ and $\tilde{w}$ by $\tilde{x}+\tilde{\epsilon}_x$ and $w_0+\tilde{\epsilon}_w$, respectively, in the equilibrium price formula, where $E\tilde{\epsilon}_x = E\tilde{\epsilon}_w = 0$ and $\tilde{x}$, $\tilde{\epsilon}_x$, and $\tilde{\epsilon}_w$ are independent. Assuming that u exhibits standardness, it easily follows from the results of Weil (1992) and Gollier and Schlesinger (2002) that two distinct background risks together reduce the equilibrium price more than any one of those risks, i.e., for all $\tilde{x}$,

$$P_{u,w_0+\tilde{\epsilon}_w}(\tilde{x}+\tilde{\epsilon}_x) < \left\{ \begin{array}{c} P_{u,w_0+\tilde{\epsilon}_w}(\tilde{x}) \\ P_{u,w_0}(\tilde{x}+\tilde{\epsilon}_x) \end{array} \right\} < P_{u,w_0}(\tilde{x}). \tag{1}$$

Weil (1992) proved the upper second inequality in (1), and interpreted background risk $\tilde{\epsilon}_w$ as private information and therefore uninsurable due to observability asymmetries. Gollier and Schlesinger (2002) attached the "noise" term $\tilde{\epsilon}_x$ to the original asset distribution $\tilde{x}$ to extend Weil's argument for the equity-premium puzzle under miscalibrated risk, and proved that $P_{u,w_0}(\tilde{x}+\tilde{\epsilon}_x) < P_{u,w_0+\tilde{\epsilon}_w}(\tilde{x})$ when $\tilde{\epsilon}_x = \tilde{\epsilon}_w$ in the middle of (1). Assume that the market analyst calculates a sampling distribution function of the true distribution for $\tilde{x}$, which is based on historical data. If consumers all possess the same distributional information as the analyst, but consumers include a spurious noise term $\tilde{\epsilon}_x$ in their estimated distribution, they argued that the lower second inequality in (1) follows from Weil's result, that is, the analyst's estimated equilibrium price $P_{u,w_0}(\tilde{x})$ is higher than the empirical equilibrium price $P_{u,w_0}(\tilde{x}+\tilde{\epsilon}_x)$. This, of course, leads to a higher empirical equity premium than the analyst's prediction. Hence, they have another potential explanation for the equity premium puzzle.

However, in general, it is not clear which of $P_{u,w_0+\tilde{\epsilon}_w}(\tilde{x})$ (Weil's effect) and $P_{u,w_0}(\tilde{x}+\tilde{\epsilon}_x)$ (GS effect) is larger than the other unless $\tilde{\epsilon}_w = \tilde{\epsilon}_x$. By the first inequality in (1), we may interpret that a market analyst who ignores the uninsurable risk $\tilde{\epsilon}_w$ and calculates price according to a sampling distribution of the true distribution for $\tilde{x}$ will overestimate the empirical equilibrium price $P_{u,w_0+\tilde{\epsilon}_w}(\tilde{x}+\tilde{\epsilon}_x)$ in a market with two distinct background risks $\tilde{\epsilon}_w$ and $\tilde{\epsilon}_x$ which is also strictly smaller than the equilibrium prices due to Weil's effect and GS effect.

3. Numerical Analyses

For simplicity, we assume that $w_0 = 0$. Then we interpret $\tilde{x}$ as the random variable representing the GDP per capita. Taking the time series of the growth rate of U.S. real GDP per capita for the period from 1871 to 2004 (average, 2.1%; standard deviation, 5.4%; maximum value, 18.7% in 1942; minimum value, -21.5% in 1946; see Maddison, 2007), we assume that agents believe that each realized growth rate in the past will occur with equal probability, and that agents have a constant relative risk aversion (CRRA) α. Then using the formula $\phi_{u,0}(\tilde{x})$, we can calculate the equity premium in the static Lucas economy. Several empirical studies about U.S. stock markets (e.g., Merton, 1980; Pindyck, 1988; Finn, et al., 1990; Klock and Phillips, 1999; Tödter, 2008) show that the range of estimated value of α may be given by an interval $[1, 8]$. Table 1 reports various equity premiums $\phi_{u,0}(\tilde{x})$ as a function of α.

Table 1. Equity premiums with CRRA and $\tilde{x}$ based on the actual growth rates of U.S. real GDP per capita, 1871-2004

CRRA(α)	Equity premium (%)	CRRA(α)	Equity premium (%)
1.0	1.2	8.0	4.0
4.0	1.5	10.0	5.2
5.0	1.9	11.0	5.9

Over the period 1871-2004, we take the real returns of S&P500 and interest rates, which are respectively regarded as risky and risk-free assets in our model. The averages of the real return of S&P500 and the interest rates are respectively 8.2% and 2.9% per year (see Shiller, 2005). Hence we obtain that the average of equity premiums over the period is around 5.3%. Clearly, this simple static version of Lucas tree economy does not fit the data[1] for $1 \leq \alpha \leq 8$.

To bridge the gap, we shall calculate the effects of background risks on the sizes of equity premium. For computational simplicity, we assume that labor income risk $\tilde{\epsilon}_w$ is distributed

[1]Surprisingly, the puzzle disappears under the static model if $\alpha \geq 10$. Mehra and Prescott (1985) used data of consumption per capita over the period 1889-1978. They obtained that, for $\alpha \leq 10$, the maximum estimated equity premium is 0.35%. On the other hand, Gollier (2001) used GDP growth rates over the period 1963-1992. He obtained that, for $\alpha \leq 10$, the maximum estimated equity premium is 0.61%. Thses estimated values are ten times smaller than the values in Table 1. This may be due to two empirical findings: smaller standard deviation of concumption data than GDP and smaller sandard deviation for shorter period.

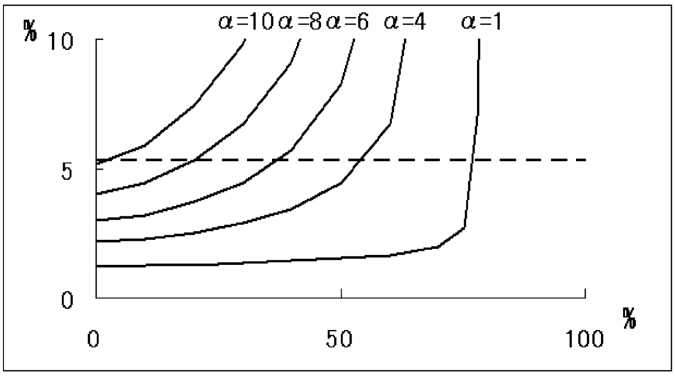

Figure 1: Equity Premium (vertical line) and labor income risk (horizontal line)

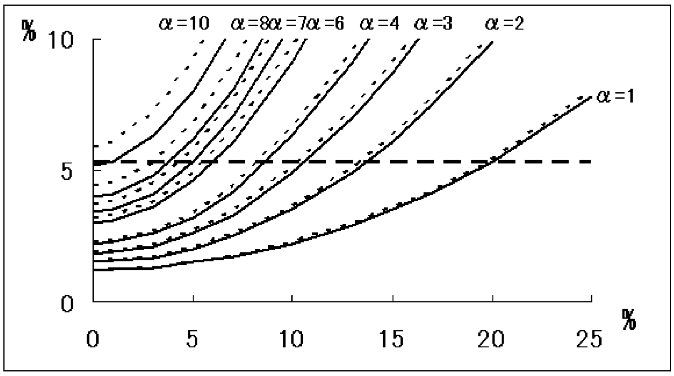

Figure 2: Equity Premium (vertical line) and miscalibrated risk (horizontal line)

as $\left\langle -k, \frac{1}{2}; +k, \frac{1}{2} \right\rangle$ for some $k > 0$, where value $+k$ obtains with probability 1/2 and value $-k$ obtains with probability 1/2. Parameter k is the standard deviation of the growth of labor incomes. Using the historical frequency of the growth of GDP per capita for $\tilde{x}$ and CRRA utility functions with $\alpha = 1 \sim 10$, we obtain the equity premium $\phi_{u,\tilde{\epsilon}_w}(\tilde{x})$ as a function of the size k of the uninsurable risk. Figure 1 shows these numerical results, where the dashed horizontal line depicts the level of equity premium obtained in the preceding paragraph. We see that a very large background risk is required to explain the puzzle, for example, with a standard deviation of the annual growth of individual labor income exceeding 20% for $\alpha \leq 8$.

Now we numerically verify our modification of Gollier and Schlesinger's static Lucas model in U.S. stock market. Although growth rates of annual earnings in U.S. manufacturing were not available over the period 1871-2004, we assume from NBER and Penn World that the standard deviation of the growth rates is 9%. Thus we let $k = 0.09$, so that

$\tilde{\epsilon}_w = \left\langle -0.09, \frac{1}{2}; +0.09, \frac{1}{2} \right\rangle$ represents labor income risk. Let us also assume that background risk $\tilde{\epsilon}_x$ is distributed as $\left\langle -n, \frac{1}{2}; +n, \frac{1}{2} \right\rangle$ for some $n > 0$. Parameter n is the standard deviation of the miscalibrated risk of the asset-payoff distribution $\tilde{x}$. As in the preceding paragraph, we can calculate the equity premium $\phi_{u,\tilde{\epsilon}_w}(\tilde{x}+\tilde{\epsilon}_x)$ as a function of the size n of the miscalibrated risk.

In Figure 2, these calculations are presented in two cases of $k = 0.09$ (dashed curves) and $k = 0$ (real curves). The latter case is exactly the results from Gollier-Schlesinger model. We see that Weil's effect is much smaller than GS effect. We may conclude that a mild background risk $\tilde{\epsilon}_x$, whose standard deviation is between 3% and 8% depending on $\alpha = 4 \sim 8$, is sufficient to explain the puzzle. For $1 \leq \alpha < 4$, however, larger parameter values n (10% $\sim$ 20%), which may blur out a sampling distribution function of $\tilde{x}$, are required to reduce the equilibrium price.

4. Conclusion

The aim of this article was to further elaborate Gollier and Schlesinger's simplified version of a static Lucas model. Introducing two types of background risk in the model, we demonstrated that our version reduces equilibrium price of risky asset more than each one of Weil's effect and GS effect. Then we analyzed U.S. stock markets during 1871-2004 to verify that the estimated equilibrium price is small enough to explain the equity premium puzzle for $\alpha = 4 \sim 8$. It also follows from our calibration that Weil's effect is much smaller than GS effect. It may be theoretical interest to investigate this large difference of effects of background risks on the equilibrium price.

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